Random Errors induced by the Superconducting Windings in the LHC Dipoles

W. Scandale, E. Todesco and R. Wolf
CERN, Geneva, Switzerland

***Abstract*—The problem of estimating the random errors in the LHC dipole is considered. The main contributions to random errors are due to random displacements of the coil position with respect to nominal design and to the variation of the magnetization of the superconducting cable. Coil displacements can be induced either by mechanical tolerances or by the manufacturing process. Analytical and numerical scaling laws that provide the dependence of the random errors due to random displacements on the multipolar order are worked out. Both simplified and more realistic models of the coil structure are analysed. The obtained scaling laws are used to extract from experimental field shape data the amplitude of the coil displacements in the magnet prototypes. Finally, random errors due to interstrand resistance variation during the ramp are estimated**

I. Introduction

The superconducting dipoles of the Large Hadron Collider [1] will be affected by field shape errors that must be minimised to avoid detrimental effects on the particle dynamics. Field shape errors can be grouped in two parts: a systematic one that is the same for all the magnets, and a random part that varies from magnet to magnet, and even along the longitudinal axis of the magnet.

A source of random errors is due to the difficulty of reaching the nominal coil position under manufacturing conditions. The geometry of the actual coil will differ from the nominal one due to thermal and mechanical stresses, and to tolerances in the magnet parts. Another source of random errors is the persistent currents in the superconductor filaments. During the field ramp additional eddy current type of errors arise. Currents induced in the strands of the superconducting cables are especially important.

In this paper we build a simple analytical model to derive scaling laws of the geometric random errors on the order of the multipolar expansion. A numerical simulation on a more realistic model confirms the validity of the analytical estimate. A review of the effect of magnetization on the random errors is also given. A cross-section of the LHC dipole coil is shown in Fig. 1.

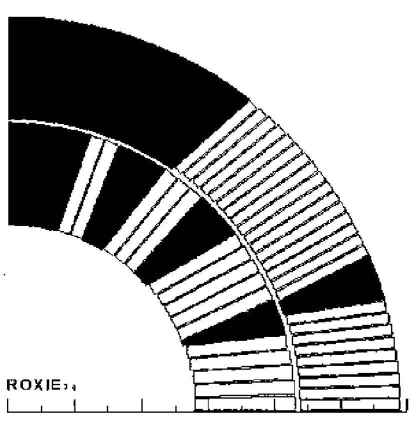

Fig. 1. Cross section of magnet coils of LHC dipole. Scale in mm. The coil consists of two layers wound with different superconducting cables.

Manuscript received September 25, 1999

II. Analytical Estimates of Random Errors due to Coil Displacement

In this section we derive analytical formulas for the effect of random coil displacements on the field-shape. We assume that a circular iron yoke of infinite permeability is present at a distance R_y. We first evaluate the effect of random displacements for a line current (section II.A). Then the case of a sector magnet is analysed (section II.B). For the analytical calculation we basically use techniques, although with a different notation, which are similar to those explained in [2]. The field is assumed to be 2-dimensional and we use complex notation.

A. Random Errors for a Current Line

The field $B_y + iB_x$ in a point $z = x + iy$ (Fig. 2) due to a line current I located in a point $z_c = x_c+iy_c = r_c\, e^{i\theta}$ can be written as:

$$B_y + iB_x = \frac{\mu_0}{2\pi} I\left(\left(z - z_c\right)^{-1} + \left(z - \frac{R_y^2}{z_c^*}\right)^{-1}\right). \tag{1}$$

The standard multipolar field expansion reads:

$$B_y + iB_x = \sum_{n=1}^{\infty} C_n z^{n-1} = \sum_{n=1}^{\infty} \left(B_n + iA_n\right) z^{n-1}, \tag{2}$$

where n is the harmonic number and B_n and A_n are the normal and the skew components respectively. *The main component* of the field is indicated by a capital letter N. The multipole field components b_n and a_n relative to the main field $B_N R_r^{N-1}$ at the reference radius R_r are given by:

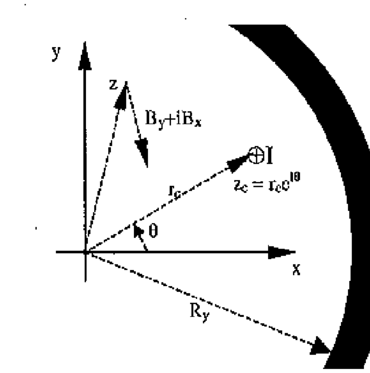

Fig. 2. Field due to a Line Current in a cylindrical yoke.

$$B_y + iB_x = B_N R_r^{N-1} \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{z}{R_r} \right)^{n-1} . \qquad (3)$$

For a current line [see (1)and (2)], the coefficient C_n is

$$C_n = -\frac{\mu_0}{2\pi} \frac{I}{z_c^n} \left(1 + \left(\frac{r_c}{R_y} \right)^{2n} \right). \qquad (4)$$

If the conductor moves by a small amount $\Delta z_c = |\Delta z_c| e^{i\alpha}$, the multipole coefficient C_n (4) changes according to:

$$\Delta C_n = n \frac{\mu_0}{2\pi} \frac{I}{z_c^{n+1}} \left(\Delta z_c - \left(\frac{r_c}{R_y} \right)^{2n} \Delta z_c^* \right). \qquad (5)$$

Let us now suppose that the conductor moves randomly in any direction from the position z_c. We furthermore suppose that Δz_c has an average $\overline{\Delta z_c} = 0$ and a standard deviation $\sigma/\sqrt{2}$ in each of its components, corresponding to a standard deviation of σ of the movement. The standard deviation in B_n is $\sigma_{Bn} = \sqrt{\overline{\Delta B_n^2} - \overline{\Delta B_n}^2} = \sqrt{\overline{\Delta B_n^2}}$. With the help of (5) and supposing that $\overline{\sin(\alpha)} = \overline{\cos(\alpha)} = 0$ we then find:

$$\sigma_{Bn} = n \frac{\mu_0}{2\pi} \frac{|I|}{r_c^{n+1}} \frac{\sigma}{\sqrt{2}} \sqrt{1 + \frac{r_c^{4n}}{R_y^{4n}} - 2\frac{r_c^{2n}}{R_y^{2n}} \cos(2n\theta)} . \qquad (6)$$

Similarly for A_n:

$$\sigma_{An} = n \frac{\mu_0}{2\pi} \frac{|I|}{r_c^{n+1}} \frac{\sigma}{\sqrt{2}} \sqrt{1 + \frac{r_c^{4n}}{R_y^{4n}} + 2\frac{r_c^{2n}}{R_y^{2n}} \cos(2n\theta)} . \qquad (7)$$

Note that σ_{An} and σ_{Bn} would be equal if there were no yoke. Their average value (over θ) is just $\sigma_c/\sqrt{2}$ where:

$$\sigma_C = \sqrt{\overline{|\Delta C_n|^2}} = n \frac{\mu_0}{2\pi} \frac{I}{r_c^{n+1}} \sqrt{1 + \left(\frac{r_c}{R_y} \right)^{4n}} \, \sigma . \qquad (8)$$

In the following we will usually assume these average values for σ_{An} and σ_{Bn}. This choice seems reasonable since:

1. In a magnet, there are often several blocks on different positions θ varying randomly. This tends to make σ_{Bn} and σ_{An} equal.
2. In practice the yoke contribution is often small, except possibly for low orders of n. This would make σ_{An} and σ_{Bn} equal for any position θ.

Note that, because of this choice, σ_{Bn} and σ_{An} are the same for any position of the conductor on a circle with radius r_c.

B. Random Errors in Simple Sector Magnets

For a sector carrying a current I with constant current density, extending between r_1 and r_2 and over an angle $2\Delta\phi$ (Fig. 3) we find in a similar way:

$$\sigma_C = \frac{\mu_0 I}{\pi (r_2^2 - r_1^2)} \frac{n}{n-1} \left(\frac{1}{r_1^{n-1}} - \frac{1}{r_2^{n-1}} \right) F_n \sigma \text{ , with} \qquad (9)$$

$$F_n(\Delta\phi) = \sqrt{\left(\frac{\sin(n+1)\Delta\phi}{(n+1)\Delta\phi} \right)^2 + \left(\frac{\sin(n-1)\Delta\phi}{(n-1)\Delta\phi} \right)^2 \left(\frac{r_2^{n+1} - r_1^{n+1}}{(n+1)R_y^{2n}} \Big/ \frac{1}{n-1} \left(\frac{1}{r_1^{n-1}} - \frac{1}{r_2^{n-1}} \right) \right)^2}$$

If we can neglect the yoke contribution and the sectors are narrow $F_n \rightarrow 1$.

We consider an $2N$ pole magnet consisting of $4N$ sectors, each carrying a current I, and extending from 0 to $\pi/3N$. We choose this angle in order to make the first allowed harmonic (b_{3N}) zero. Then the main field strength B_N can be written as :

$$B_N = -\frac{6\sqrt{3} N \mu_0 I}{\pi^2 (r_2^2 - r_1^2)} S_N \text{ , with} \qquad (10)$$

$$S_N = \left(\frac{r_1^{2-N} - r_2^{2-N}}{N-2} + \frac{r_2^{N+2} - r_1^{N+2}}{(N+2) R_y^{2N}} \right).$$

We assume that each of these sectors is divided in m equal subsectors (extending from r_1 to r_2, Fig. 3) which can move *independently* from each other. In total we thus have $4Nm$ independently moving regions in the magnet and the total random error becomes $\sqrt{4Nm}$ as large as for a single region. The standard deviation $\sigma_n (= \sigma_{bn} = \sigma_{an})$ in the error relative to the main field (b_n and a_n see (3)) at the reference radius R_r is, using (9) and (10):

$$\sigma_n = \frac{\pi R_r^{n-N}}{3 S_N \sqrt{6Nm}} \frac{n}{n-1} \left(\frac{1}{r_1^{n-1}} - \frac{1}{r_2^{n-1}} \right) F_n \left(\frac{\pi}{6Nm} \right) \sigma . \qquad (11)$$

As noted above, the approximation $F_n = 1$ is often reasonable (see Fig. 4). For moderately high n, small subsectors and $(r_1/r_2)^n \ll 1$ we find the following approximation, which justifies equation (17) used for fitting the numerical simulations:

$$\ln \sigma_n \approx \frac{1}{n} + \ln \left(\frac{\pi \sigma}{3 S_N R_r^{N-1} \sqrt{6Nm}} \right) - (n-1) \ln \left(\frac{r_1}{R_r} \right) - (n+1)^2 \left(\frac{\pi}{6Nm} \right)^2 \qquad (12)$$

C. Rotation and Deformation

To derive (11) we assumed a 'rigid coil' which, although it moves randomly, did not rotate. It is intuitively clear that if the aspect ratio of the coil is close to one the effect of rotation is small. This can be seen by considering a coil with

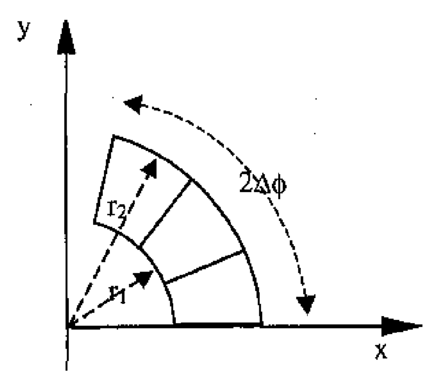

Fig. 3. Thick (dipole) sector, divided into three equal parts (m = 3) which can move randomly.

cylinder cross-section, where a rotation has no influence on the field. To make an estimation of the effect of the coil aspect ratio when it is rotated we consider a small conductor carrying a current I, with rectangular cross-section (width w and height h) which is rotated by small angle $\Delta\alpha$ around its centre z_m:

$$\Delta C_n^{(rot)} \approx -\mathrm{i}\frac{\mu_0}{2\pi} I \frac{e^{2i\alpha} n(n+1)}{(z_m)^{n+2}} \frac{\left[w^2 - h^2\right]}{12} \Delta\alpha . \qquad (13)$$

We compare this change with the change of a single line current I in the center of the block making a movement $\sqrt{w^2+h^2}\Delta\alpha/4$, which is about the average movement in the block, using (5):

$$\left|\frac{\Delta C_n^{(rot)}}{\Delta C_n^{(displacement)}}\right| \approx \frac{(n+1)}{|z_m|} \frac{\left|w^2 - h^2\right|}{3\sqrt{w^2+h^2}} . \qquad (14)$$

Therefore a nearly square current block will give little change to the field error if it is rotated compared to the effect of a similar displacement of its centre. The effect of rotation increases with n however.

Furthermore the form of the coil could change. To study this we looked at an elastic deformation of a sector in the radial and the tangential direction in such a way that the 'center of gravity' of the sector remains fixed. The resulting field change was then compared to a displacement of a rigid sector of the same magnitude as the average displacement in the deformed coil. Comparing the effect of a displacement Δr, with stretching the width of the conductor by $\pm\Delta r$ we find:

$$\left|\frac{\Delta C_n^{stretch,radial}}{\Delta C_n^{displace}}\right| \approx \frac{n(r_2 - r_1)}{3(r_2 + r_1)} . \qquad (15)$$

For a similar azimuthal stretching:

$$\left|\frac{\Delta C_n^{stretch,azimuthal}}{\Delta C_n^{displace}}\right| \approx \frac{n\Delta\phi}{3} . \qquad (16)$$

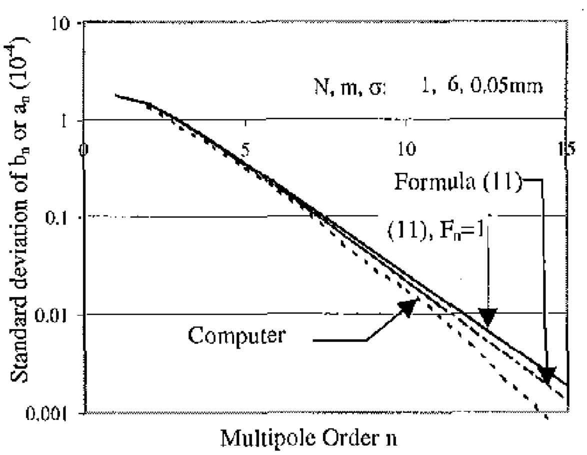

Fig. 4. Comparison of Analytical Calculation of Random errors with Monte-Carlo simulation by computer. Standard deviation of the multipoles due to random displacement of the blocks of 0.05 mm. The error is expressed relative to the main field at a reference radius R_r = 0.017m.

Equations (14)(15) and (16) show that often rotation and deformation can be neglected in comparison to a similar average movement. Therefore we can hope that (11) gives a reasonable approximation to the total random error due to the movements of the magnet coil. Indeed a comparison (see Fig. 4) with computer calculations (see Section III) gives a good agreement. For this analytical calculation we took a 60 degree sector with 6 sub-sectors. The sector has the same inner and outer coil and yoke radius as the LHC dipole.

III. Numerical Estimates of Random Errors due to Coil Displacement

A. Numerical Simulations

We carried out some numerical simulations to evaluate the effect of small random variations of the coil geometry on a more realistic model of the LHC coil [1]. The coil cross section is described using the code ROXIE [3] in terms of blocks of conductors, and a random displacement has been applied to the nominal design.

Two types of errors in the coil geometry were considered: displacements of the blocks and displacement of the conductors. In both cases we consider a random displacement of each component (block or conductor), neglecting the geometrical constraints that relate movements of neighbour parts. This is somewhat unphysical, especially for the case of the displacement of conductors, but it should give a first indication about what kind of random multipoles could be expected by incoherent coil displacements; it is also a first order modeling of the influence of tolerances on field quality.

The random displacements are assumed to belong to a Gaussian distribution with zero average and whose sigma is set to the value d, truncated at three sigma. Six different values of d have been used, ranging from 3 to 100μ m.

The multipoles arising from this modified configuration are evaluated, and differences with respect to the multipoles due to the nominal design are computed. 100 configurations with randomly generated displacements have been analyzed; one obtains the distribution of the variation of multipoles with respect to the nominal case, and averages and sigma are worked out for each multipole a_n and b_n. The averages are very close to zero, within the statistical significance. In Fig. 5 we plot the sigma of the multipoles σ_n versus the multipole order n, for different values of the sigma of the displacement d. Normal and skew components feature similar sigmas.

B. Scaling Law

Numerical data can be very well interpolated by the following three parameter formula:

$$\log\sigma_n = a + bn + cn^2 . \qquad (17)$$

or equivalently

$$\sigma_n = AB^n C^{n^2} . \qquad (18)$$

The interpolating constants for the analysed cases are shown in Table I. Indeed, multipoles obey the analytical

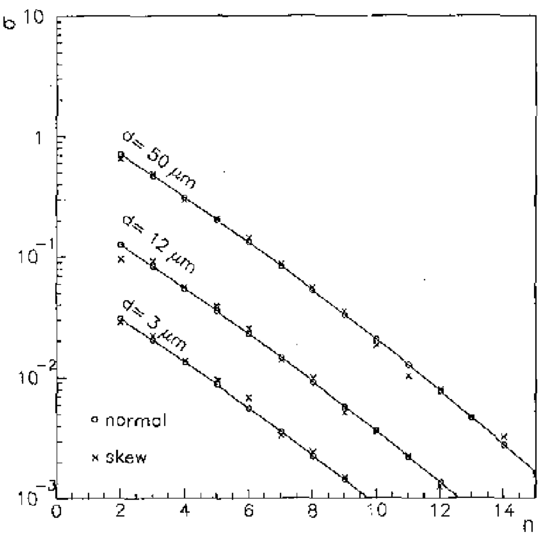

Fig. 5. Standard deviation of the multipoles σ_n versus multipole order n, for different values of the random displacements d. Numerical data: normal (circles) and skew (crosses) multipoles; interpolation through (17) (solid line). Case of conductor displacement.

estimate $\sigma_n < A\ (R_r/r_1)^n$, where in our case R_r=17 mm is the reference radius where multipoles are computed, and r_1 = 28 mm is the radius of the inner layer. In fact the series of multipoles evaluated at R_r is convergent only for $R_r < r_1$, since the Biot-Savart law features the singularity $1/r$ on the coil. One can observe that b is close to the upper bound provided by Biot-Savart law: $\log_{10}(17/28)$ = -0.216, that both b and c are independent of d, and that the only dependence on d lies in a:

$$a(d) = a_0 + a_1 \log_{10} d\ , \qquad (19)$$

with a_0 =1.9 and a_1 =1.0 . This last value implies that the effect of random displacements on multipoles is linear in the amplitude, i.e. doubling the amplitude of block displacements, the multipoles are doubled. The obtained analytical dependence on d can be rewritten in the form

$$\sigma_n(d) = dA_0 B^n C^{n^2} \qquad (20)$$

with A_0, B and C independent of d. We carried out the same computation for a six blocks design: the parametric dependence is the same, with small variations of the constants.

The same type of simulation is done using random

TABLE I

Estimated Coefficients For Dependence Of The Sigma Of The Random Errors On The Multipole Order, Simulation With Conductor Displacements.

d (μm)	a	b	c
100	0.34	-0.17	-0.0020
50	0.01	-0.16	-0.0024
25	-0.27	-0.17	-0.0019
12.5	-0.56	-0.17	-0.0020
6.125	-0.84	-0.17	-0.0021
3.0625	-1.17	-0.17	-0.0019

TABLE II.

Estimated Coefficients For Dependence Of The Sigma Of The Random Errors On The Multipole Order, Simulation With Block Displacements.

d (μm)	a	b	c
100	0.92	-0.21	-0.0041
50	0.60	-0.20	-0.0040
25	0.28	-0.20	-0.0037
12.5	-0.05	-0.20	-0.0038
6.125	-0.34	-0.19	-0.0040
3.0625	-0.64	-0.19	-0.0041

displacements of the 164 conductors that form the coils. The position of the conductors is varied randomly, and the multipoles are evaluated. The average multipoles are close to zero also in this case, and the sigmas are very well interpolated by the same equations with somewhat different constants (see Table II). One finds that c is smaller (i.e. the curve is closer to a line), that b is smaller in absolute value (i.e. the decaying of higher order multipoles is slower), and that a depends on d according to (19), with a_0 = 1.3 and a_1 = 1.0. The interpolating constants for the analysed cases are shown in Table II.

IV. Estimate of Random Displacements from Field Measurements.

The data relative to the field quality at room temperature of several magnets (four LHC dipoles [3][4] and two LHC quadrupoles [5]) have been analysed. Using the scaling laws described in the previous sections, the variation of the multipoles along the magnet axis has been interpreted as due to uncorrelated coil displacements. In all cases the variation of the multipoles is compatible with random displacements whose standard deviation d is between 12 and 25 μm.

V. Superconductor Magnetization Errors

Each pole of the LHC dipole consists of two different cables, one for the inner layer winding with 7μm NbTi filaments and one for the outer layer with 6μm filaments. According to [7] during series production, the magnetization of a superconducting cable for LHC could vary between limits of ±5% for a single manufacturer. Fig. 6 shows the calculated random field errors, at injection level (B_1 = 0.54T) of the main dipole, if the standard deviation of the magnetization is 2% in the winding blocks, and 3% of the average magnetization of a cable. Note that it is this last variation which gives the most important errors. The random field error is proportional to the magnetization change for small variation as in the above case. Programs REM [8] and ROXIE were used for these calculations.

A. Increase in random errors during magnetization decay.

It has been shown [9] that magnetization of the cable inside the LHC dipole decays in time, typically 10% within 15 minutes. This decay is not the same in all parts of the magnet and can lead to an increase in the random errors at the end of the particle injection period in the machine. According to the estimations given in [10], the additional errors could be of the same order as given below in Fig. 6.

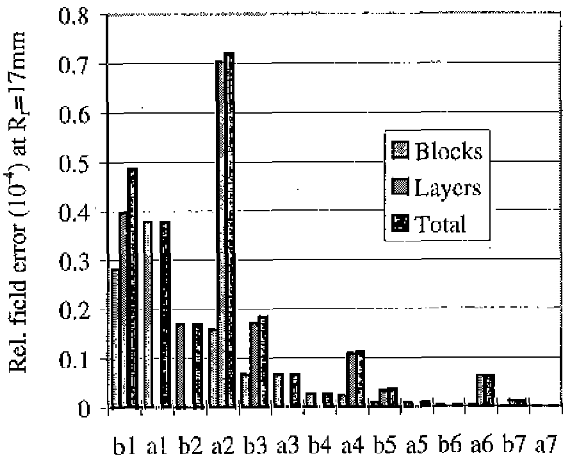

Fig. 6. Standard deviation in multipole components b_n and a_n in the LHC dipole at injection field, due to variation of superconductor magnetization

VI. Ramp Errors.

We discuss here those errors which are due to currents flowing through the finite resistance between the crossing strands [11]. The field errors for a magnet with a given cable is inversely proportional to the contact resistance R_c and proportional to the rate of change of the main field. Random errors arise because the contact resistance varies in the magnet windings. Principal causes are:

1. variation in the composition and thickness of the surface layer of the strand, which in the case of LHC is a thin SnAg layer;
2. variation in the heat treatment at the cable manufacturer;
3. variation in the coil curing temperature and time.

It is not easy to find a good estimation of the expected contact resistance, and even less for it's variation.

Studies [12] have indicated that a contact resistance R_c of

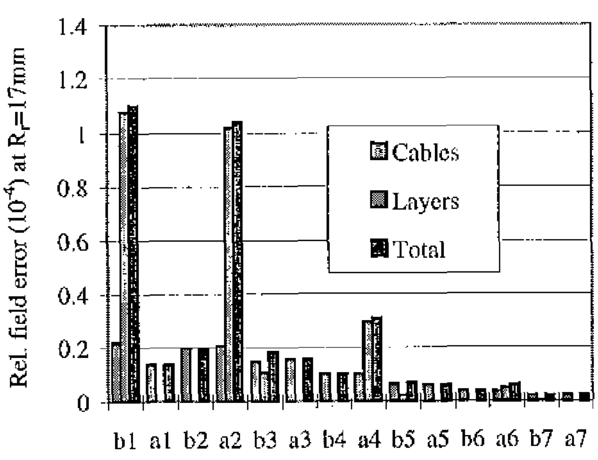

Fig. 7. Standard deviation in multipole components b_n and a_n in the LHC dipole at injection field, due to variation in the crossing strand contact resistance of the superconducting cable, with constant ramprate of 6.6 mT/s.

$20\mu\Omega$ can be reliably reached. An attempt to estimate standard deviations in prototype dipoles using the measured field [6] found standard deviations as large as 30%. We have used this value for both variation in an individual conductor and for the variation of the average of a cable. For the average contact resistance we took $15\mu\Omega$ to finally make the estimation of Fig. 7. The calculation (see [14] for the method) was performed using programs CCDI [11] and ROXIE.

VII. Conclusion

We have discussed an analytical estimate of the random errors due to imperfections of the coil geometry, and verified it through a numerical simulation with a realistic model of the LHC coil. We also showed why under usual conditions the normal and skew random multipoles are about equal. These estimates can be used to derive the size of the coil imperfection from the measured field.

The random errors due to the magnetization of the superconducting cable at LHC injection field are of the same order of magnitude as the errors due to coil movements and are expected to increase with time during the LHC particle injection period.

Random error in the LHC dipole during the LHC energy ramp can be reduced by a factor of five [13] compared to Fig. 7 by ramping slowly at the start of the ramp, without excessively increasing the total ramp time.

References

[1] The Large Hadron Collider, Conceptual Design, *CERN–AC* 95-05.

[2] K. Halbach, "Fields and first order perturbation effects in two-dimensional conductor dominated magnets", Nucl. Instr. Meth. 78 (1970) 185-198.

[3] O. Pagano, W. Scandale, E. Todesco, LHC Project Note 179.

[4] S. Gleis, J. Miles, W. Scandale, E. Todesco, these proceedings.

[5] S. Gleis, W. Scandale, E. Todesco, in preparation.

[6] R. Wolf, D. Leroy, D. Richter, A. P. Verweij, and L. Walckiers, "Determination of Interstrand Contact Resistance from Loss and Field Measurements in LHC Dipole Prototypes and Correlation with Measurements on Cable Samples", *IEEE Trans. Appl. Supercond., Vol. 7, 2, 797-800, 1997.*

[7] H. Fikis, M. Foitl, H. Kirchmayr, S. Le Naour, L. Oberli, R. Puzniak, A. Szewczyk, A. Wisniewski, R. Wolf, "Magnetization Measurements on LHC Superconducting Strands", *IEEE Trans. Appl. Supercond., Vol. 9 (2), 1763-1766, 1998.*

[8] R. Wolf, "Persistent currents in LHC magnets", IEEE Trans. Magnetics, 28 (1), 374-377, Jan. 1992.

[9] L. Bottura, L Walckiers, R. Wolf, "Field Errors Decay and Snap-Back in LHC Model Dipoles", IEEE Trans. Appl. Sup., 7 (2), 602-605, June, 1997.

[10] M. Schneider, "Decay and Snapback Studies on the LHC Dipole Model Magnets", Ph.D.Thesis, Technischen Universität Wien, Austria, 1998.

[11] A.P.Verweij, "Electrodynamics of Superconducting Cables in Accelerator Magnets", Ph.D.Thesis, Twente University (NL), 1995

[12] J. Adam, D. Leroy, L. Oberli, D. Richter, "Strand Coating for the Superconducting Cables of the LHC Main Magnets", *IEEE Trans. Appl. Supercond., Vol. 9 (2), 735-741, 1998.*

[13] L. Bottura, P. Burla, R. Wolf, "LHC Main Dipoles Proposed Baseline Current Ramping", CERN-LHC-Project-Report-172, Mar. 1998.

[14] A. P. Verweij, R. Wolf, "Field Errors due to Interstrand Coupling Currents in the LHC Dipole and Quadrupole", CERN/AT-MA internal note 94-97, March, 1994.