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The Gauging of Five-dimensional, $\mathcal{N} = 2$ Maxwell-Einstein Supergravity Theories Coupled to Tensor Multiplets¹

M. Günaydin^{†‡ 2} and M. Zagermann^{‡ 3}

[†] CERN, Theory Division
1211 Geneva 23, Switzerland

and

[‡] Physics Department
Penn State University
University Park, PA 16802, USA

Abstract

We study the general gaugings of $\mathcal{N} = 2$ Maxwell-Einstein supergravity theories (MESGT) in five dimensions, extending and generalizing previous work. The global symmetries of these theories are of the form $SU(2)_R \times G$, where $SU(2)_R$ is the R -symmetry group of the $\mathcal{N} = 2$ Poincaré superalgebra and G is the group of isometries of the scalar manifold that extend to symmetries of the full action. We first gauge a subgroup K of G by turning some of the vector fields into gauge fields of K while dualizing the remaining vector fields into tensor fields transforming in a non-trivial representation of K . Surprisingly, we find that the presence of tensor fields transforming non-trivially under the Yang-Mills gauge group leads to the introduction of a potential which does not admit an AdS ground state. Next we give the simultaneous gauging of the $U(1)_R$ subgroup of $SU(2)_R$ and a subgroup K of G in the presence of K -charged tensor multiplets. The potential introduced by the simultaneous gauging is the sum of the potentials introduced by gauging K and $U(1)_R$ separately. We present a list of possible gauge groups K and the corresponding representations of tensor fields. For the exceptional supergravity we find that one can gauge the $SO^*(6)$ subgroup of the isometry group $E_{6(-26)}$ of the scalar manifold if one dualizes 12 of the vector fields to tensor fields just as in the gauged $\mathcal{N} = 8$ supergravity.

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²murat@phys.psu.edu

³zagerman@phys.psu.edu

1 Introduction

Gauged supergravity⁴ theories in various dimensions have been studied extensively in the early and mid-eighties (see e.g. [1, 2, 3, 4, 5, 6]).

In the last few years there has been a renewed intense interest in gauged supergravity theories. This interest is driven mainly by the work on AdS/CFT (anti-de Sitter/conformal field theory) dualities [7, 8, 9, 10, 11]. For example, the IIB superstring theory on the background manifold $AdS_5 \times S^5$ with N units of five-form flux through the five-sphere, is conjectured to be equivalent (at least in a certain limit) to $4d$ $\mathcal{N} = 4$ super Yang Mills theory with gauge group $SU(N)$, which is a conformally invariant quantum field theory. In the limit of small string coupling and large N , the classical (ie. tree level) IIB supergravity approximation becomes valid and can be used to discuss the large N limit of the corresponding dual Yang Mills theory. The importance of *gauged* supergravity lies in the fact that $5d$ gauged $\mathcal{N} = 8$ supergravity [3, 4, 5] is believed to be a consistent nonlinear truncation of the lowest lying Kaluza Klein modes of IIB supergravity on $AdS_5 \times S^5$ [12, 13].⁵ Many aspects of the AdS/CFT correspondence, such as the renormalization group flows of certain non-conformal deformations of the Yang Mills theory with a smaller number of supersymmetries, can therefore be studied entirely within the framework of $5d$ gauged supergravity due to the lack of interference with the higher Kaluza-Klein modes [16, 17]. Thus, gauged supergravity theories lie at the core of AdS/CFT dualities.

On the other hand, five-dimensional $\mathcal{N} = 2$ gauged supergravity is the natural framework for so-called brane world scenarios in which our $4d$ world is realized as a domain wall in an effectively five-dimensional theory [18, 19, 20]. In fact, certain M-theory compactifications [21, 22, 23, 24] seem to suggest theories which appear five-dimensional at a certain intermediate length scale, at which the effective field theory is given by a certain $5d$ $\mathcal{N} = 2$ gauged supergravity plus $4d$ Standard Model-type matter fields on the $4d$ boundaries of this $5d$ spacetime.

Motivated by the above-mentioned applications, as well as others, we study the most general gaugings of $5d$, $\mathcal{N} = 2$ supergravity theories coupled to vector as well as *tensor* multiplets. The work presented here represents a generalization and an extension of earlier work on the gaugings of $\mathcal{N} = 2$ supergravity coupled to vector multiplets [25, 26, 27, 28, 29].

The organization of the paper is as follows. For the convenience of the reader, section 2

⁴The term “gauged supergravity” commonly refers to (usually \mathcal{N} -extended) supergravity theories in which a subgroup of the automorphism group (alias “R-symmetry group”) of the underlying supersymmetry algebra is realized as a local (Yang-Mills-type) gauge symmetry. Sometimes, this term is also used for gaugings of other global symmetry groups that are not subgroups of the R-symmetry group. In this paper, we will refer to the latter type of theories as “Yang-Mills/Einstein supergravity theories”. In contrast, “ungauged” supergravity theories are those for which the R-symmetry group is just a global symmetry group of the Lagrangian.

⁵The consistency of the nonlinear truncation of the S^7 and S^4 compactifications of 11-dimensional supergravity was shown in [14] and [15], respectively.

briefly summarizes the basic features of *ungauged* Maxwell-Einstein supergravity theories. Focussing on the global symmetries of these ungauged theories, we list the possible types of their gaugings. The subsequent four sections describe each of these gauge types in detail: Section 3 summarizes the gauging of a $U(1)_R$ subgroup of the $\mathcal{N} = 2$ R-symmetry group $SU(2)_R$. Sections 4 and 5 are devoted to the gauging of a subgroup K of the isometry group G of the scalar manifold: Section 4 summarizes the well-known case without tensor fields, whereas Section 5 covers the case when tensor fields have to be introduced. The simultaneous gauging of $U(1)_R$ and K is treated in Section 6. We conclude with a classification of possible gauge groups and the corresponding representations of tensor fields in Section 7 and a short discussion of our results in Section 8.

2 Ungauged $\mathcal{N} = 2$ Maxwell-Einstein supergravity theories and their global symmetries

In this section, we briefly recall the most relevant features of the (ungauged) $\mathcal{N} = 2$ Maxwell-Einstein supergravity theories (MESGT) constructed in [25]. Unless otherwise stated, our conventions will coincide with those of ref. [25], where further details can be found. In particular, we will use the metric signature $(-++++)$ and impose the ‘symplectic’ Majorana condition on all fermionic quantities.

The fields of the $\mathcal{N} = 2$ supergravity multiplet are the fünfbein e_μ^m , two gravitini Ψ_μ^i ($i = 1, 2$) and a vector field A_μ . An $\mathcal{N} = 2$ vector multiplet contains a vector field A_μ , two spin-1/2 fermions λ^i and one real scalar field φ . The fermions of each of these multiplets transform as doublets under the $USp(2)_R \cong SU(2)_R$ R-symmetry group of the $\mathcal{N} = 2$ Poincaré superalgebra; all other fields are $SU(2)_R$ -inert.

The $\mathcal{N} = 2$ MESGT’s constructed in [25] describe the coupling of \tilde{n} vector multiplets to supergravity. Hence, the total field content is

$$\{e_\mu^m, \Psi_\mu^i, A_\mu^{\tilde{I}}, \lambda^{i\tilde{a}}, \varphi^{\tilde{x}}\} \quad (2.1)$$

with

$$\begin{aligned} \tilde{I} &= 0, 1, \dots, \tilde{n} \\ \tilde{a} &= 1, \dots, \tilde{n} \\ \tilde{x} &= 1, \dots, \tilde{n}, \end{aligned}$$

where we have combined the ‘graviphoton’ with the \tilde{n} vector fields of the \tilde{n} vector multiplets into a single $(\tilde{n} + 1)$ -plet of vector fields $A_\mu^{\tilde{I}}$ labelled by the index \tilde{I} . The indices $\tilde{a}, \tilde{b}, \dots$ and $\tilde{x}, \tilde{y}, \dots$ should be interpreted as flat and curved indices, respectively, of the \tilde{n} -dimensional target space manifold \mathcal{M} of the scalar fields. (Our indices $(\tilde{I}, \tilde{a}, \tilde{x})$ correspond to the indices (I, a, x) in refs. [25, 26, 27].)

The generic Maxwell-Einstein supergravity Lagrangian was found to be (up to 4-fermion terms) [25]:

$$\begin{aligned}
e^{-1}\mathcal{L} = & -\frac{1}{2}R(\omega) - \frac{1}{2}\bar{\Psi}_\mu^i\Gamma^{\mu\nu\rho}\nabla_\nu\Psi_{\rho i} - \frac{1}{4}\overset{\circ}{a}_{\tilde{I}\tilde{J}}F_{\mu\nu}^{\tilde{I}}F^{\tilde{J}\mu\nu} \\
& -\frac{1}{2}\bar{\lambda}^{i\tilde{a}}\left(\Gamma^\mu\nabla_\mu\delta^{\tilde{a}\tilde{b}} + \Omega_{\tilde{x}}^{\tilde{a}\tilde{b}}\Gamma^\mu\partial_\mu\varphi^{\tilde{x}}\right)\lambda_{\tilde{b}}^i - \frac{1}{2}g_{\tilde{x}\tilde{y}}(\partial_\mu\varphi^{\tilde{x}})(\partial^\mu\varphi^{\tilde{y}}) \\
& -\frac{i}{2}\bar{\lambda}^{i\tilde{a}}\Gamma^\mu\Gamma^\nu\Psi_{\mu i}f_{\tilde{x}}^{\tilde{a}}\partial_\nu\varphi^{\tilde{x}} + \frac{1}{4}h_{\tilde{I}}^{\tilde{a}}\bar{\lambda}^{i\tilde{a}}\Gamma^\mu\Gamma^{\lambda\rho}\Psi_{\mu i}F_{\lambda\rho}^{\tilde{I}} \\
& +\frac{i}{2\sqrt{6}}\left(\frac{1}{4}\delta_{\tilde{a}\tilde{b}}h_{\tilde{I}} + T_{\tilde{a}\tilde{b}\tilde{c}}h_{\tilde{I}}^{\tilde{c}}\right)\bar{\lambda}^{i\tilde{a}}\Gamma^{\mu\nu}\lambda_{\tilde{b}}^iF_{\mu\nu}^{\tilde{I}} \\
& -\frac{3i}{8\sqrt{6}}h_{\tilde{I}}\left[\bar{\Psi}_\mu^i\Gamma^{\mu\nu\rho\sigma}\Psi_{\nu i}F_{\rho\sigma}^{\tilde{I}} + 2\bar{\Psi}^{\mu i}\Psi_i^\nu F_{\mu\nu}^{\tilde{I}}\right] \\
& +\frac{e^{-1}}{6\sqrt{6}}C_{\tilde{I}\tilde{J}\tilde{K}}\varepsilon^{\mu\nu\rho\sigma\lambda}F_{\mu\nu}^{\tilde{I}}F_{\rho\sigma}^{\tilde{J}}A_\lambda^{\tilde{K}}
\end{aligned} \tag{2.2}$$

with the supersymmetry transformation laws (to leading order in fermion fields)

$$\begin{aligned}
\delta e_\mu^m &= \frac{1}{2}\bar{\varepsilon}^i\Gamma^m\Psi_{\mu i} \\
\delta\Psi_{\mu i} &= \nabla_\mu(\omega)\varepsilon_i + \frac{i}{4\sqrt{6}}h_{\tilde{I}}(\Gamma_\mu^{\nu\rho} - 4\delta_\mu^{\nu\rho})F_{\nu\rho}^{\tilde{I}}\varepsilon_i \\
\delta A_\mu^{\tilde{I}} &= \vartheta_\mu^{\tilde{I}} \\
\delta\lambda_{\tilde{a}}^i &= -\frac{i}{2}f_{\tilde{x}}^{\tilde{a}}\Gamma^\mu(\partial_\mu\varphi^{\tilde{x}})\varepsilon_i + \frac{1}{4}h_{\tilde{I}}^{\tilde{a}}\Gamma^{\mu\nu}\varepsilon_iF_{\mu\nu}^{\tilde{I}} \\
\delta\varphi^{\tilde{x}} &= \frac{i}{2}f_{\tilde{a}}^{\tilde{x}}\bar{\varepsilon}^i\lambda_{\tilde{a}}^i,
\end{aligned} \tag{2.3}$$

where

$$\vartheta_\mu^{\tilde{I}} \equiv -\frac{1}{2}h_{\tilde{a}}^{\tilde{I}}\bar{\varepsilon}^i\Gamma_\mu\lambda_{\tilde{a}}^i + \frac{i\sqrt{6}}{4}h_{\tilde{I}}^{\tilde{I}}\bar{\Psi}_\mu^i\varepsilon_i. \tag{2.4}$$

Here, e denotes the fünfbein determinant, whereas $R(\omega)$ and $\nabla_\mu = \nabla_\mu(\omega)$ are the scalar curvature and the spacetime covariant derivative with respect to the ordinary spin connection $\omega_\mu^{mn}(e)$. $F_{\mu\nu}^{\tilde{I}}$ are the field strengths of the Abelian vector fields $A_\mu^{\tilde{I}}$. The various scalar field dependent quantities that contract the different types of indices are as follows: $f_{\tilde{x}}^{\tilde{a}}$, $g_{\tilde{x}\tilde{y}}$ and $\Omega_{\tilde{x}}^{\tilde{a}\tilde{b}}$ denote the \tilde{n} -bein, the metric and the spin connection, respectively, of the target manifold \mathcal{M} . The quantities $h_{\tilde{I}}$, $h_{\tilde{I}}^{\tilde{I}}$, $h_{\tilde{a}}^{\tilde{I}}$, $h_{\tilde{a}}^{\tilde{I}}$, $T_{\tilde{a}\tilde{b}\tilde{c}}$ and $\overset{\circ}{a}_{\tilde{I}\tilde{J}}$ are $\varphi^{\tilde{x}}$ -dependent functions that are subject to various algebraic and differential constraints (see [25] for details) as required by supersymmetry. These constraints also involve $f_{\tilde{x}}^{\tilde{a}}$, $g_{\tilde{x}\tilde{y}}$, $\Omega_{\tilde{x}}^{\tilde{a}\tilde{b}}$ and imply that *all* scalar field dependent quantities are completely determined by the *constant* symmetric tensor $C_{\tilde{I}\tilde{J}\tilde{K}}$ that appears in the $F \wedge F \wedge A$ - term in (2.2). The $C_{\tilde{I}\tilde{J}\tilde{K}}$ thus uniquely determine the whole

theory. In particular, the scalar field target manifold \mathcal{M} can be viewed as an \tilde{n} -dimensional hypersurface

$$C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{I}}h^{\tilde{J}}h^{\tilde{K}} = 1. \quad (2.5)$$

of an $(\tilde{n}+1)$ -dimensional ambient space parametrized by $(\tilde{n}+1)$ coordinates $h^{\tilde{I}}$. The resulting geometry of these theories was later referred to as “very special geometry”. In [27] it was suggested that the compactification of 11-dimensional supergravity over a Calabi-Yau threefold would lead to $d = 5$, $\mathcal{N} = 2$ MESGT’s coupled to hypermultiplets. The Calabi-Yau compactifications of 11d supergravity were later studied in [30] where it was explicitly shown that they lead to $\mathcal{N} = 2$ MESGT’s with $(h_{(1,1)} - 1)$ vector multiplets coupled to $(h_{(2,1)} + 1)$ hypermultiplets. ($h_{(1,1)}$ and $h_{(2,1)}$ are the Hodge numbers of the corresponding Calabi-Yau manifold.)

The $C_{\tilde{I}\tilde{J}\tilde{K}}$ themselves are not completely arbitrary. Going to a particular basis [25], they can be brought to the following form

$$C_{000} = 1, \quad C_{0ij} = -\frac{1}{2}\delta_{ij}, \quad C_{00i} = 0 \quad (2.6)$$

and the remaining coefficients C_{ijk} ($i, j, k = 1, 2, \dots, \tilde{n}$) may be chosen at will. We shall refer to this basis as the canonical basis.

The arbitrariness of the C_{ijk} shows that, even for a fixed number \tilde{n} of vector multiplets, various target manifolds \mathcal{M} are possible. A classification of these “very special real” manifolds has been given in [31] for the case that \mathcal{M} is a homogeneous space. This class contains the subclass of symmetric spaces, which were classified already long time ago [25, 27]. Although our further discussion is not at all restricted to symmetric (or even homogeneous) \mathcal{M} , we will look at the symmetric spaces in a somewhat greater detail in section 7. Let us therefore list the possible symmetric spaces for later reference. The symmetric spaces \mathcal{M} fall into two different categories, depending on whether they are associated with Jordan algebras or not:

(i) $\mathcal{M} = \frac{\text{Str}_0(\mathbf{J})}{\text{Aut}(\mathbf{J})}$, where $\text{Str}_0(\mathbf{J})$ and $\text{Aut}(\mathbf{J})$ are the reduced structure group and the automorphism group, respectively, of a formally real, unital, Jordan algebra, \mathbf{J} , of degree three [25, 32]. This “Jordan class” can be further divided into two subclasses:

- “Generic” or “reducible” Jordan class:

$$\mathbf{J} = \mathbf{R} \oplus \Sigma_{\tilde{n}} : \quad \mathcal{M} = \frac{\text{SO}(\tilde{n} - 1, 1) \times \text{SO}(1, 1)}{\text{SO}(\tilde{n} - 1)}, \quad \tilde{n} \geq 1. \quad (2.7)$$

Here, $\Sigma_{\tilde{n}}$ is a Jordan algebra of degree two, which can be identified as the algebra of Dirac gamma matrices in an $(\tilde{n} - 1)$ -dimensional (internal) ‘Minkowski’ space with the product being one half the anticommutator.

- “Irreducible” or “magical” Jordan class. The corresponding Jordan algebras are simple and are isomorphic to the Hermitian (3×3) -matrices over the four division algebras $\mathbf{R}, \mathbf{C}, \mathbf{H}, \mathbf{O}$ with the product being the anticommutator. They lead to the following target spaces:

$$\begin{aligned}
\mathbf{J}_3^{\mathbf{R}} : \mathcal{M} &= \mathrm{SL}(3, \mathbf{R})/\mathrm{SO}(3), & (\tilde{n} = 5) \\
\mathbf{J}_3^{\mathbf{C}} : \mathcal{M} &= \mathrm{SL}(3, \mathbf{C})/\mathrm{SU}(3), & (\tilde{n} = 8) \\
\mathbf{J}_3^{\mathbf{H}} : \mathcal{M} &= \mathrm{SU}^*(6)/\mathrm{Usp}(6), & (\tilde{n} = 14) \\
\mathbf{J}_3^{\mathbf{O}} : \mathcal{M} &= \mathrm{E}_{6(-26)}/\mathrm{F}_4, & (\tilde{n} = 26)
\end{aligned}$$

(ii) $\mathcal{M} = \frac{\mathrm{SO}(1, \tilde{n})}{\mathrm{SO}(\tilde{n})}$, $\tilde{n} > 1$. This class is not associated with Jordan algebras and will therefore be referred to as the “symmetric non-Jordan-family” [27].

We will now turn to the global symmetries of a generic MESGT (with possibly non-symmetric or non-homogeneous \mathcal{M}) described by (2.2). Two different global symmetries have to be distinguished:

- Any $\mathcal{N} = 2$ MESGT is globally invariant under the R-symmetry group $SU(2)_R$. This symmetry is inherited from the underlying supersymmetry algebra and acts exclusively on the fermions Ψ_μ^i and $\lambda^{i\tilde{a}}$ (ie. on their index i).
- Any group G of linear transformations

$$h^{\tilde{I}} \rightarrow B^{\tilde{I}}_{\tilde{J}} h^{\tilde{J}}, \quad A_\mu^{\tilde{I}} \rightarrow B^{\tilde{I}}_{\tilde{J}} A_\mu^{\tilde{J}}$$

that leaves the tensor $C_{\tilde{I}\tilde{J}\tilde{K}}$ invariant

$$B^{\tilde{I}'}_{\tilde{I}} B^{\tilde{J}'}_{\tilde{J}} B^{\tilde{K}'}_{\tilde{K}} C_{\tilde{I}'\tilde{J}'\tilde{K}'} = C_{\tilde{I}\tilde{J}\tilde{K}}$$

is automatically a symmetry of the whole Lagrangian (2.2), since the latter is uniquely determined by the $C_{\tilde{I}\tilde{J}\tilde{K}}$. In particular, these symmetries give rise to isometries of the scalar manifolds \mathcal{M} , which becomes manifest if one rewrites the kinetic energy term for the scalar fields as [25, 31]

$$-\frac{1}{2} g_{\tilde{x}\tilde{y}} (\partial_\mu \varphi^{\tilde{x}}) (\partial^\mu \varphi^{\tilde{y}}) = \frac{3}{2} C_{\tilde{I}\tilde{J}\tilde{K}} h^{\tilde{I}} \partial_\mu h^{\tilde{J}} \partial^\mu h^{\tilde{K}}$$

with the $h^{\tilde{I}}$ constrained according to (2.5).

Important (but not the only) examples with such a non-trivial symmetry group G are given by the aforementioned symmetric space cases. In the Jordan class, G coincides with the full isometry group of \mathcal{M} (ie. with the full “numerator group” $\mathrm{Str}_0(\mathbf{J})$). For the

symmetric non-Jordan family, $G = [SO(\tilde{n}-1) \times SO(1, 1)] \odot T_{(\tilde{n}-1)}$ where \odot denotes the semi-direct product and $T_{(\tilde{n}-1)}$ is the group of translations in an $(\tilde{n} - 1)$ dimensional Euclidean space. Note that for this family G is only a subgroup of the target space isometry group $SO(1, \tilde{n})$ [33].

The fact that the total global symmetry group of (2.2) factorizes into $SU(2)_R \times G$ is a consequence of the $SU(2)_R$ -invariance of the scalar fields belonging to the vector multiplets and allows to study the gaugings of the two factors separately. In general matter coupled extended supergravity theories the R-symmetry group is nontrivially embedded into some larger global symmetry group if the scalar fields are not singlets under it.

Let us now turn to the possible gaugings of subgroups of $SU(2)_R \times G$. Since the vector fields are all $SU(2)_R$ -inert, they cannot serve as non-Abelian gauge fields for the full $SU(2)_R$ ⁶. We will therefore only consider gaugings of subgroups of $U(1)_R \times G$, where $U(1)_R$ denotes the $U(1)$ subgroup of $SU(2)_R$. This obviously leaves the following possibilities:

- (i) One can simply gauge the $U(1)_R$ subgroup of $SU(2)_R$ by coupling a linear combination of the vector fields to the fermions [26], which are the only fields that transform nontrivially under $SU(2)_R$. In general, this kind of gauging (which we will refer to as “gauged MESGT”) introduces a scalar potential (see Section 3).
- (ii) Another possibility is to gauge a subgroup K of G . In this case, which we will refer to as “Yang-Mills/Einstein supergravity”, at least a subset of the vector fields has to transform in the adjoint representation of K so that these vector fields can serve as the corresponding Yang-Mills gauge fields. If there are additional vector fields (‘spectator vector fields’) beyond these gauge fields, there are two possibilities. They are either K -singlets or some of them transform non-trivially under K . In the former case, there are no technical difficulties and the gauging can be performed as described in [26] and leads to a theory *without* scalar potential (see Section 4)
- (iii) If there *are* vector fields that are charged under K , one faces the same problem that was first encountered in the context of maximally extended gauged supergravity in seven [2] and subsequently in five dimensions [3, 4, 5]. The problem is that a naive gauging of K would introduce masses for these vector fields, thereby leading to a mismatch between bosonic and fermionic degrees of freedom. The only known solution to this problem is to convert the charged vector fields into two-form fields with “self-dual” field equations [35]. In the maximally extended theories, this idea is also supported by the analysis of the spectra of the underlying Kaluza-Klein compactifications [12, 13].
- (iv) Finally, one can combine (i) and (ii), or alternatively (i) and (iii), and simultaneously gauge both $U(1)_R$ and $K \subset G$. We will refer to this type of gauging as “gauged

⁶One could, however, try to identify $SU(2)_R$ with an $SU(2)$ -subgroup of G and then gauge this diagonal subgroup, yet we have not considered such a possibility in the present paper.

Yang-Mills/Einstein supergravity”.

The first two possibilities were studied in [26, 28, 29] with special emphasis on the cases where \mathcal{M} is a symmetric space of the Jordan family. It is the purpose of this paper to extend some of the aspects that were discussed in [26, 28, 29] for the gaugings of type (i) and (ii) to more general \mathcal{M} and, moreover, study the so far uncovered gaugings of type (iii) and (iv), thereby closing a gap in the existing literature.

3 Gauged Maxwell/Einstein supergravity

In order to gauge the $U(1)_R$ -subgroup of the $SU(2)_R$ R-symmetry group, one promotes a linear combination of the $(\tilde{n} + 1)$ vector fields $A_\mu^{\tilde{I}}$ to the $U(1)_R$ -gauge field

$$A_\mu[U(1)_R] = V_{\tilde{I}} A_\mu^{\tilde{I}}, \quad (3.1)$$

where $V_{\tilde{I}}$ are $(\tilde{n} + 1)$ constants, and replaces the derivatives of the fermionic fields by $U(1)_R$ -covariant derivatives

$$\begin{aligned} \nabla_\mu \lambda^{\tilde{a}i} &\longrightarrow (D_\mu \lambda^{\tilde{a}i}) \equiv \nabla_\mu \lambda^{\tilde{a}i} + g_R V_{\tilde{I}} A_\mu^{\tilde{I}} \delta^{ij} \lambda_j^{\tilde{a}} \\ \nabla_\mu \Psi_\nu^i &\longrightarrow (D_\mu \Psi_\nu^i) \equiv \nabla_\mu \Psi_\nu^i + g_R V_{\tilde{I}} A_\mu^{\tilde{I}} \delta^{ij} \Psi_{\nu j}, \end{aligned} \quad (3.2)$$

where g_R denotes the $U(1)_R$ -coupling constant. The appearance of the δ^{ij} is due to the convention that the $SU(2)_R$ -indices i, j, \dots are raised and lowered with the antisymmetric metric $\varepsilon_{ij} = -\varepsilon_{ji}$, $\varepsilon_{12} = \varepsilon^{12} = 1$ [25, 26]. This $U(1)_R$ -covariantization in the Lagrangian (2.2) and the transformation laws (2.3) breaks the original supersymmetry. In order to restore it, some g_R -dependent gauge invariant terms have to be added. The additional terms in the Lagrangian are [26, 29] (the numerical factors are chosen for convenience)

$$\begin{aligned} e^{-1} \mathcal{L}' &= -\frac{i\sqrt{6}}{8} g_R \bar{\Psi}_\mu^i \Gamma^{\mu\nu} \Psi_\nu^j \delta_{ij} P_0(\varphi) - \frac{1}{\sqrt{2}} g_R \bar{\lambda}^{i\tilde{a}} \Gamma^\mu \Psi_\mu^j \delta_{ij} P_{\tilde{a}}(\varphi) \\ &+ \frac{i}{2\sqrt{6}} g_R \bar{\lambda}^{i\tilde{a}} \lambda^{j\tilde{b}} \delta_{ij} P_{\tilde{a}\tilde{b}}(\varphi) - g_R^2 P^{(R)}(\varphi), \end{aligned} \quad (3.3)$$

whereas the transformation laws have to be modified by

$$\begin{aligned} \delta' \Psi_\mu^i &= \frac{i}{2\sqrt{6}} g_R P_0(\varphi) \Gamma_\mu \delta^{ij} \varepsilon_j \\ \delta' \lambda^{i\tilde{a}} &= \frac{1}{\sqrt{2}} g_R P_{\tilde{a}}(\varphi) \delta^{ij} \varepsilon_j. \end{aligned} \quad (3.4)$$

The new scalar field dependent quantities P_0 , $P^{\tilde{a}}$, $P_{\tilde{a}\tilde{b}}$, and the scalar potential $P^{(R)}$ are fixed by supersymmetry

$$P^{\tilde{a}} = \sqrt{2}h^{\tilde{a}\tilde{I}}V_{\tilde{I}} \quad (3.5)$$

$$P_0 = 2h^{\tilde{I}\tilde{J}}V_{\tilde{I}} \quad (3.6)$$

$$P_{\tilde{a}\tilde{b}} = \frac{1}{2}\delta_{\tilde{a}\tilde{b}}P_0 + 2\sqrt{2}T_{\tilde{a}\tilde{b}\tilde{c}}P^{\tilde{c}} \quad (3.7)$$

$$P^{(R)} = -(P_0)^2 + P^{\tilde{a}}P_{\tilde{a}}. \quad (3.8)$$

The scalar potential $P^{(R)}$ can be written in the form [26, 29]

$$P^{(R)} = -4C^{\tilde{I}\tilde{J}\tilde{K}}V_{\tilde{I}}V_{\tilde{J}}h_{\tilde{K}}, \quad (3.9)$$

where the $\tilde{I}, \tilde{J}, \tilde{K}$ are raised with the inverse of $\tilde{a}_{\tilde{I}\tilde{J}}$. In our metric signature, a critical point φ_c of the scalar potential with $P^{(R)}(\varphi_c) < 0$ corresponds to an anti-de Sitter ground state. The critical points of the potential (3.8) have been analyzed in [26, 29] for the Jordan cases. If a critical point exists, it was found that, depending on the linear combination (3.1) of the vector fields, one either gets an $\mathcal{N} = 2$ supersymmetric anti-de Sitter ground state, or the scalar potential vanishes identically, and thus admits a Minkowski vacuum with spontaneously broken supersymmetry.

4 Yang-Mills/Einstein supergravity without tensor fields

We now consider the gauging of a subgroup K of G . As mentioned earlier, this type requires that a subset $\{A_\mu^I; I, J, \dots = 1, \dots, \dim K\}$ of the vector fields transforms in the adjoint representation of K . In this section, we assume that if there are additional spectator vector fields $\{A_\mu^M; M, N, P = 1, \dots, (\tilde{n} + 1) - \dim K\}$, they are all K -singlets (i.e., we are dealing with the gauging of type (ii)).

The only fields that transform under K are the scalar fields $\varphi^{\tilde{x}}$, the spinor fields $\lambda^{i\tilde{a}}$ and the vector fields A_μ^I , ($I = 1, \dots, \dim K$). The K -covariantization is thus achieved by replacing the corresponding derivatives/field strengths by their K -gauge covariant counterparts:

$$\begin{aligned} \partial_\mu \varphi^{\tilde{x}} &\longrightarrow \mathcal{D}_\mu \varphi^{\tilde{x}} \equiv \partial_\mu \varphi^{\tilde{x}} + g A_\mu^I K_I^{\tilde{x}} \\ \nabla_\mu \lambda^{i\tilde{a}} &\longrightarrow \mathcal{D}_\mu \lambda^{i\tilde{a}} \equiv \nabla_\mu \lambda^{i\tilde{a}} + g A_\mu^I L_I^{\tilde{a}\tilde{b}} \lambda^{i\tilde{b}} \\ F_{\mu\nu}^I &\longrightarrow \mathcal{F}_{\mu\nu}^I \equiv F_{\mu\nu}^I + g f_{JK}^I A_\mu^J A_\nu^K. \end{aligned} \quad (4.1)$$

Here, g denotes the K -coupling constant, $K_I^{\tilde{x}}$ are the Killing vectors of \mathcal{M} that correspond to the subgroup K of its isometry group G (cf. [26]), $L_I^{\tilde{a}\tilde{b}}$ are the (scalar field dependent) K -transformation matrices of the fermions $\lambda^{i\tilde{a}}$ (cf. [26, 28]) and f_{JK}^I are the structure

constants of K . These replacements in the Lagrangian (2.2) and the transformation laws (2.3) are subject to one exception: The proper gauge-covariantization of the $F \wedge F \wedge A$ -term in (2.2) leads to a Chern Simons term, i.e., $\frac{e^{-1}}{6\sqrt{6}} C_{\tilde{I}\tilde{J}\tilde{K}} \varepsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu}^{\tilde{I}} F_{\rho\sigma}^{\tilde{J}} A_{\lambda}^{\tilde{K}}$ gets replaced by

$$\frac{e^{-1}}{6\sqrt{6}} C_{\tilde{I}\tilde{J}\tilde{K}} \varepsilon^{\mu\nu\rho\sigma\lambda} \left\{ F_{\mu\nu}^{\tilde{I}} F_{\rho\sigma}^{\tilde{J}} A_{\lambda}^{\tilde{K}} + \frac{3}{2} g F_{\mu\nu}^{\tilde{I}} A_{\rho}^{\tilde{J}} (f_{\tilde{L}\tilde{M}}^{\tilde{K}} A_{\sigma}^{\tilde{L}} A_{\lambda}^{\tilde{M}}) + \frac{3}{5} g^2 (f_{\tilde{N}\tilde{P}}^{\tilde{J}} A_{\nu}^{\tilde{N}} A_{\rho}^{\tilde{P}}) (f_{\tilde{L}\tilde{M}}^{\tilde{K}} A_{\sigma}^{\tilde{L}} A_{\lambda}^{\tilde{M}}) A_{\mu}^{\tilde{I}} \right\} \quad (4.2)$$

where it is understood that $f_{\tilde{I}\tilde{J}}^{\tilde{K}}$ is zero whenever one of the indices $\tilde{I}, \tilde{J}, \tilde{K}$ corresponds to one of the spectator vector fields A_{μ}^M .

Again, supersymmetry is broken by these replacements. This time, however, its restoration requires little modification; the (covariantized) transformation laws remain unchanged, and only a Yukawa-like term has to be added to the (covariantized) Lagrangian [26, 28]

$$\mathcal{L}' = -\frac{i}{2} g \bar{\lambda}^{i\tilde{a}} \lambda_i^{\tilde{b}} K_{I[\tilde{a} h_{\tilde{b}}^I].} \quad (4.3)$$

In particular, no scalar potential is introduced so that only Minkowski ground states are possible.

5 Yang-Mills/Einstein supergravity with tensor fields

We now turn to case (iii) of our gauge type classification and consider the gauging of $K \subset G$, when *not* all the spectator vector fields are K -singlets. As mentioned earlier, consistency with supersymmetry requires that these K -charged spectator vector fields have to be dualized to “self-dual” two-form fields [35, 3, 4, 5, 12, 13]. We will therefore split the vector fields $A_{\mu}^{\tilde{I}}$ of the *ungauged* theory (2.2)-(2.3) of Section 2 into two sets. The first set contains the vector fields in the adjoint representation of the gauge group K plus possible K -singlets. The second set contains the remaining K -charged vector fields. We will use indices $I, J, K, \dots = 1, \dots, n$ for the first and $M, N, P, \dots = 1, \dots, 2m$ for the second set, where $n + 2m = \tilde{n} + 1$. The reason for the even number $2m$ is that the “self-duality”-condition of [35] requires *complex* tensor fields for $d = 5$, which we will always consider as being decomposed into their real and imaginary parts. The gauging now proceeds as follows. First, one has to replace all Abelian field strengths $F_{\mu\nu}^I$ by the corresponding non-Abelian generalizations $\mathcal{F}_{\mu\nu}^I \equiv F_{\mu\nu}^I + g f_{JK}^I A_{\mu}^J A_{\nu}^K$, with f_{JK}^I being the structure constants of K ⁷, and the $F_{\mu\nu}^M$ by the above-mentioned “self-dual” two-form fields $B_{\mu\nu}^M$:

$$F_{\mu\nu}^{\tilde{I}} \longrightarrow \mathcal{H}_{\mu\nu}^{\tilde{I}} := (\mathcal{F}_{\mu\nu}^I, B_{\mu\nu}^M). \quad (5.1)$$

⁷In the presence of K -singlets, the corresponding f_{JK}^I are again assumed to be zero (cf. Section 4)

Again, the only exception to this replacement is the $F \wedge F \wedge A$ -term of the ungauged theory. Since no ‘naked’ A_μ^M can appear anymore, we first require

$$C_{MNP} = 0 \tag{5.2}$$

and since terms of the form $B^M \wedge F^I \wedge A^J$ appear to be impossible to supersymmetrize in a gauge invariant way (except possibly in very special cases) we shall also assume that

$$C_{MIJ} = 0 \tag{5.3}$$

Hence, the only non-vanishing $C_{\tilde{I}\tilde{J}\tilde{K}}$ have the index structure C_{IJK} and C_{IMN} . The covariantization of the $C_{IJK}F^I \wedge F^J \wedge A^K$ -term again leads to a Chern-Simons term (see below). The term of the form $C_{IMN}A^I \wedge B^M \wedge B^N$ has its natural place in the gauge-invariant kinetic energy term for the tensor fields $B_{\mu\nu}^M$ (cf. eqs.(5.4) and (5.6)). The gauge covariant derivative of these tensor fields reads

$$\mathcal{D}_\mu B_{\nu\rho}^M \equiv \nabla_\mu B_{\nu\rho}^M + gA_\mu^I \Lambda_{IN}^M B_{\nu\rho}^N, \tag{5.4}$$

where the *constant* matrices Λ_{IN}^M are the corresponding representation matrices of K .

The remaining gauge covariantizations involve the scalar and spinor fields, for which we again make the replacements

$$\begin{aligned} \partial_\mu \varphi^{\tilde{x}} &\longrightarrow \mathcal{D}_\mu \varphi^{\tilde{x}} \equiv \partial_\mu \varphi^{\tilde{x}} + gA_\mu^I K_I^{\tilde{x}} \\ \nabla_\mu \lambda^{i\tilde{a}} &\longrightarrow \mathcal{D}_\mu \lambda^{i\tilde{a}} \equiv \nabla_\mu \lambda^{i\tilde{a}} + gA_\mu^I L_I^{\tilde{a}\tilde{b}} \lambda^{i\tilde{b}}. \end{aligned} \tag{5.5}$$

After all these modifications, the original supersymmetry of the ungauged theory (2.2)-(2.3) is again badly broken. This time, however, the supersymmetry breaking is not only due to the gauge covariantization alone. An additional source for the breakdown of supersymmetry is provided by the loss of the Bianchi identity for the tensor fields $B_{\mu\nu}^M$ (ie. $dB^M \neq 0$ in general). The corresponding Bianchi identity $dF^{\tilde{I}} = 0$ for the $F_{\mu\nu}^{\tilde{I}}$ in the ungauged theory is needed at several places to cancel certain supersymmetry variations in (2.2).

Remarkably enough, supersymmetry can again be restored by adding further g -dependent gauge invariant terms to the Lagrangian and the transformation laws. This procedure is very similar to what had to be done in the $\mathcal{N} = 8$ theory [3, 4, 5]. We omit the details here and quote the final result.

The Lagrangian is given by (up to 4-fermion terms)

$$\begin{aligned} e^{-1} \mathcal{L} = & -\frac{1}{2}R(\omega) - \frac{1}{2}\bar{\Psi}_\mu^i \Gamma^{\mu\nu\rho} \nabla_\nu \Psi_{\rho i} - \frac{1}{4} \overset{\circ}{a}_{\tilde{I}\tilde{J}} \mathcal{H}_{\mu\nu}^{\tilde{I}} \mathcal{H}^{\tilde{J}\mu\nu} \\ & - \frac{1}{2} \bar{\lambda}^{i\tilde{a}} \left(\Gamma^\mu \mathcal{D}_\mu \delta^{\tilde{a}\tilde{b}} + \Omega_{\tilde{x}}^{\tilde{a}\tilde{b}} \Gamma^\mu \mathcal{D}_\mu \varphi^{\tilde{x}} \right) \lambda_i^{\tilde{b}} - \frac{1}{2} g_{\tilde{x}\tilde{y}} (\mathcal{D}_\mu \varphi^{\tilde{x}}) (\mathcal{D}^\mu \varphi^{\tilde{y}}) \end{aligned}$$

$$\begin{aligned}
& -\frac{i}{2}\bar{\lambda}^{i\tilde{a}}\Gamma^\mu\Gamma^\nu\Psi_{\mu i}f_{\tilde{x}}^{\tilde{a}}\mathcal{D}_\nu\varphi^{\tilde{x}} + \frac{1}{4}h_{\tilde{I}}^{\tilde{a}}\bar{\lambda}^{i\tilde{a}}\Gamma^\mu\Gamma^{\lambda\rho}\Psi_{\mu i}\mathcal{H}_{\lambda\rho}^{\tilde{I}} \\
& + \frac{i}{2\sqrt{6}}\left(\frac{1}{4}\delta_{\tilde{a}\tilde{b}}h_{\tilde{I}} + T_{\tilde{a}\tilde{b}\tilde{c}}h_{\tilde{I}}^{\tilde{c}}\right)\bar{\lambda}^{i\tilde{a}}\Gamma^{\mu\nu}\lambda_{\tilde{I}}^{\tilde{b}}\mathcal{H}_{\mu\nu}^{\tilde{I}} \\
& - \frac{3i}{8\sqrt{6}}h_{\tilde{I}}\left[\bar{\Psi}_\mu^i\Gamma^{\mu\nu\rho\sigma}\Psi_{\nu i}\mathcal{H}_{\rho\sigma}^{\tilde{I}} + 2\bar{\Psi}^{\mu i}\Psi_i^\nu\mathcal{H}_{\mu\nu}^{\tilde{I}}\right] \\
& + \frac{e^{-1}}{6\sqrt{6}}C_{IJK}\varepsilon^{\mu\nu\rho\sigma\lambda}\left\{F_{\mu\nu}^IF_{\rho\sigma}^JA_\lambda^K + \frac{3}{2}gF_{\mu\nu}^IA_\rho^J(f_{LF}^KA_\sigma^LA_\lambda^F)\right. \\
& \quad \left.+ \frac{3}{5}g^2(f_{GH}^JA_\nu^GA_\rho^H)(f_{LF}^KA_\sigma^LA_\lambda^F)A_\mu^I\right\} \\
& + \frac{e^{-1}}{4g}\varepsilon^{\mu\nu\rho\sigma\lambda}\Omega_{MN}B_{\mu\nu}^M\mathcal{D}_\rho B_{\sigma\lambda}^N \\
& + g\bar{\lambda}^{i\tilde{a}}\Gamma^\mu\Psi_{\mu i}W^{\tilde{a}} + g\bar{\lambda}^{i\tilde{a}}\lambda_{\tilde{I}}^{\tilde{b}}W^{\tilde{a}\tilde{b}} - g^2P.
\end{aligned} \tag{5.6}$$

The transformation laws are (to leading order in fermion fields)

$$\begin{aligned}
\delta e_\mu^m &= \frac{1}{2}\bar{\varepsilon}^i\Gamma^m\Psi_{\mu i} \\
\delta\Psi_{\mu i} &= \nabla_\mu(\omega)\varepsilon_i + \frac{i}{4\sqrt{6}}h_{\tilde{I}}(\Gamma_\mu^{\nu\rho} - 4\delta_\mu^\nu\Gamma^\rho)\mathcal{H}_{\nu\rho}^{\tilde{I}}\varepsilon_i \\
\delta A_\mu^I &= \vartheta_\mu^I \\
\delta B_{\mu\nu}^M &= 2\mathcal{D}_{[\mu}\vartheta_{\nu]}^M + \frac{\sqrt{6}g}{4}\Omega^{MN}h_N\bar{\Psi}_{[\mu}^i\Gamma_{\nu]}^i\varepsilon_i + \frac{ig}{4}\Omega^{MN}h_N^{\tilde{a}}\bar{\lambda}^{i\tilde{a}}\Gamma_{\mu\nu}\varepsilon_i \\
\delta\lambda_i^{\tilde{a}} &= -\frac{i}{2}f_{\tilde{x}}^{\tilde{a}}\Gamma^\mu(\mathcal{D}_\mu\varphi^{\tilde{x}})\varepsilon_i + \frac{1}{4}h_{\tilde{I}}^{\tilde{a}}\Gamma^{\mu\nu}\varepsilon_i\mathcal{H}_{\mu\nu}^{\tilde{I}} + gW^{\tilde{a}}\varepsilon_i \\
\delta\varphi^{\tilde{x}} &= \frac{i}{2}f_{\tilde{a}}^{\tilde{x}}\bar{\varepsilon}^i\lambda_i^{\tilde{a}}
\end{aligned} \tag{5.7}$$

with

$$\vartheta_\mu^{\tilde{I}} \equiv -\frac{1}{2}h_{\tilde{a}}^{\tilde{I}}\bar{\varepsilon}^i\Gamma_\mu\lambda_i^{\tilde{a}} + \frac{i\sqrt{6}}{4}h_{\tilde{I}}^{\tilde{I}}\bar{\Psi}_\mu^i\varepsilon_i. \tag{5.8}$$

The quantities which are not already present in the ungauged theory are a (constant) real symplectic metric Ω_{MN}

$$\Omega_{MN} = -\Omega_{NM}, \quad \Omega_{MN}\Omega^{NP} = \delta_M^P, \tag{5.9}$$

two tensors $W^{\tilde{a}}(\varphi)$ and $W^{\tilde{a}\tilde{b}}(\varphi)$

$$\begin{aligned}
W^{\tilde{a}} &= -\frac{\sqrt{6}}{8}h_M^{\tilde{a}}\Omega^{MN}h_N \\
W^{\tilde{a}\tilde{b}} &= -W^{\tilde{b}\tilde{a}} = ih^{J[\tilde{a}}K_{J}^{\tilde{b}]} + \frac{i\sqrt{6}}{4}h^JK_J^{\tilde{a}\tilde{b}},
\end{aligned} \tag{5.10}$$

where the semicolon denotes covariant differentiation on the target space \mathcal{M} , and a scalar potential $P(\varphi)$

$$P = 2W^{\tilde{a}}W_{\tilde{a}}. \quad (5.11)$$

Furthermore, one finds the relation

$$\Lambda_{IM}^N = \frac{2}{\sqrt{6}}\Omega^{NP}C_{MPI} \iff \Omega_{NP}\Lambda_{IM}^P = \frac{2}{\sqrt{6}}C_{MNI}, \quad (5.12)$$

which, because of $C_{MNI} = C_{NMI}$, means that the Λ_{IM}^N have to form a symplectic representation of the gauge group K . Supersymmetry also requires the following two relations

$$W^{\tilde{a}} = \frac{\sqrt{6}}{4}h^J K_J^{\tilde{a}} \quad (5.13)$$

$$P_{,\tilde{a}} = 4iW^{\tilde{a}\tilde{b}}W_{\tilde{b}} \quad (5.14)$$

where the comma denotes partial differentiation with respect to the scalar fields. These last two conditions, however, can be shown to follow automatically from the various other constraints.

The above scalar potential $P(\varphi)$ deserves some comments.

First of all, it is a bit surprising that there is a scalar potential at all, since no minimal couplings to the gravitini have been introduced at this point, and, as we have seen in Section 4, the pure Yang-Mills/Einstein supergravity theories *without* antisymmetric tensor fields do *not* involve a scalar potential. In fact, the necessity for the scalar potential in the above Lagrangian can eventually be traced back to the loss of the Bianchi identity for the $B_{\mu\nu}^M$, which are not present in the theories considered in Section 4.

The second important point about the potential is its sign. As mentioned at the end of Section 3, in our metric signature, a critical point with $P(\varphi_c) < 0$ would correspond to an Anti-de Sitter solution. The explicit form (5.11) of our potential, however, is manifestly non-negative. Therefore, the $\mathcal{N} = 2$ Yang-Mills/Einstein supergravity theories with tensor multiplets do *not* admit an Anti-de Sitter solution. This might at first seem surprising, since it was, among other things, the representation theory of the AdS_5 -superalgebra $SU(2, 2|4)$ that hinted towards the dualization of twelve vector fields to antisymmetric tensor fields in the gauging of the $\mathcal{N} = 8$ supergravity theory in $d = 5$ [3, 4, 5]. For $\mathcal{N} = 2$ however, this argument does not apply anymore, since the $\mathcal{N} = 2$ Anti-de Sitter graviton supermultiplet also contains only one vector field and no tensor fields, giving rise to the same field content as its $\mathcal{N} = 2$ super Poincaré counterpart. Thus, for $\mathcal{N} = 2$, the antisymmetric tensor fields do not necessarily have to be associated with Anti-de Sitter spacetimes anymore.

6 Gauged Yang-Mills/Einstein supergravity with tensor fields

We will now come to case (iv) of our list of possible gaugings and simultaneously gauge the $U(1)_R$ R-symmetry subgroup and a subgroup K of G . We will do this for the most general case with tensor fields, since the case without tensor fields can easily be recovered as a special case. Our starting point will be the Yang-Mills/Einstein supergravity with tensor fields presented in the previous section, i.e. eqs. (5.6)-(5.7).

As in Section 3, we will take a linear combination of the vector fields A_μ^I as the $U(1)_R$ -gauge field

$$A_\mu[U(1)_R] = V_I A_\mu^I \quad (6.1)$$

with some constants V_I , which at this point are completely arbitrary. (Note, however, that we don't sum over \tilde{I} like in Section 3, i.e., " $V_M = 0$ ".) The gauging of $U(1)_R$ then obviously requires the $U(1)_R$ -covariantization of all fermionic derivatives:

$$\begin{aligned} \mathcal{D}_\mu \lambda^{\tilde{a}i} &\longrightarrow (\mathfrak{D}_\mu \lambda^{\tilde{a}i}) \equiv \mathcal{D}_\mu \lambda^{\tilde{a}i} + g_R V_I A_\mu^I \delta^{ij} \lambda_j^{\tilde{a}} \\ \nabla_\mu \Psi_\nu^i &\longrightarrow (\mathfrak{D}_\mu \Psi_\nu)^i \equiv \nabla_\mu \Psi_\nu^i + g_R V_I A_\mu^I \delta^{ij} \Psi_{\nu j}, \end{aligned} \quad (6.2)$$

where g_R again denotes the $U(1)_R$ -coupling constant and \mathcal{D}_μ is the K -covariant derivative introduced in (5.5). Again, this gauge covariantization breaks supersymmetry, and to restore it, new g_R -dependent terms have to be added to the Lagrangian and the transformation laws.

The additional terms in the transformation laws are

$$\begin{aligned} e^{-1} \mathcal{L}' &= -\frac{i\sqrt{6}}{8} g_R \bar{\Psi}_\mu^i \Gamma^{\mu\nu} \Psi_\nu^j \delta_{ij} P_0(\varphi) - \frac{1}{\sqrt{2}} g_R \bar{\lambda}^{i\tilde{a}} \Gamma^\mu \Psi_\mu^j \delta_{ij} P_{\tilde{a}}(\varphi) \\ &\quad + \frac{i}{2\sqrt{6}} g_R \bar{\lambda}^{i\tilde{a}} \lambda^{j\tilde{b}} \delta_{ij} P_{\tilde{a}\tilde{b}}(\varphi) - g_R^2 P^{(R)}(\varphi), \end{aligned} \quad (6.3)$$

whereas the transformation laws have to be modified by

$$\begin{aligned} \delta' \Psi_\mu^i &= \frac{i}{2\sqrt{6}} g_R P_0(\varphi) \Gamma_\mu \delta^{ij} \varepsilon_j \\ \delta' \lambda^{i\tilde{a}} &= \frac{1}{\sqrt{2}} g_R P_{\tilde{a}}(\varphi) \delta^{ij} \varepsilon_j. \end{aligned} \quad (6.4)$$

The new scalar field dependent quantities P_0 , $P_{\tilde{a}}$, $P_{\tilde{a}\tilde{b}}$, and the scalar potential $P^{(R)}$ are fixed by supersymmetry

$$P_{\tilde{a}} = \sqrt{2} h^{\tilde{a}I} V_I \quad (6.5)$$

$$P_0 = 2h^I V_I \quad (6.6)$$

$$P_{\tilde{a}\tilde{b}} = \frac{1}{2}\delta_{\tilde{a}\tilde{b}}P_0 + 2\sqrt{2}T_{\tilde{a}\tilde{b}\tilde{c}}P^{\tilde{c}} \quad (6.7)$$

$$P^{(R)} = -(P_0)^2 + P^{\tilde{a}}P_{\tilde{a}}. \quad (6.8)$$

Furthermore, the V_I are constrained by

$$V_I f_{JK}^I = 0. \quad (6.9)$$

Supersymmetry also requires the relations

$$\begin{aligned} P^{\tilde{a}}K_{\tilde{I}}^{\tilde{a}} &= 0 \\ P_{\tilde{a}\tilde{b}}W^{\tilde{b}} &= -i2\sqrt{3}W_{\tilde{a}\tilde{b}}P^{\tilde{b}} + \frac{5}{2}W_{\tilde{a}}P_0 \\ P_{;\tilde{x}}^{\tilde{a}} &= \frac{-\sqrt{3}}{4}P_0f_{\tilde{x}}^{\tilde{a}} - \frac{1}{2\sqrt{3}}P_{\tilde{a}\tilde{b}}f_{\tilde{x}}^{\tilde{b}} \\ P_{0,\tilde{x}} &= -\frac{2}{\sqrt{3}}P^{\tilde{a}}f_{\tilde{x}}^{\tilde{a}} \\ P_{,\tilde{x}}^{(R)} &= \frac{5}{2\sqrt{3}}P_0P^{\tilde{a}}f_{\tilde{x}}^{\tilde{a}} - \frac{1}{\sqrt{3}}f_{\tilde{x}}^{\tilde{a}}P_{\tilde{a}\tilde{b}}P^{\tilde{b}}, \end{aligned} \quad (6.10)$$

however, these can be shown to be consequences of the other constraints and therefore do not give rise to additional restrictions.

It should be noted that the constraints (6.5)-(6.8) are almost the same as in the case of the pure $U(1)_R$ -gauging described in Section 3. Yet there are two important differences. The first is that the (completely arbitrary) $V_{\tilde{I}}$ of Section 3 are now subject to two constraints, namely eq. (6.9) and “ $V_M = 0$ ”, which is merely a trivial consequence of (6.1).

The second difference is that (6.8) is not the full scalar potential. The latter is now a sum of the $U(1)_R$ -related potential $P^{(R)}$ and the potential P , which was due to the introduction of the 2-form fields (cf. eqs. (5.6) and (5.11)):

$$e^{-1}\mathcal{L}_{pot} = -g^2P - g_R^2P^{(R)} \quad (6.11)$$

These differences have some interesting implications:

Eq. (6.9) gives a new constraint on the possible gauge groups K , since for it to be true, the f_{JK}^I have to admit a nontrivial eigenvector V_I with eigenvalue 0. This means that either there has to be at least one spectator vector field A_{μ}^I or K has to have at least one Abelian factor (both of them together could also be true).

As for the potential, one sees that the $U(1)_R$ -gauging introduces a negative contribution to the total scalar potential so that Anti-de Sitter solutions might now be possible. In fact, the experience with the gauged MESGT's in [26, 29] and certain truncations of the $\mathcal{N} = 8$ theory [4] make this possibility quite plausible. A more detailed analysis of the potential and its critical points, however, is now complicated by the additional scalar potential term $-g^2P$ induced by the tensor field dualization and the additional constraints on the $V_{\tilde{I}}$ and will therefore be given elsewhere [34].

7 Allowed gauge groups and the corresponding representations of the tensor multiplets

In this section we will give a partial classification of the possible gauge groups and the representations under which the tensor fields transform. An attempt at a complete classification will be made elsewhere [34].

We will start our discussion of possible gauge groups with the “Magical” supergravity theories defined by simple Jordan algebras of degree 3.

- (i) The largest of the magical $\mathcal{N} = 2$ supergravity theories is defined by the exceptional Jordan algebra with the scalar manifold $E_{6(-26)}/F_4$, which we shall refer to as the exceptional supergravity theory. The exceptional supergravity theory and its counterparts in four and three dimensions share many of the remarkable properties of the maximally extended supergravity theories in the respective dimensions. In the exceptional theory one can gauge the $SO^*(6) = SU(3, 1)$ subgroup of the isometry group $E_{6(-26)}$ of the scalar manifold while dualizing twelve of the vector fields into tensor fields that form a symplectic representation $(6 + \bar{6})$ of $SO^*(6)$. The pure maximal Yang-Mills Einstein subsector of this theory is the unique unified Yang-Mills Einstein supergravity in five dimensions that was studied in [28]. To gauge a $U(1)_R$ subgroup of the R -symmetry $SU(2)_R$ one needs to break the non-Abelian gauge group $SU(3, 1)$ down to a subgroup. One possibility is to gauge the $U(1)_R$ such that the $SU(3)$ subgroup of $SU(3, 1)$ is unbroken. In this case we obtain a gauged Yang-Mills Einstein supergravity theory with the gauge group $U(1)_R \times SU(3)$ and 18 tensor multiplets in the symplectic representation $(3 + \bar{3} + 3 + \bar{3} + 3 + \bar{3})$ of $SU(3)$. The subsector of this theory involving only 6 tensor multiplets corresponds to the $\mathcal{N} = 2$ truncation of the gauged $\mathcal{N} = 8$ theory with the gauge group $SU(3) \times U(1)_R$ [4] which admits an AdS ground state. One can also gauge the subgroup $U(1)_R$ such that one has a vanishing potential $P^{(R)}$. In this case the unbroken non-Abelian symmetry is the $SU(2, 1)$ subgroup of $SU(3, 1)$ with 18 tensor multiplets.
- (ii) The magical $\mathcal{N} = 2$ MESGT defined by the Jordan algebra of 3×3 Hermitian matrices over the quaternions has the scalar manifold $SU^*(6)/USp(6)$. One can gauge the $SO^*(6)$ subgroup of the isometry group resulting in the unique unified Yang-Mills Einstein supergravity [28] with no tensor multiplets. To obtain a Yang-Mills Einstein supergravity with tensor multiplets one has to gauge a subgroup of $SO^*(6)$. One can gauge the maximal compact subgroup $SU(3) \times U(1)$ of $SO^*(6)$ by dualizing 6 of the vector fields to tensor fields transforming in the symplectic representation $(3 + \bar{3})$ of $SU(3) \times U(1)$. One can similarly gauge the non-compact subgroup $SU(2, 1) \times U(1)$ of $SO^*(6)$. In both cases one can use the gauge field associated with the Abelian factor to gauge the $U(1)_R$ symmetry thereby obtaining gauged Yang-Mills/Einstein

supergravity theories with the non-Abelian gauge groups $SU(3)$ and $SU(2, 1)$ and six tensor multiplets, respectively. We expect the generic $SU(2, 1) \times U(1)_R$ gauging to lead to a vanishing potential $P^{(R)}$.

- (iii) The magical MESGT defined by the Jordan algebra of 3×3 Hermitian matrices over the complex numbers has the scalar manifold $SL(3, \mathbf{C})/SU(3)$. In this theory one can gauge the full compact symmetry group to obtain a Yang-Mills/Einstein supergravity theory with the gauge group $SU(3)$ [26]. The remaining vector field (graviphoton) can be used to gauge the $U(1)_R$ symmetry with a non-vanishing potential and an AdS ground state. To obtain a Yang-Mills Einstein supergravity with tensor multiplets one needs to gauge a subgroup of $SU(3)$. One can, for example, gauge the $SU(2) \times U(1)$ subgroup while dualizing four of the vector fields to tensor fields in the symplectic representation $(2 + \bar{2})$. One can then use the graviphoton to obtain a $U(1)_R$ gauged version of this theory. Noncompact analogs of these theories also exist with $SU(3)$ and $SU(2)$ replaced by $SL(3, \mathbf{R})$ and $SL(2, \mathbf{R})$, respectively.
- (iv) The smallest of the magical MESGT's has the scalar manifold $SL(3, \mathbf{R})/SO(3)$. In this case one can gauge the $SL(2, \mathbf{R})$ subgroup of the isometry group while dualizing two of the vector fields into tensor fields. The remaining vector field can be used to gauge the $U(1)_R$ symmetry.
- (v) For the generic Jordan family the scalar manifold of the $\mathcal{N} = 2$ MESGT is

$$SO(\tilde{n} - 1, 1) \times SO(1, 1)/SO(\tilde{n} - 1) \tag{7.1}$$

On the other hand the scalar manifold of the generic symmetric non-Jordan family is of the form

$$SO(\tilde{n}, 1)/SO(\tilde{n}) \tag{7.2}$$

For the latter family, not all the isometries of the scalar manifold can be extended to symmetries of the Lagrangian [33]. Only the subgroup $[SO(\tilde{n} - 1) \times SO(1, 1)] \odot T_{(\tilde{n}-1)}$ (i.e the Euclidean group in $(\tilde{n} - 1)$ dimensions times dilatations) of $SO(\tilde{n}, 1)$ extends to a full symmetry of the action. This can simply be understood by the fact that there is no irreducible symmetric invariant tensor of rank three of $SO(\tilde{n}, 1)$.

One can treat the generic Jordan and non-Jordan families in a unified manner as was shown in [27]. Consider a vector \mathbf{m} in an \tilde{n} dimensional Euclidean space with components m_i . Then the non-vanishing components of the tensor $C_{\tilde{I}\tilde{J}\tilde{K}}$ can be written in the form (cf. eq. (2.6))

$$C_{000} = 1 \tag{7.3}$$

$$C_{0ij} = -\frac{1}{2}\delta_{ij}$$

$$C_{ijk} = \frac{3}{2}m_{(i}\delta_{jk)} - m_i m_j m_k$$

where $i, j, .. = 1, 2, \dots, \tilde{n}$. For the generic Jordan family the length squared of the vector \mathbf{m} is two

$$\mathbf{m} \cdot \mathbf{m} = 2 \tag{7.4}$$

while for the non-Jordan family one has

$$\mathbf{m} \cdot \mathbf{m} = \frac{1}{2}. \tag{7.5}$$

It is easy to verify that the above C_{ijk} can provide a symplectic representation of *only* an Abelian subgroup of the compact symmetry group of $\mathcal{N} = 2$ MESGT. Therefore, if we are to have tensor fields transforming nontrivially under the compact gauge group then only products of $U(1)$'s are allowed in the Yang-Mills Einstein supergravity with tensor multiplets. Of course one still has the option to gauge a non-Abelian subgroup of the compact symmetry group so long as the tensor fields are inert under it.

- (vi) The scalar manifolds listed above exhaust the list of $\mathcal{N} = 2$ MESGT's whose scalar manifolds are symmetric spaces. In addition there is a large set of other theories whose scalar manifolds admit isometries that extend to symmetries of the full action. These include theories whose scalar manifolds are homogeneous spaces as well as those that are not homogeneous. A complete list of possible homogeneous spaces was given in [31]. This classification was achieved by showing that the requirement of a transitive isometry group allows one to bring the most general solution for the symmetric tensor given in the ‘‘canonical basis’’ above to the form:

$$C_{011} = 1 \tag{7.6}$$

$$C_{0\bar{\mu}\bar{\nu}} = -\delta_{\bar{\mu}\bar{\nu}}$$

$$C_{1\bar{i}\bar{j}} = -\delta_{\bar{i}\bar{j}}$$

$$C_{\bar{\mu}\bar{i}\bar{j}} = \gamma_{\bar{\mu}\bar{i}\bar{j}}$$

where the indices \tilde{I} are now split such that $\tilde{I} = 0, 1, \bar{\mu}, \bar{i}$ with $\bar{\mu} = 1, 2, \dots, q + 1$ and $\bar{i} = 1, 2, \dots, r$. The coefficients $\gamma_{\bar{\mu}\bar{i}\bar{j}}$ are $(q + 1)$ real $r \times r$ matrices that generate a real Clifford algebra of positive signature $\mathcal{C}(q + 1, 0)$. The allowed homogeneous (but not symmetric) spaces are, in general, quotients of ‘‘parabolic groups’’ G modded out by their maximal compact subgroups H . The Lie algebra g of the group G is a semi-direct sum:

$$g = g_0 \oplus g_{+1} \tag{7.7}$$

$$\begin{aligned}
g_0 &= so(1,1) \oplus so(q+1,1) \oplus \mathcal{S}_q(P,Q) \\
g_{+1} &= (spinor, vector) ,
\end{aligned}
\tag{7.8}$$

where *spinor* denotes a spinor representation of $so(q+1,1)$ (of dimension \mathcal{D}_{q+1}) and *vector* denotes the vector representation of $\mathcal{S}_q(P,Q)$ which is of dimension $(P+Q)$.⁸ The isotropy group H is

$$H = SO(q+1) \otimes \mathcal{S}_q(P,Q) \tag{7.9}$$

The possible groups $\mathcal{S}_q(P,Q)$ and the associated real Clifford algebras were given in [31] which we list in Table 1.

q	$\mathcal{C}(q+1,0)$	\mathcal{D}_{q+1}	$\mathcal{S}_q(P,Q)$
-1	\mathbb{R}	1	$SO(P)$
0	$\mathbb{R} \oplus \mathbb{R}$	1	$SO(P) \otimes SO(Q)$
1	$\mathbb{R}(2)$	2	$SO(P)$
2	$\mathbb{C}(2)$	4	$U(P)$
3	$\mathbb{H}(2)$	8	$USp(2P)$
4	$\mathbb{H}(2) \oplus \mathbb{H}(2)$	8	$USp(2P) \otimes USp(2Q)$
5	$\mathbb{H}(4)$	16	$USp(2P)$
6	$\mathbb{C}(8)$	16	$U(P)$
7	$\mathbb{R}(16)$	16	$SO(P)$
$n+8$	$\mathbb{R}(16) \otimes \mathcal{C}(n+1,0)$	$16 \mathcal{D}_n$	as for $q=n$

Table 1: Real Clifford algebras $\mathcal{C}(q+1,0)$. \mathbb{R} , \mathbb{C} and \mathbb{H} are the division algebras of real, complex numbers and quaternions, respectively, while \mathcal{D}_{q+1} denotes the real dimension of an irreducible representation of the Clifford algebra. The $\mathcal{S}_q(P,Q)$ is the metric preserving group in the centralizer of the Clifford algebra in the $(P+Q)\mathcal{D}_{q+1}$ dimensional representation.

Now the gamma matrices $\gamma_{\bar{\mu}\bar{j}}$ provide a symplectic representation of a group only for $q=1$ or $q=2$ i.e for $U(1)$ or $SU(2)$. Hence one can gauge $SU(2)$ symmetry of the $\mathcal{N}=2$ MESGT for $q=2$ while dualizing the $2P$ vector fields to tensor fields. One can then use the remaining two $SU(2)$ singlet vector fields to gauge the $U(1)_R$ symmetry and/or the Abelian $U(1)$ factor in $U(P) = U(1) \times SU(P)$. For $q=1$ one can gauge the $SO(2,1)$ symmetry while dualizing the $2P$ vector fields into tensor fields. The remaining $SO(2,1)$ singlet vector field can then be used to gauge the $U(1)_R$ symmetry of these theories.

⁸We should note that in case the scalar manifold is a symmetric space the above Lie algebra gets extended by additional symmetry generators belonging to grade -1 space transforming in the conjugate representation of g_{+1} with respect to g_0 .

- (vii) As is clear from above, the coupling to the tensor fields restricts the possible non-Abelian symmetry groups greatly for those theories whose scalar manifolds are symmetric spaces or homogeneous spaces. We would like to point out that there does exist a novel class of (gauged) $\mathcal{N} = 2$ Yang-Mills Einstein supergravity theories coupled to tensor multiplets with a rich set of possible non-Abelian groups that admit symplectic representations. To construct these theories one simply chooses the *arbitrary* tensor C_{ijk} in the canonical basis (2.6) as follows. Split the indices $i, j, k..$ as $i = (\bar{i}, M), j = (\bar{j}, N), ..$ where $\bar{i}, \bar{j} = 1, 2, .., n - 1$ and $M, N, .. = 1, .., 2m$ and identify

$$C_{\bar{i}\bar{j}\bar{k}} = d_{\bar{i}\bar{j}\bar{k}} \quad (7.10)$$

$$C_{iMN} = C_{M\bar{i}N} = C_{MN\bar{i}} = \sqrt{\frac{3}{2}}(\Lambda_{\bar{i}})_{MN} = \sqrt{\frac{3}{2}}(\Lambda_{\bar{i}})_{NM} \quad (7.11)$$

where $d_{\bar{i}\bar{j}\bar{k}}$ are the completely symmetric Gell-Mann d -symbols of a Lie group K and the $(\Lambda_{\bar{i}})$ are the matrices of $2m$ dimensional symplectic representation of K . Now the d -symbols vanish for all simple groups except for the groups $SU(N), N > 2$ and $Spin(6)$ which is isomorphic to $SU(4)$. For vanishing d -symbols and $m = 0$ the cubic form reduces to that of the generic Jordan family. Thus the theories defined by non-vanishing $C_{ijk} = d_{ijk}$ can be considered as the non-trivial generalizations of the generic Jordan family. The first non-trivial example i.e the case of $K = SU(3)$ d -symbols (with $m = 0$) lead to the magical $\mathcal{N} = 2$ MESGT with the scalar manifold $SL(3, \mathbf{C})/SU(3)$. The scalar manifold obtained by taking C_{ijk} to be the d -symbols of $SU(N)$ for $N > 3$ cannot be a symmetric or homogeneous space. This follows from the fact that for such theories $SU(N)$ act as isometries of the scalar manifold that extend to symmetries of the full Lagrangian. However, it is clear from the list of possible homogeneous spaces [31] that it does not include manifolds with such properties. Hence the isometries of the scalar manifolds corresponding to $C_{ijk} = d_{ijk}$ for $N > 3$ in the canonical basis cannot act transitively. This is perhaps expected from the fact that the number of independent invariants of a group in its adjoint representation is equal to its rank i.e. the number of Casimir operators. The term involving δ_{ij} in the cubic form corresponds to the quadratic invariant and the term involving d_{ijk} corresponds to the third order Casimir. Only for $SU(3)$ do they form a complete set of invariants and the resulting scalar manifold is a symmetric space. For higher $SU(N)$ ($N > 3$) one has invariants of order up to N .

As for the symplectic representations $\Lambda_{\bar{i}}$ of $SU(N)$, one can, for example, choose the reducible $(N + \bar{N})$ representations corresponding to the standard embedding of $U(N)$ in $USp(2N)$ by taking $m = N$ and $\bar{n} = N^2$.

8 Conclusions

Our results imply several interesting conclusions.

Whereas the R-symmetry group and the isometry group G of the scalar manifold \mathcal{M} are entangled with each other for $\mathcal{N} > 2$ MESGT's, and for simple supergravities for $\mathcal{N} > 4$ the case $\mathcal{N} = 2$ allows a separate discussion of the gaugings of subgroups of these two groups. In particular, the issues of the tensor field dualization and the gravitino coupling to gauge fields can be completely separated. It turns out that both mechanisms require their own scalar potential. The potential due to the introduction of the tensor fields is manifestly non-negative and does therefore not admit an anti-de Sitter solution. This is in contrast to the pure $U(1)_R$ -gauging, which involves minimal coupling to the gravitini and leads to an indefinite potential which can sustain anti-de Sitter vacua.

Combining both types of gauging, one observes surprisingly little interference. In particular, the scalar potential is just a sum of the two potentials of the individual gaugings. Nevertheless, the analysis of the critical points seems to be more complicated, but is, in general, expected to allow anti-de Sitter solutions [34]. A particular example of such a theory obtained by a truncation of the gauged $N = 8$ supergravity does admit an AdS vacuum [4].

The introduction of the tensor fields leads to strong constraints on the possible gauge groups $K \subset G$ and the representations under which the tensor fields transform. The simultaneous $U(1)_R$ -gauging further restricts these gauge groups K , which is one of the few places where these two types of gaugings interfere with each other. We gave a list of possible gauge groups and the corresponding representations of the tensor fields using the known classification of $\mathcal{N} = 2$ MESGT's whose scalar manifolds are symmetric or homogeneous spaces. We also pointed out the existence of a novel family of $\mathcal{N} = 2$ MESGT's whose scalar manifolds are, in general, not homogenous, but admit $SU(N)$ isometries. The latter class of theories lead to a richer class of gaugings with some of the vector fields dualized to tensor fields.

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References

- [1] B. de Wit and H. Nicolai, Nucl. Phys. **B208** (1982) 323
- [2] M. Pernici, K. Pilch and P. van Nieuwenhuizen, Phys. Lett. **143B** (1984) 103
- [3] M. Günaydin, L.J. Romans and N.P. Warner, Phys. Lett. **154B** (1985) 268
- [4] M. Günaydin, L.J. Romans and N.P. Warner, Nucl. Phys. **B272** (1986) 598

- [5] M. Pernici, K. Pilch and P. van Nieuwenhuizen, Nucl. Phys. **B259** (1985) 460
- [6] For further references on the vast subject of gauged supergravity theories see the reprint volumes, *Supergravity in Different Dimensions*, ed. by A. Salam and E. Sezgin, World Scientific (1989).
- [7] J. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231; hep-th/9711200
- [8] S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Lett. **428B** (1998) 105; hep-th/9802109
- [9] E. Witten, Adv. Theor. Math. Phys. **2** (1998) 253-291; hep-th/9802150; *ibid*, **2** (1998) 505-532, hep-th/9803131.
- [10] For the relationship between the earlier work on Kaluza-Klein supergravity theories and the Maldacena conjecture see M. Günaydin and D. Minic, Nucl. Phys. **B253** (1998) 145-157, hep-th/9802047 and M.J. Duff, H. Lü and C. Pope, hep-th/9803061.
- [11] For an extensive list of references on AdS/CFT dualities see the recent review paper O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, hep-th/9905111
- [12] M. Günaydin and N. Marcus, Class. and Quantum Gravity, **2** (1985) L11
- [13] H.J. Kim, L.J. Romans and P. van Nieuwenhuizen, Phys. Rev **D32** (1985) 389
- [14] B. de Wit and H. Nicolai, Nucl. Phys. **B281** (1987) 211
- [15] H. Nastase, D. Vaman and P. van Nieuwenhuizen, hep-th/9905075
- [16] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, hep-th/9904017
- [17] For a recent overview see N.P. Warner, hep-th/9911240.
- [18] L. Randall and R. Sundrum, hep-th/9905221
- [19] L. Randall and R. Sundrum, hep-th/9906064
- [20] K. Behrndt and M. Cvetič, hep-th/9909058
- [21] P. Hořava and E. Witten, Nucl. Phys. **B460** (1996) 506; hep-th/9510209 ; *ibid* **B475** (1996) 94; hep-th/9603142
- [22] E. Witten, Nucl. Phys. **B471** (1996) 135; hep-th/9602070
- [23] A. Lukas, B.A Ovrut, K.S. Stelle and D Waldram, hep-th/9803235 and hep-th/9806051

- [24] J. Ellis, Z. Lalak, S. Pokorski and W. Pokorski, Nucl. Phys. **B540** (1999) 149-186, hep-ph/9805377
- [25] M. Günaydin, G. Sierra and P.K. Townsend, Nucl. Phys. **B242** (1984), 244; Phys. Lett. **133B** (1983) 72.
- [26] M. Günaydin, G. Sierra and P.K. Townsend, Nucl. Phys. **B253** (1985), 573
- [27] M. Günaydin, G. Sierra and P.K. Townsend, Class. Quantum Grav. **3** (1986) 763
- [28] M. Günaydin, G. Sierra and P.K. Townsend, Phys. Rev. Lett. **53**(1984) 332.
- [29] M. Günaydin, G. Sierra and P.K. Townsend, Phys. Lett. **144B** (1984) 41
- [30] A.C. Cadavid, A. Ceresole, R. D'Auria and S. Ferrara, Phys. Lett. **B 357** (1995) 76; hep-th/9506144
- [31] B. de Wit and A. van Proeyen, Commun. Math. Phys. **149** (1992) 307; hep-th/9112027
- [32] K. Mc Crimmon, Pacific J. Math. **15** (1965)925
- [33] B. de Wit and A. van Proeyen, Phys. Lett. **B 293** (1992) 94; hep-th/9207091.
- [34] M. Günaydin and M. Zagermann, work in progress
- [35] K. Pilch, P.K. Townsend and P. van Nieuwenhuizen, Phys. Lett. **136B** (1984) 38