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The equivalence theorem and the production of gravitinos after inflation

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ABSTRACT

We study the application of the high-energy equivalence between helicity $\pm 1/2$ gravitinos and goldstinos in order to calculate the production of helicity $\pm 1/2$ gravitinos in time-dependent scalar and gravitational backgrounds. We derive this equivalence for equations of motion, paying attention to several subtleties that appear in this context and are not present in the standard derivations of the theorem, mainly because of the presence of external sources. We also propose the Landau gauge as an alternative to the usual gauge choices given in the standard proofs at the Lagrangian level.

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1 Introduction

In supergravity theories [1, 2] the superpartner of the graviton field is a spin 3/2 particle called the gravitino. This particle couples only with gravitational strength to the rest of matter fields, and accordingly its lifetime can be very long, with a decay rate of $\Gamma_{3/2} \simeq m_{3/2}^3/M_P^2$. In particular, gravitinos lighter than $m_{3/2} < 100\,\mathrm{MeV}$ will live longer than the age of the Universe. This fact can have important consequences in cosmology and imposes stringent constraints on supergravity models. Owing to their weak couplings, gravitinos freeze out very early when they are still relativistic; therefore their primordial abundance can be estimated as $n_{3/2}/s \simeq 10^{-3}$ [3]. Considering only the case of unstable gravitinos, such a primordial abundance would give rise to an enormous amount of entropy, in conflict with the standard cosmology. In particular, gravitinos decaying during the nucleosynthesis can destroy the nuclei created in that era. A possible way out of this *gravitino problem*, within supergravity scenarios, is the existence of a period of inflation that dilutes any primordial density [4]. Unfortunately the problem can be re-created if, after inflation, gravitinos are produced by some mechanism. In fact, this could be the case if during the period of inflaton oscillations, at the end of inflation, the reheating temperature was sufficiently high. A successful nucleosynthesis era then requires (we give some conservative bounds [5]): $n_{3/2}/s < 10^{-15}$ for a gravitino mass $m_{3/2} \simeq 100\,\mathrm{GeV}$, $n_{3/2}/s < 10^{-14}$ for $m_{3/2} \simeq 1\,\mathrm{TeV}$ and $n_{3/2}/s < 10^{-13}$ for $m_{3/2} \simeq 10\,\mathrm{TeV}$. The production of gravitinos during reheating depends on the temperature T_R as [6]: $n_{3/2}/s \simeq 10^{-14} T_R/(10^9\ \mathrm{GeV})$, which implies that, for a typical mass $m_{3/2} \simeq 1\,\mathrm{TeV}$, the reheating temperature has to be $T_R < 10^9$ GeV. Another constraint appears in supergravity models where the gravitino mass is determined by the scale of supersymmetry breaking. In order to solve the hierarchy problem, it is then suggested that $m_{3/2} < 1\,\mathrm{TeV}$ [2].

As we have just mentioned, during the reheating period gravitinos can be created by the decay of other particles produced from the inflaton oscillations. However, in recent years, the standard picture of reheating has changed dramatically, as a consequence of some works [7] in which it was realized that, during the first inflaton oscillations, reheating can not be studied by the standard perturbative techniques. This *preheating* period can give rise to an explosive production of bosons due to the phenomenon of parametric resonance. In this period, the energy of the coherent oscillations of the inflaton field is very efficiently converted into particles. In the case of fermions, the limit imposed by the Pauli exclusion principle avoids the explosive production, although the results still deviate from the perturbative expectations [8, 9]. This fact will be of the utmost importance in the case when gravitinos are directly coupled to the inflaton: during the new preheating period they could indeed be produced in excess, thus imposing new constraints on the particular supergravity inflationary model.

In previous works [10, 11] it was shown that in fact the production of helicity $\pm 3/2$ gravitinos can take place during the preheating era and that the results deviate from the perturbative expectations by several orders of magnitude (see also [12]). In the case of helicity $\pm 1/2$ gravitinos, the production is in general more abundant, depending on the specific supergravity model [13]. Some other works dealing with this topic can be found in [14].

In the present work we are interested in the production of helicity $\pm 1/2$ gravitinos during preheating. The relative difficulty of the calculations in the unitary gauge, used in the above references, suggests that we should explore alternative methods. In particular we will exploit the relation between helicity $\pm 1/2$ gravitinos and goldstinos suggested by the Equivalence Theorem (ET) [15] (see [13, 14] where this possibility was pointed out).

The ET was first introduced, in the framework of non-abelian gauge theories [16, 17], as a way to calculate processes involving longitudinal gauge bosons, but using only Goldstone bosons, which are scalars and therefore much easier to handle. The first formal proof in terms of S-matrix elements was given in [18], and that is basically the derivation followed in the proof of the ET in the supergravity scenario. In the last few years, and still within the framework of the non-abelian gauge theory, several works have appeared which complete the proof of the theorem [19], including renormalization effects, but also raise some questions about its Lorentz non-invariance ambiguity and applicability [20]. In this paper we also discuss briefly how these new considerations may affect the gravitino-goldstino ET applied to the production of gravitinos during the preheating period.

Intuitively, the ET tells us that, since the goldstinos disappear from the spectrum through the super-Higgs mechanism, giving rise to physical helicity $\pm 1/2$ gravitinos, it is possible to use goldstinos in the calculation of observables instead of the complicated $\pm 1/2$ gravitinos. Of course, this identification can only be carried out at energies high enough to neglect the effect of the masses.

Rigorously, this theorem has only been proved for S-matrix elements containing initial or final helicity $\pm 1/2$ gravitinos and in the absence of external backgrounds. This would provide a good approximation for gravitino production during the reheating period, but only at the perturbative level, where the rate of production is given by the decay of inflaton quanta [7, 15].

However, as we mentioned before, preheating is a non-perturbative (and out of equilibrium) process and it is not obvious that the same proof still holds in the presence of external sources, such as the inflaton field or the space-time curvature, which couple to the matter fields. In particular, the presence of a source that creates particles makes different the initial and final vacua that appear in the generating functional of Green functions. In contrast to the standard proofs of the ET, this source is present in the R_ξ gauge-fixing function, which is the starting relation in the derivations. In order to take into account these new effects, one possibility would be to include, in the calculation of S-matrix elements, the terms corresponding to the change from one vacuum to the other, which are given by Bogolyubov coefficients (for a detailed presentation see [21]); in this case therefore a complete proof of the ET would also require the equivalence at the level of Bogolyubov coefficients. Alternatively, one could try to derive the equivalence in the in-in (out-out) formalism [22], which is more appropriate for the calculation of expectations values of physical observables in the initial or final regions. However, one would find the same kind of difficulties related to the sum over intermediate states and the calculation of Bogolyubov coefficients. As far as we are interested in the calculation of particle production from external sources, we will not follow any of these two approaches, but, instead, we will concentrate on the simpler canonical quantization formalism (see [21]).

Of course, we still expect that the intuitive relation suggested by the ET should hold, but since it is not the same to establish an equality at the level of matrix elements as at the operator (fields, indeed) level, we present in this paper a derivation more suited for the formalism in terms of equations of motion. In this way we can also identify the physical conditions on the sources that we need for this theorem to hold. In addition we will obtain some insight about the equivalence for Bogolyubov coefficients that could be useful for the formalisms mentioned above. Finally, we will propose the Landau gauge as the best choice to perform the calculations, although, probably, it is not the most intuitive. In this gauge, not only the proof of the theorem, but also the final equations that govern gravitino production are greatly simplified.

All the previous considerations basically concern the production process of gravitinos. But we also have to take into account the fact that we are producing very many gravitinos (out of equilibrium) which have a distribution in energies. Some of them will satisfy the physical conditions to apply the ET, whereas some others will not. Hence, we also present an additional condition on the number of those gravitinos not satisfying the applicability conditions, in order to obtain reliable calculations with the ET.

2 Supergravity Lagrangian

Let us consider $N = 1$ minimal supergravity [1, 2, 23] coupled to a single chiral superfield Φ, which describes a complex scalar field ϕ and a Majorana spinor η satisfying $\eta = C\bar{\eta}^T = \eta^C$, with the charge-conjugation matrix given by $C = i\gamma^2\gamma^0$. In principle, the derivation could be extended to more than one chiral multiplet in a similar way. The scalar component will play the role of the inflaton field and it will therefore be considered as an external background. The corresponding Lagrangian is defined by the superpotential $W(\Phi)$ and the Kähler potential $G(\Phi, \Phi^\dagger) = \Phi^\dagger\Phi + \log|W|^2$. We will define: $G_{,\phi} = \partial G/\partial\phi$, $G_{,\phi^*} = \partial G/\partial\phi^*$, $G_{,\phi\phi^*} = \partial^2 G/\partial\phi\partial\phi^*$, etc. In this case we will have $G_{,\phi\phi^*} = 1$. The bosonic part of the Lagrangians is given by

$$g^{-1/2}\mathcal{L}_B = -\frac{1}{2}R + g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi^* + e^G\left(3 - |G_{,\phi}|^2\right), \tag{1}$$

where we are working in units $M = M_P/\sqrt{8\pi} = 1$. In the fermionic part of the Lagrangians, we are only interested in those terms that are quadratic in the fermionic fields (gravitinos and goldstinos), since we are going to work with the linearized equations of motion. For the sake of simplicity we will assume that the scalar field ϕ is real. With this assumption those terms are:

$$\begin{aligned} g^{-1/2}\mathcal{L}_F \;\; = \;\; & -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu D_\rho\psi_\sigma + \frac{i}{2}\bar{\eta}\,\not{D}\eta + e^{G/2}\left(\frac{i}{2}\bar{\psi}_\mu\sigma_{\mu\nu}\psi^\nu + \frac{1}{2}\left(-G_{,\phi\phi} - G_{,\phi}^2\right)\bar{\eta}\eta\right. \\ + \;\; & \left.\frac{i}{\sqrt{2}}G_{,\phi}\bar{\psi}_\mu\gamma^\mu\eta\right) + \frac{1}{\sqrt{2}}\bar{\psi}_\mu(\not{\partial}\phi)\gamma^\mu\eta, \end{aligned} \tag{2}$$

with $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. Since we will be mainly concerned with the production of gravitinos after inflation, we will assume in the following that our scalar field depends only on time and that the space-time metric is of the Friedmann-Robertson-Walker (FRW) form. In particular, it will be very useful to work in conformal time, for which the FRW metric with flat spatial sections reads:

$$ds^2 = a^2(t)(dt^2 - d\vec{x}^2), \tag{3}$$

where $a(t)$ is the Universe scale factor and the non-vanishing gravitational field is assumed to be created by the scalar field.

In contrast with the usual proof of the ET, we see that there are two mixing terms between gravitinos and goldstinos in eq. (2). When the scalar field has settled down in the minimum of the potential, $\phi = \phi_0$, the last term does not contribute, and this is why it is not considered in the discussions about the spontaneous breaking of supersymmetry. However, since we are interested in the effects of the evolution of ϕ, such a term cannot be ignored any longer. In flat space-time, with $\phi = \phi_0$, and when supersymmetry is not broken, i.e. $G_{,\phi_0} = 0$, the mixing terms are absent and the equations of motion describe the evolution of a gravitino field with only two independent degrees of freedom, namely, the two helicity $\pm 3/2$ states. However, when supersymmetry is broken spontaneously, the gravitino acquires two more degrees of freedom with helicity $\pm 1/2$, because of the interaction with the goldstinos, giving rise to much more complicated evolution equations.

Nevertheless, using the unitary gauge, and projecting the gravitino equations along the $\pm 1/2$ components, it has been possible to calculate the production of helicity $\pm 1/2$ gravitinos in preheating after inflation [13]. By absorbing all the goldstino dependent terms in a redefinition of the gravitino field, this gauge shows explicitly the super-Higgs mechanism in which the goldstino becomes the helicity $\pm 1/2$ components of the gravitino field. There are no mixing terms but, still, we have to deal with $\pm 1/2$ helicity states of a Rarita-Schwinger field, which can be rather involved.

If the chiral superfield we have introduced is responsible for supersymmetry breaking, then the inflatino will also play the role of the goldstino that is absorbed by the gravitino in the super-Higgs mechanism, as commented before. However, this is not always the case, and the inflaton sector may not be responsible for supersymmetry breaking. In that case, although supersymmetry is broken during and after inflation, it is restored at the minimum of the potential so that the super-Higgs mechanism does not take place. Accordingly, the gravitino does not have a $\pm 1/2$ component. In order to avoid this problem, we will assume in the following that supersymmetry is broken at the minimum of the scalar field potential. We will also assume that at the minimun the cosmological constant is zero, then $G^2_{,\phi_0} = 3G_{,\phi_0\phi_0^*}$, and we have $G_{,\phi_0} = \sqrt{3}$.

The equations of motion for gravitinos and goldstinos derived from the Lagrangians in eq. (2) are

$$\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu D_\rho\psi_\sigma + \frac{1}{2}e^{G/2}[\gamma^\mu, \gamma^\nu]\psi_\nu - \frac{i}{\sqrt{2}}G_{,\phi}e^{G/2}\gamma^\mu\eta - \frac{1}{\sqrt{2}}(\partial\!\!\!/\phi)\gamma^\mu\eta = 0 \tag{4}$$

and

$$i\, D\!\!\!\!/\,\eta + e^{G/2}\left(-G_{,\phi\phi} - G^2_{,\phi}\right)\eta - \frac{i}{\sqrt{2}}e^{G/2}G_{,\phi}\gamma^\mu\psi_\mu + \frac{1}{\sqrt{2}}\gamma^\mu(\partial\!\!\!/\phi)\psi_\mu = 0 \tag{5}$$

If we consider only helicity $\pm 3/2$ gravitinos, then it can be shown that the equations of motion reduce to a very simple form [10]:

$$(i \not{D} - e^{G/2})\psi_\mu^{\pm 3/2} = 0 \tag{6}$$

If we want to study the full equation, we see that its form is rather involved and, since we are not using the unitary gauge, the goldstino terms are present. However, if we are only interested in the high-energy behaviour of helicity $\pm 1/2$ gravitinos, $E \gg m_{3/2}$, then it is possible to simplify the calculations by means of the ET. This limit is sensible in most of the models of supergravity inflation with one chiral supermultiplet, since the typical energy of the particles created during preheating is of the order of the inflaton mass, which is usually several orders of magnitude larger than the gravitino mass. For instance in the model discussed in [24, 25], the inflaton mass is $m_\phi \simeq 10^{10}\,\mathrm{GeV}$, whereas the gravitino mass is smaller than $1\,\mathrm{TeV}$. Note also that all these scales are well below the scale M_P at which supergravity breaks down as an effective theory.

As it was commented in the introduction, the ET has been rigorously derived in terms of S-matrix elements. At sufficiently high energies, it identifies those S-matrix elements containing helicity $\pm 1/2$ gravitinos with those where each gravitino has been replaced by its associated goldstino. However, to calculate the non-perturbative production of gravitinos during the preheating epoch, we use a formalism in terms of equations of motion and fields. In order to show how the gravitino-goldstino high-energy equivalence can be used in this context, we will follow these steps:

i) Introduce a gauge-fixing term corresponding to a certain generalization of the R_ξ gauges, which allows us to cancel the mixing gravitino-goldstino terms in the equations of motion.

ii) Assuming that in the asymptotic regions $t \to \pm\infty$, the external sources are static, i.e, $\phi \to \phi_0$ and $g_{\mu\nu} \to \eta_{\mu\nu}$, then using the equations of motion in those regions we will show that $\partial^\mu \psi_\mu = N(\xi) m_{3/2} \eta$.

iii) Using the equivalence between helicity $\pm 1/2$ gravitinos and goldstinos in the asymptotic regions we will obtain for the $\pm 1/2$ projectors $P^\mu_{\pm 1/2} = p^\mu / m_{3/2} + \mathcal{O}(m_{3/2}/E)$, and therefore $\psi_{\pm 1/2} = P^\mu_{\pm 1/2}\psi_\mu = \tilde{N}(\xi)\eta + \mathcal{O}(m_{3/2}/E) -$ suppressed.

iv) The Landau gauge, corresponding to $\xi \to \infty$, provides an additional simplification that makes it easier to solve the problem of goldstino production; then, using (iii), we will obtain the production of $\psi_{\pm 1/2}$ particles.

3 Gauge fixing

In the model we have just presented, helicity $\pm 1/2$ gravitinos are physical degrees of freedom. In contrast, goldstinos do not belong to the physical spectrum, and in the unitary gauge we can even get rid of them in the equations of motion. The total production of helicity $\pm 1/2$ gravitinos during reheating is a gauge-invariant quantity, and it is only related to the goldstino production in R_ξ gauges, in which both kinds of fields appear simultaneously in the Lagrangian. Let us then consider the following gauge-fixing condition, which is a generalization of the R_ξ gauge used in [15, 26]:

$$\gamma^\mu\psi_\mu - \frac{1}{\sqrt{2}\xi\,\not{D}}e^{G/2}G_{,\phi}\eta + \frac{i}{G_{,\phi}}e^{-G/2}\gamma^\mu(\not{\partial}\phi)\psi_\mu = 0. \tag{7}$$

When ϕ is constant we recover the gauge-fixing term in [15]. Note that in our case, *because of the presence of external sources*, all the coefficients in the gauge-fixing function are no longer constants. The limit $\xi \to 0$ corresponds to the unitary gauge. The above equation provides us with a relation between gravitinos and goldstinos, but we want to extract only the helicity $\pm 1/2$ gravitinos for which we need a relation between $\partial^\mu\psi_\mu$ and η. In the following we will use the equations of motion to obtain a relation of the desired form.

If we assume that in the asymptotic regions $t \to \pm\infty$ the space-time is flat and the scalar field settles down at the minimum of the potential ϕ_0, then, in those regions, the above condition reduces to

$$a_{in,out}^{-1}\ \not{\partial}\gamma^\mu\psi_\mu = \sqrt{\frac{3}{2}}\frac{m_{3/2}}{\xi}\eta, \tag{8}$$

where $m_{3/2} = e^{G_0/2}$ and $a_{in,out}$ are the scale factor values in the asymptotic past and future. In order to simplify the notation, we will absorb the scale factor into the mass: $m_{in,out} \equiv a_{in,out}m_{3/2}$; to avoid the proliferation of indices, we will denote $m_{in,out}$ simply by m. With this notation, the gauge-fixing condition reads:

$$\not{\partial}\gamma^\mu\psi_\mu = \frac{m}{\xi}\sqrt{\frac{3}{2}}\eta. \tag{9}$$

Let us recall that it is only in the static regions that the definition of particle and the separation between different helicities is unambiguous. However, in the strict sense, within the inflationary cosmology neither the initial ($t \to -\infty$) nor the final ($t \to \infty$) regions can be considered static, since there is a period of inflation before preheating and today we know that the Universe is expanding. Nevertheless, for practical purposes, we can still consider the initial and final regions as static, since the particle production will mainly take place during the first inflaton oscillations. Accordingly, we will define our initial vacuum by imposing such initial conditions on our fields that they behave as plane waves before preheating starts; also, since the rate of expansion decreases with time, the vacuum at the end of preheating could be defined as an adiabatic vacuum [21], and therefore we will not expect additional gravitino production to be coming from the Universe expansion.

Let us then consider first the equations of motion for gravitinos eq. (4) and goldstinos eq. (5) in the initial and final regions with the notation that we have just introduced. Since the inflaton is in the minimum, they read

$$\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\partial_\rho\psi_\sigma + \frac{1}{2}m[\gamma^\mu,\gamma^\nu]\psi_\nu - i\sqrt{\frac{2}{3}}m\gamma^\mu\eta = 0, \tag{10}$$

and

$$i\,\partial\!\!\!/\eta - 2m\eta - i\sqrt{\frac{3}{2}}m\gamma^\mu\psi_\mu = 0, \tag{11}$$

where we have used the condition in the minimum $G_{,\phi_0^*}G_{,\phi_0\phi_0} = -G_{,\phi_0}$. If we now fix the gauge using eq.(9) in the above equations, they can be rewritten as

$$\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\partial_\rho\psi_\sigma + \frac{1}{2}m[\gamma^\mu,\gamma^\nu]\psi_\nu - i\xi\gamma^\mu\,\partial\!\!\!/\gamma^\nu\psi_\nu = 0, \tag{12}$$

and

$$i\,\partial\!\!\!/\eta - 2m\eta - i\frac{3}{2}\frac{m^2}{\xi}\frac{1}{\partial\!\!\!/}\eta = 0, \tag{13}$$

In the following we will rewrite the equations of motion for goldstinos and gravitinos in the asymptotic regions in a more appropriate form and also the gauge-fixing condition in a way useful to derive the equivalence between goldstinos and gravitinos. Contracting the gravitino equation with ∂_μ, we obtain [27]

$$\frac{1}{2}m(\partial\!\!\!/\gamma^\nu\psi_\nu - \gamma^\nu\,\partial\!\!\!/\psi_\nu) - i\xi\,\partial\!\!\!/\,\partial\!\!\!/\gamma^\nu\psi_\nu = 0, \tag{14}$$

whereas contracting with $\gamma_\lambda\gamma_\mu$, we find

$$2i(\partial_\lambda\gamma^\sigma\psi_\sigma - \partial\!\!\!/\psi_\lambda) + m(\gamma_\lambda\gamma^\nu\psi_\nu + 2\psi_\lambda) - 2i\xi\gamma_\lambda\,\partial\!\!\!/\gamma^\nu\psi_\nu = 0, \tag{15}$$

which can be contracted again with γ_λ to get

$$i(\partial\!\!\!/\gamma^\sigma\psi_\sigma - \gamma^\lambda\,\partial\!\!\!/\psi_\lambda) + 3m\gamma^\mu\psi_\mu - 4i\xi\,\partial\!\!\!/\gamma^\nu\psi_\nu = 0. \tag{16}$$

Substituting eq. (14) into eq. (16), we obtain

$$-i\xi\,\partial\!\!\!/\,\partial\!\!\!/\gamma^\nu\psi_\nu + \frac{3}{2}im^2\gamma^\nu\psi_\nu + 2\xi m\,\partial\!\!\!/\gamma^\nu\psi_\nu = 0, \tag{17}$$

which, finally, can be rewritten as:

$$(i\,\partial\!\!\!/ - m_+)(i\,\partial\!\!\!/ - m_-)\gamma^\nu\psi_\nu = 0, \tag{18}$$

where we have defined $m_\pm = m(1 \pm \sqrt{1-3/(2\xi)})$. Note that, in the perturbative sense, the poles in the propagator are exactly those obtained in [15]. In addition we can derive the very same equation for the goldstino, just by multiplying eq. (13) by $i\,\partial\!\!\!/$:

$$(i\,\partial\!\!\!/ - m_+)(i\,\partial\!\!\!/ - m_-)\eta = 0 \tag{19}$$

The implications of these equations are clearer if we recall that the physical fields, i.e. the $\pm 3/2$ and $\pm 1/2$ helicity modes in the asymptotic regions, are those satisfying both $\gamma^\mu\psi_\mu = 0$

and $\partial^\mu \psi_\mu = 0$. We can see that they still satisfy $(i\ \partial\!\!\!/ - m)\psi_\mu^{Phy} = 0$, and thus have the correct physical mass m. In contrast, the unphysical spin-1/2 modes present poles in the propagator at $m_\pm$, exactly as happens with the goldstinos. Therefore, by fixing the gauge we have only modified the poles of the unphysical modes, but, of course, the physical modes remain unchanged because of the gauge invariance of the theory. From (14) and by means of the gauge-fixing condition, we obtain

$$\frac{1}{2}m(2\ \partial\!\!\!/\gamma^\nu\psi_\nu - 2\partial^\nu\psi_\nu) - i\ \partial\!\!\!/\sqrt{\frac{3}{2}}m\eta = 0 \tag{20}$$

Since η satisfies eq. (19), we have two possible solutions: $(i\ \partial\!\!\!/ - m_+)\eta = 0$ and $(i\ \partial\!\!\!/ - m_-)\eta = 0$; together with the gauge-fixing condition (20), they yield

$$\sqrt{\frac{3}{2}}\frac{m}{\xi}\eta - \partial^\mu\psi_\mu - \sqrt{\frac{3}{2}}m_\pm\eta = 0. \tag{21}$$

From this expression we get:

$$\partial^\mu\psi_\mu = \sqrt{\frac{3}{2}}\frac{m}{\xi}\left(1 - \xi\frac{m_\pm}{m}\right)\eta. \tag{22}$$

At first sight, this equation relates the unphysical gravitino $\partial^\mu\psi_\mu$ with the goldstino; however, the key observation is that, as we will show, at high energy, $\partial^\mu\psi_\mu$ tends to the physical helicity $\pm 1/2$ gravitino. Note that, apparently, there are two relations, one for goldstinos that correspond to the m_- solution and another for those with m_+.

4 The equivalence theorem

In the asymptotic initial and final regions, we expect that a general solution of the equations of motion for gravitinos and goldstinos will be written as a linear superposition of on-shell positive and negative frequency plane waves [27]. In particular, let us consider a positive frequency mode solution in the initial region with momentum $p^\mu = (\omega_{in}, 0, 0, p)$ (there is no loss of generality in this choice), with $p_\mu p^\mu = m_{in}^2$ and $p = |\vec{p}|$, such that $p \gg m_{in}$:

$$\psi_\mu^p(x) \quad = \quad \frac{1}{a_{in}^{3/2}\sqrt{2\omega_{in}}}e^{ipx}\tilde{\psi}_\mu(\vec{p}) + \mathcal{O}\left(\frac{m_{in}}{p}\right), \tag{23}$$

where $\tilde{\psi}_\mu(\vec{p})$ is the corresponding Fourier component. For the unphysical goldstino we have

$$\eta^p(x) \quad = \quad \frac{1}{a_{in}^{3/2}\sqrt{2\omega_{in}}}e^{ipx}\tilde{\eta}(\vec{p}) + \mathcal{O}\left(\frac{m_{in}}{p}\right). \tag{24}$$

Similar expressions can be written for the negative-frequency solutions and for solutions in the final region. Note that the on-shell conditions for gravitinos and goldstinos are different because of the different positions of the poles. In particular, for physical gravitinos we have $p_\mu p^\mu = m_{in}^2$ and for goldstinos and unphysical gravitinos ($\gamma^\mu\psi_\mu$ and $\partial^\mu\psi_\mu$) we have $p_\mu p^\mu = (m_+^{in})^2$ (we will not use m_- for reasons that will become clear below). We have thus included the $\mathcal{O}(m_{in}/p)$ term at the end of (23) and (24). Strictly, it should not be there for physical gravitinos, but at this level we keep a compact notation.

In the unitary gauge, $\xi = 0$, only the four helicities $\pm 3/2$ and $\pm 1/2$, which are the physical degrees of freedom, are present in the Lagrangian. However, all the degrees of freedom appear explicitly in the R_ξ gauge, although, owing to the above constraint equations and gauge-fixing condition, only four are independent.

The spin-1 polarization vectors are given by $\epsilon_\mu(\vec{p}, m) = a(t)\delta_\mu^a \epsilon_a(\vec{p}, m)$, where

$$\epsilon_a(\vec{p},1) = \frac{1}{\sqrt{2}}(0,1,i,0) \;\; ,\epsilon_a(\vec{p},0) = \frac{1}{m_{in}}(p,0,0,\omega_{in}) \;\; ,\epsilon_a(\vec{p},-1) = -\frac{1}{\sqrt{2}}(0,1,-i,0). \tag{25}$$

If $u(\vec{p}, s)$ are spinors with definite helicity $s = \pm 1/2$, then $P_\pm u(\vec{p}, \pm 1/2) = u(\vec{p}, \pm 1/2)$, where $P_\pm = (1/2)(1 \pm \gamma_5 \gamma^\mu \epsilon_\mu(\vec{p}, 0))$ are the helicity projectors. Accordingly, the helicity $\pm 3/2$ and $\pm 1/2$ projectors are nothing but

$$\begin{aligned} P_\mu^{\pm 3/2} &= P_\pm \epsilon_\mu(\vec{p}, \pm 1), \\ P_\mu^{\pm 1/2} &= \sqrt{\frac{1}{3}} P_\mp \epsilon_\mu(\vec{p}, \pm 1) + \sqrt{\frac{2}{3}} P_\pm \epsilon_\mu(\vec{p}, 0). \end{aligned} \tag{26}$$

We see that, *at high energy*, the $\pm 1/2$ projector behaves as

$$P_\mu^{\pm 1/2} = \sqrt{\frac{2}{3}} P_\pm \frac{p_\mu}{m_{in}} + \mathcal{O}\left(\frac{m_{in}}{p}\right), \tag{27}$$

where we have neglected $\epsilon_\mu(\vec{p}, \pm 1)$ with respect to $\epsilon_\mu(\vec{p}, 0)$. Let us then define the helicity $\pm 1/2$ components of the gravitino field in the asymptotic initial regions and in momentum space as

$$\tilde{\psi}_{\pm 1/2}(\vec{p}) \equiv P_{\pm 1/2}^\mu \tilde{\psi}_\mu(\vec{p}) = \sqrt{\frac{2}{3}} P_\pm \frac{p^\mu}{m_{in}} \tilde{\psi}_\mu(\vec{p}) + \mathcal{O}\left(\frac{m_{in}}{p}\right) - \text{suppressed}. \tag{28}$$

At high energies, we see that *the helicity $\pm 1/2$ gravitino tends to the unphysical $\partial^\mu\psi_\mu$ field* and therefore we can use the gauge-fixing condition in (22) to obtain a relation between each Fourier mode of the goldstino and the helicity $\pm 1/2$ gravitino fields (For that, as pointed out in [15], it is essential that both $\partial^\mu\psi_\mu$ and η have the same poles)

For arbitrary values of the ξ parameter it would be necessary to take into account both solutions, with m_+ and with m_-. In this case, the solution of the goldstino equation can be written as

$$\eta_p(x) = \eta_p^+(x) + \eta_p^-(x). \tag{29}$$

From (22), we see that each solution is related to the 1/2 helicity gravitino with different proportionality constants, i.e. for positive frequency solution we will have in the initial region

$$\tilde{\psi}_{\pm 1/2}(\vec{p}) = \sum_{+,-}\left[-i\frac{1}{\xi}\left(1-\xi\frac{m^{in}_{+,-}}{m_{in}}\right)P_{\pm 1/2}+\mathcal{O}\left(\frac{m_{in}}{p}\right)\right]\tilde{\eta}^{+,-}(\vec{p}). \tag{30}$$

In the derivations of the ET in the S-matrix formalism, the proportionality constant between the helicity $\pm 1/2$ gravitinos and the goldstinos disappears in the S-matrix once the external lines of the Green functions have been removed, the momenta are on-shell, and the tensor indices are contracted with the corresponding polarization vectors [15]. However, this is not so straightforward in the proofs based on the generating functional formalism, be it for supergravity [15] or even in the non-abelian context [16]. These "classical" proofs are given for the clever choice $\xi = 3/2$, where $m_- = m_+$ and the proportionality constant disappears (in the non-abelian case the choice is $\xi = 1$ and the proportionality constant is unity).

However, for our purposes, it is much more appropriate to choose the Landau gauge. Indeed, in an arbitrary generalized R_ξ gauge, eq. (5) will be written

$$i\,\not{D}\eta + e^{G/2}\left(-G_{,\phi\phi}-G^2_{,\phi}\right)\eta - e^{G/2}G_{,\phi}\frac{i}{2\xi\,\not{D}}e^{G/2}G_{,\phi}\eta = 0 \tag{31}$$

The presence of the last term makes it very difficult to obtain solutions even at the numerical level. However, we get a dramatic simplification by using the Landau gauge, $\xi \to \infty$, in which the last term, which is the most complicated, vanishes. Thus we have:

$$i\,\not{D}\eta + e^{G/2}\left(-G_{,\phi\phi}-G^2_{,\phi}\right)\eta = 0 \tag{32}$$

Note that now this last expression corresponds to the m_+ case in (19) (the $m_- = 0$ solution is just an artefact due to the multiplication by $i\,\not{\partial}$). There is only one solution in this case, which gives rise to the gauge condition: $\partial^\mu\psi_\mu = -2\sqrt{\frac{3}{2}}m\eta$. For that reason we do not have to consider the m_- case in the previous formulae. Thus, in this gauge, (28) reads

$$\tilde{\psi}_{\pm 1/2}(\vec{p}) = \left[2\,i\,P_{\pm}+\mathcal{O}\left(\frac{m_{in}}{p}\right)\right]\eta(\vec{p}). \tag{33}$$

This is the relation we were looking for. Note that this is an equality at the level of fields and not for S-matrix elements. This result shows that *although the helicity* $\pm 1/2$ *gravitinos and the goldstinos can evolve differently* during the oscillations of the scalar field, *they approach each other in the asymptotic regions* (up to a constant). The fact that the gauge-fixing condition imposes the equality of helicity $\pm 1/2$ gravitinos and goldstinos only in the asymptotic regions, is sufficient for our purposes, since the particle number is evaluated only there.

Therefore, the simplest way to calculate the helicity $\pm 1/2$ gravitinos production is to study the much simpler equations of motion of the goldstino field in the Landau gauge.

We should stress once more that such a result is only useful when the energy of the particles we are producing is much larger than their masses. In our case, we are in a frame where the inflaton is a homogeneous field, which only depends on t, oscillating with a typical frequency m_ϕ; thus, we expect the gravitinos to have a typical physical momentum of order m_ϕ, so that we can use eq. (33) if $m_{3/2} \ll m_\phi$. Usually we will evaluate a_{out} right after the preheating ends and then $m_{in,out} \ll m_{3/2}$. The fact that we are in an homogeneous background is technically relevant owing to the remarks about the ET and Lorentz invariance done in [20] in the context of non-abelian gauge theories. Indeed the ET is not Lorentz invariant, since not only is the helicity decomposition frame-dependent, but also terms like $\mathcal{O}(m/E)$, $\mathcal{O}(m/p)$... do have very different values depending on the reference frame. For non-abelian gauge theories, the ET is used for processes at accelerators, where the longitudinal gauge bosons are produced, in practice, at a fixed energy and in their centre-of-mass frame; the ET is applicable to all of them since they have similar, high, energies. The fact that the inflaton field is homogeneous ensures that all our gravitinos are produced from rather similar conditions and we can apply the ET to the vast majority of them. Notice that (32) is just the standard equation of motion for a fermion field in curved space-time with a time-dependent mass and therefore we can apply the standard techniques of particle production in a straightforward way.

5 Particle production

Up to now our discussion has been purely classical. We have shown that within the mentioned approximations, the classical solution for the helicity $\pm 1/2$ gravitinos behaves exactly like the solution for the goldstinos (up to a normalization constant). In order to interpret these solutions in terms of particle number, we have to quantize them [21, 28]. The quantization procedure has been performed previously in the literature in the unitary gauge [13], $\xi = 0$. However, our gauge-fixing condition only affects the unphysical modes, but not those with helicity 1/2. Accordingly we expect our classical solution to be gauge invariant, at least in the class of R_ξ gauges we are considering and therefore the quantization of these modes can be done in a straightforward way. As we mentioned before, in the Landau gauge, $\xi \to \infty$, the equation of motion of the goldstino reduces to the simple form (32). We are then left with an equation for a Majorana fermion coupled to a scalar field in a curved space-time. But this problem is well-known and particle production has been studied before in this case [8, 9].

Let us then consider a classical solution to the goldstino equation (32) with helicity l, such that in the past ($t \to -\infty$), it behaves as a positive energy plane-wave, i.e.:

$$\eta_l^p(x) \to \frac{1}{a_{in}^{3/2}\sqrt{2\omega_{in}^+}} e^{i\omega_{in}^+ t - i\vec{p}\vec{x}} u(\vec{p}, l), \tag{34}$$

where a_{in} is the scale factor at the end of inflation. Note that because of (33) the solution for the helicity $\pm 1/2$ gravitino will be proportional to the solution for the goldstino. In the asymptotic future ($t \to \infty$), because of the presence of the time-dependent background fields, this solution will no longer behave as a mode of positive energy; rather, it will be a linear superposition of positive and negative frequency modes:

$$\eta_l^p(x) \rightarrow \frac{1}{a_{out}^{3/2}\sqrt{2\omega_{out}^+}} \left(\alpha_{p,l}^G e^{i\omega_{out}^+ t - i\vec{p}\vec{x}} u(\vec{p},l) + \beta_{-p,l}^G e^{-i\omega_{out}^+ t - i\vec{p}\vec{x}} u^C(-\vec{p},l)\right), \tag{35}$$

where, for a given $\vec{p}$, we have $(\omega_{in,out}^+)^2 = (m_{in,out}^+)^2 + p^2$. The Bogolyubov coefficients satisfy

$$|\alpha_{p,l}^G|^2 + |\beta_{p,l}^G|^2 = 1. \tag{36}$$

Using again our previous result in (33) we can identify each Fourier mode above with the corresponding Fourier mode for a helicity $\pm 1/2$ gravitino (up to a constant). In particular we will find that the Bogolyubov coefficient for the helicity $\pm 1/2$ gravitino $\beta_{p,l}^L$ will be the same as the above coefficient for the goldstino $\beta_{p,l}^G$, up to $\mathcal{O}(m/p)$, i.e $\beta_{p,l}^L = (1 + \mathcal{O}(m/p))\beta_{p,l}^G$. Notice that because of the different masses of goldstinos and physical gravitinos, the correction to the Bogolyubov coefficients for gravitinos can depend on time as $\exp(i\Delta_\omega t)$, where $\omega_{out}^+ = \omega_{out} + \Delta_\omega$. However, such a term will be relevant only to $t \simeq 1/\Delta_\omega$, that is, many orders of magnitude after preheating ends. Moreover, remember that the Bogolyubov coefficients are normalized according to (36) and therefore *the proportionality constant is irrelevant.*

As a remark, let us note that if we had a renormalizable theory whose low-energy limit is supergravity, we could still use our estimates with the ET, irrespective of the renormalization corrections [19] needed in the complete proof of the theorem. The reason is that we do not need the proportionality constant to obtain the Bogolyubov coefficients.

Therefore the number of gravitinos created with helicity $l = \pm 1/2$ and momentum p, $N_{p,l}^L$ will be given by

$$N_{p,l}^L = \left[1 + \mathcal{O}\left(\frac{m}{p}\right)\right] \; |\beta_{p,l}^G|^2 \tag{37}$$

In conclusion, solving the equation of motion for the goldstinos in the presence of the external backgrounds, and using (37), we will obtain the number of helicity $\pm 1/2$ gravitinos created during the preheating. In the above expression it is explicit that only the knowledge of the solutions in the asymptotic regions is relevant to the particle number calculation.

In order to check these results with those obtained in the unitary gauge in [13], we can see that in the limit $|\phi| \ll M_P$, the equation for the goldstinos in the Landau gauge (32) can be approximated by

$$i \not{D}\eta - (\partial_\phi \partial_\phi W)\, \eta = 0, \tag{38}$$

which is the same equation as obtained in [13] for the helicity $\pm 1/2$ components of the gravitino (in the global supersymmetric limit). Therefore, the number of created goldstinos, calculated in the Landau gauge, will be the same as the number of helicity $\pm 1/2$ gravitinos, calculated in the unitary gauge. Notice that the condition $|\phi| \ll M_P$, which is equivalent to the global supersymmetry limit, is not necessary to prove the ET and is independent of the high-energy requirements of the ET. It is just a particular simple case in which the results given by the ET can be compared with previous works. In principle, the results obtained from (38) should be valid only at high energies; however, the exact equality between (38) and the helicity $\pm 1/2$ equation in the unitary gauge suggests that the global supersymmetric limit includes information that is not contained in the high-energy limit.

6 Numerical example

As a possible application of the previous results and in order to compare with other works [13, 14], we will study a simple supergravity model based on the superpotential

$$W = \sqrt{\lambda}\frac{\Phi^3}{3}, \tag{39}$$

The corresponding inflaton potential is known to give rise to the problems mentioned before, that is, at the minimum supersymmetry is restored. We will then assume that (39) is only valid far from the minimum and that close to $\phi = 0$ it is modified in such a way as to satisfy the requirements imposed in our derivation of the ET. For this superpotential it is also known that the amplitude of the oscillations of the effective gravitino mass is not damped in time. But still we can apply the ET to estimate the production of gravitinos if we take $\phi(t) = 0$ for $t \leq 0$ and $t \geq nT$, where n is an integer number (in our example $n = 6$) and T is the period of the inflaton oscillations. Despite these problems, we consider that this model is useful to illustrate numerically the equivalence comparing with previous results in the literature. Thus, we have taken the initial amplitude of the inflaton oscillation to be $\tilde{\phi}_0 \simeq 0.2 M_P$ (where $\tilde{\phi} = \sqrt{2}\mathrm{Re}\phi$ is the canonically normalized inflaton), which implies that the effective mass of the goldstino is oscillating with amplitude $m_G \simeq \sqrt{2\lambda}M$. With these initial and final conditions we have calculated numerically the number of goldstinos produced using eq. (32) in the Landau gauge. For this equation we can use the standard results for the production of fermions obtained in [8]. We thus look for solutions of the goldstino equation with momentum p and helicity l of the form:

$$\eta^{pl}(x) = a^{-3/2}(t) e^{i\vec{p}\cdot\vec{x}} U^{\vec{p}l}(t) \tag{40}$$

with

$$U^{\vec{p}l}(t) = \frac{1}{\sqrt{\omega_{in} + m_{in}}}\left[i\gamma^0\partial_0 - \vec{p}\cdot\vec{\gamma} + a(t)\left(e^{G/2}\left(-G_{,\phi\phi} - G^2_{,\phi}\right)\right)\right] f_{pl}(t) u(\vec{p}, s) \tag{41}$$

Using the above ansatz, we can write the equation as follows:

$$\left[\frac{d^2}{d\tilde{t}^2} + \kappa^2 + \frac{i}{\sqrt{\lambda}}\frac{d}{d\tilde{t}}\left(b e^{G/2}\left(-G_{,\phi\phi} - G^2_{,\phi}\right)\right) + \frac{b^2}{\lambda} e^G \left(-G_{,\phi\phi} - G^2_{,\phi}\right)^2\right] f_{\kappa l}(\tilde{t}) = 0 \tag{42}$$

with $\kappa = p/(a_{in}\sqrt{\lambda})$ and $\tilde{t} = a_{in}\sqrt{\lambda}t$ and the new scale factor is defined as $b(\tilde{t}) = a(\tilde{t})/a_{in}$. The initial conditions are $f_{\kappa l}(0) = 1$ and $\dot{f}_{\kappa l}(0) = -i\kappa$. In particular, for the goldstino occupation number we have:

$$N^G_{\kappa l}(nT) = \frac{1}{4\kappa}\left(2\kappa + i[\dot{f}^*_{\kappa l}(nT) f_{\kappa l}(nT) - f^*_{\kappa l}(nT)\dot{f}_{\kappa l}(nT)]\right) \tag{43}$$

Using our previous result, we can get the occupation number of helicity $\pm 1/2$ gravitinos directly from (43). The production of helicity $\pm 3/2$ gravitinos can be obtained from eq. (6) following the same steps as before [10]. In Fig. 1, we have plotted both spectra and we see that for this particular model, the production of helicity $\pm 3/2$ gravitinos is suppressed by two to three orders of magnitude with respect to the $\pm 1/2$ gravitinos. Because of the dependence of N_{kl} on the number of oscillations at which it is evaluated and on the initial conditions for the inflaton, the comparison with the estimations obtained in [13, 14] in the unitary gauge, (and therefore without the ET, although they have checked that the formulas hold in one particular case) cannot be carried out for the whole spectrum. In any case, the ET tells us that, since the mass of the goldstino will be no larger than $a(t)\sqrt{\lambda}$, both results should agree for $p \gg a(nT)\sqrt{\lambda}$, i.e. with the above definitions $\kappa \gg b(nT)$ (in our example $b(nT) \simeq 12$), irrespectively of the initial conditions. We see that, indeed, the orders of magnitude are in good agreement with the previous works.

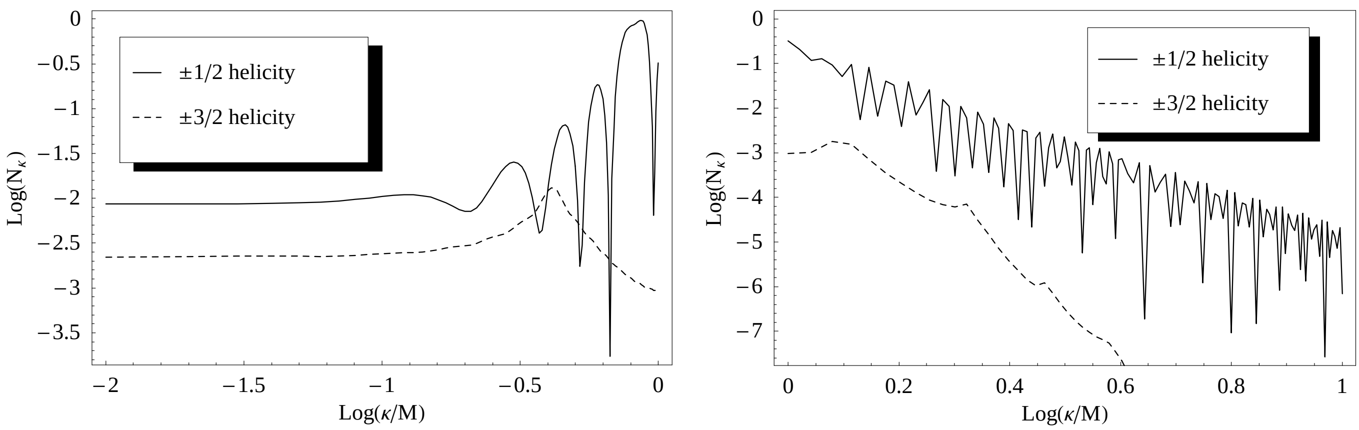

Figure 1: Spectrum of helicity $\pm 3/2$ and helicity $\pm 1/2$ gravitinos for different ranges of momentum. The helicity $\pm 1/2$ production has been obtained using the ET.

Finally, there is an additional condition to obtain reliable predictions of helicity $\pm 1/2$ gravitino production during reheating, since we are producing them in large numbers, with vastly different energies. In order to know the potential applicability of the ET, we have to estimate, out of the total number of gravitinos, what fraction of them does not satisfy the ET conditions. For that purpose we define the number density of gravitinos produced with both helicities as

$$n(t) = \frac{1}{\pi^2 a^3(t)} \int_0^\infty N_{p,l} p^2 dp. \tag{44}$$

Gravitinos with energy lower than their mass are excluded from the ET conditions, i.e. their comoving momenta satisfy $p^2 < a_{out}^2 m_{3/2}^2$, where a_{out} is the scale factor at the end of preheating.

Thus we have for the number density of excluded gravitinos:

$$n(t)_{exc} \leq \frac{1}{3\pi^2 a^3(t)} a_{out}^3 m_{3/2}^3 \tag{45}$$

In addition, we can estimate the total number density of gravitinos produced as:

$$n(t)_{tot} \simeq \frac{1}{3\pi^2 a^3(t)} a_{in}^3 m_\phi^3 \tag{46}$$

Thus we obtain that the fraction of gravitinos that do not satisfy the ET applicability conditions is

$$\frac{n_{exc}}{n_{tot}} \leq \left(\frac{a_{out} m_{3/2}}{a_{in} m_\phi}\right)^3 \tag{47}$$

Accordingly, the condition for the applicability of the TE is: $a_{out}m_{3/2} << a_{in}m_\phi$. We see that the result depends on the duration of the preheating era and the ratio of gravitino and inflaton masses. Typically the production takes place in a few inflaton oscillations, which implies that the scale factor only grows by a few orders of magnitude, not enough to overcome the mass difference. Therefore, in these models, the ET safely describes the production of the vast majority of helicity $\pm 1/2$ gravitinos.

7 Conclusions

We have studied the production of helicity $\pm 1/2$ gravitinos using the equivalence of goldstinos and gravitinos at high energies. We have shown that choosing the appropriate gauges, it is possible to disentangle the goldstino and gravitino equations of motion and also that the gauge condition, in the asymptotic static regions, imposes the classical solutions for the goldstino equation to be proportional to the solutions for the helicity $\pm 1/2$ gravitinos. This result is sufficient to relate the production of goldstinos to the production of helicity $\pm 1/2$ gravitinos. The Landau gauge seems to be the most appropriate, since the equation of motion for the goldstino is considerably simpler. As a check, we have compared our results with previous ones obtained in the unitary gauge and we have found good agreement in the equivalence theorem applicability regions.

To obtain this result, the only conditions we have imposed on the external sources have been: i) the frequency of the inflaton field oscillations should be larger than the gravitino mass, $m_\phi \gg m_{3/2}$ and if we are interested in the pure gravitational production of gravitinos, then one should also require $H \gg m_{3/2}$, these conditions ensuring that the typical energy of the particles produced will be larger than their masses , ii) the sources should vanish asymptotically, which implies that the space-time curvature should decrease with time and also that the amplitude of the inflaton oscillations should be damped.

Concerning the potential applications of our results, we have shown that cubic superpotentials, with a slight modification in their form, together with appropriate initial conditions, could satisfy the above applicability requirements. However, there are some other cases in which the application of the ET is somewhat restricted, such as a pure quadratic superpotential $W = m_\phi \Phi^2$. In this case, at the minimum of the potential, supersymmetry is restored and therefore the definition of helicity $\pm 1/2$ gravitinos is meaningless. Ignoring this problem, although the amplitude of the inflaton oscillations is damped in this model, the goldstino mass term, which contains the second derivative of the superpotential, tends to a constant of order m_ϕ, typically much larger than $m_{3/2}$. Then, the ET will be useful to calculate only that portion of the spectrum with energy much higher than the inflaton mass.

The completion of the proof presented in this work will require its extension to the case of expanding asymptotic regions, and also inclusion of other supergravity models with more than one chiral multiplet and non-minimal supergravity.

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References

[1] H.P. Nilles, *Phys. Rep.* **110**, 1 (1984)

[2] D. Bailin and A. Love, *Supersymmetric Gauge Field Theory and String Theory*, IOP, Bristol, (1994)

[3] E.W. Kolb and M.S Turner, *The Early Universe* (Addison-Wesley, New York, 1990)

[4] J. Ellis, A. Linde and D. Nanopoulos, *Phys. Lett.* **118B**, 59 (1982)

[5] S. Sarkar, *Rep. Prog. Phys.* **59**, 1493 (1996)

[6] J. Ellis, J.E. Kim and D.V. Nanopoulos, *Phys. Lett.* **145B**, 181 (1984)

[7] L. Kofman, A. D. Linde and A.A. Starobinsky, *Phys. Rev. Lett.* **73**, 3195 (1994); L.Kofman, A.D. Linde and A. A. Starobinsky, *Phys. Rev.* **D 56**, 3258 (1997); J.H. Traschen, R.H. Brandenberger, *Phys. Rev.* **D42**, 2491 (1990); Y. Shtanov, J. Traschen and R. Brandenberger, *Phys. Rev.* **D51**, 5438 (1995)

[8] J. Baacke, K. Heitmann and C. Patzold, *Phys. Rev.* **D58**, 125013 (1998)

[9] P.B. Greene and L. Kofman, *Phys. Lett.* **B448**, 6 (1999)

[10] A.L. Maroto and A. Mazumdar, hep-ph/9904206

[11] A.L. Maroto, hep-ph/9906388

[12] M. Lemoine, *Phys. Rev.* **D60**, 103522 (1999)

[13] R. Kallosh, L. Kofman, A. Linde and A. Van Proeyen, hep-th/9907124

[14] D.H. Lyth, D. Roberts and M. Smith, *Phys. Rev.* **D57** 7120 (1998); D.H. Lyth, hep-ph/9909387 and hep-ph/9911257; G.F. Giudice, A. Riotto and I. Tkachev, *JHEP* (1999) 9908:009 and hep-ph/9911302

[15] R. Casalbuoni, S. De Curtis, D. Dominici, F. Feruglio and R. Gatto, *Phys. Lett.* **B215**, 313 (1988) and *Phys. Rev.* **D39**, 2281 (1989).

[16] J.M. Cornwall, D.N. Levin and G. Tiktopoulos, *Phys. Rev.* **D10**, 1145 (1974)

[17] C.E. Vayonakis, *Lett. Nuovo. Cim.* **19**, 383 (1976)
B.W. Lee, C. Quigg and H. Thacker, *Phys. Rev.* **D16**, 1519 (1977)

[18] M.S. Chanowitz and M.K. Gaillard, *Nucl. Phys.* **B261**, 379 (1985)

[19] Y.P.Yao and C.P.Yuan, *Phys. Rev.* **D38**, 2237 (1988)
J.Bagger and C.Schmidt, *Phys. Rev.* **D41**, 2237 (1990)
H.J.He, Y.P. Kuang and X. Li, *Phys. Rev. Lett.* **69**, 2619 (1992).

[20] H.J.He, Y.P. Kuang and X. Li, *Phys. Rev.* **D69**, 6463 (1994).

[21] N.D. Birrell and P.C.W. Davies *Quantum Fields in Curved Space*, Cambridge University Press (1982)

[22] E. Calzetta and B.L. Hu, *Phys. Rev.* **D35**, 495 (1987) and **D40**, 656 (1989)

[23] E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, *Nucl. Phys.* **147**, 105 (1979)

[24] P. Holman, P. Ramond and G.G. Ross, *Phys. Lett.* **137B**, 343 (1984)

[25] G.G. Ross and S. Sarkar, *Nucl. Phys.* **B461**, 597 (1996)

[26] L. Baulieu, A. Georges and S. Ouvry, *Nucl. Phys.* **B273**, 366 (1986)

[27] T. Moroi, PhD Thesis (Tohoku University) (1995), hep-ph/9503210

[28] V.M. Mostepanenko and V.M Frolov, *Sov. J. Nucl. Phys.* **19** 451 (1974); A.A. Grib, S.G. Mamayev and V.M. Mostepanenko *Vacuum Quantum Effects in Strong Fields*, Friedmann Laboratory Publishing, St. Petersburg (1994)