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PHYSICS AND ALGORITHMS OF THE HADRONIC MONTE-CARLO EVENT GENERATORS[1]. Notes for a developer.

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Abstract

This report is a collection of notes written during the work on physics and algorithms of theory-driven hadronic models for the GEANT4 toolkit. These notes are "practical answers" for hadronic model code implementors on their numerous questions about the choice of suitable physical models, about the derivations of formulas, about model parameter values, about needed numerical algorithms etc.

Contents

Chapter 1

Introduction.

The hadronic interaction Monte-Carlo (MC) models have twofold application: (1) as event generators with aim to predict an information about hadronic interactions that is needed to suggest or to build an experiment as well as to analyse and store experimental information, and (2) as auxiliary model codes with the aim to predict missing information needed for more general hadron transport code.

The large energy region considered includes different interaction regimes. For example, the production of pions and Δ's, as well as heavier mesons and higher baryon states, is abundant above 1 GeV energies. String excitations are the characteristic feature of hadronic collisions above 5 GeV/A energies. Here, hadrons are produced essentially in the soft QCD collisions with relatively small momentum transfer through the excitation and decay of colored strings. A change in the hadron production mechanism is expected at RHIC and LHC collider energies, when the so-called semi-hard and hard parton scattering processes with momentum transfer more than 1 GeV will dominate. For theoretical predictions of the total production cross sections the different reaction mechanisms (such as preequilibrium and equilibrium decays) have to be considered. Presently, there is no unique hadronic model available, which is able to cover all these reaction mechanisms simultaneously.

We describe physics and algorithms of the hadronic event generator models listed below:

1. The Resonance Decayer. It models a resonance decay according to the relativistic phase-space.

2. The Cluster Decayer. It models a colourless cluster decay into two hadrons.

3. The String Decayer. It models longitudinal or kinky string decay into hadrons.

4. The Elastic Scatterer. It can be used to perform low and high energy hadron elastic scatterings.

5. The Resonance Interaction Model. It can be used at intermediate energies to perform hadron inelastic interaction with excitation and deexcitation of the meson and baryon resonances.

6. The Baryon Annihilator. It models baryon-antibaryon annihilation.

7. The Reggeon Parton String Model. It can be used at high energies to generate final states of γ-nucleon and hadron-hadron inelastic collisions.

8. The Multiple Parton Hard Scattering Model. It generates final states of the inelastic nucleon-nucleon collision at collider energies.

9. The Pomeron Parton String Model. It generates final states of the γ-nucleus, hadron-nucleus and nucleus-nucleus inelastic collisions at high energies.

10. The Hadron Transport Model. It generates final states of the γ-nucleus, hadron-nucleus and nucleus-nucleus inelastic collisions at intermediate energies and performs the secondary hadron rescatterings after the "string" stage of a reaction.

11. The Intranuclear Transport model. It generates final states of the γ-nucleus, hadron-nucleus and stopped particle-nucleus inelastic collisions at low and intermediate energies.

12. The Preequilibrium Exciton Model. It generates final states of the nucleon-nucleus inelastic collision and provides a transition of an excited nucleus to the equilibrium state after "kinetic" stage of a reaction.

13. The Equilibrium Evaporation Model. It can be used to model the evaporation of photons, nucleons as well as light fragments from the excited compound nucleus.

14. The Symmetric and Asymmetric Fissions Model. It models symmetric or asymmetric fission of heavy excited compound nucleus.

15. The Light Nucleus Multifragmentation Model. It performs decay (explosion) of the very excited light nucleus into the nuclear fragments.

16. The Heavy Nucleus Multifragmentation Model. It performs decay (explosion) of the very excited heavy nucleus into the nuclear fragments.

Energy Projectile	0	> 20 MeV/A	$> 100 - 200$ MeV/A	$> 800 - 1000$ MeV/A	$> 5 - 500$ GeV/A	> 500 GeV/A
γ		X	X	X	X	X
Stopped: π^-, K^-,$\bar{p}$	X					
Hadrons: π , K, n, $\bar{n}$, ...		X	X	X	X	X
A		X	X	X	X	X

Table 1.1: MC hadronic models. Allowed projectiles and bombarding energy ranges for the interaction with nucleon and nucleus targets.

We also describe the Hadron Cross Section Model and the Nuclear Cross Section Model are devoted to the calculation of total and elastic hadron interaction cross sections and total and elastic hadron-nucleus and nucleus-nucleus interaction cross sections at high energies, respectively

Most of the enumerated models are currently under implementation by the GEANT4 collaboration (see [1], [3]).

From the above list there are physically simplified models like the Baryon Annihilation Model, which considers the annihilation reaction as the only diquark annihilation. To be more realistic their physics should be essentially improved. Of course, each model, by definition, should be continuously improved.

Besides the model descriptions we also present here particle properties, nuclear properties as well as the kinematic relations.

The high energy Monte Carlo models, which will be capable to predict final states of reactions with a large variety of projectiles and nucleon and nuclear targets in the energy region above approximately 20 MeV as well as for π^-, K^- and $\bar{p}$ stopped particles (see Table. 1.1) are considered as an essential part of the high energy transport codes, e.g. the GEANT4 [2].

One of the peculiarities in using hadronic models for applied purposes is that they are not used separately. As a rule they are successive applied to predict as many as possible details of hadronic interaction. For example, to model hadron-nucleus interaction at collider energies we can apply at first the Pomeron Parton String Model to give a prediction for the particles mostly produced in the central rapidity region. Then the Intranuclear Hadron Transport Model should be applied to improve predictions for particles produced in the nucleus fragmentation rapidity region, as well as to predict residual nucleus parameters. After we should apply the Preequilibrium Exciton Model to predict production of nuclear fragments and to provide the parameters of residual nucleus at equilibrium state. Finally different statistical models (the Equilibrium Evaporation Model, the Sym-

Bombarding energy	> 20 MeV/A	$> 100 - 200$ MeV/A	$> 800 - 1000$ MeV/A	$> 5 - 500$ GeV/A	> 500 GeV/A
Nucleus deexcitation models:					
Equilibrium Evaporation	XX	XX XXX	XX XXX	XXX XXXX	XXX XXXX
Light Nucleus Multifragmentation	XX	XX XXX	XX XXX	XXX XXXX	XXX XXXX
Symmetric and Asymmetric Fission	XX	XX XXX	XX XXX	XXX XXXX	XXX XXXX
Heavy Nucleus Multifragmentation	XX	XX XXX	XX XXX	XXX XXXX	XXX XXXX
Intranuclear Transport		X	X	XX	XX
Preequilibrium Exciton	X	XX	XXX	XXX	
Pomeron Parton String Model:				X	X

Table 1.2: Applicability of hadronic models as a function of bombarding energy. Direct applicability is marked by X. Next order indirect applicability is marked by XX etc.

metric and Asymmetric Fissions Model as well as the Light and the Heavy Nucleus Multifragmentation Models) should be applied recursively to perform the residual excited nucleus decay and to predict evaporated photons, neutrons, protons and nuclear fragments. The models are participating in such inelastic collision generation chains are presented in the Table 1.2.

It is expected that in the intermediate and high energy regimes the inclusion of these models will allow for reliable extrapolation of the simulation results beyond the test-beam energy, projectile and target ranges.

Bibliography

[1] GEANT4: Physics Reference Manual, Theory Driven Models, December 8, 1998.

[2] Wenaus T. *et al.*, GEANT4: An Object-Oriented Toolkit for Simulation in HEP, CERN/LHCC/97-40.

[3] Amelin N., GEANT4 high energy hadronic theoretical models: physics and algorithms. Invited talk presented at the GEANT4 workshop, Niigata, July 17, 1998.

Chapter 2

Particle properties.

2.1 Particle tables.

The description of particle properties, i. e. characteristics of light and heavy quarks/antiquarks, light diquarks/antidiquarks, photons, gluons, leptons as well mesons and baryons, can be found in publications of the Particle Data Group (PDG96) [1] and (PDG98)[2].

2.1.1 Baryon and meson species.

Within the UrQMD approach [3] we can tabulate properties of the 55 different baryon species and 32 different meson species, which are supplemented by their anti-particle and all isospin-projected states. The full baryon-antibaryon symmetry can be included.

The $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ baryon and baryon resonance states as well as the J^{PC} meson and meson resonance states are listed below, where J, P and C denote a spin, a parity and a charge conjugation, respectively.

2.1.2 Tabulated baryon and meson masses, widths, branching ratios and encodings.

The particle masses are given in GeV and widths in MeV.

The meson and baryon resonances decay branching ratios can be taken from [3].

The encodings for different baryons and mesons can be taken from the PDG96 [1]. It should be mentioned that there are no encodings recommended by the PDG96 for some of resonances: $h_1'(1380)$, N^*_{1900}, N^*_{1990}, N^*_{2080}, N^*_{2220} and N^*_{2250}, Ξ^*_{1690}, Ξ^*_{1950}, Ξ^*_{2030}, which spins have been not experimentally established. Their encodings can be taken from the PDG98 [2].

nucleon	delta	lambda	sigma	xi	omega
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1192}	Ξ_{1315}	Ω_{1672}
N_{1440}	Δ_{1600}	Λ_{1405}	Σ_{1385}	Ξ_{1530}	
N_{1520}	Δ_{1620}	Λ_{1520}	Σ_{1660}	Ξ_{1690}	
N_{1535}	Δ_{1700}	Λ_{1600}	Σ_{1670}	Ξ_{1820}	
N_{1650}	Δ_{1900}	Λ_{1670}	Σ_{1750}	Ξ_{1950}	
N_{1675}	Δ_{1905}	Λ_{1690}	Σ_{1775}	Ξ_{2030}	
N_{1680}	Δ_{1910}	Λ_{1800}	Σ_{1915}		
N_{1700}	Δ_{1920}	Λ_{1810}	Σ_{1940}		
N_{1710}	Δ_{1930}	Λ_{1820}	Σ_{2030}		
N_{1720}	Δ_{1950}	Λ_{1830}			
N_{1900}		Λ_{1890}			
N_{1990}		Λ_{2100}			
N_{2080}		Λ_{2110}			
N_{2190}					
N_{2200}					
N_{2250}					

Table 2.1: Baryons and baryon resonances.

0^{-+}	1^{--}	0^{++}	1^{++}	1^{+-}	2^{++}	$(1^{--})^*$	$(1^{--})^{**}$
π	ρ	a_0	a_1	b_1	a_2	ρ_{1450}	ρ_{1700}
K	K^*	K_0^*	K_1^*	K_1	K_2^*	K_{1410}^*	K_{1680}^*
η	ω	f_0	f_1	h_1	f_2	ω_{1420}	ω_{1662}
η'	ϕ	f_0^*	f_1'	h_1'	f_2'	ϕ_{1680}	ϕ_{1900}

Table 2.2: Meson and meson resonance states.

resonance	**m**	Γ
N^*_{1440}	1.440	200
N^*_{1520}	1.520	125
N^*_{1535}	1.535	150
N^*_{1650}	1.650	150
N^*_{1675}	1.675	140
N^*_{1680}	1.680	120
N^*_{1700}	1.700	100
N^*_{1710}	1.710	110
N^*_{1720}	1.720	150
N^*_{1900}	1.870	500
N^*_{1990}	1.990	550
N^*_{2080}	2.040	250
N^*_{2190}	2.190	550
N^*_{2220}	2.220	550
N^*_{2250}	2.250	470
Δ_{1232}	1.232	115
Δ^*_{1600}	1.700	200
Δ^*_{1620}	1.675	180
Δ^*_{1700}	1.750	300
Δ^*_{1900}	1.850	240
Δ^*_{1905}	1.880	280
Δ^*_{1910}	1.900	250
Δ^*_{1920}	1.920	150
Δ^*_{1930}	1.930	250
Δ^*_{1950}	1.950	250

Table 2.3: Masses and widths for non-strange baryon resonances.

resonance	**m**	Γ
Λ^*_{1405}	1.407	50
Λ^*_{1520}	1.520	16
Λ^*_{1600}	1.600	150
Λ^*_{1670}	1.670	35
Λ^*_{1690}	1.690	60
Λ^*_{1800}	1.800	300
Λ^*_{1810}	1.810	150
Λ^*_{1820}	1.820	80
Λ^*_{1830}	1.830	95
Λ^*_{1890}	1.890	100
Λ^*_{2100}	2.100	200
Λ^*_{2110}	2.110	200
Σ^*_{1385}	1.384	36
Σ^*_{1660}	1.660	100
Σ^*_{1670}	1.670	60
Σ^*_{1750}	1.750	90
Σ^*_{1775}	1.775	120
Σ^*_{1915}	1.915	120
Σ^*_{1940}	1.940	220
Σ^*_{2030}	2.030	180

Table 2.4: Masses and widths for single-strange baryon resonances.

resonance	**m**	Γ
Ξ^*_{1530}	1.532	9
Ξ^*_{1690}	1.700	50
Ξ^*_{1820}	1.823	24
Ξ^*_{1950}	1.950	60
Ξ^*_{2030}	2.025	20

Table 2.5: Masses and widths for double-strange baryon resonances.

resonance	**m**	Γ
ω	0.782	8
ρ	0.769	151
$f_0(980)$	0.980	100
η'	0.958	0.2
K^*	0.893	50
ϕ	1.019	4
K^*_0	1.429	287
a_0	0.984	100
$f_0(1370)$	1.370	200
$K_1(1270)$	1.273	90
a_1	1.230	400
f_1	1.282	24
$f_1(1510)$	1.512	350
$K_2(1430)$	1.430	100
$a_2(1320)$	1.318	107
$f_2(1270)$	1.275	185
$f'_2(1525)$	1.525	76
$K_1(1400)$	1.400	174
$b_1(1235)$	1.235	142
$h_1(1170)$	1.170	360
$h'_1(1380)$	1.380	80
$K^*(1410)$	1.410	227
$\rho(1465)$	1.465	310
$\omega(1419)$	1.419	174
$\phi(1680)$	1.680	150
$K^*(1680)$	1.680	323
$\rho(1700)$	1.700	235
$\omega(1662)$	1.662	280
$\phi(1900)$	1.900	400

Table 2.6: Masses and widths for meson-resonances.

resonance	$N\gamma$	$N\pi$	$N\eta$	$N\omega$	$N\varrho$	$N\pi\pi$	$\Delta_{1232}\pi$	$N^*_{1440}\pi$	ΛK
N^*_{1440}		0.70				0.05	0.25		
N^*_{1520}		0.60				0.15	0.25		
N^*_{1535}	0.001	0.55	0.35			0.05		0.05	
N^*_{1650}		0.65	0.05			0.05	0.10	0.05	0.10
N^*_{1675}		0.45					0.55		
N^*_{1680}		0.65				0.20	0.15		
N^*_{1700}		0.10	0.05		0.05	0.45	0.35		
N^*_{1710}		0.15	0.20		0.05	0.20	0.20	0.10	0.10
N^*_{1720}		0.15			0.25	0.45	0.10		0.05
N^*_{1900}		0.35		0.55	0.05		0.05		
N^*_{1990}		0.05			0.15	0.25	0.30	0.15	0.10
N^*_{2080}		0.60	0.05		0.25	0.05	0.05		
N^*_{2190}		0.35			0.30	0.15	0.15	0.05	
N^*_{2220}		0.35			0.25	0.20	0.20		
N^*_{2250}		0.30			0.25	0.20	0.20	0.05	
Δ_{1232}	0.01	1.00							
Δ^*_{1600}		0.15					0.55	0.30	
Δ^*_{1620}		0.25					0.60	0.15	
Δ^*_{1700}		0.20			0.10		0.55	0.15	
Δ^*_{1900}		0.30			0.15		0.30	0.25	
Δ^*_{1905}		0.20			0.60		0.10	0.10	
Δ^*_{1910}		0.35			0.40		0.15	0.10	
Δ^*_{1920}		0.15			0.30		0.30	0.25	
Δ^*_{1930}		0.20			0.25		0.25	0.30	
Δ^*_{1950}	0.01	0.45			0.15		0.20	0.20	

Table 2.7: Branching ratios for non-strange baryon resonances.

resonance	NK	NK^*_{892}	$\Sigma\pi$	$\Sigma^*\pi$	$\Lambda\gamma$	$\Lambda\eta$	$\Lambda\omega$	$\Lambda\pi$	$\Sigma\eta$	$\Lambda^*\pi$	ΔK
Λ^*_{1405}			1.00								
Λ^*_{1520}	0.45		0.43	0.11	0.01						
Λ^*_{1600}	0.35		0.65								
Λ^*_{1670}	0.20		0.50			0.30					
Λ^*_{1690}	0.25		0.45	0.30							
Λ^*_{1800}	0.40	0.20	0.20	0.20							
Λ^*_{1810}	0.35	0.45	0.15	0.05							
Λ^*_{1820}	0.73		0.16	0.11							
Λ^*_{1830}	0.10		0.70	0.20							
Λ^*_{1890}	0.37	0.21	0.11	0.31							
Λ^*_{2100}	0.35	0.20	0.05	0.30		0.02	0.08				
Λ^*_{2110}	0.25	0.45	0.30								
Σ^*_{1385}			0.12					0.88			
Σ^*_{1660}	0.30		0.35					0.35			
Σ^*_{1670}	0.15		0.70					0.15			
Σ^*_{1750}	0.40		0.05						0.55		
Σ^*_{1775}	0.40		0.04	0.10				0.23		0.23	
Σ^*_{1915}	0.15		0.40	0.05				0.40			
Σ^*_{1940}	0.10	0.15	0.15	0.15				0.15		0.15	0.15
Σ^*_{2030}	0.20	0.04	0.10	0.10				0.20		0.18	0.18

Table 2.8: Branching ratios for single-strange baryon resonances.

resonance	$\Xi\pi$	$\Xi\gamma$	ΛK	ΣK
Ξ^*_{1530}	0.98	0.02		
Ξ^*_{1690}	0.10		0.70	0.20
Ξ^*_{1820}	0.15		0.70	0.15
Ξ^*_{1950}	0.25		0.50	0.25
Ξ^*_{2030}	0.10		0.20	0.70

Table 2.9: Branching ratios for double-strange baryon resonances.

meson	$\gamma\pi$	$\gamma\rho$	$\gamma\omega$	$\gamma\eta$	γK	$\pi\pi$	$\pi\rho$	3π	$\pi\eta$	4π	KK^*	KK^*
ω	0.09					0.02		0.89				
ρ						1.00						
$f_0(980)$						0.70						
η'		0.30	0.05									
K^*												
ϕ				0.01			0.13	0.02				
K^*_0												
a_0									0.90			
$f_0(1370)$						0.10				0.70		
$K_1(1270)$												
a_1	0.10						0.90					
f_1		0.07								0.20		
$f_1(1510)$											0.50	0.50
$K_2(1430)$												
$a_2(1320)$							0.70		0.14			
$f_2(1270)$						0.50				0.30		
$f'_2(1525)$						0.01						
$K_1(1400)$												
$b_1(1235)$												
$h_1(1170)$							0.90	0.10				
$h'_1(1380)$											0.50	0.50
$K^*(1410)$												
$\rho(1465)$						0.50				0.50		
$\omega(1419)$							1.00					
$\phi(1680)$											0.40	0.40
$K^*(1680)$												
$\rho(1700)$						0.10				0.20		
$\omega(1662)$							0.50					
$\phi(1900)$											0.40	0.40

Table 2.10: Branching ratios for meson resonances.

meson	$\eta\pi\pi$	$\eta\rho$	$\rho\pi\pi$	$\omega\pi\pi$	$\eta\eta$	KK	$KK\pi$	$K\pi$	$K^*\pi$	$K\rho$	$K\omega$	$K^*\pi\pi$	$\omega\pi$
ω													
ρ													
$f_0(980)$						0.30							
η'	0.65												
K^*								1.00					
ϕ						0.84							
K_0^*								1.00					
a_0						0.10							
$f_0(1370)$						0.20							
$K_1(1270)$									0.47	0.42	0.11		
a_1													
f_1	0.54		0.10				0.09						
$f_1(1510)$													
$K_2(1430$								0.50	0.25	0.09	0.03	0.13	
$a_2(1320)$				0.11		0.05							
$f_2(1270$						0.20							
$f_2'(1525)$					0.10	0.89							
$K_1(1400)$									0.96	0.03	0.01		
$b_1(1235)$		0.10											0.90
$h_1(1170)$													
$h_1'(1380)$													
$K^*(1410)$								0.30	0.65	0.05			
$\rho(1465)$													
$\omega(1419)$													
$\phi(1680)$						0.10	0.10						
$K^*(1680)$								0.40	0.30	0.30			
$\rho(1700)$			0.70										
$\omega(1662)$				0.50									
$\phi(1900)$						0.10	0.10						

Table 2.11: Branching ratios for meson resonances.

resonance	**encoding**
N^*_{1440}	12112, 12212
N^*_{1520}	1214, 2124
N^*_{1535}	22112, 22212
N^*_{1650}	32112, 32212
N^*_{1675}	2116, 2216
N^*_{1680}	12116,12216
N^*_{1700}	21214, 22124
N^*_{1710}	42112, 42212
N^*_{1720}	31214, 32124
N^*_{1900}	41214, 42124
N^*_{1990}	11218, 12128
N^*_{2080}	51214, 52124
N^*_{2190}	1218, 2128
N^*_{2220}	12110, 21210
N^*_{2250}	112110, 121210
Δ_{1232}	1114, 2114, 2214, 2224
Δ^*_{1600}	31114, 32114, 32214, 32224
Δ^*_{1620}	1112, 1212, 2122, 2222
Δ^*_{1700}	11114, 12114, 12214, 12224
Δ^*_{1900}	11112, 11212, 12122, 12222
Δ^*_{1905}	1116, 1216, 2126, 2226
Δ^*_{1910}	21112, 21212, 22122, 22222
Δ^*_{1920}	21114, 22114, 22214, 22224
Δ^*_{1930}	11116, 11216, 12126, 12226
Δ^*_{1950}	1118, 2118, 2218, 2228

Table 2.12: Encodings for non-strange baryon resonances.

resonance	encoding
Λ^*_{1405}	13122
Λ^*_{1520}	3124
Λ^*_{1600}	23122
Λ^*_{1670}	33122
Λ^*_{1690}	13124
Λ^*_{1800}	43122
Λ^*_{1810}	53122
Λ^*_{1820}	3126
Λ^*_{1830}	13126
Λ^*_{1890}	23124
Λ^*_{2100}	3128
Λ^*_{2110}	23126
Σ^*_{1385}	3114, 3214, 3224
Σ^*_{1660}	13112, 13212, 13222
Σ^*_{1670}	13114, 13214, 13224
Σ^*_{1750}	23112, 23212, 23222
Σ^*_{1775}	3116, 3216, 3226
Σ^*_{1915}	13116, 13216, 13226
Σ^*_{1940}	23114, 23214, 23224
Σ^*_{2030}	3118, 3218, 3228

Table 2.13: Encodings for single-strange baryon resonances.

resonance	encoding
Ξ^*_{1530}	3314, 3324
Ξ^*_{1690}	23314, 23324
Ξ^*_{1820}	13314, 13324
Ξ^*_{1950}	33314, 33324
Ξ^*_{2030}	13316, 13326

Table 2.14: Encodings for double-strange baryon resonances.

resonance	**encoding**
ω	223
ρ	-213, 113, 213
$f_0(980)$	10221
η'	331
K^*	313, 323
ϕ	333
K_0^*	10313, 10323
a_0	-10211, 10111, 10211
$f_0(1370)$	20221
$K_1(1270)$	10313, 10323
a_1	-20213, 20113, 20213
f_1	20223
$f_1(1510)$	40223
$K_2(1430)$	315, 325
$a_2(1320)$	-215, 115, 215
$f_2(1270)$	225
$f_2'(1525)$	335
$K_1(1400)$	20313, 20323
$b_1(1235)$	-10213, 10113, 10213
$h_1(1170)$	10223
$h_1'(1380)$	10333
$K^*(1410)$	30313, 30323
$\rho(1465)$	-40213, 40113, 40213
$\omega(1419)$	50223
$\phi(1680)$	10333
$K^*(1680)$	40313, 40323
$\rho(1700)$	-30213, 30113, 30213
$\omega(1662)$	60223
$\phi(1900)$	337

Table 2.15: Encodings for meson resonances.

2.2 Parton initial state simulation.

We assume that a nucleon consists only from u, d, s quarks and anti-quarks as well as gluons g.

The nucleon initial state is simulated according to the procedure, which described in [4]. It is assumed there is no correlation between momentum and spatial parton distributions inside a nucleon.

2.2.1 Spatial parton distribution.

Partons are distributed randomly inside the nucleon sphere with radius R_N according to

$$\rho_N(\mathbf{r}) = \frac{3}{4\pi R_N^3}\theta(R_N - |\mathbf{r}|), \tag{2.1}$$

where $\theta(x)$ is the unit step function and $\mathbf{r}$ is a parton radius vector.

2.2.2 Parton flavours, momenta and their virtualities.

The centre of mass of the two colliding nucleons is chosen as the frame, where we model nucleon collision evolution. For this frame nucleons have zero transverse momenta $\mathbf{P}_t = 0$ and balancing longitudinal momenta P_z and $-P_z$.

The parton flavours f, parton momenta $\mathbf{p}$ as well as their space-like virtualities q are sampled using the momentum distribution:

$$P_f(\mathbf{p}, \mathbf{P}, \mathbf{Q_0^2}) = \frac{\mathbf{x}}{\mathbf{x_e}}\mathbf{F_f}(\mathbf{x}, \mathbf{Q_0^2})\delta(\mathbf{P_z} - \mathbf{P})\delta^{\mathbf{2}}(\mathbf{P_t}), \tag{2.2}$$

where $\mathbf{P}$ is the nucleon c.m. momenta, Q_0^2 is initial resolution scale and $x = p_z/\mathbf{P}$ and

$$x_e = \frac{E}{\mathbf{P}} = \sqrt{x^2 + \frac{p_t^2 + m_f^2 + q^2}{P^2}} \tag{2.3}$$

are the momentum and energy fractions. The total number $n(\mathbf{P}, \mathbf{Q_0^2})$ of partons in the nucleon with momentum $\mathbf{P}$ at Q_0^2 can be obtained from the momentum distribution function as follows

$$\sum_f \int d|q^2| \frac{d^3p}{(2\pi)^3 2E} e P_f(\mathbf{p}, \mathbf{P}, \mathbf{Q_0^2}) = \mathbf{n}(\mathbf{P}, \mathbf{Q_0^2}). \tag{2.4}$$

The u and d valence quark longitudinal momenta p_z are sampled using the proton or/and neutron valence structure functions $F_v(x, Q_0^2)$. The longitudinal momenta of gluons and sea quarks are sampled according to the

summed structure function

$$F(x,Q_0^2) = F_g(x,Q_0^2) + \sum_{q_s}[F_{q_s}(x,Q_0^2) + F_{\bar{q}_s}(x,Q_0^2)]. \tag{2.5}$$

Then a particular flavour of quark or gluon is selected according to relative probabilities

$$P_{g,q_s} = F_{g,q_s}(x,Q_0^2)/F(x,Q_0^2). \tag{2.6}$$

The nucleon scale-dependent structure functions are normalised according to

$$\sum_i \int_0^1 xF_i(x,Q_0^2)dx = 1 \tag{2.7}$$

and The number of gluons and sea quarks (antiquarks) $n_{g,s}$ are determined from the condition:

$$\sum_{i=1}^{n_{v,g,s}} x_i = 1. \tag{2.8}$$

The transverse momentum $\mathbf{p_t}$ of each parton is selected independently according to the Gaussian distribution:

$$P(\mathbf{p_t}) = \frac{\mathbf{1}}{\mathbf{2\pi a}} \exp\,[-\frac{\mathbf{p^2}}{\mathbf{a}}] \tag{2.9}$$

with the normalisation

$$\int_0^\infty P(\mathbf{p}_t)d^2p_t = 1. \tag{2.10}$$

The valence quarks are considered as real particles, while the gluon or sea parton energy is calculated as $E = vp_z$, where v is the nucleon velocity. From the relation: $E(q^2) = \sqrt{p^2+m^2+q^2}$ we calculate gluon and sea parton virtualities $q^2 < 0$. The width of the parton $\mathbf{p}_t$ distribution is constrained by the requirement that total invariant mass of partons is equal to the nucleon mass. We require that total 4-momentum of partons, which belong to the system of colliding nucleons, is equal the total nucleon 4-momentum.

The initial resolution scale $Q_0^2 = 1GeV^2$ can be chosen.

2.2.3 Boosted parton positions.

The positions of valence quarks are boosted into the nucleon c.m. frame. The spread of longitudinal positions of gluons and see quarks depends on their x:

$$\Delta z = \frac{\hbar}{xP_z} \tag{2.11}$$

at $\Delta z \leq 2R_N$, i.e. $x > x_{min} = \hbar/(2R_N P_z)$.

2.3 Nucleon structure functions.

The different parametrisations of the nucleon structure functions can be found in the PDFLIB [5]. Recently the Regge based successful parametrization of Capella *et al.* [6] for the nucleon structure functions valid at $Q^2 < 5\ GeV^2$ and covering the full x region was suggested. We can use this parametrization adding some parton phenomenology arguments to separate impact of different quarks and gluons.

At $Q^2 > 5\ GeV^2$ we can employ parametrisation of Glück *et al.*[7].

2.3.1 Regge based parametrisation of the nucleon structure functions.

The authors of [6] suggested the following parametrisation of the structure functions in the region of small Q^2 ($0 \leq Q^2 \leq 5\ GeV^2$):

$$\begin{aligned} F_2(x,Q^2) = Ax^{-\Delta(Q^2)}(1-x)^{n(Q^2)+4}[\tfrac{Q^2}{Q^2+a}]^{1+\Delta(Q^2)}+ \\ +B(x)x^{1-\alpha_R}(1-x)^{n(Q^2)}[\tfrac{Q^2}{Q^2+b}]^{\alpha_R}. \end{aligned} \tag{2.12}$$

The first term of the above parametrisation corresponds to the Pomeron contribution. Its behaviour at $x \to 0$ is

$$F_2(x,Q^2) \sim x^{-\Delta(Q^2)}, \tag{2.13}$$

where

$$\Delta(Q^2) = \Delta_0[1 + \frac{2Q^2}{Q^2+d}]. \tag{2.14}$$

Δ_0 and d are free parameters. It was found [6] from the data fit that $\Delta(Q^2=0) \approx 0.08$ and $\Delta(Q^2 \to \infty) \approx 0.24$. The second term of Eq. (2.12) is the contribution of the reggeons. Its behaviour at $x \to 0$ determined by the reggeon intercept $\alpha_R \approx 0.4 - 0.6$. $n(Q^2)$ determines structure functions behaviour at $x \to 1$:

$$n(Q^2) = \frac{3}{2}[1 + \frac{2Q^2}{Q^2+c}]. \tag{2.15}$$

Thus, the second term of Eq. (2.12) behaves as $(1-x)^{1.5}$ at $Q^2 = 0$ and as $(1-x)^3$ at $Q^2 \to \infty$. The last factors in two terms of the Eq. (2.12) have been introduced to calculate the real photon-nucleon total cross section. The parameter $B(x) = B_u + B_d(1-x) = 0.754 + 0.4495(1-x)$ was fixed using the normalisation condition for valence quarks. Two terms in $B(x)$ are due to the relation $d(x) = u(x)(1-x)$, which is used for the valence quark distribution in the proton.

The values of parameters: $A = 0.1502$, $\alpha_R = 0.4150$, $a = 0.2631\ GeV^2$, $b = 0.6452\ GeV^2$, $c = 3.5489\ GeV^2$, $d = 1.1170\ GeV^2$ and $\Delta_0 = 0.07684$ were obtained from the fits of experimental structure functions and total gamma-proton cross section.

To define gluon distribution it was assumed [6] that the difference between gluon and sea-quark distributions is the $x \to 1$ behaviour:

$$F_g(x,Q^2) = G(Q^2)F_{q_s}(x,Q^2)/(1-x), \tag{2.16}$$

where

$$G(Q^2) = G_g\left[\frac{Q^2}{Q^2+a}\right]^{1+\Delta(Q^2)} \tag{2.17}$$

and $G_g = 1.84$ was determined from the energy-momentum conservation sum rule at chosen $Q_0^2 = 2\ GeV^2$[6]. Thus, for gluon distribution in the nucleon we obtain

$$F_g(x,Q^2) = G(Q^2)\frac{x^{-\Delta(Q^2)}}{x}(1-x)^{n(Q^2)+3}. \tag{2.18}$$

Applying parton phenomenology, i.e.

$$F_2(x,Q^2) = \sum_i e_i^2 x F_i(x,Q^2), \tag{2.19}$$

we can find valence and see quark distribution functions $F_i(x,Q^2)$. To find the strange sea component we can use the parameter $\delta = 0.06$, which is defined as follows

$$\delta = \frac{\int_0^1 dx x[F_s(x) + F_{\bar{s}}(x)]}{\int_0^1 dx x[F_u(x) + F_{\bar{u}}(x) + F_d(x) + F_{\bar{d}}(x)]}. \tag{2.20}$$

2.3.2 Leading order parton distributions [LO].

The authors of [7] have suggested to use the non-singlet distribution in the form:

$$xv(x,Q^2) = Nx^a(1 + Ax^b + Bx + Cx^{3/2})(1-x)^D. \tag{2.21}$$

For $v = u_v$

$$\begin{aligned} a &= 0.590 - 0.024s,\ b = 0.131 + 0.063s,\\ N &= 2.284 + 0.802s + 0.055s^2,\\ A &= -0.449 - 0.138s - 0.076s^2,\\ B &= 0.213 + 2.669s - 0.728s^2,\\ C &= 8.854 - 9.135s + 1.979s^2,\\ D &= 2.997 + 0.753s - 0.076s^2, \end{aligned} \tag{2.22}$$

for $v = d_v$

$$\begin{aligned} a &= 0.376,\ b = 0.486 + 0.062s,\\ N &= 0.371 + 0.083s + 0.039s^2,\\ A &= -0.509 + 3.310s - 1.248s^2,\\ B &= 12.41 - 10.52s + 2.267s^2,\\ C &= 6.373 - 6.208s + 1.418s^2,\\ D &= 3.691 + 0.799s - 0.071s^2, \end{aligned} \tag{2.23}$$

and for $v = \bar{d} - \bar{u}$

$$\begin{gathered} a = 0.409 - 0.005s,\ b = 0.799 + 0.071s, \\ N = 0.082 + 0.014s + 0.008s^2, \\ A = -38.07 + 36.13s - 0.656s^2, \\ B = 90.31 - 74.15s + 7.645s^2, \\ C = 0, \\ D = 7.486 + 1.217s - 0.159s^2 \end{gathered} \tag{2.24}$$

were found. The gluon and sea $\bar{u} + \bar{d}$ distributions have been parametrised as follows

$$xw(x,Q^2) = \left[x^a (A + Bx + Cx^2)(\ln \frac{1}{x})^b + s^\alpha \exp\left(-E + \sqrt{E' s^\beta \ln \frac{1}{x}} \right) \right] (1-x)^D. \tag{2.25}$$

For $w = g$

$$\begin{gathered} \alpha = 0.524,\ \beta = 1.088, \\ a = 1.742 - 0.930s,\ b = -0.399s^2, \\ A = 7.486 - 2.185s, \\ B = 16.69 - 22.74s + 5.779s^2, \\ C = -25.59 + 29.71s - 7.296s^2, \\ D = 2.792 + 2.215s + 0.422s^2 - 0.104s^3, \\ E = 0.807 + 2.005s,\ E' = 3.841 + 0.316s, \end{gathered} \tag{2.26}$$

and for $w = \bar{u} + \bar{d}$

$$\begin{gathered} \alpha = 1.451,\ \beta = 0.271, \\ a = 0.410 - 0.232s,\ b = 0.534 - 0.457s, \\ A = 0.890 - 0.140s,\ B = -0.981 \\ C = 0.320 + 0.683s, \\ D = 4.752 + 1.164s + 0.286s^2, \\ E = 4.119 + 1.713s,\ E' = 0.682 + 2.978s \end{gathered} \tag{2.27}$$

were found. The strange sea $\bar{s} = s$ distribution has been parametrised by

$$xw'(x,Q^2) = \frac{s^\alpha}{(\ln \frac{1}{x})^a} \left[1 + A\sqrt{x} + Bx \right] \exp\left(-E + \sqrt{E' s^\beta \ln \frac{1}{x}} \right) (1-x)^D. \tag{2.28}$$

For $w' = \bar{s} = s$

$$\begin{gathered} \alpha = 0.914,\ \beta = 0.577, \\ a = 1.798 - 0.596s, \\ A = -5.548 + 3.669\sqrt{s} - 0.616s, \\ B = 18.98 - 16.73\sqrt{s} + 5.168s, \\ D = 6.379 - 0.350s + 0.142s^2, \\ E = 3.981 + 1.638s,\ E' = 6.402 \end{gathered} \tag{2.29}$$

were found. The variable s is defined as

$$s = \ln \frac{\ln [Q^2/\Lambda_{LO}]}{\ln [\mu^2_{LO}/\Lambda_{LO}]}, \tag{2.30}$$

where $\mu^2_{LO} = 0.23\ GeV^2$, $\Lambda_{LO} = 0.232\ GeV^2$.

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Chapter 3

Resonance decay model.

3.1 Resonance decay simulation.

Resonance decay channels are sampled using the partial decay widths $\Gamma(m)$, which depend from masses m of resonances. All resonances are considered as non-polarised particles. If a resonance is among of the outgoing particles, its mass is determined according to a Breit-Wigner mass distribution:

$$F(m) = \frac{1}{2\pi} \frac{\Gamma(m_R)}{(m_R - m)^2 + \Gamma(m_R)^2/4}. \tag{3.1}$$

All pole masses m_R and partial decay widths Γ_R are taken from the Review of Particle Properties [1]. If the exit channel contains two, three or four particles, then the respective N-body phase-space is taken into account for their momenta.

The Kopylov's algorithm[2] can be used to sample product particle 4-momenta. We would stress that Kopylov's algorithm gives a possibility to take into account the resonance decay matrix elements.

3.1.1 Kopylov's algorithm for N-body decay generation.

It allows us to simulate also nuclear fragment production as result of an excited nucleus decay (see the light nucleus multifragmentation chapter). Using this algorithm one can also calculate decay widths and different particle distributions.

We can write the n particles state probability

$$dW = \prod_{i=1}^{n} \frac{d^3\mathbf{p}_i}{2\omega_i} \delta(\sum_{i=1}^{n} p_i - P_n)|M|^2, \tag{3.2}$$

where $P_n = (\mathbf{P}_n, E_n)$ is the decaying particle 4-momentum and $M(p_1, p_2, ..., p_n)$ is the reaction amplitude, which depends on the 4-momenta p_i of products.

We know the sum of produced particle masses

$$\mu_n = \sum_{i=1}^{n} m_i \tag{3.3}$$

and c.m. system kinetic energy

$$T_n = M_n - \mu_n, \tag{3.4}$$

where $M_n = \sqrt{E_n^2 - P_n^2}$ is the system invariant mass.

Performing integrations of the probability expressed by the (3.2), one can obtain different particle distributions. The integration over all set of variables S can be shortly written as

$$W = \int_D \Phi(S) dS, \tag{3.5}$$

where $\Phi(S)$ can be considered as the weight of n-particles production event.

The Kopylov's algorithm is based on the choice of a set of variables S_1: T_k, η_k and ϕ_k instead of particle 4-momenta to perform the integration of dW. The integration range D_1 over these new variables is simple. The angular variables are inside $1 \geq \eta_k \geq -1$ and $2\pi\phi_k \geq 0$, where $k = n, n-1, ..., 2$, and kinetic energies of systems with some numbers of particles are ordered according to $T_n \geq T_{n-1} \geq ... \geq T_2 \geq T_1 = 0$. Further substitution of the variables T_k, η_k and ϕ_k by the set variables S_2: ξ_k, γ_k and β_k, which are limited between 0 and 1, creates an unit hypercube from the integration range D_2:

$$W \sim \int_0^1 ... \int_0^1 \prod_2^n d\beta_k d\gamma_k \prod_2^{n-1} d\xi_k \xi_k^{\frac{3k-5}{2}} \sqrt{1-\xi_k} |M|^2. \tag{3.6}$$

Two sets of variables S_1 and S_2 are related to each other:

$$\begin{gathered} T_k = T_{k+1}\xi_k, k = n-1, ..., 2 \\ \eta_k = 2\beta_k - 1, k = n, ..., 2 \\ \phi_k = 2\pi\gamma_k, k = n, ..., 2. \end{gathered} \tag{3.7}$$

The event weight $\Phi(S_1)$ is [2]:

$$\Phi(S_1) = \frac{\pi^{\frac{3}{2}(n-1)} T_n^{\frac{3n-5}{2}}}{2M_n \Gamma(\frac{3}{2}(n-1))} \prod_{k=2}^{n} \frac{\hat{p}_k}{\sqrt{T_k - T_{k-1}}} |M|^2, \tag{3.8}$$

where $\Gamma(x)$ is the gamma function and $\hat{p}_k$ is the 4-momentum of the k-th particle in the rest frame 0_k, where $T_k = 0$.

As one can see from the Eq. (3.6) the variables ξ_k, γ_k and β_k can be considered as the random numbers distributed between 0 and 1. Thus to generate particle momenta we can apply the next recursive procedure [2].

1. Sample ξ_{k-1} according to the distribution

$$F(\xi_k) = \xi_k^{\frac{3k-5}{2}}\sqrt{1-\xi_k}, \tag{3.9}$$

where functions $F(\xi_k)$ have maxima $F_{max} = F((3k-5)/(3k-4))$, and compute $T_{k-1} = T_k\xi_{k-1}$. If $k = 2$ then T_{k-1} should be set to 0 in accordance with the definition of the rest frame 0_{k-1}.

The functions $F(x)$ are sharp functions at the large values of k. At large k we can use another method of sampling with tabulation [2]. In this case for the uniformly distributed between 0 and 1 random number α_{k-1} we have to solve equation

$$C_{k-1}(\xi_{k-1}) = \alpha_{k-1}, \tag{3.10}$$

where

$$C_k(\xi_k) = \frac{\int_0^{\xi_k} d\xi_k \xi_k^{\frac{3k-5}{2}}\sqrt{1-\xi_k}}{\int_0^1 d\xi_k \xi_k^{\frac{3k-5}{2}}\sqrt{1-\xi_k}} \tag{3.11}$$

and find value of ξ_{k-1}. $C_k(\xi_k)$ are tabulated smooth functions and the linear interpolation can be applied to find function values at the intermediate points.

2. Compute the total energy

$$\hat{\omega}_k = (M_k^2 + m_k^2 - M_{k-1}^2)/2M_k \tag{3.12}$$

and the absolute value of the 3-momentum

$$\hat{p}_k = \sqrt{\hat{\omega}_k^2 - m_k^2} \tag{3.13}$$

for the k-th particle in the rest frame 0_k, where

$$\mu_{k-1} = \mu_k - m_k \tag{3.14}$$

and

$$M_{k-1} = \mu_{k-1} - T_{k-1}. \tag{3.15}$$

3. Sample direction of the k-th momentum $\hat{\mathbf{p}}_k$, i.e. sample the azimuthal and polar angles according to $\phi_k = 2\pi\gamma_k$ and $\cos\theta_k = 2\beta_k - 1$, respectively. The γ_k and β_k are uniformly distributed between 0 and 1 random numbers.

4. Calculate the k-th particle energy

$$\omega_k = \frac{E_k\hat{\omega}_k + \mathbf{P}_k\hat{\mathbf{p}}_k}{M_k} \tag{3.16}$$

and its 3-momentum

$$\mathbf{p}_k = \hat{\mathbf{p}}_k + \hat{\mathbf{P}}_k\frac{\omega_k + \hat{\omega}_k}{E_k + M_k} \tag{3.17}$$

in the observer frame.

5. Calculate the total energy E_{k-1} and 3-momentum $\mathbf{P}_{k-1}$ the system with $k-1$ particles:

$$E_{k-1} = E_k - \omega_k, \tag{3.18}$$

$$\mathbf{P}_{k-1} = \mathbf{P}_k - \mathbf{p}_k. \tag{3.19}$$

If $k = 2$, then we can check that $m_1 = \sqrt{E_1^2 - P_1^2}$, where $\mathbf{p}_1 = \mathbf{P}_1$ and $\omega_1 = E_1$.

As result of the outlined recursive procedure we know momenta of all products and can calculate the event weight (3.8).

Finally, if $\Phi(S_1) < \xi\Phi_{max}$, where ξ is uniformly distributed between 0 and 1 random number and Φ_{max} is the maximal possible value from the $\Phi(S_1)$ set of values, we will accept the event.

We should note that in the non-relativistic case and for $|M|^2 = 1$ the last step, i.e. the test $\Phi(S_1) < \xi\Phi_{max}$, is not needed [2].

We also should note that the outlined procedure can be used to calculate the relativistic phase space volume W, i.e. to perform the integral over the event weight (3.5) at $|M|^2 = 1$. To perform integrations we need to find the average value of the event weights for N events, where $N \to \infty$. This method to perform integration is also efficient at $|M|^2 \neq 1$, if $|M|^2$ has no pole behaviour.

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Chapter 4

Cluster decay model.

4.1 Cluster decay simulation.

The cluster decay model is originated from [1], [2]. Each cluster with mass M_c can decay isotropically in its rest frame into hadron pair with masses M_1 and M_2 and spins J_1 and J_2 by pulling a quark-antiquark or a diquark-antidiquark pair with a quark (diquark) mass m_q. For hadron containing u, d and s quarks the $J^P = 0^-, 1^\pm$ meson and $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ baryon states are allowed. The decay channel is selected according to the probability:

$$P_{decay} = P_{flavor}(M_c, m_q) P_{spin}(J_1, J_2) P_{ps}(M_c, M_1, M_2), \tag{4.1}$$

where the flavour factor is

$$P_{flavor}(M_c, m_q) = 1 + \frac{2m_q^2}{M_c^2}\sqrt{1 - \frac{4m_q^2}{M_c^2}}, \tag{4.2}$$

the spin factor is

$$P_{spin}(J_1, J_2) = (2J_1 + 1)(2J_2 + 1) \tag{4.3}$$

and the phase space factor is

$$P_{ps} = \frac{\sqrt{M_c^4 + M_1^4 + M_2^4 - 2(M_c^2 M_1^2 + M_c^2 M_2^2 + M_1^2 M_2^2)}}{M_c^2}. \tag{4.4}$$

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Chapter 5

String decay model.

5.1 String types.

The string decay model is capable to predict final states, i.e. produced hadrons, which belong to the scalar and vector meson nonets and the baryon/antibaryon octet and decuplet, as result of the longitudinal $q-qbar$ or $qq-qqbar$ or $q-qq$ or $qbar-qqbar$ string or kinky $q-g-qbar$ or $qq-g-qqbar$ or $q-g-qq$ or $qbar-g-qqbar$ string decay.

5.2 Longitudinal string decay.

5.2.1 Hadron production by string fragmentation.

A coloured string is stretched between flying away constituents: quark and antiquark or quark and diquark or diquark and antidiquark or antiquark and antidiquark. From knowledge of the constituent longitudinal $p_{3i} = p_{zi}$ and transversal $p_{1i} = p_{xi}$, $p_{2i} = p_{yi}$ momenta as well as their energies $p_{0i} = E_i$, where $i = 1, 2$, we can calculate string or string with quark ends mass squared:

$$M_S^2 = p^\mu p_\mu = p_0^2 - p_1^2 - p_2^2 - p_3^2, \tag{5.1}$$

where $p_\mu = p_{\mu 1} + p_{\mu 2}$ is the string four momentum and $\mu = 0, 1, 2, 3$.

The fragmentation of a string follows the iterative scheme:

$$string \Rightarrow hadron + new\ string, \tag{5.2}$$

i. e. a quark-antiquark (or diquark-antidiquark) pair is created and placed between leading quark-antiquark (or diquark-quark or diquark-antidiquark or antiquark-antidiquark) pair.

The values of the strangeness suppression and diquark suppression factors are

$$u : d : s : qq = 1 : 1 : 0.35 : 0.1. \tag{5.3}$$

As an option the possibility of a diquark breaking during the string fragmentation can be considered.

A hadron is formed randomly on one of the end-points of the string. The quark content of the hadrons determines its species and charges. In the chosen fragmentation scheme we can produce not only the groundstates of baryons and mesons, but also their lower excited states. If for baryons the quark-content does not determine whether the state belongs to the lowest octet or to the lowest decuplet, then octet or decuplet are chosen with probability which defined by spin and isospin of diquark (antidiquark). In the case of mesons the multiplet must be also determined before a type of hadron can be assigned. The probability to choose a certain multiplet depends on the spin of the multiplet.

In case of resonances the mass m is determined according to the Breit-Wigner distribution:

$$F(m) = 2\pi \frac{\Gamma(m_R)}{(m_R - m)^2 + \Gamma(m_R)^2/4}, \tag{5.4}$$

All pole masses m_R and total decay widths $\Gamma(m_R)$ can be taken from the Review of Particle Properties [1].

The zero transverse momentum of created quark-antiquark (or diquark-antidiquark) pair is defined by the sum of an equal and opposite directed transverse momenta of quark and antiquark.

The transverse momentum of a created quark is randomly sampled according to probability (9.18) with the parameter $a = 0.55$ GeV^{-2}. Then a hadron transverse momentum $\mathbf{p_t}$ is determined by the sum of the transverse momenta of its constituents.

The fragmentation function $f^h(z, p_t)$ represents the probability distribution for hadrons with the transverse momenta $\mathbf{p_t}$ to curry the light cone momentum fraction $z = z^{\pm} = (E^h \pm p_z^h)/(E^q \pm p_z^q)$, where E^h and E^q are the hadron and fragmented quark energies, respectively. p_z^h and p_z^q are hadron and fragmented quark longitudinal momenta, respectively. The values of z are limited between $z_{min}^{\pm}$ and $z_{max}^{\pm}$. $z_{min,max}^{\pm}$ are determined by hadron m_h and constituent transverse masses and the available string mass. One of the most common fragmentation function is used in the LUND model [2]:

$$f^h(z, p_t) \sim \frac{1}{z}(1 - z)^a \exp\left[-\frac{b(m_h^2 + p_t^2)}{z}\right]. \tag{5.5}$$

One can use this fragmentation function for the decay of the excited string.

One can use also the fragmentation functions are derived in [3]:

$$f_q^h(z, p_t) = [1 + \alpha_q^h(< p_t >)](1 - z)^{\alpha_q^h(<p_t>)}. \tag{5.6}$$

The advantage of last functions as compared to the LUND fragmentation function is that they have correct three-reggeon behaviour at $z \to 1$ [3].

5.2.2 Times and coordinates of produced hadrons.

To calculate produced hadron formation times and longitudinal coordinates we consider the $(1+1)$-string with mass M_S and string tension κ, which decays into hadrons at string rest frame. The i-th produced hadron has energy E_i and its longitudinal momentum p_{zi}, respectively. Introducing light cone variables $p_i^{\pm} = E_i \pm p_{iz}$ and numerating string breaking points consecutively from the right to the left we obtain $p_0^+ = M_S$, $p_i^+ = \kappa(z_{i-1}^+ - z_i^+)$ and $p_i^- = \kappa x_i^-$.

We can identify the hadron formation point coordinate and time as the point in space-time, where the quark lines of the quark-antiquark pair forming the hadron meet for the first time (the so-called "yo-yo" formation point [2]):

$$t_i = \frac{1}{2\kappa}[M_S - 2\sum_{j=1}^{i-1} p_{zj} + E_i - p_{zi}] \tag{5.7}$$

and coordinate

$$z_i = \frac{1}{2\kappa}[M_S - 2\sum_{j=1}^{i-1} E_j + p_{zi} - E_i]. \tag{5.8}$$

5.2.3 Lorentz boost and rotation of string.

The simulation of string decay is considered in the rest of string frame, with string end quarks are moving along z-axis. We can perform Lorentz transformation to the c.m. of string of the constituent momenta:

$$p_{\mu 1,\mu 2} \to L_\mu p_{\mu 1,\mu 2}, \tag{5.9}$$

where

$$L_0 = \beta^\nu p_\nu \tag{5.10}$$

and

$$L_k = \beta_k p_0 + \sum_{l=1}^{3}(\delta_{lk} + \frac{\beta_k \beta_l}{1+\beta_0})p_l. \tag{5.11}$$

β_μ is defined as follows:

$$\beta_\mu = \frac{p_\mu}{M_S}. \tag{5.12}$$

The string orientation respecting z-axis is determined by two Euler angles α and β, which can be calculated according to

$$\cos\alpha = \frac{p_{32}}{\sqrt{p_{22}^2 + p_{32}^2}} \tag{5.13}$$

and

$$\cos\beta = \frac{\sqrt{p_{22}^2 + p_{32}^2}}{\sqrt{p_{12}^2 + p_{22}^2 + p_{32}^2}}. \tag{5.14}$$

Then, by string rotation:

$$p_{k1,k2} \rightarrow R_{kl} p_{l1,l2}, \tag{5.15}$$

we can obtain the motion of constituents along the z-axis. Here the matrix R_{kl} is given by

$$R_{kl} = \begin{vmatrix} \cos\beta & -\sin\alpha\cos\beta & -\cos\alpha\sin\beta \\ 0 & \cos\alpha & -\sin\alpha \\ \sin\beta & -\sin\alpha\cos\beta & -\cos\alpha\cos\beta \end{vmatrix} \tag{5.16}$$

and $k, l = 1, 2, 3$.

Finally after string decay, using Eq. (5.9) and Eq. (5.15) we have to perform the backward Lorentz boost with $-\beta_\mu$ of hadron 4-momenta and 4-coordinates and the backward rotation of hadron 3-momenta and 3-coordinates with R^{-1}_{kl}.

5.3 Kinky string decay simulation.

For kinky string decay simulation we have assumed the two steps process:

1. Split gluon $g \rightarrow q_1\bar{q}_1$ and create two longitudinal strings;
2. Decay longitudunal strings $q\bar{q}_1 \rightarrow h$ and $\bar{q}q_1 \rightarrow h$ into hadrons h.

The production of $q_1\bar{q}_1$ is considered similar (the same sampling of quark flavors and the same p_t-distribution for the quarks) as a production of $q\bar{q}$-pairs during a longitudinal string decay. The $g \rightarrow q_1\bar{q}_1$ splitting function [4]:

$$f^q_g(z) = z^2 + (1-z)^2, \tag{5.17}$$

where $z = \frac{E_q + p^z_q}{E_g + p^z_g}$, have been derived by Altarelli and Parisi [5].

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Chapter 6

Hadron elastic scattering model.

6.1 Applicability of the hadron elastic scattering model.

The hadron elastic scattering model is capable to predict final states of the hadron elastic collisions.

The elastic angular distribution can be obtained from the Pomeron eikonal model[1]. It gives a possibility to sample scattering angles at initial energies $\sqrt{s} > 2$ GeV. At lower energies sampling of scattering angles are based on the parametrisation of experimental data for nucleon-nucleon interactions.

6.2 Hadron elastic scatterings at low and high energies.

At high energies, when string production becomes possible, the angular distributions of the two-body hadron elastic scatterings are well described by

$$\frac{dW(s)}{dt} = B_{el}(s) \exp\left[B_{el}(s)t\right], \tag{6.1}$$

where $t = -2p_{cm}^2(1 - \cos\theta)$ is the four momentum transfer, which is expressed through the particle centre of mass momentum p_{cm} and the particle centre of mass scattering angle θ. Thus $t_{min} \leq t \leq t_{max}$, where $t_{max} = 0$ and $t_{min} = -4p_{cm}^2$. s is the colliding hadrons c.m. energy squared. The slope parameter $B_{el}(s)$ is defined in the Regge-Gribov approach [1] as follows

$$B_{el}(s) = 2\lambda(s), \tag{6.2}$$

where

$$\lambda(s) = R_P^2 + \alpha_P^{'} \ln(s/s_0). \tag{6.3}$$

As was explained below (see the reggeon based parton string model chapter) the Pomeron parameters R_P^2, $\alpha_P^{'}$ and s_0 can be found from a global fit of the total, elastic, differential elastic and diffractive cross sections for the nucleon-nucleon, pion-nucleon and kaon-nucleon interactions at different values of s. Performing elastic scattering simulations we can use the Eq. (6.1) to sample t and to calculate scattering angle θ. The Eq. (6.2) is used, when initial energy $\sqrt{s} > \sqrt{s_{th}}$, where the threshold energy $\sqrt{s_{th}} = 1.6$ GeV for pion-nucleon collisions, $\sqrt{s_{th}} = 2.0$ GeV for kaon-nucleon collisions and $\sqrt{s_{th}} = 2.5$ GeV for nucleon-nucleon collisions, respectively.

To sample $\cos\theta$ at lower energies we can use the parametrisation:

$$B_{el}(s) = \frac{2.15 R_P^2 [3.65(\sqrt{s} - m_1 - m_2)]^6}{1 + [3.65(\sqrt{s} - m_1 - m_2)]^6}, \tag{6.4}$$

where m_1 and m_2 are masses of colliding particles. This form of angular distribution was obtained by modifying the parametrisation [2] to make smooth transition from the high energy region described by the Regge motivated slopes.

6.3 Hadron elastic scattering MC procedure.

We can use the following MC procedure to sample elastic scattering angle $\cos\theta$:

1. Sample transferred momentum squared according to

$$t = \frac{1}{B_{el}(s)} \ln \xi, \tag{6.5}$$

   where ξ are random numbers, which are uniformly distributed between 0 and 1, $t_{min} \leq t \leq t_{max}$;

2. Calculate

$$\cos\theta = 1 - \frac{t}{2p_{cm}^2}. \tag{6.6}$$

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Chapter 7

Hadronic cross section model.

7.1 Calculation of the total and elastic cross sections.

The hadron interaction cross sections are functions of the incoming and outgoing particle types and their c. m. energy $\sqrt{s_{i,j}} = \sqrt{(p_i + p_j)^2}$, where $p_i = (E_i, \mathbf{p_i})$ and $p_j = (E_j, \mathbf{p_j})$ denote the four momenta of the incoming particle i with mass $m_i = \sqrt{E_i^2 - \mathbf{p_i}^2}$ and the incoming particle j with mass $m_j = \sqrt{E_j^2 - \mathbf{p_j}^2}$, respectively. The hadron interaction cross sections can either be tabulated or parametrised according to algebraic functions. These cross sections can be estimated from the additive quark model, the Regge theory or extracted from other cross sections via general principles, such as isotopic invariance and detailed balance (see the resonance interaction model chapter).

7.1.1 Tabulated and parametrised hadron interaction cross sections.

The total and elastic pp, pn, $\bar{p}p$, $\bar{p}n$, $\pi^{\pm}p$ and $K^{\pm}p$ cross sections are well known [1], [2], [3]. We can tabulate experimentally known total and elastic cross sections at low energies $\sqrt{s_{i,j}} < 4$ GeV.

At higher energies we can use the CERN/HERA parametrisations [1], [2]. The parametrisation

$$\sigma(p) = A + Bp^n + C \ln^2 p + D \ln p, \tag{7.1}$$

where $\sigma(p)$ in mb and the initial laboratory momentum p in GeV/c can be used both for the total and elastic cross section descriptions[1]. The fit

Reaction	A	B	n	C	D
$\sigma^{tot}_{\gamma p}$	0.147 ± 0.001	0.	0.	0.0022 ± 0.0001	-0.0170 ± 0.0007
$\sigma^{tot}_{\pi^+ p}$	16.4 ± 1.2	19.3 ± 0.8	-0.42 ± 0.05	0.19 ± 0.02	0.
$\sigma^{el}_{\pi^+ p}$	0.	11.4 ± 0.3	-0.4 ± 0.2	0.079 ± 0.005	0.
$\sigma^{tot}_{\pi^- p}$	33.0 ± 1.2	14.0 ± 1.8	-1.36 ± 0.29	0.456 ± 0.049	-4.03 ± 0.48
$\sigma^{el}_{\pi^- p}$	1.76 ± 0.42	11.2 ± 0.3	-0.64 ± 0.07	0.043 ± 0.011	0.
$\sigma^{tot}_{K^+ p}$	18.1 ± 0.1	0.	0.	0.26 ± 0.03	-1.0 ± 0.1
$\sigma^{el}_{K^+ p}$	5.0 ± 1.2	8.1 ± 1.5	-1.8 ± 0.7	0.16 ± 0.06	-1.3 ± 0.5
$\sigma^{tot}_{K^- p}$	32.1 ± 0.2	0.	0.	0.66 ± 0.01	-5.6 ± 0.1
$\sigma^{el}_{K^- p}$	7.3 ± 0.1	0.	0.	0.29 ± 0.01	-2.40 ± 0.09
σ^{tot}_{pp}	48.1 ± 0.1	0.	0.	0.522 ± 0.005	-4.51 ± 0.05
σ^{el}_{pp}	11.9 ± 0.8	26.9 ± 1.7	-1.21 ± 0.11	0.169 ± 0.021	-1.85 ± 0.26
σ^{tot}_{pn}	47.3 ± 0.17	0.	0.	0.513 ± 0.023	-4.27 ± 0.15
σ^{el}_{pn}	91.3 ± 0.2	0.	0.	1.05 ± 0.33	-8.8 ± 0.02
$\sigma^{tot}_{p\bar{p}}$	38.4 ± 4.4	77.6 ± 2.8	-0.64 ± 0.07	0.26 ± 0.05	-1.2 ± 0.9
$\sigma^{el}_{p\bar{p}}$	10.2 ± 0.7	52.7 ± 1.8	-1.16 ± 0.05	0.125 ± 0.014	-1.28 ± 0.20
$\sigma^{tot}_{\bar{p}n}$	0.	133.6 ± 4.6	-0.70 ± 0.03	-1.22 ± 0.13	13.7 ± 0.7
$\sigma^{el}_{\bar{p}n}$	36.5 ± 1.5	0.	0.	0.	-11.9 ± 1.8

Table 7.1: The fit and exponent parameters of the parametrised total and elastic cross sections.

coefficients A, B, C, D and exponent n are tabulated below. The range of momentum over the fit was done is also given in the table.

For momenta $p < 5$ GeV/c other parametrisations for $\bar{p}p$ collisions can be applied [4]:

$$\sigma^{tot}(p) = 75.0 + 43.1p^{-1} + 2.6p^{-2} - 3.9p, \ 0.3 < p < 5, \tag{7.2}$$

$$\sigma^{tot}(p) = 271.6 \exp(-1.1p^2), \ p < 0.3, \tag{7.3}$$

$$\sigma^{el}(p) = 31.6 + 18.3p^{-1} - 1.1p^{-2} - 3.8p, \ 0.3 < p < 5, \tag{7.4}$$

$$\sigma^{el}(p) = 78.6, \ p < 0.3. \tag{7.5}$$

The functional dependence on the total particle c.m. energy $\sqrt{s_{i,j}}$ of the total $\pi^{\pm}p$ and K^-p cross sections shows the resonance behaviour at low energies. These cross sections can be calculated. We can calculate also nucleon-nucleon cross sections for resonance excitation processes (see the next chapter). The calculation of other cross sections using general principles, such as isotopic invariance, detailed balance and additive quark model is explained below.

7.1.2 The isotopic invariance.

As known from the experimental observations of charge or isotopic invariance of nuclear forces the hadron interaction scattering amplitudes depend only from the total isospin T, but not from its projection T^z. It gives a possibility to separate cross sections according to the isospin and to find relations between different hadron interaction cross sections.

Each hadrons h can be characterised by the isospin T and its projection T^z, i. e. using Dirac's notations we can write for the hadron state $|h>= |T, T^z>$. Thus, e. g. nucleon and pion states can be defined as follows: $|p>= |1/2, 1/2>$, $|n>= |1/2, -1/2>$, $|\pi^+>= |1, 1>$, $|\pi^->= |1, -1>$ and $|\pi^0>= |1, 0>$. The $|np>$, $|pp>$, $|nn>$ and different pion-nucleon states are given by

$$|h_1 h_2>= \sum_{|T_{h_1}-T_{h_2}|}^{T_{h_1}+T_{h_2}} |T_{h_1h_2}T^z_{h_1h_2}>< T_{h_1}T^z_{h_1}T_{h_2}T^z_{h_2}||T_{h_1h_2}T^z_{h_1h_2}>, \qquad (7.6)$$

where $< T_{h_1}T^z_{h_1}T_{h_2}T^z_{h_2}||T_{h_1h_2}T^z_{h_1h_2}>$ are Clebsch-Gordan coefficients, T^{h_1},$T^{h_1}_z$ and T^{h_2},$T^{h_2}_z$ are the isospin and its projection for hadron h_1 and hadron h_2, respectively. $T^{h_1h_2}$, $T^z_{h_1h_2} = T^z_{h_1} + T^z_{h_2}$ are the total isospin and its projection for the h_1, h_2 system. The Clebsch-Gordan coefficient squared gives the probability to find hadron system in a state $|T_{h_1h_2}T^z_{h_1h_2}>$. In the nucleon-nucleon ($T_{NN} = 0$ or $T_{NN} = 1$) or in the pion-nucleon ($T_{\pi N} = 1/2$ or $T_{\pi N} = 3/2$) system only two values of isospin are allowed. If we consider reactions

$$\begin{array}{c} p + p \to p + p, \\ p + n \to p + n, \\ n + n \to n + n, \end{array} \qquad (7.7)$$

then corresponding amplitudes are related to the isospin amplitudes:

$$\begin{array}{c} A_{p+p\to p+p} =< 1, 1|1, 1>= A_1, \\ A_{p+n\to p+n} = 1/2 < 1, 0|1, 0> +1/2 < 0, 0|0, 0>= 1/2A_1 + 1/2A_0, \\ A_{n+n\to n+n} =< 1, -1|1, -1>= A_1, \end{array} \qquad (7.8)$$

where A_1 and A_0 are the $1 \to 1$ and $0 \to 0$ isospin amplitudes. If we consider reactions

$$\begin{array}{c} \pi^+ + p \to \pi^+ + p, \\ \pi^- + p \to \pi^0 + n, \\ \pi^0 + p \to \pi^0 + p, \end{array} \qquad (7.9)$$

then corresponding scattering amplitudes are also related to the isospin am-

plitudes:

$$A_{\pi^+ + p \to \pi^+ + p} =< 3/2, 3/2 | 3/2, 3/2 >= A_{3/2},$$

$$A_{\pi^- + p \to \pi^0 + n} =$$
$$= \sqrt{2}/3 < 3/2, -1/2 | 3/2, -1/2 > -\sqrt{2}/3 < 1/2, -1/2 | 1/2, -1/2 >=$$
$$= \sqrt{2}/3 A_{3/2} - \sqrt{2}/3 A_{1/2},$$

$$A_{\pi^0 + p \to \pi^0 + p} =$$
$$= 2/3 < 3/2, 1/2 | 3/2, 1/2 > +1/3 < 1/2, 1/2 | 1/2, 1/2 >=$$
$$= 2/3 A_{3/2} + 1/3 A_{1/2}, \tag{7.10}$$

where $A_{3/2}$ and $A_{1/2}$ are the $3/2 \to 3/2$ and $1/2 \to 1/2$ isospin amplitudes. The optical theorem links the total particle interaction cross section $\sigma_{tot}^{ij}(s_{ij})$ and the imaginary part of the scattering amplitude $A_{ij}(0)$ at zero scattering angle:

$$\sigma_{tot}(s_{ij}) = \frac{1}{s_{ij}} Im A_{ij}(0). \tag{7.11}$$

Thus, e. g. for the nucleon-nucleon system isotopic invariance leads to the relation $\sigma_{tot,el}^{nn} = \sigma_{tot,el}^{pp}$. For the pion-nucleon system one has to measure only the first two of the above considered elastic and charge-exchange cross sections, the third one and all other possible elastic and charge-exchange cross sections can be determined by them. The similar consideration can be done for kaon-nucleon and other hadronic systems.

7.1.3 The additive quark model cross sections.

In the additive quark model (AQM) cross section depends only on the quark content of the colliding hadrons [5], [6],[4]:

$$\sigma_{tot}^{AQM} = 40(\frac{2}{3})^{n_M}(1 - 0.4x_1^s)(1 - 0.4x_2^s) \tag{7.12}$$

and

$$\sigma_{el}^{AQM} = 0.039\sigma_{tot}^{\frac{2}{3}}, \tag{7.13}$$

where n_M is the number of colliding mesons and x_i^s is the ratio of strange quarks to non-strange quarks in the i-th hadron. Thus we can use the AQM to obtain unknown cross sections by the "scaling" of known cross sections.

7.1.4 High energy hadron-nucleon interaction cross sections.

The hadron-nucleon cross sections at given c.m. energy squared s can be calculated from the corresponding interaction probabilities $p_{ij}(b_{ij}^2, s)$ by

integration over impact parameter b. Particularly, the hadron-nucleon total and inelastic cross section can be calculated according to

$$\sigma_{tot} = 2\pi \int_0^\infty bdbp_{ij}^{tot}(b_{ij}^2, s) = \sigma_P f(\frac{z}{2}) \tag{7.14}$$

and

$$\sigma_{in} = 2\pi \int_0^\infty bdbp_{ij}^{in}(b_{ij}^2, s) = \sigma_P f(z), \tag{7.15}$$

where

$$\sigma_P = 4\pi z(s)\lambda(s), \tag{7.16}$$

and

$$f(z) = \sum_{\nu=1}^{\infty} \frac{(-z)^{\nu-1}}{\nu\nu!}. \tag{7.17}$$

The probabilities $p_{ij}(b_{ij}^2, s)$, the quantities z(s) and $\lambda(s)$ are expressed through the parameters of the Pomeron trajectory, $\alpha_P^{'}$ and $\alpha_P(0)$, and the parameters of the Pomeron-nucleon vertex R_P^2 and γ_P (see the reggeon based parton string model chapter).

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Chapter 8

Resonance interaction model.

8.1 Allowed projectiles, targets and bombarding energy range.

The hadron resonance interaction model based on the UrQMD approach[1]. It is capable to predict wide spectrum (see the particle properties chapter) of baryons as well as wide spectrum of meson and baryon resonances. These particles are result of the excitation and deexcitation of baryons. They can also be produced in the meson-meson and meson-baryon interactions by means of the quark-antiquark annihilations.

The allowed bombarding energy is limited by chosen set (see also the particle properties chapter) meson and baryon resonances and their excitation cross sections.

This model is also able to predict baryon resonance excitation in the photon-nucleon inelastic collisions.

8.2 Hadron resonance interactions.

The modelling of the inelastic meson-meson, meson-baryon (fusion into one particle in the final state) and baryon-baryon (two particles in the final state) interactions with resonance excitation as well as the inverse baryon-baryon inelastic interactions with resonance deexcitation is based on the resonance excitation and deexcitation cross sections.

The hadron interaction cross sections are a function of the incoming and outgoing particle types and their c. m. energies $\sqrt{s_{i,j}} = \sqrt{(p_i + p_j)^2}$, where $p_i = (E_i, \mathbf{p_i})$ and $p_j = (E_j, \mathbf{p_j})$ are denoting the four momenta of the incoming particle i with mass $m_i = \sqrt{E_i^2 - \mathbf{p_i}^2}$ and the incoming particle j with mass $m_j = \sqrt{E_j^2 - \mathbf{p_j}^2}$, respectively.

8.2.1 Interaction cross sections for meson-baryon collisions with quark annihilation.

The meson-baryon interactions going through the quark annihilation are dominated by the formation of an intermediate resonance up to c.m. energies of 2.2 GeV. Thus the total meson-baryon cross section with quark annihilation can be approximated as follows [1]

$$\sigma_{tot}^{MB}(\sqrt{s_{MB}}) = \\ = \sum_{R=\Delta,N^*} < j_M, m_M, j_B, m_B || J_R, M_R > \frac{2T_R+1}{(2T_M+1)(2T_B+1)} \times \\ \times \frac{\pi}{p_{c.m.}^2} \frac{\Gamma_{tot}\Gamma_{R\to MB}}{(m_R-\sqrt{s_{MB}})^2+1/4\Gamma_{tot}^2} \qquad (8.1)$$

with the total and partial $\sqrt{s}$-dependent decay widths Γ_{tot} and $\Gamma_{R\to MB}$.

In Eq. (8.1) $< j_M, m_M, j_B, m_B || J_R, M_R >$ denotes the Clebsch-Gordan coefficient determined by meson j_M, baryon j_B and resonance J_R angular momenta and their projections m_M, m_B and M_R, respectively. T_M, T_B and T_R are meson, baryon and resonance isospins, respectively, and $p_{c.m.}$ is c.m. momentum of incoming particles. Similarly, we able to calculate the meson-meson total interaction cross section going through the quark annihilation [1].

8.2.2 Nucleon-nucleon resonance interaction cross sections.

Up to incident beam energies of $4-5$ GeV/nucleon particle production in nucleon-nucleon collisions is dominated by resonance decays. To obtain resonance excitation cross sections we can employ the practical approach of [1]. In this approach the resonance excitation cross sections are determined by

$$\sigma_{i,j\to k,l}(\sqrt{s_{i,j}}) = (2S_k+1)(2S_l+1)\frac{p_{k,l}}{p_{i,j}}\frac{1}{s_{i,j}}|M(\sqrt{s_{i,j}}, m_k, m_l)|^2, \qquad (8.2)$$

where S_k and S_l are spins of outgoing particles and $p_{k,l}$ and $p_{i,j}$ are incoming and outgoing c.m. momenta. The matrix element $|M|^2$ is assumed to have no spin-dependence with free parameters are tuned [3] to the experimental measurements.

8.2.3 Detailed balance cross sections for resonance-nucleon interactions.

The principle of detailed balance is based on the time-reversal invariance of the reaction matrix element. It gives a possibility to determine differential and total resonance-nucleon and resonance-resonance cross sections, e. g. the cross sections for the processes: $\Delta_{1232}N \to NN$ and $\Delta_{1232}\Delta_{1232} \to NN$. It should be mentioned that inelastic baryon-resonance deexcitations are very important for nuclear absorption of mesons, which are "bound" in

resonances. According to the detailed balance the cross sections of inverse $f \to i$ and direct $i \to f$ reactions, where i and f denote initial and final states, respectively, are related each other as follows:

$$\sigma_{f\to i} = \frac{\mathbf{p_i^2}}{\mathbf{p_f^2}} \frac{g_i}{g_f} \sigma_{i\to f}, \tag{8.3}$$

where $\mathbf{p_{i,f}^2}$ are the c. m. momenta of the initial and final states,

$$g_{i,f} = (2S_{i1,f1}+1)(2S_{i2,f2}+1)(2T_{i1,f1}+1)(2T_{i2,f2}+1) \tag{8.4}$$

are denoting the spin-isospin degeneracy factors.

However, the Eq. (8.3) is exactly valid in the case of stable particles with well-defined masses. For the case of one incoming resonance this equation should be modified as it has been derived in [2].

8.2.4 Simulation of hadron resonance interactions.

To simulate two-body scatterings with resonance excitation and deexcitation we have to select a proper channel (according to the calculated cross sections) and sample resonance masses (according to the Breit-Wigner distribution with corresponding pole mass and pole width) and sample scattered angle.

We can assume [1] that angular distributions for all relevant resonance excitation and deexcitation processes can be approximated by the angular of NN elastic scattering with the replacement $s \to s - (m_1+m_2)^2 + 4m_N^2$, where m_1 and m_2 denote the incoming hadron masses.

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Chapter 9

Reggeon based parton string model.

9.1 Allowed inelastic hadron interactions.

The Reggeon based Parton String Model (RPSM) [1], [2], [3] is capable to predict final states (produced hadrons which belong to the scalar and vector meson nonets and the baryon/antibaryon octet and decuplet) of inelastic hadron collisions.

The allowed bombarding energy in the hadron-nucleon collision should be above 2-pion production threshold.

This model is also able to predict final states in the photon-nucleon inelastic collisions at the initial energy above 2-pion production. The last case is described in the separate chapter.

9.2 Sampling of the longitudinal strings.

9.2.1 MC procedure to define the number of produced strings.

In the Regge-Gribov approach[4] the probability for an inelastic collision of nucleons i and j as a function of the squared impact parameter difference $b_{ij}^2 = (\vec{b}_i - \vec{b}_j)^2$ and s is given by

$$p_{ij}(\vec{b}_i - \vec{b}_j, s) = c^{-1}[1 - \exp\{-2u(b_{ij}^2, s)\}] = \sum_{n=1}^{\infty} p_{ij}^{(n)}(\vec{b}_i - \vec{b}_j, s), \qquad (9.1)$$

where

$$p_{ij}^{(n)}(\vec{b}_i - \vec{b}_j, s) = c^{-1} \exp\{-2u(b_{ij}^2, s)\} \frac{[2u(b_{ij}^2, s)]^n}{n!}. \qquad (9.2)$$

is the probability to find the n cut Pomerons[5] or the probability for $2n$ string produced in an inelastic nucleon-nucleon collision, if we assume that each cut Pomeron represents two strings. These probabilities are defined in

terms of the (eikonal) amplitude of nucleon-nucleon elastic scattering with the Pomeron exchange:

$$u(b_{ij}^2, s) = \frac{z(s)}{2} \exp[-b_{ij}^2/4\lambda(s)]. \tag{9.3}$$

The quantities $z(s)$ and $\lambda(s)$ are expressed through the parameters of the Pomeron trajectory, $\alpha_P^{'} = 0.25\ GeV^{-2}$ and $\alpha_P(0) = 1.0808$, and the parameters of the Pomeron-nucleon vertex $R_P^2 = 3.56\ GeV^{-2}$ and $\gamma_P = 3.96\ GeV^{-2}$:

$$z(s) = \frac{2c\gamma_P}{\lambda(s)}(s/s_0)^{\alpha_P(0)-1} \tag{9.4}$$

$$\lambda(s) = R_P^2 + \alpha_P^{'} \ln(s/s_0), \tag{9.5}$$

respectively, where $s_0 = 3.0\ GeV^2$ is a dimensional parameter.

In Eqs. (9.1,9.2, 9.4) the so-called shower enhancement coefficient $c = 1.4$ can be introduced to determine the contribution of diffractive dissociation[4]. Thus, the probability for diffractive dissociation of a pair of nucleons can be computed as

$$p_{ij}^d(\vec{b}_i - \vec{b}_j, s) = \frac{c-1}{c}[p_{ij}^{tot}(\vec{b}_i - \vec{b}_j, s) - p_{ij}(\vec{b}_i - \vec{b}_j, s)], \tag{9.6}$$

where

$$p_{ij}^{tot}(\vec{b}_i - \vec{b}_j, s) = (2/c)[1 - \exp\{-u(b_{ij}^2, s)\}]. \tag{9.7}$$

The Pomeron parameters are found from a global fit of the total, elastic, differential elastic and diffractive cross sections for the nucleon-nucleon interaction at different energies.

The above equations are suitable for the pion-nucleon and the kaon-nucleon collisions, but the Pomeron vertex parameters and shower enhancement coefficients should be changed, e. g. $R_{P^2}^{\pi} = 2.36\ GeV^{-2}$, $\gamma_P^{\pi} = 2.17\ GeV^{-2}$, $s_0^{\pi} = 1.5\ GeV^2$, $c^{\pi} = 1.6$ and $R_{P^2}^{K} = 1.96\ GeV^{-2}$, $\gamma_P^{K} = 1.92\ GeV^{-2}$, $s_0^{K} = 2.3\ GeV^2$, $c^{\pi} = 1.8$ are obtained to describe properly the total, elastic and diffractive cross sections.

9.2.2 Separation of hadron diffraction excitation.

For each pair of hadrons i and j with chosen impact parameters $\vec{b}_i$ and $\vec{b}_j$ we should check whether they interact inelastically or not using the probability

$$p_{ij}^{in}(\vec{b}_i - \vec{b}_j, s) = p_{ij}(\vec{b}_i - \vec{b}_j, s) + p_{ij}^d(\vec{b}_i - \vec{b}_j, s). \tag{9.8}$$

If an interaction will be realized, then we have to consider it to be diffractive or nondiffractive with the probabilities

$$\frac{p_{ij}^d(\vec{b}_i - \vec{b}_j, s)}{p_{ij}^{in}(\vec{b}_i - \vec{b}_j, s)} \tag{9.9}$$

and

$$\frac{p_{ij}(\vec{b}_i - \vec{b}_j, s)}{p^{in}_{ij}(\vec{b}_i - \vec{b}_j, s)}. \tag{9.10}$$

9.3 Sampling of the kinky strings.

To determine the number of kinky strings are produced in hard hadron collisions we assume [6], [7] that each cut Pomeron can be substituted either by the two longitudinal strings as result of soft hadron interaction or by the two kinky strings as result of hard hadron interactions.

At the moment it is not completely clear how to choose which cut pomeron should be substituted by longitudinal and which one should be substituted by kinky strings.

One recipe is based on the eikonal model [8], [7]

$$u(b^2_{ij}, s) = u_{soft}(b^2_{ij}, s) + u_{hard}(b^2_{ij}, s). \tag{9.11}$$

The soft eikonal part is defined as

$$u_{soft}(b^2_{ij}, s) = \frac{\gamma_{soft}}{\lambda_{soft}(s)}(s/s_0)^{\Delta_{soft}} \exp[-b^2_{ij}/4\lambda_{soft}(s)]. \tag{9.12}$$

The hard part is calculated according to

$$u_{hard}(b^2_{ij}, s) = \frac{\sigma_{jet}}{8\pi\lambda_{hard}(s)}(s/s_0)^{\Delta_{hard}} \exp[-b^2_{ij}/4\lambda_{hard}(s)]. \tag{9.13}$$

The $\sigma_{jet} = 0.027$ mbarn and $\Delta_{hard} = 0.47$ were found [8] from the fit of the two–jet experimental cross section [9]. Then from the global fit of the total and elastic cross sections for pp collisions the values of $\gamma_{soft} = 35.5$ mbarn, $\Delta_{soft} = 0.07$ and $R^2_{hard} = R^2_{soft} = 3.56$ GeV^{-2} were found.

Thus we can examine each cut Pomeron and substitute it by two kinky strings with probability

$$P_{hard}(b^2_{ij}, s) = \frac{u_{hard}(b^2_{ij}, s)}{u_{soft}(b^2_{ij}, s) + u_{hard}(b^2_{ij}, s)}. \tag{9.14}$$

9.4 Longitudinal string excitation.

9.4.1 Hadron inelastic collision.

Let us consider collision of two hadrons with their c. m. momenta $P_1 = \{E^+_1, m^2_1/E^+_1, \mathbf{0}\}$ and $P_2 = \{E^-_2, m^2_2/E^-_2, \mathbf{0}\}$, where the light-cone variables $E^{\pm}_{1,2} = E_{1,2} \pm P_{z1,2}$ are defined through hadron energies $E_{1,2} = \sqrt{m^2_{1,2} + P^2_{z1,2}}$, hadron longitudinal momenta $P_{z1,2}$ and hadron masses $m_{1,2}$, respectively. Two hadrons collide by means of two partons with momenta $p_1 = \{x^+E^+_1, 0, \mathbf{0}\}$ and $p_2 = \{0, x^-E^-_2, \mathbf{0}\}$, respectively.

9.4.2 Diffractive string excitation.

In the diffractive string excitation (the Fritiof approach [10]) only momentum can be transferred:

$$\begin{array}{l} P_1' = P_1 + q \\ P_2' = P_2 - q, \end{array} \tag{9.15}$$

where

$$q = \{-q_t^2/(x^- E_2^-), q_t^2/(x^+ E_1^+), \mathbf{q_t}\} \tag{9.16}$$

is parton momentum transferred and $\mathbf{q_t}$ is its transverse component.

9.4.3 String excitation by parton exchange.

For this case (the QGSM approach [1]) the parton exchange (rearrangement) and the momentum exchange are allowed [1],[2],[3]:

$$\begin{array}{l} P_1' = P_1 - p_1 + p_2 + q \\ P_2' = P_2 + p_1 - p_2 - q, \end{array} \tag{9.17}$$

where $q = \{0, 0, \mathbf{q_t}\}$ is parton momentum transferred.

9.4.4 Transverse and longitudinal momenta generation.

The transverse component of the parton momentum transferred is sampled according to the probability

$$P(\mathbf{q_t}) d\mathbf{q_t} = \sqrt{\frac{a}{\pi}} \exp\left(-a q_t^2\right) d\mathbf{q_t}, \tag{9.18}$$

where parameter $a = 0.6$ GeV^{-2}.

The light cone parton quantities x^+ and x^- are generated independently and according to distribution:

$$u(x) \sim x^\alpha (1-x)^\beta, \tag{9.19}$$

where $x = x^+ > x_{min}$ or $x = x^- > x_{min}$. The parameters $\alpha = -1$ and $\beta = 0$ are chosen for the FRITIOF approach [10]. In the case of the QGSM approach [1] the parameters α and β are explained below. Masses of the excited strings should satisfy the kinematic constraints:

$$P_1'^+ P_1'^- \geq m_{h1}^2 + q_t^2 \tag{9.20}$$

and

$$P_2'^+ P_2'^- \geq m_{h2}^2 + q_t^2, \tag{9.21}$$

where hadronic masses m_{h1} and m_{h2} are defined by string quark contents. Thus, the random selection of the values x^+ and x^- is limited by the above constraints.

9.4.5 String excitation by quark or diquark annihilation.

We consider also hadron-hadron inelastic processes, when antiquark from hadron projectile annihilate with suitable quark from hadron target. In this case excitation of a baryon (string with quark and diquark ends) or a meson (string with quark and antiquark ends) string is occurred, respectively. These processes in the Regge theory correspond to the cut reggeon exchange diagrams. Initial energy $\sqrt{s}$ dependences of these processes cross sections are defined by the intercepts of reggeon exchange trajectories. For example, $\sigma_{\pi^+p\to S(s)} \sim s^{\alpha_\rho(0)-1}$, S denotes a string and $\alpha_\rho(0)$ is the intercept of the ρ reggeon trajectory. Thus $\sigma_{\pi^+p\to S(s)}$ decreases with the energy rise. Thus quark annihilation processes are important at relative low initial energies. Another example of diquark annihilation processes is $\bar{p}p \to S$, which can be used, e.g. in the hadron transport model to describe final states of the $\bar{p}p$ annihilation is explained in the baryon annihilation model chapter.

Simulation of such kind process is rather simple. We should randomly (according to weights are calculated using hadron wave functions) choose quark/antiquark from projectile and find suitable (with the same flavour content) partner for annihilation from target. The created string four-momentum

$$P = P_1 + P_2 \tag{9.22}$$

is approximated by the total reaction four-momentum, an annihilated system has small neglected momentum (only low momenta quarks are able to annihilate).

To determine statistical weights for quark annihilation processes are followed a string production and separate them from processes, when two or more strings can be produced, we can use the Regge motivated expression for the total cross section:

$$\sigma_{tot}(s) = \sum_i \sigma_i(s), \tag{9.23}$$

where $\sigma_i(s)$ is the cross section of the i-th subprocess. It can be the elastic cross section $\sigma_{el}(s)$ or the annihilation cross section with one string production $\sigma_S(s)$ or the diffraction cross section $\sigma_D(s)$ or the multistring production cross section $\sigma_{MS}(s)$.

From knowledge of the total and elastic cross sections we can calculate inelastic cross section:

$$\sigma_{in}(s) = \sigma_{tot}(s) - \sigma_{el}(s) = \sigma_S(s) + \sigma_D(s) + \sigma_{MS}(s). \tag{9.24}$$

The diffraction and multistring cross sections are calculated according to the Regge eikonal model as it was described above. Thus the statistical weight for the annihilation process can be calculated as follows

$$W_S(s) = \frac{\sigma_{in}(s) - \sigma_D(s) - \sigma_{MS}(s)}{\sigma^{in}(s)}. \tag{9.25}$$

9.4.6 String excitation by the parton exchange and hadron structure functions.

In the QGSM approach [3] strings (as result of parton rearrangement) should be spanned not only between valence quarks of colliding hadrons, but also between valence and sea quarks and between sea quarks. Each participant hadron should be split into set of partons: valence quark and antiquark for meson or valence quark/antiquark and diquark/antidiquark for baryon/antibaryon and additionally the $(n-1)$ sea quark-antiquark pairs (their flavours are selected according to probability ratios $u:d:s=1:1:0.35$), if a hadron is participating in the n inelastic collisions.

As an option the possibility to split a hadron additionally into the sea diquark-antidiquark pairs can be included. Such assumption enhances the antibaryon production in the central rapidity region.

Thus for each participant hadron we have to generate a set of light cone variables x_{2n}, where $x_{2n}=x_{2n}^{+}$ or $x_{2n}=x_{2n}^{-}$ according to the distribution:

$$f^h(x_1,x_2,...,x_{2n})=f_0\prod_{i=1}^{2n}u_{q_i}^h(x_i)\delta(1-\sum_{i=1}^{2n}x_i), \tag{9.26}$$

where f_0 is the normalisation constant. Here, the quark structure functions $u_{q_i}^h(x_i)$ for valence quark/antiquark q_v, sea quark and antiquark q_s and valence diquark/antidiquark qq are

$$u_{q_v}^h(x_v)=x_v^{\alpha_v},\ u_{q_s}^h(x_s)=x_s^{\alpha_s},\ u_{qq}^h(x_{qq})=x_{qq}^{\beta_{qq}}, \tag{9.27}$$

where $\alpha_v=-0.5$ and $\alpha_s=-0.5$ [1] for the non-strange quarks/antiquarks and $\alpha_v=0$ and $\alpha_s=0$ for strange quarks/antiquarks, $\beta_{uu}=1.5$ and $\beta_{ud}=2.5$ for proton/antiproton and $\beta_{dd}=1.5$ and $\beta_{ud}=2.5$ for neutron/antineutron. Usually x_i is selected between $x_i^{min}\le x_i\le 1$, where the model parameter $x^{min}=0.3/\sqrt{s}$ is a function of initial c.m. energy $\sqrt{s}$. It prevents the production of strings with low masses (smaller than hadron masses), while the whole selection procedure should be repeated. Then the transverse momenta of partons $\mathbf{q_{it}}$ are generated according to the Gaussian probability Eq. (9.18) with $a=1/4\lambda(s)$ and under the constraint: $\sum_{i=1}^{2n}\mathbf{q_{it}}=0$. The partons are considered as the off-shell or virtual partons.

9.4.7 Baryon and meson splitting.

To perform a simulation of the string excitation we need to split hadron and choose valence quark/antiquark and diquark/antidiquark. In the case of a meson we split it into valence quark and antiquark (for the neutral mixed mesons this sampling is performed according to the quark mixing probabilities.) In the case of baryon we determine probabilities of baryon/antibaryon state (a quark/antiquark and a diquark/antidiquark with given spin and

Baryon type	Quark content
p	$1/3uu_{11}d + 1/6ud_{11}u + 1/2ud_{00}u$
n	$1/6ud_{11}d + 1/2ud_{00}d + 1/3dd_{11}u$
Σ^+	$1/3uu_{11}s + 1/6us_{11}u + 1/2us_{00}u$
Σ^0	$1/3ud_{11}s + 1/12us_{11}d + 1/4us_{00}d + 1/12ds_{11}u + 1/4ds_{00}u$
Σ^-	$1/3dd_{11}s + 1/6ds_{11}d + 1/2ds_{00}d$
Ξ^-	$1/6ds_{11}s + 1/2ds_{00}s + 1/3ss_{11}d$
Ξ^0	$1/6us_{11}s + 1/2us_{00}s + 1/3ss_{11}u$
Λ^0	$1/3ud_{00}s + 1/4us_{11}d + 1/12us_{00}d + 1/4ds_{11}u + 1/12ds_{00}u$
Δ^{++}	$uu_{11}u$
Δ^+	$1/3uu_{11}d + 2/3ud_{11}u$
Δ^0	$2/3ud_{11}d + 1/3dd_{11}u$
Δ^-	$dd_{11}d$
Σ^{*+}	$1/3uu_{11}s + 2/3us_{11}u$
Σ^{*0}	$1/3ud_{11}s + 1/3us_{11}d + 1/3ds_{11}u$
Σ^{*-}	$1/3dd_{11}s + 2/3ds_{11}d$
Ξ^{*0}	$1/3us_{11}s + 2/3ss_{11}u$
Ξ^{*-}	$2/3ds_{11}s + 1/3ss_{11}d$
Ω^-	$ss_{11}s$

Table 9.1: Baryon quark contents. Diquark indices indicate spin-isospin states.

isospin) from $SU(6)$ symmetric baryon wave functions. These probabilities are given below in the Table

We also can use these probabilities to sample baryon or baryon resonance in the string fragmentation, assuming that a valence diquark/antidiquark keeps its spin and isospin during reaction.

9.5 Kinky string excitation.

Having sampled configuration of kinky strings we can generate outgoing gluon-kink momenta.

We assume that kinky strings are produced as result of $gg \to gg$ hard interactions. Our generation of the outgoing gluons (kinks) momenta is based on the two-jets inclusive production cross section:

$$\frac{d\sigma_{gg}}{dx_g^+ dx_g^- d\cos\theta} = f(x_g^+, Q^2) f(x_g^-, Q^2) \frac{d\sigma_{gg}(\hat{s})}{d\cos\theta}, \tag{9.28}$$

where we take $\hat{s} = Q^2 = x_g^+ x_g^- s$ and s is the total centre of mass energy squared for the colliding system, which is calculated using x_i and q_{ti} and m_i^2 string end partons. The value of s should be large enough to produce gluons with the transverse momentum above the chosen cutoff $Q_0^2 = 2\ GeV^2$. The QCD gluon-gluon interaction cross section

$$\frac{d\sigma_{gg}(\hat{s})}{d\cos\theta} = \frac{9\pi\alpha_s^2(Q^2)}{32s} \frac{(3+\cos^2\theta)^3}{(1-\cos^2\theta)^2} \tag{9.29}$$

was calculated in the Born approximation [11]. The scattering angle can be expressed through the transferred momentum squared $\hat{t}$:

$$\cos\theta = 1 + \frac{2\hat{s}\hat{t}}{[\hat{s} - (m_i + m_j)^2)(\hat{s} - (m_i - m_j)^2)]}, \tag{9.30}$$

where m_i and m_j refer initial parton masses. The sampling of $\hat{t}$ (see the multiple parton hard scattering model chapter) is much simple than sampling of the $\cos\theta$. The θ is the scattering angle in the centre of mass of the parton-parton system and $-z_0 < \cos\theta < z_0$ with

$$z_0 = \sqrt{1 - \frac{4Q_0^2}{sx^+x^-}}. \tag{9.31}$$

The QCD running coupling constant

$$\alpha_s(Q^2) = \frac{12\pi}{25\ln(Q^2/\Lambda^2)}, \tag{9.32}$$

which is corresponding to four flavours and $\Lambda^2 = 0.01$ GeV^2 can be taken.

$f(x, Q^2)$ is the momentum fraction distribution of gluons in hadron. It can be chosen from[12] (see the particle properties chapter).

Thus the MC procedure to build the kinky strings can be outlined as follows:

- Sample x_i, $\mathbf{q_{it}}$ and m_i^2, where $i = 1, 2, ..., 2n$, for partons, which will be on the $2n$ string ends for both soft and kinky strings.
- For each pair of kinky string calculate total centre of mass energy s and sample $x^+ > x^+_{min}$ and $x^- > x^-_{min}$, where $x_{min} = 2Q_0/\sqrt{s}$ using gluon distribution function (see the particle properties chapter).
- Sample the outgoing gluon centre of mass scattering angle θ using Eq. (9.29).
- For each kinky string recalculate parton string end energies and momenta.

This procedure should be improved taking into account initial and final state gluon radiation (see the multiple parton hard scattering model chapter).

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Chapter 10

Baryon annihilation model.

10.1 Allowed hadronic reactions.

The baryon annihilation model is able to predict final states (produced hadrons) with zero net baryon number in the baryon-antibaryon collision at different energies. The produced hadrons belong to the scalar and vector meson nonets and the baryon/antibaryon octet and decuplet.

The baryon annihilation process is considered in the diquark-antidiquark annihilation approximation.

10.2 Baryon annihilation.

10.2.1 Baryon annihilation cross sections.

The baryon annihilation cross section is a function of the incoming and outgoing particle types and their c. m. energy $\sqrt{s_{i,j}} = \sqrt{(p_i + p_j)^2}$. Here $p_i = (E_i, \mathbf{p_i})$ and $p_j = (E_j, \mathbf{p_j})$ denote the four momenta of the incoming particle i with mass $m_i = \sqrt{E_i^2 - \mathbf{p_i}^2}$ and the incoming particle j with mass $m_j = \sqrt{E_j^2 - \mathbf{p_j}^2}$, respectively. We follow the approach [1] to calculate baryon-antibaryon annihilation cross sections. In this approach the same initial energy dependence is used for all baryon-antibaryon cross sections and different baryon quark content is taken into account by scaling factor obtained from the additive quark model (AQM)[2], [3] (in the additive quark model cross section depends only from the quark content of the colliding hadrons):

$$\sigma_{\bar{B}B}(\sqrt{s}) = \frac{\sigma_{BB}^{AQM}}{\sigma_{NN}^{AQM}} \sigma_{\bar{N}N}(\sqrt{s}), \tag{10.1}$$

where antiproton-proton annihilation cross section $\sigma_{\bar{p}p}^{ann}$ is parametrised as [4]

$$\sigma_{\bar{p}p}^{ann}(s) = \sigma_0 \frac{s_0}{s} \left[\frac{A^2 s_0}{(s - s_0)^2 + A^2 s_0} + B \right] \tag{10.2}$$

and antiproton-neutron cross annihilation cross section is treated identically. The values of parameters are $\sigma_0 = 120$ mb, $s_0 = 4m_N^2$, $A = 50$ MeV and $B = 0.6$.

10.2.2 Baryon annihilation simulation.

Simulation of such kind of processes is rather simple. One should randomly, according to the weight calculated using hadron wave function (see the baryon content table in the reggeon based parton string model chapter), choose diquark/antidiquark from projectile and find suitable (with the same flavour content) partner for annihilation from target. The created string four-momentum

$$P = P_1 + P_2 \tag{10.3}$$

is approximated by the total reaction four-momentum since annihilated system has small neglected momentum (only low momenta quarks are able to annihilate), where P_1 and P_2 are interacting baryon and antibaryon momenta, respectively.

The baryon annihilation cross section $\sigma_{ann}(s)$ is used to determine statistical weights for diquark annihilation process:

$$W_{ann}(s) = \frac{\sigma_{ann}(s)}{\sigma_{tot}(s)}, \tag{10.4}$$

where $\sigma_{tot}(s)$ is the total baryon-antibaryon interaction cross section as a function of the total c.m. energy squared.

The baryon annihilation model can be improved by adding another competing subprocesses like the string junction annihilation, where three meson strings can be produced (see papers[5], [6]).

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Chapter 11

Multiple parton hard scattering model.

11.1 Allowed hadronic reactions.

The Multiple Hard Scattering Model (MHSM) is capable to predict final state produced hadrons, which contain only u, d and s flavours and belong to $J^P = 0^-, 1^\pm$ meson and $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ baryon states.

The allowed bombarding energies for the nucleon-nucleon collisions should be above 100 GeV in the laboratory system.

The counting of multiple hard parton rescatterings to explain the properties of the the high energy hadron and nucleus was suggested by E. Levin, M. Ryskin and L. Gribov [1]. This particular model is based on the K. Geiger's approach [2], [3].

The reaction initialisation procedure was described in the particle properties chapter. This procedure have many similarities with the initialisation procedure for nucleus-nucleus collision (see the hadron transport model chapter). Even more, we would stress that the multiple scattering model as the model based on the relativistic particle kinetics has many algorithms, which are similar to the algorithms are used in the hadron transport model.

11.2 Parton propagation.

The propagation of partons obeys the equation:

$$\mathbf{r}(t_i) = \mathbf{r}(t_{i-1}) + \frac{\mathbf{p}}{E}\Delta t, \tag{11.1}$$

where Δt is a time step defined as $\Delta t = t_i - t_{i-1}$ and $\mathbf{p}$ is parton 3-momentum and E is parton energy. For on-shell partons with masses m, their energies are $E = \sqrt{\mathbf{p}^2 + m^2}$. For time-like partons with virtualities $q^2 > 0$, their energies are calculated according to $E = \sqrt{\mathbf{p}^2 + m^2 + q^2}$ and space-like partons are considered as massless, i. e. $E = |\mathbf{p}|$.

11.3 Search of parton collisions.

For each pair (i, j) of partons their total interaction cross section $\sigma_{tot}(s_{ij})$ is calculated. This cross sections obtained by evaluation (see below) of the parton $2 \to 2$ differential cross sections and by sum over all possible final states. The parton collision cross section is a function of the total parton c.m. energy squared $\hat{s}_{ij} = (p_i + p_j)^2$, where $p_i = (E_i, \mathbf{p_i})$ and $p_j = (E_j, \mathbf{p_j})$ are four momenta of partons. The cross section is interpreted geometrically as an interaction area with the radius

$$b_{ij} = \sqrt{\frac{\sigma_{tot}(s_{ij})}{\pi}}. \tag{11.2}$$

It was assumed that two partons will collide, if their transverse (with regards to the relative velocity vector of the partons) distance d_{ij} fulfils the conditions:

$$d_{ij} \leq b_{ij} \tag{11.3}$$

and

$$\left| \frac{(\mathbf{r_i^*} - \mathbf{r_j^*})(\mathbf{p_i^*} - \mathbf{p_j^*})}{\mathbf{p_i^*} - \mathbf{p_j^*}} \right| < (v_i + v_j)\frac{\Delta t}{2}. \tag{11.4}$$

The distance d_{ij} is the relative distance between two partons at time of the closest approach calculated in their c.m. frame. These partons have c.m. coordinates $\mathbf{r_i^*}$ and $\mathbf{r_j^*}$ and c.m. momenta $\mathbf{p_i^*}$ and $\mathbf{p_j^*}$, respectively. v_i and v_j are the parton c.m. velocities. It is determined by the equation:

$$d_{ij}^2 = (\mathbf{r_i^*} - \mathbf{r_j^*})^2 - \frac{[(\mathbf{r_i^*} - \mathbf{r_j^*})(\mathbf{p_i^*} - \mathbf{p_j^*})]^2}{(\mathbf{p_i^*} - \mathbf{p_j^*})^2}. \tag{11.5}$$

For chosen parton i we can have more than one potential collision partner j. For this case we choose as a collision partner the j-th parton with the minimal distance d_{ij}^{min}.

11.4 Parton collision cross section.

We consider the following $2 \to 2$ parton collision processes:

$$\begin{gathered}
(1) qq' \to qq',\ q\bar{q}' \to q\bar{q}' \\
(2) q\bar{q} \to q'\bar{q}', \\
(3) qq \to qq, \\
(4) q\bar{q} \to q\bar{q}, \\
(5) q\bar{q} \to gg \\
(6) gg \to q\bar{q}, \\
(7) qg \to qg, \\
(8) gg \to gg.
\end{gathered} \tag{11.6}$$

The total parton collision cross section is calculated from

$$\hat{\sigma}_{ij}(\hat{s}) = \sum_{kl}\left(\int_0^{p_{tcut}^2} dp_t^2 \frac{d\hat{\sigma}_{ij\to kl}^{soft}(\hat{s})}{dp_t^2} + \int_{p_{tcut}^2}^{\infty} dp_t^2 \frac{d\hat{\sigma}_{ij\to kl}^{hard}(\hat{s})}{dp_t^2}\right), \tag{11.7}$$

where $p_t^2 \approx \hat{t}\hat{u}/\hat{s} \approx -\hat{t}$, $\hat{s} = (p_i + p_j)^2$, $\hat{t} = (p_i - p_k)^2$, $\hat{u} = (p_i - p_l)^2$ are Mandelstam variables. They are connected by relation: $\hat{s} + \hat{u} + \hat{t} = \sum_i m_i^2$. The total parton collision cross section is represented by two terms [4]: the soft collision at $p_t \leq p_{tcut}$ term $\hat{\sigma}_{ij\to kl}^{soft}(\hat{s})$ and the hard collision at $p_t > p_{tcut}$ term $\hat{\sigma}_{ij\to kl}^{hard}(\hat{s})$. The collision resolution scale is defined as $Q_0^2 = p_t^2$.

Hard differential collision cross section is determined by

$$\frac{d\hat{\sigma}_{ij\to kl}^{hard}(\hat{s})}{d\hat{t}} = \frac{\pi\alpha^2(Q_0^2)}{\hat{s}^2}|M(\hat{s},\hat{t},\hat{u})|^2, \tag{11.8}$$

where the squared matrix element is calculated in the lowest order perturbative QCD and has been averaged over spin and colour [5], [6]. The matrix elements for above enumerated parton processes are

$$|M^{(1)}(\hat{s},\hat{t},\hat{u})|^2 = \frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right), \tag{11.9}$$

$$|M^{(3)}(\hat{s},\hat{t},\hat{u})|^2 = \frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} + \frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right) - \frac{8}{27}\frac{\hat{s}^2}{\hat{t}\hat{u}}, \tag{11.10}$$

$$|M^{(5)}(\hat{s},\hat{t},\hat{u})|^2 = \frac{32}{27}\left(\frac{\hat{u}^2+\hat{t}^2}{\hat{t}\hat{u}}\right) - \frac{8}{3}\left(\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right), \tag{11.11}$$

$$|M^{(6)}(\hat{s},\hat{t},\hat{u})|^2 = \frac{1}{6}\left(\frac{\hat{u}^2+\hat{t}^2}{\hat{t}\hat{u}}\right) - \frac{3}{8}\left(\frac{\hat{u}^2+\hat{t}^2}{\hat{s}^2}\right), \tag{11.12}$$

$$|M^{(7)}(\hat{s},\hat{t},\hat{u})|^2 = -\frac{9}{4}\left(\frac{\hat{u}^2+\hat{s}^2}{\hat{u}\hat{s}}\right) + \left(\frac{\hat{u}^2+\hat{s}^2}{\hat{t}^2}\right), \tag{11.13}$$

$$|M^{(8)}(\hat{s},\hat{t},\hat{u})|^2 = \frac{9}{2}\left(3 - \frac{\hat{u}\hat{t}}{\hat{s}^2} - \frac{\hat{u}\hat{s}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2}\right), \tag{11.14}$$

respectively. The squared matrix element for the process (2) can be described by the same equation as for the process (1) with substitution $\hat{s} \leftrightarrow \hat{t}$. The squared matrix element for the process (4) can be expressed by the same equation as for the process (3) with $\hat{s} \leftrightarrow \hat{u}$.

The running coupling strength is determined by

$$\alpha(Q^2) = \frac{12\pi}{(33 - 2n_f)\ln(Q^2/\Lambda^2)}, \tag{11.15}$$

where n_f is the number of quark flavours and Λ is the QCD renormalisation scale.

The soft collision differential cross section is assumed in the form [4]:

$$\frac{d\hat{\sigma}^{soft}_{ij\to kl}(\hat{s})}{d\hat{t}} = W_0(P)\exp\left[\frac{\hat{t}-\hat{t}_{cut}}{\hat{t}_0}\right], \tag{11.16}$$

where $\hat{t}_{cut} = -\hat{s}/2(1-\sqrt{1-4p^2_{tcut}/\hat{s}})$ and slope parameter $\hat{t}_0 = 1\ GeV^2$ is chosen. The normalisation

$$W_0(P_z) = \frac{\hat{\sigma}^{inel}(P_z) - \sigma^{hard}(P_z)}{\exp[-\hat{t}_{cut}/\hat{t}_0] - 1} \tag{11.17}$$

depends from the initial nucleon momentum P_z. The $\hat{\sigma}^{inel} = \hat{\sigma}^{hard} + \hat{\sigma}^{soft}$ is required to describe measured inelastic nucleon-nucleon cross section.

11.5 Parton scattering simulation.

We can select $\hat{t}$ (see also the hadron elastic scattering model chapter) using the probability distribution:

$$W(\hat{t}) = \frac{1}{\hat{\sigma}(\hat{s})}\int_{\hat{t}_{min}}^{\hat{t}} \frac{d\hat{\sigma}(\hat{s})}{d\hat{t}} d\hat{t}, \tag{11.18}$$

where $\hat{s}$ and $\hat{t}$ are the total c.m. parton energy squared and parton momentum transferred $\hat{t}_{min}$, respectively. $\hat{t}_{min}$ is defined by the transverse momentum cut. In the above equation the normalisation is done according to

$$\hat{\sigma}(\hat{s}) = \int_{\hat{t}_{min}}^{\hat{t}_{max}} \frac{d\hat{\sigma}(\hat{s})}{d\hat{t}} d\hat{t}. \tag{11.19}$$

The azimuthal symmetry is assumed. The scattering angle $\cos\theta$ is determined by the equation:

$$\cos\theta = 1 + \frac{2\hat{s}\hat{t}}{[\hat{s}-(m_i+m_j)^2)(\hat{s}-(m_i-m_j)^2)]}, \tag{11.20}$$

where m_i and m_j refer initial parton masses.

11.6 Initial state radiation.

The partons with negative virtualities can participate in space-like cascades, when their virtualities increase from initial values $q_0^2 = -Q^2$ at which they were originally resolved in the nucleon structure function until virtualies $q^2 = -Q_0^2$. At each decay step we will also have the parton are produced

with the time-like virtuality. It can initiate a time-like cascade. The probability for a space-like parton i_{n-1} to be involved into decay $i_{n-1} \to i_n k_n$ between Q^2 and $Q^2 + dQ^2$ is [7]

$$dW_{i_{n-1}i_n}(x_{i_{n-1}}, x_{i_n}, Q^2) = \frac{\alpha(Q^2)}{2\pi Q^2} \frac{F_{i_{n-1}}(x_{i_{n-1}}, Q^2)}{F_{i_n}(x_{i_n}, Q^2)} \frac{1}{\hat{z}} P_{i_{n-1}\to i_n k_n}(\hat{z}) d\hat{z} dQ^2, \tag{11.21}$$

where parton $x = p_z/P_z$, $\hat{z} = x_{i_n}/x_{i_{n-1}}$ and Q^2 is taken at scattering vertex of parton i_n. Here $F_i(x_i, Q^2)$ presents the nucleon structure function.

The backward evolution procedure [7] can be applied to simulate space-like cascades. We consider process starting from the scattering vertex, i.e. we are moving from virtuality $q_n^2 < 0$, where $|q_n^2| > |q_{n-1}^2$, to virtuality $q_{n-1}^2 < 0$.

The Sudakov form factor for space-like decay, i.e. probability that decay will not occurred, is defined by

$$S_s(x_{i_n}, Q^2, Q_0^2) = \exp\left[- \sum_{i_{n-1}} \int_{Q_0^2}^{Q^2} dQ'^2 \int_{\hat{z}_{min}(Q'^2)}^{\hat{z}_{max}(Q'^2)} d\hat{z} dW_{i_{n-1}i_n}(x_{i_{n-1}}, x_{i_n}, Q'^2) \right]. \tag{11.22}$$

From the Sudakov form factor we can find the probability distribution for parton i_n to be produced in the branching $i_{n-1} \to i_n + k_n$ between Q^2 and $Q^2 + dQ^2$:

$$P_s^{decay}(x_{i_n}, Q^2_{max}, Q^2) = S_s(x_{i_n}, Q^2_{max}, Q^2_{min}) \frac{d}{dQ^2} \left(\frac{1}{S_s(x_{i_n}, Q^2_{max}, Q^2_{min})} \right), \tag{11.23}$$

where Q^2_{max} is the maximal value of virtuality for the parton i and $Q^2_{min} = Q_0^2$.

From the above distribution we can select Q^2 for the generated decay by solving the equation:

$$S_s(x_{i_n}, Q^2, Q_0^2)) = \frac{S_s(x_{i_n}, Q^2_{max}, Q_0^2)}{\varsigma}, \tag{11.24}$$

where ς is random number, which is uniformly distributed between 0 and 1. The situation when $\varsigma < S_s(x_{i_n}, Q^2, Q_0^2)$ means that was no decay occurred. If decay was occurred, the next step is to specify partons according to the relative probabilities of allowed channels:

$$\int dz \gamma_{i_{n-1}\to i_n k_n} \frac{F_{i_{n-1}}(x_{i_{n-1}}, Q^2)}{F_{i_n}(x_{i_n}, Q^2)}, \tag{11.25}$$

where $\gamma_{i_{n-1}\to i_n k_n}$ is branching functions [8], [9]. Having Q^2 and parton flavours, we can select $\hat{z}$ according to the probability given by integrand of Eq. (11.25). Then decay kinematics, i.e. daughter parton 4-momenta and virtualities can be constructed. The outlined procedure is repeated by replacing $x_{i_n} \to x_{i_{n-1}}$ $Q^2_{max} \to Q^2$ until the virtuality becomes nearly Q_0^2.

11.7 Final state radiation.

The time-like parton produced in the hard scattering has virtuality $m_i^2 \approx Q_0^2 > 0$, where Q_0^2 is the resolution scale at hard parton collision. It will decay with probability

$$dW_{i\to kl}(\xi, z) = P_{LP}(m_i^2, \Delta t)\frac{\alpha(Q^2)}{2\pi\xi}P_{i\to kl}(z)dzd\xi. \tag{11.26}$$

The first factor $P_{LP}(m_i^2, \Delta t)$ takes into account the virtual parton life-time $1/m_i$ [11]. It is a probability for virtual parton to decay at given time step Δt. The probability $P_{LP}(m^2, \Delta t)$, which was introduced in [4] effectively describes the Landau-Pomeranchuk effect [10] by the time delay of parton emission. The next two factors determine the QCD Altarelli-Parisi branching probability [9] with the branching factor $P_{i\to kl}(z)$. The variables $z = E_k/E_i$, $1-z = E_l/E_i$, $\xi = (p_k \cdot p_l)/(E_k E_l) \approx 1 - \cos\theta_{kl}$ [12] and m^2 are connected with each other through the relation

$$m_i^2 = m_k^2 + m_l^2 + 2E_i z(1-z)\xi, \tag{11.27}$$

where E_i, E_k, E_l and m_k, m_l are parton energies and masses, respectively.

The Q^2 scale is chosen as

$$Q^2 = 2z^2(1-z)^2E_i^2\xi. \tag{11.28}$$

$Q^2 \approx k_t^2$, where k_t the relative transverse momentum between partons k and l at the decay vertex.

The Sudakov form factor for time-like decay, i. e. probability that decay will not occurred between ξ and ξ_{min}, is defined as follows

$$S_t(\xi_{min}, \xi) = \exp[-\int_{\xi_{min}}^{\xi} d\xi' \int_{z_-}^{z_+} dz W_{i\to kl}(\xi', z)], \tag{11.29}$$

where value ξ_{min} regularizes infrared divergences and values $z_\pm$ regularize collinear divergences. These values are represented by

$$\xi_{min} = \frac{4m_{cut}^2}{E_i^2}, \tag{11.30}$$

$$z_- = \frac{m_{cut}}{\sqrt{\xi}E_i} \tag{11.31}$$

and

$$z_+ = 1 - z_-, \tag{11.32}$$

where m_{cut}^2 is a cutoff virtuality.

From the Sudakov form factor we can find the probability distribution for parton i to decay into partons k and l between ξ and $\xi+d\xi$ with $\xi < \xi_{min}$:

$$P_t^{decay}(\xi, \xi_{max}) = S_t(\xi_{min}, \xi_{max}) \frac{d}{d\xi} \left(\frac{1}{S_t(\xi, \xi_{min}}) \right), \tag{11.33}$$

where $\xi_{max} = (p_i \cdot p_j)/(E_i E_j)$ is the value was calculated at parton i produced vertex, i.e. at a parton-parton collision $n + m \to i + j$. The above defined probability distribution is used for time-like cascades generation [13]. The decay is simulated by solving the equation

$$S_t(\xi, \xi_{min}) = \frac{S_t(\xi_{max}, \xi_{min})}{\varsigma}, \tag{11.34}$$

where ς is random number uniformly distributed between 0 and 1. From the equation we find the virtuality of parton i. If this virtuality is less than the cutoff virtuality m_{cut}^2, then time like branching will be stopped.

In the case of decay we can select decay channel, i.e. specify the flavour of the daughter partons using the relative probabilities given by integrals over z for branching functions $\gamma_{i\to kl}(z)$ [8], [9]:

$$\gamma_{q\to qg} = C_q \left(\frac{1+z^2}{1-z} \right), \tag{11.35}$$

$$\gamma_{q\to gq} = C_q \left(\frac{1+(1-z)^2}{z} \right), \tag{11.36}$$

$$\gamma_{g\to gg} = 2C_g \left(z(1-z) + \frac{z}{1-z} + \frac{1-z}{z} \right) \tag{11.37}$$

and

$$\gamma_{g\to q\bar{q}} = \frac{1}{2} \left(z^2 + (1-z)^2 \right), \tag{11.38}$$

where $C_q = (n_c^2 - 1)/2n_c$, $C_g = n_c$ with $n_c = 3$. The last step for known virtuality of i and k and l flavours is to select variable z according to the probability distribution $\gamma_{i\to kl}(z)$.

The decay kinematics, i.e. daughter parton 4-momenta and virtualities can be constructed [13] from the values of ξ and z. The decay chain is generated until virtualities of partons drop bellow m_{cut}^2. Then partons can be put on mass shell with the effective masses [13] $m_g = 0.5 m_{cut}$ and $m_q = \sqrt{0.25 m_{cut}^2 + m_q^2}$ for gluons and quarks, respectively.

11.8 Colour neutral cluster formation.

The hadronization of coloured partons can be done by means of the colour neutral cluster formation [14]. The created clusters will subsequently decay

into stable hadrons and hadron resonances (see the cluster decay chapter). After partonic stage of the collision evolution we will have the time-like produced partons, as well as the space-like and valence mass shell spectator partons. These two types of partons will arise two types of clusters: produced clusters from the time-like partons and spectator clusters from the space-like and mass shell partons.

Before to look for produced cluster parton candidates, the time-like gluons are forced to split into collinear $q\bar{q}$ pairs. The q and $\bar{q}$ have random (with uniform probability) momenta, which collinear with the parent gluon momentum. If the invariant mass of pair exceeds the kaon mass, their flavours are assigned to the u, d or s randomly with the equal probabilities.

Each time-like quark can be combined with an time-like or mass shell antiquark, if they have no common gluon parent. The quark combining procedure can be outlined as follows:

1. The phase-space distances

$$d_{ij} = \sqrt{x_{ij}^{\nu} x_{ij,\nu}} \sqrt{p_{ij}^{\mu} p_{ij,\mu}}, \tag{11.39}$$

   where x denotes 4-vector $(\mathbf{r}, t)$, $x_{ij} = x_i - x_j$ and $p_{ij} = p_i - p_j$ are calculated for all possible antiquark partners;

2. The chosen quark will form a cluster with an antiquark having minimal phase space distance. The cluster momentum and cluster radius vector are determined according to

$$P_c^{\nu} = p_i^{\nu} + p_j^{\nu}, \tag{11.40}$$

$$\mathbf{R}_c = \frac{m_i \mathbf{r}_i + m_j \mathbf{r}_j}{m_i + m_j}, \tag{11.41}$$

   respectively.

The spectator clusters are created from the spectator partons having the same sign of the longitudinal momenta. Their four momenta are calculated according to

$$P_c^{\nu} = \sum_i p_i^{\nu} \tag{11.42}$$

and their radius vectors are

$$\mathbf{R}_c = \frac{1}{n} \sum_i \mathbf{r}_i, \tag{11.43}$$

where n is number of partons included into a cluster. To fulfil the charge conservation law, if it was broken during spectator cluster assignment procedure, one of the lowest energy charge unbalanced quark can be put on mass shell and assigned to a produced cluster. If it is necessary this procedure is repeated until the restoration of charge conservation law.

11.9 Hadronization procedure.

The produced clusters decay into stable or resonance hadrons. We can use the cluster decay model described in the cluster decay chapter. For hadron containing u, d and s quarks the $J^P = 0^-, 1^\pm$ meson and $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ baryon states are allowed. If mass of a produced cluster is small to decay into 2 hadrons, then it is substituted by the lightest hadron having the same quark content and its energy-momentum is adjusted keeping the energy-momentum of produced cluster system.

If mass of a cluster is larger than $M_s^2 = 4\ GeV$ than cluster is considered as a string and string decay model (see the string decay model chapter) can be invoked to decay it.

The spectator clusters are pulled on the mass shells of two barions through the pairwise interaction [14] with squared momentum transfer t, which is distributed according to

$$W(t) \sim \exp\left(At + Bt^2\right), \tag{11.44}$$

with parameters A and B are taken from the single and double nucleon-nucleon diffractive scattering measurements.

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Chapter 12

Nuclear properties.

12.1 Nuclear masses and binding energies.

We can tabulate [1] values of nuclear masses and binding energies. We also have a possibility to determine nuclear masses as well as binding energies from the liquid drop model formulas.

12.1.1 Cameron's formula.

According to the Cameron's liquid drop model formula [2] values of mass defect $dM(A,Z)$ can be determined from the equation:

$$\begin{gathered} dM(A,Z) = M_{Volume}(A,Z) + M_{Surface}(A,Z) + M_{Coulomb}(A,Z) + \\ + M_{Exchange}(A,Z) + \Delta^{C}_{Shell}(A,Z) + \Delta^{C}_{Pair}(A,Z) + \\ + (A-Z)m_N + Zm_H. \end{gathered} \tag{12.1}$$

Here

$$M_{Volume}(A,Z) = -17.035\left[1 - 1.846\left(\frac{A-2Z}{A}\right)^2\right]A \tag{12.2}$$

is the volume energy,

$$M_{Surface}(A,Z) = 25.8357\left[1 - 1.712\left(\frac{A-2Z}{A}\right)^2\right]\left[1 - \frac{0.62025}{A^{2/3}}\right]^2 A^{2/3} \tag{12.3}$$

is the surface energy,

$$M_{Coulomb}(A,Z) = 0.779\frac{Z(Z-1)}{A^{1/3}}\left[1 - \frac{1.5849}{A^{2/3}} + \frac{1.2273}{A} + \frac{1.5772}{A^{4/3}}\right] \tag{12.4}$$

is the Coulomb energy and

$$M_{Exchange}(A,Z) = -0.4323\frac{Z^{4/3}}{A^{1/3}}\left[1 - \frac{0.57811}{A^{1/3}} - \frac{0.14518}{A^{2/3}} + \frac{0.49597}{A}\right] \tag{12.5}$$

is the exchange energy. The masses $m_N = 8.07169$ MeV and $m_H = 7.8922$ MeV are the mass excesses of neutron and the hydrogen atom, respectively. The values of $\Delta^C(A,Z) = \Delta^C_{Shell}(A,Z) + \Delta^C_{Pair}(A,Z)$ are corrections connected with influences of shell structure and pairing effect. These values are tabulated in [2].

12.1.2 Bethe-Weizsäcker's formula.

In this approach[3] a nucleus binding energy $B(A,Z)$ is given by

$$B(A,Z) = \\ = -0.01587A + 0.01834A^{2/3} + 0.09286\left(Z - \frac{A}{2}\right)^2 + 0.00071\frac{Z^2}{A^{1/3}}, \quad (12.6)$$

i.e. volume, surface and Coulomb energy terms are summed.

12.2 Nuclear potential for nucleons.

For i-th nucleon this potential is defined by sum of the Fermi-potential

$$T^F(r_i) = \frac{[p^F(r_i)]^2}{2m_i}, \quad (12.7)$$

the binding energy $B(A,Z)$ and the Coulomb potential $V_C(Z,r_i)$ (for protons only):

$$V(A,Z,r_i) = T^F(r_i) + B(A,Z) + V_C(Z,r_i). \quad (12.8)$$

The Fermi momentum in the local Thomas-Fermi approximation [4] depends from the proton or neutron density $\rho_{Z,N}(r)$ as follows

$$p^F(r) = \hbar(3\pi^2\rho_{Z,N})^{1/3}, \quad (12.9)$$

were $\hbar = 197.327$ MeVfm.

The Coulomb barrier is (in MeV) given by

$$V_C(A,Z) = C\frac{Z}{r_0(1+A^{1/3})}, \quad (12.10)$$

where $C = 1.44$ MeVfm and $r_0 = 1.3$ fm.

12.3 Fission barriers.

The fission barriers are determined as differences between the saddle-point and ground state nuclear masses. In the general case fission barriers are functions of the charges Z, atomic mass numbers A, excitation energies of fissioning nuclei E^*, their angular momenta L and their deformations α.

Shell structure effects play an essential role at the fission barrier. The height of fission barrier can be approximated as follows

$$B_{fis} = B^0_{fis} + \Delta_{Shell} + \Delta_{SP}, \tag{12.11}$$

where B^0_{fis} is the so-called liquid drop component of the fission barrier, Δ_{Shell} is the shell correction for the mass of a nucleus in the ground state and Δ_{SP} is the shell correction for the mass of nucleus in the saddle point. The last correction is very important for the actinide nuclei. It leads to a double-humped shape of the fission barrier.

There are many models for fission barriers: the phenomenological approach of Barashenkov et al. [6], the semiphenomenological approach of Barashenkov and Gereghi [7], the liquid-drop model (LDM) with Myers and Swiatecki's parameters [8], the LDM with Pauli and Ledergerber's parameters [9], the single-Yukawa modified LDM of Krappe and Nix [10], the Yukawa-plus-exponential modified LDM [11], the subroutine BARFIT of Sierk [12], which provides macroscopic fission barriers of rotating nuclei in the Yukawa-plus-exponential modified LDM [11], double-humped fission barriers for transuranium nuclides have been proposed in [13]. As the first approximation we can implement the simple semiphenomenological approach, which has been suggested by Barashenkov and Gereghi [7]. In their approach fission barriers $B_{fis}(A, Z)$ are approximated by

$$B_{fis} = B^0_{fis} + \Delta^C_{Shell} + \Delta^C_{Pair} + \delta(A, Z), \tag{12.12}$$

where shell and pairing corrections are taken from the Cameron's liquid drop mass formula [2]. $\delta(A, Z) = 0$ for even Z and even $N = A - Z$, $\delta(A, Z) = 1.248$ MeV for odd A and $\delta(A, Z) = 2.496$ MeV for odd Z and odd N. According to the Eq. (12.12) fission barrier height $B^0_{fis}(x)$ varies with the fissility parameter x. $B^0_{fis}(x)$ is determined by

$$B^0_{fis}(x) = a_S A^{2/3} 0.83(1 - x)^3 \tag{12.13}$$

for $2/3 \leq x \leq 1$ and

$$B^0_{fis}(x) = a_S A^{2/3} 0.38(3/4 - x) \tag{12.14}$$

for $1/3 \leq x \leq 2/3$. The fissility parameter x is given by

$$x = \frac{E^0_C}{2E^0_S} = \frac{(a_C/2a_S)Z^2/A}{1 - k[(N - Z)/A]^2}, \tag{12.15}$$

where E^0_C and E^0_S are Coulomb and surface energies of a spherical nucleus, respectively.

The liquid drop model parameters $a_S = 17.9439$ MeV, $a_C = 0.7053$ MeV and $k = 1.7826$ are taken from the paper[5].

The fission barrier height is a function of the excitation energy. There is a possibility to use empirical relation has been proposed in [6] to estimate the excitation energy E^* dependence:

$$B_{fis}(E^*) = \frac{B_{fis}}{1 + \sqrt{\frac{E^*}{2A}}}. \tag{12.16}$$

12.4 Level density parameter.

This parameter plays a major role in the level density models. The parametrization $a = A/k$ MeV^{-1}, where $k \approx 8$, is the approximation, which is frequently employed in equilibrium decay calculations. It is not adequate in the neighborhood of the magic nuclei. Instead, marked shell effects appear for these nuclei and these effects manifest themselves by an associated decrease of the level density parameter at the binding energy. It has been argued by Ignatyuk [14] that these shell effects disappear with the rise of excitation energy. So at sufficiently high excitation energies a simple linear mass dependence of the level density parameter is recovered. The Iljinov's systematics for $a(Z, A, E^*)$ [15] can be implemented. The authors of [15] used a functional form proposed by Ignatyuk [14]:

$$a(Z, A, E^*) = a_0(A) \left[1 + \Delta^C_{Shell}(Z, A) \frac{f(E^* - \Delta_{Pair})}{E^* - \Delta_{Pair}}\right], \tag{12.17}$$

where

$$a_0(A) = \alpha A + \beta A^{2/3} B_s \tag{12.18}$$

is the Fermi-gas value of the level density parameter at high excitation energies and

$$f(U) = 1 - \exp(-\gamma E^*). \tag{12.19}$$

B_s is the ratio of the surface of a nucleus to the surface of the sphere with the same volume. It was taken $B_s = 1$. $\Delta_{Shell}(Z, A)$ is the shell correction in the nuclear mass formula [2]. The parameters $\alpha = 0.072$, $\beta = 0.257$ and $\gamma = 0.052$ MeV^{-1} were found in [15].

12.5 Nucleus initial state simulation.

12.5.1 Sampling of the nucleon coordinates and momenta.

The initialisation of each nucleus, consisting from A nucleons and Z protons ($N = A - Z$ neutrons) with coordinates $\mathbf{r}_i$ and momenta $\mathbf{p}_i$, where $i = 1, 2, ..., A$ should be performed in the case of nuclear collision simulation. For the hadron transport model, the intranuclear hadron transport model and for the pomeron based parton string model the MC initialisation procedures are similar except of some peculiarities, which can be taken into account (see the corresponding chapters):

- Nucleon radii r_i are selected randomly in the rest of nucleus according to proton or neutron density $\rho(r_i)$. For heavy nuclei with $A > 16$ [16] normalised nucleon density is

$$\rho(r_i) = \frac{\rho_0}{1 + \exp\left[(r_i - R)/a\right]} \tag{12.20}$$

where

$$\rho_0 \approx \frac{3}{4\pi R^3}\left(1 + \frac{a^2\pi^2}{R^2}\right)^{-1}. \tag{12.21}$$

Here $R = r_0 A^{1/3}$ fm and $r_0 = 1.16(1 - 1.16A^{-2/3})$ fm and $a \approx 0.545$ fm. For light nuclei with $A < 17$ normalised on unity nucleon density is given by a harmonic oscillator shell model [17], e. g.

$$\rho(r_i) = (\pi R^2)^{-3/2} \exp\left(-r_i^2/R^2\right), \tag{12.22}$$

where $R^2 = 2/3 < r^2 >= 0.8133A^{2/3}$ fm^2 or

$$\rho(r_i) = \frac{4}{\pi^{3/2}R^3}\left[1 + \frac{A-4}{6}\left(\frac{r_i}{R}\right)^2\right] \exp\left(-r_i^2/R^2\right), \tag{12.23}$$

where

$$R^2 = \left(\frac{5}{2} - \frac{4}{A}\right)^{-1} (< r_{ch}^2 >_A - < r_{ch}^2 >_p) \tag{12.24}$$

The mean charge radii squared of the nucleus $< r_{ch}^2 >_A$ and proton $< r_{ch}^2 >_p$ are taken from the lepton-nucleus scattering experiments [18]. To take into account nucleon repulsive core it is assumed that internucleon distance $d > 0.8$ fm;

- The initial momenta of the nucleons are randomly chosen between 0 and $p_{max}^F(r)$, where $p_{max}^F(r)$ is calculated according to the Eq. (12.9).

- To obtain coordinate and momentum components, it is assumed that nucleons are distributed isotropically in configuration and momentum spaces;

  We assume [19] that nucleons are not points in configuration space and they are represented by Gaussian shaped density distributions:

$$\phi(\mathbf{x_i}, t) = \left(\frac{2\alpha}{\pi}\right)^{3/4} \exp\left\{-\alpha[\mathbf{x_i} - \mathbf{r_i}(t)]^2 + \frac{i}{\hbar}\mathbf{p_i}(t)\mathbf{x_i}\right\}, \tag{12.25}$$

where $\alpha = 0.25$ fm^{-2} is a model parameter. It gives us a possibility to take into account the Pauli blocking (the Pauli blocking procedure is described in the hadron transport chapter), i. e. if the phase space of i-th nucleon is already occupied by other nucleons, its sampling is deserted;

- A nucleus must be centred in configuration space around $\mathbf{0}$, *i. e.* $\sum_i \mathbf{r}_i = \mathbf{0}$ and it must be at rest, i. e. $\sum_i \mathbf{p_i} = \mathbf{0}$ and $\sum_i \mathbf{r_i} \times \mathbf{p_i} = \mathbf{0}$. To provide the first two conditions we can perform shifts of nucleon coordinates $\mathbf{r'_i} = \mathbf{r_i} - 1/A \sum_i \mathbf{r_i}$ and momenta $\mathbf{p'_i} = \mathbf{p_i} - 1/A \sum_i \mathbf{p_i}$ of nucleon momenta.

- There is a possibility to unpack a nucleus into "free" streaming nucleons by computing the energy per nucleon as $e = E/A = m_N + B(A,Z)/A$, where m_N is nucleon mass and a nucleus binding energy $B(A,Z)$ is given by the Bethe-Weizsäcker formula (Eq. (12.6)) and assign the effective mass $m_i^{eff} = \sqrt{(E/A)^2 - p_i^{2\prime}}$ for each nucleon.

12.5.2 Random rotation of a nucleus.

The nucleus initialisation can be performed at once, even if one needs to simulate multiple nuclear interactions, when the same nucleus is participating. Each time before interaction the nucleus is randomly rotated in coordinate and momentum spaces. To perform this rotation three Euler angles: $0 < \theta_1 < 2\pi$, $0 < \theta_2 < 2\pi$ and $0 < \theta_3 < 2\pi$ are selected (with uniform probabilities) randomly. Then the nucleon coordinates $\mathbf{r_i} = \{r_{xi}, r_{yi}, r_{zi}\}$ and momenta $\mathbf{p_i} = \{p_{xi}, p_{yi}, p_{zi}\}$ are rotated sequentially:

$$\mathbf{r_i} \rightarrow \mathbf{r_i} R_1 \rightarrow \mathbf{r_i} R_2 \rightarrow \mathbf{r_i} R_3, \tag{12.26}$$

and

$$\mathbf{p_i} \rightarrow \mathbf{p_i} R_1 \rightarrow \mathbf{p_i} R_2 \rightarrow \mathbf{p_i} R_3, \tag{12.27}$$

where the rotation matrices are detrmined as follows

$$R_1 = \begin{vmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \tag{12.28}$$

and

$$R_2 = \begin{vmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 \end{vmatrix} \tag{12.29}$$

and

$$R_3 = \begin{vmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{vmatrix} \tag{12.30}$$

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Chapter 13

Nuclear cross section model.

13.1 Glauber model at high energies.

We can use the Glauber approach [1] to calculate total, elastic and differential elastic hadron-nucleus and nucleus-nucleus interaction cross sections at high (more than hundreds of MeV) energies.

13.1.1 Hadron-nucleus and nucleus-nucleus total and elastic interaction cross sections.

The knowledge of the nuclear elastic scattering amplitude $F(\vec{q}, s)$, where s is total hadron-nucleon or nucleon-nucleon c.m. energy squared and $\vec{q}$ is the momentum transfer vector, gives us a possibility to calculate total cross section (the optical theorem)

$$\sigma_{tot}(s) = \frac{4\pi}{k} ImF(0, s), \tag{13.1}$$

where k is a hadron or nucleon projectile momentum in the target nucleus rest frame. Using this amplitude we are also able to calculate differential elastic cross section

$$\frac{d\sigma_{el}(s)}{d\Omega} = |F(\vec{q}, s)|^2 \tag{13.2}$$

or

$$\frac{d\sigma_{el}(s)}{dt} = \frac{\pi}{k^2} |F(\vec{q}, s)|^2 \tag{13.3}$$

and total elastic cross section

$$\sigma_{el}(s) = \int d\Omega |F(\vec{q}, s)|^2 = \frac{1}{k^2} \int dq |F(\vec{q}, s)|^2. \tag{13.4}$$

The elastic scattering amplitude can be expressed through the profile function

$$\Gamma(\vec{B}, s) = 1 - S(\vec{B}, s) \tag{13.5}$$

as follows

$$F(\vec{q},s)=\frac{ik}{2\pi}\int d^2\vec{B}\exp\left[i\vec{q}\Gamma(\vec{B},s)\right], \tag{13.6}$$

where $S(\vec{B},s)$ is the S-matrix and $\vec{B}$ is the impact parameter vector perpendicular to the incident momentum $\vec{k}$.

The total and elastic cross sections can be obtained from the profile function $\Gamma(\vec{B},s)$:

$$\sigma_{tot}(s)=2\int d^2\vec{B}Re\Gamma(\vec{B},s) \tag{13.7}$$

and

$$\sigma_{el}(s)=\int d^2\vec{B}|\Gamma(\vec{B},s)|^2. \tag{13.8}$$

Thus to calculate the total, elastic and differential cross sections we need to know S-matrix $S(\vec{B},s)$.

13.1.2 Hadron-nucleus and nucleus-nucleus S-matrix.

Let us consider nucleus-nucleus scattering at given impact parameter $\vec{B}$ and at given squared total c.m. nucleon-nucleon energy s.

In the Glauber approach [1] an elastic nucleus-nucleus interaction is a result of the interactions between nucleons from the projectile and target nuclei. The S-scattering matrix $S^{AB}(\vec{B},s)$ for nucleus A on nucleus B collision in the impact parameter representation can be expressed as follows:

$$S^{AB}(\vec{B},s)=\left\langle\prod_{i=1}^{A}\prod_{j=1}^{B}[1-\Gamma_{ij}(\vec{B}+\vec{b}_i^A-\vec{b}_j^B,s)]\right\rangle \tag{13.9}$$

where $<...>$ means integration over the sets $\{\vec{b}_i^A\}$ and $\{\vec{b}_j^B\}$ with weight functions $T_A(\{\vec{b}^A\})$ and $T_B(\{\vec{b}^B\})$. These functions

$$T_{A,B}(\vec{b}_i^{A,B})=\int\rho((\vec{b}_i^{A,B}z_i)dz_i \tag{13.10}$$

can be obtained from the nucleon densities $\rho((\vec{b}_i^{A,B},z_i)$. The nucleon profile function is

$$\Gamma_{ij}(\vec{B}+\vec{b}_i^A-\vec{b}_j^B,s)=\frac{\sigma_{ij}(s)}{4\pi\beta_{ij}(s)}\exp\left[-\frac{(\vec{B}+\vec{b}_i^A-\vec{b}_j^B)^2}{2\beta_{ij}(s)}\right]. \tag{13.11}$$

The last equation has been obtained by means of the following nucleon-nucleon amplitude parametrisation:

$$f_{ij}(q,s)=\frac{ik\sigma_{ij}(s)}{4\pi}\exp\left[-\frac{1}{2}\beta_{ij}(s)q^2\right]. \tag{13.12}$$

The equation (13.9) is a result of the assumptions that the AB-scattering phase is sum of the nucleon–nucleon scattering phases and no correlations between nucleons inside a nucleus are taken into account.

The hadron-nucleus S-matrix is calculated in similar way using Eq. (13.9) for $i = 1$ and $\vec{b}_i = 0$. In this case we need to apply the parameter $\sigma_{hN}(s)$ and $\beta_{hN}(s)$ in the nucleon profile function.

As we will show below the hadron-nucleon and nucleon-nucleon elastic scattering amplitudes at high energies can be expressed through the reggeon-nucleon vertex parameters and the parameters of the reggeon trajectory[2].

13.2 High energy MC cross section algorithm.

To obtain total (see Eq. (13.7)) and elastic (see Eq. (13.8)) hadron-nucleus or nucleus-nucleus cross section at given initial energy we have to integrate the nucleon profile function $\Gamma(\vec{B}, s) = 1-S(\vec{B}, s)$. This can be with help of the Monte Carlo averaging procedure [3], [4] are used to obtain the S-matrix values from the Eq. (13.9). These values depend on the squared total c.m. energy of the colliding (i, j) pair $s_{ij} = (p_i + p_j)^2$, where p_i and p_j are the particle 4-momenta, respectively. Performing Monte Carlo averaging one has to pick up projectile and target nucleons randomly according to the weight functions $T([\vec{b}_i^A])$ and $T([\vec{b}_j^B])$, respectively. The last functions represent probability densities to find sets of the nucleon impact parameters $[\vec{b}_i^A]$, where $i = 1, 2, ..., A$ and $[\vec{b}_j^B]$, where $j = 1, 2, ..., B$. Then by integration over $\vec{B}$ we find the total and elastic cross sections. To obtain the elastic differential cross section from the Eqs. (13.2) and (13.3) we have at first to perform the back Fourier transform of the nucleon profile function (see Eq. (13.6)).

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Chapter 14

Hadron transport model.

14.1 Reaction initial state simulation.

14.1.1 Allowed hadron reactions.

The Hadron Transport Model (HTM) is capable to predict final states, i. e. produced hadrons, which belong to the pseudoscalar meson nonet and the baryon/antibaryon octet, for the reactions on nucleon and nuclear targets with a large variety of hadron (the same types as final state hadrons) and nuclear projectiles. The allowed bombarding kinetic energy in the hadron-nucleon, hadron-nucleus and nucleus-nucleus collisions is recommended to be more than 200 MeV in the laboratory frame. The upper limit of initial kinetic energy is approximately 10 GeV per initial particle.

This model is also able to predict final states in the photon-nucleon and photon-nucleus inelastic collisions. The last case is described in the separate chapter.

One can also apply this model to simulate hadron kinetics after the "parton string" stage of nuclear interaction, taking account secondary hadron rescatterings.

We can simulate initial nuclear state, when a nucleus becomes "unpacked" into the free streaming nucleons, according to the procedure described in the nuclear properties chapter.

14.1.2 Impact parameter sampling.

The impact parameter $0 \leq b \leq R_p + R_t$ is randomly selected according to the distribution:

$$P(\mathbf{b}) = b, \tag{14.1}$$

where R_p and R_t are the target and projectile radius, respectively. In the case of nuclear projectile or target the nuclear radius is determined from the

condition:

$$\frac{\rho(R)}{\rho(0)} = 0.01. \tag{14.2}$$

Then one should update the transversal components of nucleon coordinates:

$$r_{xi} \rightarrow r_{xi} + b_x \tag{14.3}$$

and

$$r_{yi} \rightarrow r_{yi} + b_y. \tag{14.4}$$

14.1.3 Lorentz boost of nucleon longitudinal momenta and energies.

In the case of fast moving nucleus with initial momentum per nucleon $\mathbf{P_0} = \{0, 0, P_{z0}\}$ one should perform Lorentz transformation of the nucleon longitudinal momenta

$$p_{zi} \rightarrow \gamma_i(p_{zi} - \beta_i e_i) \tag{14.5}$$

and the nucleon energies

$$e_i \rightarrow \gamma_i(e_i - \beta_i p_{zi}), \tag{14.6}$$

where β_i is defined as

$$\beta_i = \frac{P_{z0}}{\sqrt{P_{z0}^2 + m_i^{eff2}}} \tag{14.7}$$

and γ_i is given by

$$\gamma_i = \frac{1}{\sqrt{1 - \beta_i^2}}. \tag{14.8}$$

The m_i^{eff} denote an effective i-th nucleon mass (see the nuclear properties chapter).

14.1.4 Lorentz contraction of the fast moving nucleus.

For the fast moving nucleus one needs also to perform its Lorentz contraction, *i. e.*

$$r_{zi} \rightarrow r_{zi}/\gamma_i. \tag{14.9}$$

14.1.5 Longitudinal shift of the target nucleus.

For target nucleus, if a projectile is centred around the origin of coordinate system, one should perform a shift of the nucleon longitudinal coordinates:

$$r_{zi} \rightarrow r_{zi} + \Delta r_z/\gamma_i, \tag{14.10}$$

where $\Delta r_z = R_p + 1.5\ fm + R_t + 1.5\ fm$ can be taken.

14.1.6 Initial angular momentum.

In the case of hadron-nucleus collision we can determine the initial angular momentum of the target nucleus:

$$\vec{L} = \vec{p_0} \times \vec{r_0}, \tag{14.11}$$

where $\vec{p_0}$ and $\vec{r_0}$ are initial momentum and enter radius vector of incoming hadron in the target nucleus rest frame.

14.2 Hadron propagation.

The hadrons are transported along the straight line paths (cascade approach), i. e. particle coordinates $\mathbf{r}$ for the time "shift" $\Delta t = t_1 - t_0$ between t_0 and t_1 are updated according to

$$\mathbf{r}(t_1) = \mathbf{r}(t_0) + \frac{\mathbf{p}(t_0)}{E(t_0)}\Delta t, \tag{14.12}$$

where $\mathbf{p}$ and E denote particle momenta and energies. The intranuclear nucleons are assumed to be "frozen" before collision, i. e. they are allowed to move only with the c. m. nucleus velocity. However, to find interaction cross sections or characteristics of the secondary particles produced in the course of the intranuclear collision their Fermi momenta are taken into account.

14.3 Selection of particle collisions and particle decays.

For each pair (i, j) of particles their total interaction cross section $\sigma_{tot}(s_{ij})$ is calculated. This cross section depends on the particle quantum numbers. It is also a function of the total particle c.m. energy squared $s_{ij} = (p_i + p_j)^2$, where $p_i = (E_i, \mathbf{p_i})$ and $p_j = (E_j, \mathbf{p_j})$ are four momenta of particle i with mass $m_i = \sqrt{E_i^2 - \mathbf{p_i}^2}$ and particle j with mass $m_j = \sqrt{E_j^2 - \mathbf{p_j}^2}$, respectively. The cross section is interpreted geometrically as an interaction area with radius

$$b_{ij} = \sqrt{\frac{\sigma_{tot}(s_{ij})}{\pi}}. \tag{14.13}$$

It is assumed that two particles will collide, if their transverse (with regards to the relative velocity vector of the particles) distance d_{ij} fulfils the condition:

$$d_{ij} \leq b_{ij}. \tag{14.14}$$

For the distance d_{ij} we choose the relative distance between two particles at the time of the closest approach calculated in the c.m. frame of two

particles with coordinates $\mathbf{x_i^*}$ and $\mathbf{x_j^*}$ and momenta $\mathbf{p_i^*}$ and $\mathbf{p_j^*}$, respectively. It is determined by

$$d_{ij}^2 = (\mathbf{x_i^*} - \mathbf{x_j^*})^2 - \frac{[(\mathbf{x_i^*} - \mathbf{x_j^*})(\mathbf{p_i^*} - \mathbf{p_j^*})]^2}{(\mathbf{p_i^*} - \mathbf{p_j^*})^2}. \tag{14.15}$$

For the distance d_{ij} one can choose also the distance of the closest approach at the time in the rest frame of particle i or in the rest frame of particle j. In the rest frame of one or another particle it can be calculated using the Lorentz invariant expression [1]:

$$d_{ij}^2 = -(\Delta x_{ij})^2 - \frac{(\Delta x_{ij}p_i)^2 p_j^2 + (\Delta x_{ij}p_j)^2 p_i^2 - 2(\Delta x_{ij}p_i)(\Delta x_{ij}p_j)(p_i p_j)}{(p_i p_j)^2 - p_i^2 p_j^2}, \tag{14.16}$$

where notation $\Delta x_{ij} = x_j - x_i$ and $x_i = (t_i, \mathbf{x_i})$ and $x_j = (t_j, \mathbf{x_j})$ are four coordinates of particles. Because the particles are distance d_{ij} apart, in an reference frame the collision event will have two different times t_{ij} and t_{ji}.

These times are given by

$$t_{ij} - t_i = \frac{E_i[(\Delta x_{ij}p_i)p_j^2 - (\Delta x_{ij}p_j)p_i p_j]}{(p_i p_j)^2 - p_i^2 p_j^2} \tag{14.17}$$

and

$$t_{ji} - t_j = \frac{E_j[(\Delta x_{ij}p_j)p_i^2 - (\Delta x_{ij}p_i)p_i p_j]}{(p_i p_j)^2 - p_i^2 p_j^2}. \tag{14.18}$$

The particle j is moving away from particle i at the time t_i if $t_{ij} < t_i$.

If we will consider more than two particles, each particle will have a set of possible collision times. Therefore the time ordering of the individual binary collisions strongly varies with the respective reference frame. The final result does depend on this criterion [1], [2].

The common recipe is to define the average of the two times $\tau_{coll} = \frac{t_{ij}+t_{ji}}{2}$ as the collision time τ_{coll} for particles i and j. On the other hand one could choose for τ_{coll} either the earliest or the latest of the two times.

We order collisions in the observer reference frame for the hadron-nucleus or nucleus-nucleus interaction. Thus using the particle locations $\mathbf{x}$ and the particle momenta $\mathbf{p}$ in the observer reference frame one can obtain for the time of the closest approach for two colliding particles:

$$\tau_{coll} = t_{ij} = t_{ji} = -\frac{(\mathbf{x_i} - \mathbf{x_j})(\mathbf{p_i}/E_i - \mathbf{p_j}/E_j)}{(\mathbf{p_i}/E_i - \mathbf{p_j}/E_j)^2}. \tag{14.19}$$

14.3.1 Hadron resonance lifetime.

The lifetime t for each particle with mass m can be sampled according to the probability:

$$P(t) = 1 - \exp\left(-\frac{t\Gamma_{tot}(m)}{\gamma\hbar}\right), \tag{14.20}$$

where $\gamma = E/m$ is the Lorentz factor, defined for the particle with total energy E. The particle resonance width $\Gamma_{tot}(m)$ depends from particle mass m.

The full decay width $\Gamma_{tot}(m)$ of a resonance can be defined as the sum of all partial decay widths and depends on the mass m of the excited resonance:

$$\Gamma_{tot}(m) = \sum_{br=\{i,j\}}^{N_{br}} \Gamma_{i,j}(m), \tag{14.21}$$

where the partial decay widths $\Gamma_{i,j}(m)$ for the decay into the exit channel with particles i and j can can be parametrised as follows [3]

$$\Gamma_{i,j}(m) = \Gamma_{i,j}(m_R)\frac{m_R}{m}\left(\frac{p_{i,j}(m)}{p_{i,j}(m_R)}\right)^{2l+1}\frac{1.2}{1+0.2\left(\frac{p_{i,j}(m)}{p_{i,j}(m_R)}\right)^{2l}}, \tag{14.22}$$

where m_R is the pole mass of a resonance, $\Gamma_{i,j}(m_R)$ its partial width for decay into the channel i and j at the pole and l the decay angular momentum of the exit channel and

$$p_{i,j}(m) = p_{c.m.}(m, m_i, m_j) = \frac{1}{2m}\sqrt{(m^2-(m_i+m_j)^2)(m^2-(m_i-m_j)^2)}. \tag{14.23}$$

All pole masses and partial decay widths can be taken from the Review of Particle Properties [10].

14.4 Hadron collision simulation.

In the accordance with the chosen elastic, total and annihilation hadron interaction cross sections we need to simulate two-body hadron elastic scatterings, two-body inelastic scatterings, which include the diffractive scatterings as well as the antibaryon-baryon annihilations.

14.4.1 High energy non-resonance hadron kinetics.

To simulate these processes we can use the elastic scattering model and the Reggeon based parton string model are described above. We can call this version of our hadron transport model as the high energy non-resonance hadron transport model to stress that we neglect the resonance hadron interactions. In spite of that the production of resonances mostly through string decay and decay of resonances are taken into account. This simplified version of the hadron transport model makes sense, when we need to perform fast simulations.

The final state of a baryon-antibaryon annihilation can be simulated using the baryon annihilation model, where only diquark annihilation subprocess is taken into account. Another subprocesses as initial quark redistribution, production of two and more strings can also be implemented (see

[9]). All these subprocesses have strong initial energy dependencies. Particularly, initial quark redistribution plays crucial role at stopped baryon annihilation and three string production process is main process at high energies.

14.4.2 Intermediate energy resonance kinetics.

The more advanced version of the hadron transport model should also include baryon-baryon, meson-baryon and meson-meson resonance interactions as well as pion absorptions.

We can use the resonance interaction model described before to simulate two-body baryon scatterings with resonance excitations and deexcitations. Within this model we select a proper channel (according to the calculated cross sections), then sample resonance masses (according to th Breit-Wigner distributions with corresponding pole masses and pole widths) and finally sample scattering angles. It is assumed that angular distributions for all relevant two-body processes without string production are similar and can be approximated by the angular of NN elastic scattering [3]. For the scattering angle $\cos\theta$ distribution parametrisation we can use the angular distribution of NN elastic scattering. It can be extended to all two-body collisions by the replacement $s \to s - (m_1 + m_2)^2 + 4m_N^2$, where m_1 and m_2 denote the masses of incoming hadrons.

The meson-baryon and meson-meson resonance interaction can also be simulated using the resonance interaction model. From the knowledge of resonance interaction meson-baryon and meson-meson cross sections we are able to select a resonance, which should be created at given initial energy.

14.4.3 π-absorption simulation.

It can be performed by means of the two-steps (resonant) absorption, which is going through the baryon resonance excitation and deexcitation, e. g. (1) $\pi + N \to \Delta$ and (2) $\Delta \to NN$.

14.5 Counting of the hadron formation time.

As it was discussed in the string decay chapter the creation of hadrons requires formation times. During the formation times the interaction hadron cross sections are reduced [6], [3] as compared with the free hadron interaction cross sections. The cross section reductions depend on hadron contents. If a hadron does not include its parent (colliding hadron) valence quarks, then its cross section is considered to be zero. If a hadron includes valence quarks of its parent, then its cross section can be estimated from the additive quark model [7], [8]: $\sigma_{qh} = 1/3\sigma_{Bh}$ for baryons keeping a valence quark from

its parents, $\sigma_{qqh} = 1/2\sigma_{Bh}$ for baryons keeping a valence diquark from its parents, $\sigma_{qh} = 1/2\sigma_{Mh}$ for mesons keeping a valence quark from its parents.

14.6 Pauli blocking simulation.

A free fermion particle interaction cross section is also reduced to an effective cross section by the Pauli-blocking due to the Fermi statistics. For each collision the phase-space densities f_i, where i means fermion, in the final states should be checked in order to assure, that the final distribution in phase space is in agreement with the Pauli principle, which rules out the possibility of finding more than one fermion in a single quantum state. The Quantum Molecular Dynamics (QMD) Pauli blocking procedure (see e.g., [4]) can be applied for any final state fermions.

14.6.1 Pauli blocking probability.

We consider nucleons (and other fermions) are not points in phase space. They are represented by Gaussian shaped density distributions [3]:

$$\phi(\mathbf{x_i}, t) = \left(\frac{2\alpha}{\pi}\right)^{3/4} \exp\{-\alpha[\mathbf{x_i} - \mathbf{r_i}(t)]^2 + \frac{i}{\hbar}\mathbf{p_i}(t)\mathbf{x_i}\}, \tag{14.24}$$

where $\alpha = 0.25$ fm^{-2} is a model parameter and $\hbar = 197.327$ MeVfm. The total wave function is assumed to be a direct product of these functions. The phase-space density can be obtained by the Wigner transform of the wave function:

$$f(\mathbf{r}, \mathbf{p}) = \sum_i f_i(\mathbf{r}, \mathbf{p}), \tag{14.25}$$

where

$$f_i(\mathbf{r}, \mathbf{p}) = \frac{1}{(\pi\hbar)^3} \exp\{-2\alpha[\mathbf{r} - \mathbf{r}_i(t)]^2 - \frac{1}{2\alpha\hbar^2}[\mathbf{p} - \mathbf{p}_i(t)]^2\} \tag{14.26}$$

with normalisation

$$\int d\mathbf{r}d\mathbf{p} f_i(\mathbf{r}, \mathbf{p}) = 1. \tag{14.27}$$

The normalised on the number of particles density is

$$\rho(\mathbf{r}) = \sum_i \rho_i(\mathbf{r}), \tag{14.28}$$

where

$$\rho_i(\mathbf{r}) = \int \frac{d\mathbf{p}}{(\pi\hbar)^3} f_i(\mathbf{r}, \mathbf{p}) = \left(\frac{\pi}{2\alpha}\right)^{-3/2} \exp\{-2\alpha[\mathbf{r} - \mathbf{r}_i]^2\}. \tag{14.29}$$

The normalised on the number of particles momentum density is

$$g(\mathbf{p}) = \sum_i g_i(\mathbf{p}), \tag{14.30}$$

where

$$g_i(\mathbf{p}) = \int \frac{d\mathbf{r}}{(\pi\hbar)^3} f_i(\mathbf{r},\mathbf{p}) = \hbar^{-3}(2\pi\alpha)^{-3/2} \exp\{-\frac{1}{2\alpha\hbar^2}(\mathbf{p}-\mathbf{p}_i)^2\}. \tag{14.31}$$

The overlap phase-space density f_i^{ovp} and particle density ρ_i^{ovp} of particle i with other particles are given by

$$\begin{array}{c} f_i^{ovp} = \sum_{j\neq i} \int d\mathbf{r} d\mathbf{p} f_i(\mathbf{r},\mathbf{p}) f_j(\mathbf{r},\mathbf{p}) = \\ = \frac{1}{8(\pi\hbar)^3} \sum_{j\neq i} \exp\{-\alpha(\mathbf{r}_i-\mathbf{r}_j)^2 - \frac{1}{4\alpha\hbar^2}(\mathbf{p}_i-\mathbf{p}_j)^2\} \end{array} \tag{14.32}$$

and

$$\rho_i^{ovp} = \sum_{j\neq i} \int d\mathbf{r} \rho_i(\mathbf{r}) \rho_j(\mathbf{r}) = \sum_{j\neq i} (\frac{\pi}{\alpha})^{-3/2} \exp\{-\alpha(\mathbf{r}_i-\mathbf{r}_j)^2\}. \tag{14.33}$$

Thus the phase-space fermion overlapping densities f_i^{ovp} at the final states can be directly calculated and used for simulation of Pauli-blocking. For two indistinguishable nucleons i and j the function

$$\begin{array}{c} F_i^{block} = \sum_{j\neq i} 8(\pi\hbar)^3 \delta_{\sigma_i\sigma_j} \delta_{\tau_i\tau_j} \int d\mathbf{r} d\mathbf{p} f_i(\mathbf{r},\mathbf{p}) f_j(\mathbf{r},\mathbf{p})] = \\ = \delta_{\sigma_i\sigma_j} \delta_{\tau_i\tau_j} \exp\{-\alpha(\mathbf{r}_i-\mathbf{r}_j)^2 - \frac{1}{4\alpha\hbar^2}(\mathbf{p}_i-\mathbf{p}_j)^2\} \end{array} \tag{14.34}$$

can be interpreted as the Pauli-blocking probability. Here $\sigma_{i,j} = \pm 1$ and $\tau_{i,j} = \pm 1$ denote spin and isospin indices of nucleons, respectively. For example the Pauli-blocking of the two-body collisions can be checked by the blocking-probability $1-(1-F_i^{block})(1-F_j^{block})$.

14.6.2 The QMD Pauli-blocking algorithm.

For each produced baryon, which is located at position $\mathbf{r}$ and has momentum $\mathbf{p}$, the value of F_i^{block} and at the same time the value of

$$d_i = \sum_{j\neq i} \exp\{-2\alpha(\mathbf{r}-\mathbf{r}_i)^2\} \tag{14.35}$$

are calculated. As was found in [4], there is the approximately straight line dependence:

$$F_i^{block} = a_{fit} + b_{fit} d_i, \tag{14.36}$$

where $a_{fit} = 1.49641$ and $b_{fit} = 0.208736$, which divides $(F_i^{block} - d_i)$-plane into two the Pauli-blocked and the Pauli-allowed domains. Thus a collision is only allowed, if computed values fulfil the conditions:

$$F_i^{block} \leq a_{fit} + b_{fit} d_i \tag{14.37}$$

for each outgoing baryon i.

14.7 Calculation of the residual nucleus characteristics.

If we need to consider inelastic interaction after the pomeron parton string model work completion or after hadron-nucleus and nucleus-nucleus collisions applying the preequilibrium exciton or equilibrium model, we have to know characteristics of a residual nucleus or residual nuclei. It should be remembered for above cases we do not use any potentials since we have "unpacked" free streaming nucleons from the participating nucleus at the beginning of reaction.

Calculation of the atomic mass and charge, number of holes (see the preequilibrium exciton model chapter), the energy-momentum for residual nucleus is trivial since we are able to separate nucleon participants and nucleon spectators.

For the particle-nucleus interaction we can estimate also the angular momentum of a residual nucleus:

$$\mathbf{L}_{res} = \mathbf{L}_0 - \sum_{i=1}^{N_{out}} \mathbf{l_i^c}. \tag{14.38}$$

Here $\mathbf{L}_0$ is initial "particle-nucleus" angular momentum and $\mathbf{l_i^c}$ denote the angular carried away by the ith outgoing particle¿ N_{out} is total number of the outgoing particles.

The angular momentum of emitted particle is treated as a classical vector $\mathbf{l_i^c} = [\mathbf{p_i r_i}]$, where $\mathbf{r_i^c}$ is the radius vector at the exit of the transported particle i from the nucleus and $\mathbf{p_i^c}$ is its momentum.

The excitation energy calculation is not so straightforward. For this we can apply the procedure suggested in the paper[5].

14.7.1 Calculation of nucleus excitation energy.

The excitation energy U of a residual nucleus is defined as the energy above the ground state energy (mass) $M_{res}(A_{res}, Z_{res})$ of residual nucleus with mass number A_{res} and charge Z_{res}:

$$U = E_{res}(A_{res}, Z_{res}) + E_{recoil} - M_{res}(A_{res}, Z_{res}), \tag{14.39}$$

where $E_{res}(A_{res}, Z_{res})$ is the c. m. total energy of the residual nucleus and E_{recoil} is the so-called recoil energy [5]. The residual nucleus recoil energy is calculated summing the recoil energies of particles leaving the nucleus. If particle leaves a nucleus, we reduce its energy by the effective potential (the recoil energy) and rescale its momentum.

14.8 Search collision and decay algorithm [6].

We have neglected by the field interactions and kept only particle collisions and decays. The hadrons move along straight trajectories. Thus, at the beginning of simulation we look for all possible collision candidates and create an ordered in time list of potential candidates for collisions. After each realized collision we only have to look for the collisions between new produced particles and the existing particles, as well as decay candidates from produced resonances. Then the number of computations at given time "shift" will be $n_{prod}n$, where n_{prod} is the number of produced particles during the last collision or decay and n is the total number of particles under consideration (not including n_{prod}. We also do not need to use fixed time step. We "shift" particle system on times between events of collisions or decays. It should be noted that in this case we have to maintain the list of potential particle collision or decay events. For this algorithm we have the criterion to stop system of particles evolution process: the list of potential particle collision candidates and decay candidates is empty.

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Chapter 15

Intranuclear hadron transport model.

15.1 Allowed hadronic reactions.

The Intranuclear Hadron Transport Model (IHTM) is capable to predict final states, i. e. produced hadrons, which belong to the pseudoscalar meson nonet and the baryon/antibaryon octet, of reactions on nucleon and nuclear targets with a large variety of hadron projectiles. The allowed bombarding kinetic energies in the hadron-nucleon and hadron-nucleus collision is recommended to be more than 20 MeV in the laboratory frame. The upper limit of initial kinetic energy is approximately 10 GeV.

This model is also able to predict final states in the photon-nucleon and photon-nucleus inelastic collisions as well as the nuclear absorption of the stopping π^-, K^- and $\bar{p}$ particles.

γ-nucleus and hadron-nucleus interactions are considered in the target nucleus rest frame. It gives us a possibility to take into account intranuclear potentials, as well as absorption of photons and pions by nuclear medium of the target. Thus preparing initial reaction state as it was described in the hadron transport model chapter we do not need to boost and contract a target nucleus. For this case we also do not need to modify nucleon masses by fixing nucleon energies: $e = m_N + B(A, Z)/A$ (see the nuclear properties chapter).

The reaction initialisation procedure is described in the hadron transport model chapter as well other simulation details. Here we only describe some peculiarities of this model as compared to the hadron transport model.

15.2 Phenomenological potentials.

We can take into account the mean field nuclear potentials for nucleons, antinucleons, pions, kaons, lambdas and sigmas particles. Thus we will have

a possibility to consider nuclear absorption of these particles and therefore to calculate residual nucleus excitation energy as well as the number of excitons (see the preequilibrium exciton model chapter). The hadron potential energy $V_A(p,r)$ is in general case a function of momentum p and position r of a hadron in the field of a nucleus as a whole. At the moment we do not consider the momentum dependent potential to avoid problem connected with energy conservation within this approach. The nucleon potential was already described in the nuclear properties chapter. For other particles we can use the real parts of the corresponding optical potential.

15.2.1 Nucleon potential.

The i-th nucleon potential is defined as the sum of Fermi-potential $T^F(r)$, binding energy $B(A,Z)$ and Coulomb potential $V_C(Z,r)$ (only for protons):

$$V(A,Z,r_i) = T^F(r_i) + B(A,Z) + V_C(Z,r_i), \tag{15.1}$$

where r_i is the i-th nucleon radius vector, A is the nucleus atomic number and Z is the nucleus charge, respectively.

15.2.2 Optical potentials.

The optical potential represents all particle-nucleon interactions between incident particle and target nucleus. Its real part describes the scattering and its imaginary part describes the absorption. The real part is often used to describe bound states. The simplest form of optical potential is

$$V_{opt} = U(r) + iW(r), \tag{15.2}$$

where the real part of optical potential is defined as $U(r) = -U_0 f(r)$. Here U_0 is depth and $f(r)$ is the radial form factor. This form factor is often taken in the Woods-Saxon form

$$f(r) = \frac{1}{1 + \exp\left(\frac{r-R}{a}\right)} \tag{15.3}$$

where R and a are the radius and surface diffuseness parameters, which are different for different nuclei. The shape of imaginary part of the optical potential $W(r)$ varies, essentially depending on the energy of incident particle. At low incident energies the absorption takes place mainly near the nuclear surface, whereas the energy rises absorption inside nuclear volume becomes important. Such form of optical potential is used mainly for description of nucleon, antiproton, kaon as well as lambda and sigma nucleus interactions. The optical potential is used for pions has more complicated form [3]:

$$V_{opt} = \beta(k,r) - \nabla\alpha(k,r) - \nabla^2 q(k,r), \tag{15.4}$$

where the functions β, α, and q depend on the coordinate r and momentum p of an incident particle.

15.2.3 Pion-nucleus interaction potential.

The optical πA-potential of the second order [3] defined by Eq. (15.4), is a rather complicated function of the pion momentum and of the pion radius r. This potential can be either attractive or repulsive. Such potential is difficult to implement. To simplify situation for pions we can use

$$V_\pi(r) = -V_0^\pi \theta(r), \tag{15.5}$$

where $V_0^\pi = 25$ MeV and $\theta(x)$ is the unit step function. The value $V_0^\pi = 25$ is often used in the cascade models [6]. Also the value $V_0^\pi = 0$ MeV is recommended by authors of [7].

15.2.4 Kaon-nucleus interaction potential.

This optical potential is used for an analysis of kaon atoms and can be written in the form

$$V_{opt}(r) = -\frac{2\pi}{\mu}\left(1 + \frac{m_K}{m}\right)[a^{eff}_{K^-n}\rho_n(r) + a^{eff}_{K^-p}\rho_p(r)], \tag{15.6}$$

where μ is K-nucleus reduced mass, m_K is kaon mass and m is mass of the nucleon. $\rho_n(r)$ and $\rho_p(r)$ are the neutron and proton density distributions. They are normalised to give the number of nucleons. $a^{eff}_{K^-n}$ and $a^{eff}_{K^-p}$ are complex effective scattering lengths for kaon-neutron and kaon-proton interactions, respectively.

The simplest form of this optical potential is often used together with Coulomb potential:

$$V_{opt}(r) = -\frac{2\pi}{\mu}\left(1 + \frac{m_K}{m}\right)a\rho(r), \tag{15.7}$$

where $\rho(r)$ is the nuclear density distribution normalised to the number of nucleons. The complex coefficient $a = 0.35 \pm 0.03 + i(0.82 \pm 0.03$ fm can be found in [4] and $a = 0.63 \pm 0.06 + i(0.89 \pm 0.05$ fm can be found in [5]. These coefficients are determined by fit of the K-atom data.

15.2.5 Antiproton- and sigma-nucleus interaction potential.

For antiproton and sigma atoms an optical potential with the form similar to the kaon-nucleus potential can be used:

$$V_{opt}(r) = -\frac{2\pi}{\mu}\left(1 + \frac{m_{\bar{p}}}{m}\right)a\rho(r), \tag{15.8}$$

where μ is $\bar{p}$-nucleus reduced mass and $m_{\bar{p}}$ is the antiproton mass. The complex coefficient $a = 1.53 \pm 0.27 + i(2.50 \pm 0.25$ fm is determined by fit of the $\bar{p}$-atom data [4]. To fit the sigma-atom data the value of $a = 0.36 \pm 0.05 + i(0.19 \pm 0.03$ fm was applied [4].

15.3 Hadron propagation.

We can take into account the curvatures of hadron (except pions) trajectories due to particle potentials.

Usually the curvature effects and other potential effects are simplified in a cascade approach by taken into account the refraction and reflection of nucleons due to a jump of potential [6], [7]. The influence of intranuclear nucleons on the incoming nucleon is taken into account by adding to its kinetic energy this effective nuclear potential, i.e. the kinetic energy of a nucleon is calculated from the bottom of a potential well. For a proton leaving the nucleus the kinetic energy of this proton has to be sufficiently high to overcome Coulomb barrier is given by

$$V_C(A,Z) = C\frac{Z}{r_0(1+A^{1/3})}, \tag{15.9}$$

where $C = 1.44$ MeVfm, $r_0 = 1.3$ fm.

To consider a potential influence at each time step Δt we have to solve equations of motion. The equations of motion of the i-th hadron with 3-coordinate $\mathbf{r}_i$ and 3-momentum $\mathbf{p}_i$ are the Newtonian equations:

$$\dot{\mathbf{r}_\mathbf{i}} = \frac{\partial H}{\partial \mathbf{p}_i},\ \dot{\mathbf{p}_\mathbf{i}} = -\frac{\partial H}{\partial \mathbf{r}_i}. \tag{15.10}$$

Hamiltonian H is given by

$$H = T(p_i) + V_i(r_i), \tag{15.11}$$

where $T(p_i)$ and $V_i(r_i)$ are i-th hadron kinetic energy and potential energy, respectively.

Numerical integration of the Newtonian's equations of motion (15.10) can be done by the Heun's method. This is an improved version of the second order Runge-Kutta method [8]. Thus, $\mathbf{r}_i$ and $\mathbf{p}_i$ for particle i are calculated according to

$$\mathbf{r}_i(t+\Delta t) = \mathbf{r}_i(t) + \Delta t\left[c_1\frac{\partial H(t)}{\partial \mathbf{p}_i} + c_2\frac{\partial H(t+c_3\Delta t)}{\partial \mathbf{p}_i(t+c_3\Delta t)}\right], \tag{15.12}$$

$$\mathbf{p}_i(t+\Delta t) = \mathbf{p}_i(t) - \Delta t\left[c_1\frac{\partial H(t)}{\partial \mathbf{r}_i} + c_2\frac{\partial H(t+c_3\Delta t)}{\partial \mathbf{r}_i(t+c_3\Delta t)}\right], \tag{15.13}$$

where time step $\Delta t = 0.01$ fm/c can be chosen. Parameters $c_2 = 0.75$, because this value is known [8] to reduce numerical errors (e.g. $c_2 = 0.5$ gives results the second order Runge-Kutta method), and $c_1 = 1-c_2$, $c_3 = 1/(2c_2)$.

15.4 Pion absorption by the nuclear meadium.

15.4.1 Cross section of π-absorption by a quasi-deutron pair.

We consider two mechanisms of the pion absorption. For low energy the s-wave (non-resonant) absorption is dominated, while the energy rises the two-steps p-wave (resonant) absorption, which is going through the resonance excitations and deexcitations, e. g. $\pi + N \to \Delta$ then $\Delta \to NN$, plays an important role.

The s-wave absorption is also important for the stopped negative pions. The s-wave absorption cross section of pion with energy E_π and momentum p in the laboratory frame is derived from the optical model [1] as

$$\sigma_s^A(E_\pi) = \frac{4\pi}{p}\left[1 + \frac{E_\pi}{2m} ImB_0(E_\pi)\right]\rho(r), \tag{15.14}$$

where $\rho(r)$ is nuclear density and $ImB_0 = 0.14$ fm^4 can be taken.

15.4.2 π-absorption simulation.

From Eq. (15.14) we can find the absorption mean free path

$$\lambda_{abs}(p, r) = 1/[\sigma_s^A(E_\pi)\rho(r)] \tag{15.15}$$

and use it to sample pion absorption point r.

Sampling of the absorption point can be done according to the next algorithm [2]. The number of mean free paths of travel particle is

$$n_\lambda = \int \frac{dr}{\lambda(r)}. \tag{15.16}$$

n_r is a random variable denoting the number of mean free paths from a given point until the interaction point. It is distributed according to

$$P(n_r < n_\lambda) = 1 - \exp(-n_\lambda). \tag{15.17}$$

Thus, n_λ can be sampled according to

$$n_\lambda = -\ln \xi, \tag{15.18}$$

where ξ are random numbers uniformly distributed between 0 and 1. n_λ is updated after each step Δr using the relation:

$$n'_\lambda = n_\lambda - \frac{\Delta r}{\lambda(r)}, \tag{15.19}$$

until $n'_\lambda \leq 0$.

This algorithm can also be used to find interaction point in the case of the photon absorption.

The s-wave (non-resonant) pion absorption is determined mainly by the the simplest cluster consisting of two nucleons. Once a pion has been absorbed by a nucleon pair, the pion mass is converted into kinetic energy of nucleons. Each nucleon has energy $E_N = m_\pi/2$ in the centre of mass system of the pair. In this system nucleons fly away isotropically in the opposite directions. The initial momentum of the pair is taken as the sum of the nucleon Fermi momenta.

15.5 Pauli blocking simulation.

A free particle interaction cross section is reduced to an effective cross section by the Pauli-blocking due to the Fermi statistics. For each collision the phase-space densities f_i, where i means fermion, in the final states should be checked in order to assure that the final distribution in phase space is in agreement with the Pauli principle, which rules out the posibility of finding more than one fermion in a single quantum state. We can apply the so-called cascade Pauli blocking procedure [6], [7] for the nucleon final states.

15.5.1 Cascade Pauli blocking procedure.

In this procedure, a nucleus with atomic number A and charge Z is treated as an ideal local completely degenerated Fermi gas of nucleons with coordinates $\mathbf{r}$, momenta $\mathbf{p}$. The nucleon phase-space density is approximated by

$$f_i(\mathbf{r}, \mathbf{p}) = \Theta[\mathbf{p}_i^F(\mathbf{r}) - \mathbf{p}]. \tag{15.20}$$

Because all states below Fermi-level are already occupied, after each interaction one should check that the momenta $\mathbf{p'}_i$ of all secondary nucleons are above the Fermi-level, i. e.

$$p'_i > p_i^F(r). \tag{15.21}$$

If among the secondary nucleons there is a nucleon with momentum lower the the Fermi-level, then such collision is considered as prohibited (Pauli-blocked).

15.6 Residual nucleus parameters.

The number of nucleons A_{res} as well as the number of protons Z_{res} and total momentum P_{res} of the residual nucleus are determined by the relations:

$$A_{res} = A + q_p - \sum_{i=1}^{N_{out}} q_i^c, \tag{15.22}$$

$$Z_{res} = Z + e_p - \sum_{i=1}^{N_{out}} e_i^c, \tag{15.23}$$

$$\mathbf{P}_{res} = \mathbf{p}_p - \sum_{i=1}^{N_{out}} \mathbf{p_i^c}, \tag{15.24}$$

$$\mathbf{L}_{res} = \mathbf{L}_0 - \sum_{i=1}^{N_{out}} \mathbf{l_i^c}. \tag{15.25}$$

Here A, Z are the numbers of target nucleons and protons, $q_p, e_p, \mathbf{p}_p$ are the baryon number, the charge and momentum of incoming particle, respectively. $\mathbf{L}_0$ is initial particle-nucleus angular momentum. $q_i, e_i, \mathbf{p}_i, \mathbf{l_i^c}$ denote the baryon number, charge, momentum and angular momentum, respectively, carried away by the ith outgoing particle. N_{out} is the total number of outgoing particles. As in the case of the hadron transport model angular momentum of emitted particle can be treated as a classical vector $\mathbf{l_i^c} = [\mathbf{p_i r_i}]$, where $\mathbf{r_i^c}$ is the radius vector at the exit of the transported particle i from the nucleus and $\mathbf{p_i^c}$ is its momentum.

The excitation energy U of residual nucleus is

$$U = E_{in} - \sum_{i=1}^{N_{out}} E_i, \tag{15.26}$$

where E_{in} is the initial excitation energy and E_i is the energy of outgoing particle, if it is not a nucleon. In the case of nucleon projectile the initial excitation energy E_{in} is defined by nucleon kinetic energy T_{in} and nuclear binding energy $B(A, Z)$:

$$E_{in} = T_{in} + B(A, Z). \tag{15.27}$$

In the case of non-nucleon projectile E_{in} is simple total projectile energy. If nucleon is an outgoing particle, then $E_i = T_i + B(A, Z)$, where T_i is the nucleon kinetic energy.

For the preequilibrium exciton model (see the corresponding chapter) we need also to know the number of excitons, which consists from the number of "particles" and the number of "holes". All cascade nucleons, which were not able to leave a nucleus are called "particles". The "holes" are results of intranuclear interactions, when particles stroke nucleons from a nucleus and they occupy states below the Fermi level $T_F(r)$.

15.7 Stopped particle absorption.

15.7.1 Mechanism of the stopped particle absorption by a nucleus.

The absorptions of the stopped π^--meson, K^--meson and $\bar{p}$ by a nucleus proceeds in several steps [7]:

- Particle is captured by the Coulomb field of a nucleus forming a pion or a kaon or $\bar{p}$-atom;
- Such atom deexcites through the emission of Auger-electrons and X-rays;
- Stopped particle from the atomic orbit is captured by the nucleus (by two or more of intranuclear nucleons in the case of a stopped pion or by reaction on a quasifree nucleon producing the pion and Λ or Σ hyperon in the case of the stopped kaon or by annihilation on a quasifree nucleon in the case of $\bar{p}$-capture);
- Rescatterings of fast hadrons produced in a stopped particle absorption (intranuclear hadron transport);
- Decay of the excited residual nucleus.

Thus absorption processes for the stopped pion, kaon and antiproton are similar. However, there are some peculiarities for a particular absorption process.

15.7.2 Absorption of the stopped π^- by a nucleus.

As follows from calculations within the framework of the optical model [9] with the Kisslinger potential [10] the capture a pion from an orbit of atom takes place at radius r in the nuclear surface and absorption probability $P_{abs}(r)$ can be approximated by

$$P_{abs}(r) = P_0 \exp\left[-0.5\left(\frac{r - R_\pi}{D_\pi}\right)^2\right], \tag{15.28}$$

where parameter $R_\pi \approx R_{1/2}$ and $R_{1/2}$ is the half-density radius. The values of another parameter D_π for different nuclei can be found in [7].

The absorption of the pion can be considered as the s-wave (non-resonant) absorption mainly by the the simplest cluster consisting of two nucleons (np) or (pp).

15.7.3 Absorption of the stopped K^- by a nucleus.

In this case the absorption probability can be chosen the same as in annihilation of the stopped antiprotons.

15.7.4 Annihilation of the stopped $\bar{p}$ by a nucleus.

In this case the absorption probability is also given by Eq. (15.28) with the values of $R_{\bar{p}} = R_\pi$ and dispersion $D^2 = 1$ fm^2 [11].

The annhihilation of antiproton on a quasifree nucleon can be modelled via the annihilation of a diquark-antidiquark with subsequent fragmentation of the meson string within the baryon annihilation model (see the corresponding chapter).

15.8 MC procedure for the stopped particle absorption.

The stopped particle absorption MC procedure consists from several steps:

1. Sample position inside nucleus, where absorption takes place, according to phenomenological probability distribution;
2. Simulate particle absorption according to particular particle absorption mechanism;
3. Using the intranuclear hadron transport model perform rescatterings of final state (after absorption) particles;
4. Using corresponding deexcitation model (see the corresponding chapters) perform decay of a residual excited nucleus.

15.9 Intranuclear transport model algorithm.

The hadron transport algorithm can be considered as a step by step updating of a particle vector:

1. Create a vector of particles: assign initial particle types, their coordinates and momenta etc. Assign initial value for time evolution parameter;
2. For chosen evolution parameter (time step) find pairs of particles (according to a collision criterion), which are assumed to collide and particles (according to their life times), which are assumed to decay;
3. Perform particle collisions and particle decays. Collisions and decays convert incoming particles into outgoing particles. During this step one should update also particle coordinates and momenta (particle propagation);
4. Starting from (2) perform the next step.

At any evolution parameter step one needs a possibility to interrupt "updating" process. Thus, at each evolution time step for the system consisting

from n particles we are looking for the particles, which will decay according to their lifetimes and the pairs (i, j), where $i \neq j$ and $i, j = 1, 2, ..., n$, of particles, which will collide according to the chosen collision criterion. The found pairs are ordered chronologically using their collision times. This algorithm is very simple and general enough, but it is the CPU time consuming, if n is large. At each time step we need to perform $n(n-1)/2 \sim n^2$ computations.

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Chapter 16

Pomeron based parton string model.

16.1 Reaction initial state simulation.

16.1.1 Allowed hadron high energy reactions.

The Pomeron based Parton String Model (PPSM) is capable to predict final states, i. e. produced hadrons, which belong to the pseudoscalar meson nonet and the baryon/antibaryon octet, for the reactions on nucleon and nuclear targets with a large variety of hadron (the same types as final state hadrons) and nuclear projectiles. The allowed bombarding energies in the hadron-nucleon collision should be above 2-pion production threshold. In the case of hadron-nucleus or nucleus-nucleus collisions the initial energies $\sqrt{s} > 5$ AGeV is recommended. This model is also able to predict final states in the photon-nucleon and photon-nucleus inelastic collisions at the initial energies $\sqrt{s} > 5$ AGeV. The last case is described in separate chapter.

The procedure to generate initial reaction state is the same as for the hadron transport model (see the hadron transport model chapter). The chosen "working" frames, where interaction process is simulated, for this particular model is the c.m. frame of colliding particles and the particle equal velocities frame in the case of nuclear interaction.

16.2 Nuclear collision participants.

16.2.1 MC procedure to define collision participants.

The inelastic hadron-nucleus or nucleus-nucleus interactions at ultra-relativistic energies are considered as independent hadron-nucleon or nucleon-nucleon inelastic collisions. It was shown long time ago [1] for the hadron-nucleus collision that such picture can be obtained starting from the Regge-Gribov approach [2], when one assumes that the hadron-nucleus elastic scat-

tering amplitude is the result of reggeon exchanges between initial hadron and nucleons from target-nucleus. This result can be extended for the nucleus-nucleus collision case and leads to simple and efficient MC procedure [3] to define the interaction cross sections and the number of the nucleons participating in the inelastic nucleus-nucleus collision:

- We randomly distribute A nucleons from the projectile-nucleus and B nucleons from the target-nucleus inside the impact parameter plane according to the weight functions $T([\vec{b}_i^A])$ and $T([\vec{b}_j^B])$, respectively. These functions represent probability densities to find sets of the nucleon impact parameters $[\vec{b}_i^A]$ and $[\vec{b}_j^B]$, where $i = 1, 2, ..., A$ and $j = 1, 2, ..., B$, respectively.
- For each pair of nucleons i and j with chosen impact parameters $\vec{b}_i^A$ and $\vec{b}_j^B$ we check whether they interact inelastically or not using the probability $p_{ij}(\vec{b}_i^A - \vec{b}_j^B, s)$, where $s_{ij} = (p_i + p_j)^2$ is the total c.m. energy squared of the given nucleons with the 4-momenta p_i and p_j, respectively.

The described MC procedure is based on the probability $P(\vec{B}, s)$ at given impact parameter $\vec{B}$ and at given total c.m. nucleon-nucleon energy squared s (to simplify notations, we assume, that all interacting nucleon pairs have the same s) of a such configuration, when several pairs of nucleons from projectile and target nuclei interact inelastically and the rest of the nucleons do not participate in collisions:

$$P(\vec{B}, s) = \left\langle \prod_{i,j=1} p_{ij}(\vec{b}_i^A - \vec{b}_j^B, s) \prod_{k,l=1} [1 - p_{kl}(\vec{b}_k^A - \vec{b}_k^B, s)] \right\rangle . \tag{16.1}$$

The last equation can be rewritten more explicitly as follows

$$\begin{aligned} P(\vec{B}, s) = \int \prod_{i,j=1} p_{ij}(\vec{b}_i^A - \vec{b}_j^B, s) \prod_{k,l=1} [1 - p_{kl}(\vec{b}_k^A - \vec{b}_l^B, s)] \times \\ \times T_A(\vec{b}_1^A) T_A(\vec{b}_2^A) ... T_A(\vec{B} - \vec{b}_B^B) d\vec{b}_1^A d\vec{b}_2^A ... d\vec{b}_B^B . \end{aligned} \tag{16.2}$$

In the Regge-Gribov approach [4], [2] the probability for an inelastic collision of nucleons i and j as a function of the squared impact parameter difference $b_{ij}^2 = (\vec{b}_i^A - \vec{b}_j^B)^2$ and s is given by

$$p_{ij}(\vec{b}_i^A - \vec{b}_j^B, s) = c^{-1}[1 - \exp\{-2u(b_{ij}^2, s)\}] = \sum_{n=1}^{\infty} p_{ij}^{(n)}(\vec{b}_i^A - \vec{b}_j^B, s), \tag{16.3}$$

where

$$p_{ij}^{(n)}(\vec{b}_i^A - \vec{b}_j^B, s) = c^{-1} \exp\{-2u(b_{ij}^2, s)\} \frac{[2u(b_{ij}^2, s)]^n}{n!}. \tag{16.4}$$

is the probability to find the n cut Pomerons or the probability for $2n$ string produced in an inelastic nucleon-nucleon collision, if we assume that each cut

Pomeron can be substituted by two strings. These probabilities are defined in terms of the (eikonal) amplitude of nucleon–nucleon elastic scattering with Pomeron exchange:

$$u(b_{ij}^2, s) = \frac{z(s)}{2} \exp[-b_{ij}^2/4\lambda(s)]. \tag{16.5}$$

The quantities $z(s)$ and $\lambda(s)$ are expressed through the parameters of the Pomeron trajectory $\alpha_P^{'} = 0.25\ GeV^{-2}$ and $\alpha_P(0) = 1.0808$, and the parameters of the Pomeron-nucleon vertex $R_P^2 = 3.56\ GeV^{-2}$ and $\gamma_P = 3.96\ GeV^{-2}$:

$$z(s) = \frac{2c\gamma_P}{\lambda(s)}(s/s_0)^{\alpha_P(0)-1} \tag{16.6}$$

$$\lambda(s) = R_P^2 + \alpha_P^{'} \ln(s/s_0), \tag{16.7}$$

respectively, where $s_0 = 3.0\ GeV^2$ is a dimensional parameter.

In Eqs. (16.3,16.4) the so-called shower enhancement coefficient $c = 1.4$ can be introduced to determine the contribution of diffractive dissociation[2]. Thus, the probability for diffractive dissociation of a pair of nucleons can be computed as

$$p_{ij}^d(\vec{b}_i^A - \vec{b}_j^B, s) = \frac{c-1}{c}[p_{ij}^{tot}(\vec{b}_i^A - \vec{b}_j^B, s) - p_{ij}(\vec{b}_i^A - \vec{b}_j^B, s)], \tag{16.8}$$

where

$$p_{ij}^{tot}(\vec{b}_i^A - \vec{b}_j^B, s) = (2/c)[1 - \exp\{-u(b_{ij}^2, s)\}]. \tag{16.9}$$

The Pomeron parameters are found from a global fit of the total, elastic, differential elastic and diffractive cross sections of the nucleon-nucleon interaction at different energies.

For the pion-nucleon and kaon-nucleon collisions the Pomeron vertex parameters and shower enhancement coefficient should be changed, e. g. $R_{P^2}^{\pi} = 2.36\ GeV^{-2}$, $\gamma_P^{\pi} = 2.17\ GeV^{-2}$, $s_0^{\pi} = 1.5\ GeV^2$, $c^{\pi} = 1.6$ and $R_{P^2}^{K} = 1.96\ GeV^{-2}$, $\gamma_P^{K} = 1.92\ GeV^{-2}$, $s_0^{K} = 2.3\ GeV^2$, $c^{\pi} = 1.8$ can be used to describe properly the total, elastic and diffractive cross sections (see the hadron cross section chapter).

Describing this model we consider only the Pomeron exchanges in the hadron elastic scattering amplitude. With such amplitude we are not able to predict properly the hadron interaction cross sections and also the corresponding interaction probabilities in the whole energy range under investigation. We will strongly underestimate the low energy cross section parts and the corresponding interaction probabilities.

At this energy region neglecting by the real part of hadron scattering amplitude we can express the eikonal $u(b_{ij}^2, s)$ through experimental values of the total $\sigma_{tot}(s)$, elastic $\sigma_{el}(s)$ and single diffraction $\sigma_d(s)$ cross sections:

$$\frac{z}{2} = \frac{\sigma_{tot}(s)}{4\pi B_{el}(s)} \tag{16.10}$$

$$\lambda(s) = 2B_{el}(s), \tag{16.11}$$

where

$$B_{el}(s) = \frac{\sigma_{tot}^2(s)}{8\pi[\sigma_{el}(s) + \sigma_d(s)]}. \tag{16.12}$$

16.2.2 Single diffraction cross section.

The proton-proton single diffraction cross section was parametrised in paper [5]

$$\sigma_{SD}(s) = (0.68 \pm 0.05)\left(1 + \frac{36 \pm 8}{s}\right)\ln(0.6 + 0.1s), \tag{16.13}$$

where s is the total c.m. energy squared.

16.2.3 Separation of hadron diffractive excitation.

For each pair of nucleons i and j with chosen impact parameters $\vec{b}_i^A$ and $\vec{b}_j^B$ we should check whether they interact inelastically or not using the probability

$$p_{ij}^{in}(\vec{b}_i^A - \vec{b}_j^B, s) = p_{ij}(\vec{b}_i^A - \vec{b}_j^B, s) + p_{ij}^{d}(\vec{b}_i^A - \vec{b}_j^B, s). \tag{16.14}$$

If interaction will be realized, then we have to consider it to be diffractive or nondiffractive with the probabilities

$$\frac{p_{ij}^{d}(\vec{b}_i^A - \vec{b}_j^B, s)}{p_{ij}^{in}(\vec{b}_i^A - \vec{b}_j^B, s)} \tag{16.15}$$

and

$$\frac{p_{ij}(\vec{b}_i^A - \vec{b}_j^B, s)}{p_{ij}^{in}(\vec{b}_i^A - \vec{b}_j^B, s)}. \tag{16.16}$$

We should note that, if we will consider all inelastic nucleon or hadron-nucleon collisions as the diffractive collisions (see the string excitation mechanism in the reggeon based parton string model chapter), we will obtain (as an option) the model version, which is very similar to the FRITIOF model[8].

16.3 Sampling of the kinky strings.

To determine the number of kinky strings are produced in hard hadron collisions we assume [6], [7] that each cut Pomeron can be substituted either by the two longitudinal strings as result of soft hadron interaction or by the two kinky strings as result of hard hadron interactions. This procedure was described in the reggeon based parton string model chapter.

16.4 Longitudinal and kinky string excitations.

The longitudinal and kinky string excitations for hadron-nucleon inelastic collision as well as hadron-nucleus and nucleus-nucleus inelastic collisions were described in the reggeon based parton string model chapter. It should be noted that in the hadron-nucleus and nucleus-nucleus cases we will produce also string spanned between valence and sea quarks.

16.5 Calculation of the residual nucleus characteristics.

It was described in the hadron transport model chapter.

16.6 MC procedure.

The pomeron based parton string model algorithm can be considered as a set of steps should be performed:

1. Create two vectors (projectile and target) of particles: assign initial projectile and target particle types, their coordinates and momenta. In the case of hadron-nucleus (or nucleus-nucleus) interaction one should perform target nucleus (or projectile and target nuclei) initial state simulation and sample impact parameter;

2. Sample collision participants and separated them into diffractive and non-diffractive. Store the total interaction four momentum of participants;

3. For non-diffractive inelastic collisions sample the number of the soft longitudinal and hard kinky string can be produced;

4. Excite and reexcite colliding particles in the case of diffractive collisions and create diffractive longitudinal strings, if particles are not participate in further soft or hard collisions;

5. Perform longitudinal and kinky string excitations;

6. Perform string decay simulation;

7. Correct energies and momenta of produced particles, if it is needed.

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Chapter 17

Hadron interaction with photons.

17.1 Reaction initial state.

The γ-nucleon and γ-nucleus interaction model is capable to predict final states, i.e. produced hadrons which belong to the pseudoscalar meson nonet and the baryon/antibaryon octet, of photon reactions on nucleon and nuclear targets.

The recommended bombarding energies in the photon-nucleon or photon–nucleus interaction have to be more than 20 MeV in the laboratory frame.

17.2 Photon-nucleon and photon-nucleus interactions at intermediate energies.

We can use the above described intranuclear hadron transport model to perform simulation of the γ-nucleus inelastic collisions at intermediate energies. In this particular model interaction of a particle with target nucleus is reduced to the interaction of a particle with intranuclear nucleons. We can realize several different mechanisms of the γ-nucleon interaction.

The first one is the absorption of γ-quantum by a quasideutron pair

$$\gamma + (np) \rightarrow np. \tag{17.1}$$

The photoabsorption cross section can be calculated according to quasideutron model [1], [2]

$$\sigma_A = kZ(1 - Z/A)\sigma_D, \tag{17.2}$$

where σ_D is the cross section for deutron photodisintegration and A, Z are the mass and charge numbers of the nucleus in question, respectively. The model parameter $k = 10$ can be taken. Using this cross section we are able to calculate the mean free path length $\lambda(e, r) = 1/\sigma_A(e)\rho(r)$, where e is the

photon energy and $\rho(r)$ is nucleon density at radius vector r. The algorithm to find absorption point inside nucleus is described in the intranuclear hadron transport model chapter.

The second one is the excitation of resonance and production of mesons:

$$\gamma N \to \Delta(1232), \gamma N \to N^*(1535) \tag{17.3}$$

At high energies the Regge approach can be used to calculate γ-nucleon cross sections (see the hadronic cross section model chapter) and the inelastic γ-nucleon interactions can be simulated using the reggeon based parton string model as described below.

17.3 Photon-nucleon and photon-nucleus interactions at high energies.

To simulate high energy photon interactions with nucleon and nucleus we can use the approach[4]. We consider following kinematics variables for γ-nucleon scattering: the Bjorken-x variable defined as $x = Q^2/2m\nu$ with Q^2, ν and m the photon virtuality, the photon energy and nucleon mass, respectively. The total energy squared of the γ-nucleon system is given by $s = Q^2(1-x)/x + m^2$. We restrict consideration to the range of small x-values and Q^2 is much less than s.

The Generalised Vector Dominance Model (GVDM) [3] assumes that the virtual photon fluctuates into intermediate $q\bar{q}$-states V of mass M, which subsequently may interact with a nucleon N. Thus, the total photon-nucleon cross section can be expressed by the relation [4]:

$$\sigma_{\gamma N}(s,Q^2) = 4\pi\alpha_{em}\int_{M_0^2}^{M_1^2} dM^2 D(M^2)\left(\frac{M^2}{M^2+Q^2}\right)^2\left(1+\epsilon\frac{Q^2}{M^2}\right)\sigma_{VN}(s,Q^2), \tag{17.4}$$

where integration over M^2 is performed between $M_0^2 = 4m_\pi^2$ and $M^2 = s$. Here $\alpha_{em} = e^2/4\pi = 1/137$ and the density of $q\bar{q}$-system per unit mass squared is given by

$$D(M^2) = \frac{R_{e^+e^-}(M^2)}{12\pi^2 M^2}, \tag{17.5}$$

$$R_{e^+e^-}(M^2) = \frac{\sigma_{e^+e^-\to hadrons}(M^2)}{\sigma_{e^+e^-\to\mu^+\mu^-}(M^2)} \approx 3\Sigma_f e_f^2, \tag{17.6}$$

where e_f^2 the squared charge of quark with flavour f. ϵ is the ratio between the fluxes of longitudinally and transversally polarised photons.

Similarly the inelastic cross section for the scattering of a photon with virtuality Q^2 with a nucleus of A nucleons at impact parameter B and the

γ-nucleon c.m. energy squared s is given by [5]:

$$\sigma_{\gamma A}(s,Q^2,B) = 4\pi\alpha_{em}\int_{M_0^2}^{M_1^2} dM^2 D(M^2)\left(\frac{M^2}{M^2+Q^2}\right)^2\left(1+\epsilon\frac{Q^2}{M^2}\right)\sigma_{VA}(s,Q^2,B), \tag{17.7}$$

To calculate γ-nucleon or γ-nucleus inelastic cross sections we need a model for the M^2-, Q^2- and s-dependence of the σ_{VN} or σ_{VA}. We can apply the Gribov-Regge approach, similarly as it was done for h-nucleon or h-nucleus inelastic cross sections (see the pomeron based string parton model chapter).

The effective cross section for the interaction of a $q\bar{q}$-system with squared mass M^2 with nucleus while the coherence length

$$d = \frac{2\nu}{M^2+Q^2} \tag{17.8}$$

exceeds the average distance between two nucleons can be written as follows

$$\sigma_{VA}(s,Q^2,B) = \int\prod_{i=1}^{A} d^3r_i\rho_A(\mathbf{r}_i)\left(1-\left|\prod_{i=1}^{A}[1-u(s,Q^2,M^2,b_i^2)]\right|^2\right). \tag{17.9}$$

Here the amplitude (eikonal) $u(s,Q^2,M^2,b_i^2)$ for the interaction of the hadron fluctuation with i-th nucleon is given by [5]

$$u(s,Q^2,M^2,\mathbf{b}_i) = \frac{\sigma_{VN}(s,Q^2,M^2)}{8\pi\lambda(s,Q^2,M^2)}\left(1-i\rho\exp\left[-\frac{b^2}{4\lambda(s,Q^2,M^2)}\right]\right), \tag{17.10}$$

where $\rho\approx 0$ is the ratio of real and imaginary parts of scattering amplitude at zero angle. The amplitude parameters are an effective $q\bar{q}$-nucleon cross section

$$\sigma_{VN}(s,Q^2,M^2) = \frac{\tilde{\sigma}_{VN}(s,Q^2)}{M^2+Q^2+C^2}, \tag{17.11}$$

where $C^2 = 2$ GeV^2, and

$$\lambda(s,Q^2,M^2) = 2+\frac{m_\rho^2}{M^2+Q^2}+\alpha'_P\ln\left(\frac{s}{M^2+Q^2}\right). \tag{17.12}$$

The values of $\tilde{\sigma}_{VN}(s,Q^2)$ are calculated in paper [5]. It was shown [5] that Q^2 dependence of $\sigma_{VN}(s,Q^2)$ is very week at $Q^2 < m_\rho^2+C^2$, where m_ρ is the ρ-meson mass. We can omit this dependence and we also can use $\sigma_{VN}(s,Q^2)$ has been calculated in [5] at $M^2 = m_\rho^2$.

If coherence length is smaller that the integrated over B distance between two nucleons, then cross section $\sigma_{VA} = A\sigma_{VN}$.

17.4 MC procedure.

At intermediate energies γ-nucleon and γ-nucleus interactions can be performed within the intranuclear hadron transport model similarly as the hadron-nucleon and hadron-nucleus interactions.

At high energies the Monte Carlo procedure in the case of γ-nucleon collision can be outlined as follows:

- At given c.m. energy squared and at given virtuality Q^2 sample mass M^2 of hadron $q\bar{q}$ fluctuation according to the Eq.(17.5) and sample its flavour according to statistical weights: $\omega_{u\bar{u}} = 1/2$, $\omega_{d\bar{d}} = 1/4$ and $\omega_{s\bar{s}} = 1/4$ are derived from (17.6);

- Sample the momentum fraction x of a valence quark inside a hadron fluctuation according to

$$\rho(x) \sim \frac{1}{\sqrt{x(1-x)}} \tag{17.13}$$

  and transverse momentum of a quark according to the Gaussian distribution as for hadrons;

- Split nucleon into quark and diquark as it was described for hadron-nucleon interaction in the reggeon based parton string model chapter;

- Create two strings spanned between quark from a hadron fluctuation and diquark from nucleon and between antiquark from a hadron fluctuation and quark from nucleon;

- Decay string into hadrons as it was described in the string decay chapter.

In the case of γ-nucleus collision the MC procedure is following:

- At given c.m. energy squared and at given virtuality Q^2 sample mass M^2 of hadronic $q\bar{q}$ fluctuation and sample its flavour as it was described for the γ-nucleon collision;

- Calculate coherence length d;

- If coherence length less than intranuclear distance, then simulate inelastic hadron fluctuation-nucleon collision at chosen impact parameter B as was described above;

- If coherence length more than distance between nucleons, then perform simulation of hadron fluctuation-nucleus collision at chosen impact parameter B using the Pomeron based parton string model similarly as for meson-nucleus interactions. For this case the probability of

inelastic collision of hadron fluctuation with nucleon i at given impact parameter $\mathbf{b}_i$ is calculated according to

$$p_{VN}(s, b^2) = 1 - exp[-2u(s, b^2)]; \tag{17.14}$$

with the eikonal $u(s, b^2)$ defined by the Eq. (17.10) at $Q^2 = 0$ and $M^2 = M_\rho$.

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Chapter 18

Preequilibrium exciton model.

18.1 Reaction initial state.

The preequilibrium exciton model based on the approach[1]. It is considered as an extension of a hadron transport model. It gives a possibility to extend the low energy range of the intranuclear hadron transport model applicability for nucleon-nucleus inelastic collision and it provides a "smooth" transition from kinetic stage of reaction described by the intranuclear transport model to the equilibrium stage of reaction, which can be described by equilibrium deexcitation models (see the next chapters).

The initial information for calculation of preequilibrium nuclear stage consists from the atomic mass number A, charge Z of residual nucleus, its four momentum P_0, angular momentum $\vec{L}_0$, its excitation energy U and number of excitons n equals the sum of number of particles p (p_Z from them are charged) and number of holes h.

The exciton energies are calculated in the intranuclear hadron transport model from the Fermi energy level T_F. All cascade nucleons with kinetic energies above T_F and absorbed by the target nucleus are called particles. The holes are results of cascade interactions, when particles stroke nucleons from nucleus. The holes occupy states below T_F.

There are some peculiarities to get an initial information about preequilibrium stage for different kind of reactions. Particularly, the excitation energy of nucleus in the case of nucleon-nucleus interaction can be calculated as

$$U = T_{init} + B(A, Z), \tag{18.1}$$

where T_{init} is initial nucleon kinetic energy and $B(A, Z)$ is the binding energy of a nucleon. The initial number of exciton is a model parameter and usually defined from comparison with experimental data. The recommended initial configuration is $2p1h$, i. e. $n = 3$.

Another way to obtain this information is to invoke the intranuclear hadron transport model. As it was described in the intranuclear transport model chapter this model is able to predict the excitation energy and the exciton numbers of residual nucleus as well as its other parameters.

At the preequilibrium stage of reaction we take into account all possible nuclear transition, when the number of excitons n is changed on the $\Delta n = +2, -2, 0$ [1]. These transitions are described by the corresponding transition probabilities. Only emission of neutrons, protons, deutrons, thritium and helium nuclei are taken into account.

18.2 Simulation of the preequilibrium reaction.

The preequilibrium stage of nuclear reaction is modelled until nuclear system is not an equilibrium state. Further emission of nuclear fragments or photons from excited nucleus can be simulated using an equilibrium emission model (see the next chapter).

18.2.1 Equilibrium condition.

In the state of statistical equilibrium, which is characterised by the equilibrium number of excitons n_{eq}, all three type of transitions are equiprobable. Thus, n_{eq} is fixed by $\omega_{+2}(n_{eq}, U) = \omega_{-2}(n_{eq}, U)$. From this condition we can obtain

$$n_{eq} = \sqrt{0.5 + 2gU}. \tag{18.2}$$

18.2.2 Level density of the excited (n-exciton) states.

To obtain Eq. (18.2) it was also assumed an equidistant scheme of single-particle levels with the density $g \approx 0.595aA$, where a is the level density parameter (see the nuclear properties chapter). The level density of the n-exciton state is

$$\rho_n(U) = \frac{g(gU)^{n-1}}{p!h!(n-1)!}. \tag{18.3}$$

18.2.3 Transition probabilities.

The partial transition probabilities to change the exciton number by Δn are determined by the squared matrix element averaged over allowed transitions $< |M|^2 >$ and the density of final states $\rho_{\Delta n}(n, U)$, which are really accessible in this transition. They can be defined as follows:

$$\omega_{\Delta n}(n, U) = \frac{2\pi}{h} < |M|^2 > \rho_{\Delta n}(n, U). \tag{18.4}$$

The density of final states $\rho_{\Delta n}(n, U)$ were derived in paper [2] using the Eq. (18.3) for the level density of the n-exciton state and later corrected for the

Pauli principle and indistinguishability of identical excitons in paper [3]:

$$\rho_{\Delta n=+2}(n,U) = \frac{1}{2}g\frac{[gU - F(p+1,h+1)]^2}{n+1}[\frac{gU - F(p+1,h+1)}{gU - F(p,h)}]^{n-1}, \tag{18.5}$$

$$\rho_{\Delta n=0}(n,U) = \frac{1}{2}g\frac{[gU - F(p,h)]}{n}[p(p-1) + 4ph + h(h-1)] \tag{18.6}$$

and

$$\rho_{\Delta n=-2}(n,U) = \frac{1}{2}gph(n-2), \tag{18.7}$$

where $F(p,h) = (p^2 + h^2 + p - h)/4 - h/2$ was taken to be equal zero. To avoid calculation of the averaged squared matrix element $< |M|^2 >$ it was assumed [1] that transition probability $\omega_{\Delta n=+2}(n,U)$ is the same as the probability for quasi-free scattering of a nucleon above the Fermi level on a nucleon of the target nucleus, i. e.

$$\omega_{\Delta n=+2}(n,U) = \frac{< \sigma(v_{rel})v_{rel} >}{V_{int}}. \tag{18.8}$$

In Eq. (18.8) the interaction volume is estimated as $V_{int} = \frac{4}{3}\pi(2r_c + \lambda/2\pi)^3$, with the De Broglie wave length $\lambda/2\pi$ corresponding to the relative velocity $< v_{rel} >= \sqrt{2T_{rel}/m}$, where m is nucleon mass and $r_c = 0.6$ fm.

The averaging in $< \sigma(v_{rel})v_{rel} >$ is further simplified by

$$< \sigma(v_{rel})v_{rel} >=< \sigma(v_{rel}) >< v_{rel} > . \tag{18.9}$$

For $\sigma(v_{rel})$ we take approximation:

$$\sigma(v_{rel}) = 0.5[\sigma_{pp}(v_{rel}) + \sigma_{pn}(v_{rel}]P(T_F/T_{rel}), \tag{18.10}$$

where factor $P(T_F/T_{rel})$ was introduced to take into account the Pauli principle. It is given by

$$P(T_F/T_{rel}) = 1 - \frac{7}{5}\frac{T_F}{T_{rel}} \tag{18.11}$$

for $\frac{T_F}{T_{rel}} \leq 0.5$ and

$$P(T_F/T_{rel}) = 1 - \frac{7}{5}\frac{T_F}{T_{rel}} + \frac{2}{5}\frac{T_F}{T_{rel}}(2 - \frac{T_{rel}}{T_F})^{5/2} \tag{18.12}$$

for $\frac{T_F}{T_{rel}} > 0.5$.

The free-particle proton-proton $\sigma_{pp}(v_{rel})$ and proton-neutron $\sigma_{pn}(v_{rel})$ interaction cross sections are estimated using the equations [4]:

$$\sigma_{pp}(v_{rel}) = \frac{10.63}{v_{rel}^2} - \frac{29.93}{v_{rel}} + 42.9 \tag{18.13}$$

and

$$\sigma_{pn}(v_{rel}) = \frac{34.10}{v_{rel}^2} - \frac{82.2}{v_{rel}} + 82.2, \tag{18.14}$$

where cross sections are given in mbarn.

The mean relative kinetic energy T_{rel}, which is needed to calculate $< v_{rel} >$ and the factor $P(T_F/T_{rel})$, was computed as $T_{rel} = T_p + T_n$, where mean kinetic energies of projectile nucleons $T_p = T_F + U/n$ and target nucleons $T_N = 3T_F/5$, respectively.

Combining Eqs. (18.4) - (18.8) and assuming that $< |M|^2 >$ are the same for transitions with $\Delta n = 0$ and $\Delta n = \pm 2$ we obtain for other transition probabilities:

$$\omega_{\Delta n=0}(n,U) = \\ = \frac{<\sigma(v_{rel})v_{rel}>}{V_{int}} \frac{n+1}{n} [\frac{gU-F(p,h)}{gU-F(p+1,h+1)}]^{n+1} \frac{p(p-1)+4ph+h(h-1)}{gU-F(p,h)} \tag{18.15}$$

and

$$\omega_{\Delta n=-2}(n,U) = \\ = \frac{<\sigma(v_{rel})v_{rel}>}{V_{int}} [\frac{gU-F(p,h)}{gU-F(p+1,h+1)}]^{n+1} \frac{ph(n+1)(n-2)}{[gU-F(p,h)]^2}. \tag{18.16}$$

18.2.4 Emission probability for nucleons.

Emission process probability has been chosen similar as in the classical Weisskopf-Ewing evaporation theory [5] (see also the next chapter). Probability to emit nucleon b in the energy interval $(T_b, T_b + dT_b)$ is given by

$$W_b(n,U,T_b) = \sigma_b(T_b) \frac{(2s_b+1)\mu_b}{\pi^2 h^3} R_b(p,h) \frac{\rho_{n-b}(E^*)}{\rho_n(U)} T_b, \tag{18.17}$$

where $\sigma_b(T_b)$ is the inverse (absorption of nucleon b) reaction cross section. Here s_b and m_b are nucleon spin and reduced mass,respectively. The factor $R_b(p,h)$ takes into account the condition for the exciton to be a proton or neutron. $\rho_{n-b}(E^*)$ and $\rho_n(U)$ are level densities of nucleus after and before nucleon emission are defined in the evaporation model (ses the quilibrium evaporation model chapter), respectively. $E^* = U - Q_b - T_b$ is the excitation energy of nucleus after fragment emission.

18.2.5 Emission probabilities for complex fragments.

It was assumed [1] that nucleons inside excited nucleus are able to "condense" and form complex fragment. The condensation probability to create fragment consisting from N_b nucleons inside nucleus with A nucleons is given by

$$\gamma_{N_b} = N_b^3 (V_b/V)^{N_b - 1} = N_b^3 (N_b/A)^{N_b - 1}, \tag{18.18}$$

where V_b and V are fragment b and nucleus volumes, respectively. The last equation was estimated [1] as the overlap integral of (constant inside a volume) wave function of independent nucleons with that of the fragment.

During the preequilibrium stage a condensed fragment can be emitted. The probability to emit the fragment can be written as [1]

$$W_b(n,U,T_b) = \gamma_{N_b} R_b(p,h) \frac{\rho(N_b,0,T_b+Q_b)}{g_b(T_b)} \sigma_b(T_b) \frac{(2s_b+1)\mu_b}{\pi^2 h^3} \frac{\rho_{n-b}(E^*)}{\rho_n(U)} T_b, \tag{18.19}$$

where

$$g_b(T_b) = \frac{V_b(2s_b+1)(2\mu_b)^{3/2}}{4\pi^2 h^3}(T_b+Q_b)^{1/2} \tag{18.20}$$

is the single-particle density for complex fragment b, which is obtained by assuming that complex fragment moves inside volume V_b in the uniform potential well whose depth is equal to be Q_b. The factor $R_b(p,h)$ provides correct isotopic composition of the fragment b.

18.2.6 Total probability.

This probability is defined as

$$W_{tot}(n,U) = \sum_{\Delta n=+2,0,-2} \omega_{\Delta n}(n,U) + \sum_{b=1}^{6} W_b(n,U), \tag{18.21}$$

where total emission $W_b(n,U)$ probability to emit fragment b can be obtained from Eqs. (18.17) and (18.19) by integration over T_b:

$$W_b(n,U) = \int_{V_b}^{U-Q_b} W_b(n,U,T_b) dT_b. \tag{18.22}$$

18.2.7 Calculation of kinetic energies for emitted particles.

The equations (18.17) and (18.19) can used to sample kinetic energies of emitted fragment.

18.2.8 Angular distribution of emitted fragments.

The formulation of the preequilibrium exciton models in the term of master equation for population probability for the n-exciton states does not give a possibility to obtain angular distribution of emitted fragments[1]. The assumption about isotropic emission of fragments, which is used in the equilibrium emission models (see the next chapters) contradicts experimental fragment angular distributions. To improve isotropic approximation the next simple prescription [1] can be applied. It is considered that the particle emission will be isotropic in the proper n-exciton system and the incoming nuclear momentum is shared only by the n-exciton system rather than whole excited nucleus. Thus, in the nucleus rest frame the angular distribution of emitted particles will be anisotropic.

The next algorithm can be used to obtain anisotropic angular distributions. After each preequilibrium transition:

- We sample the absolute value of nucleon Fermi momentum **p** according to
$$p = p_F^{max} \xi^{1/3}, \tag{18.23}$$
where p_F^{max} is maximal Fermi momentum of a nucleon has been calculated at the maximal value of the nucleon density and ξ is a random number uniformly distributed between 0 and 1;

- The vector **p** considered as an isotropic vector;

- We calculate a new momentum vector
$$\mathbf{p}' = \mathbf{p} + \frac{\mathbf{P_0}}{N_p}, \tag{18.24}$$
N_p is the number of particles and $\mathbf{P_0}$ is a momentum of residual nucleus at the moment of transition.

Finally, the isotropically sampled momentum of outgoing particle produced at preequilibrium emission is rotated on the angle between two vectors: $\mathbf{p}'$ and residual nucleus velocity **V** vector.

There is another approach [6], [7] related on the fast particle approximation. In this model the angular orientation of nucleus at each exciton collision step is defined by the direction of the fast particle, which changes gradually in series of two-body collisions. The transition rate between different exciton states and the emission probability are assumed to be factored in angle dependent and energy dependent factors.

18.2.9 Angular momenta of the emitted fragments.

The angular momenta of and emitted particles are considered as classical vectors $\mathbf{l_i}$ and estimated in the sharp cut-off approximation [8], [9] according to
$$P(l_b)dl_b \sim l_b dl_b, 0 \leq l_b \leq l_b^{max}, \tag{18.25}$$
where
$$l_b^{max} = \sqrt{2\mu_b(E_b - V_b)} R_b/\hbar. \tag{18.26}$$
Here R_b is radius of the interaction of the b-th emitted particle with the residual nucleus, E_b,V_b and μ_b are the energy in the centre of mass system, Coulomb barrier and reduced mass of particle, respectively.

18.2.10 Parameters of residual nucleus.

After fragment emission we update parameters of decaying nucleus:
$$\begin{array}{c} A_f = A - A_b; Z_f = Z - Z_b; P_f = P_0 - p_b; \\ E_f^* = \sqrt{E_f^2 - \vec{P}_f^2} - M(A_f, Z_f); \vec{L}_f = \vec{L}_0 - \vec{l_b}. \end{array} \tag{18.27}$$

Here p_b is the evaporated fragment four momentum. The spins of the emitted particles are not taken into account. Angular momenta of residual nuclei are calculated without taking into account the spin of initial target nucleus and intermediate nuclei during the precompound emission.

18.2.11 MC procedure.

The Monte Carlo simulation of the preequlibrium process is outlined as follows:

1. For given excitation energy U, atomic number A and number of excitons n calculate the equilibrium number n_{eq} of excitons according to the Eq. (18.2). If exciton number $n \geq n_{eq}$, then further emission of fragments are simulated using an equilibrium model. If $n < n_{eq}$, then perform next step.

2. Taking into account reaction threshold and $R_b(p,h)$ factor calculate transition (according to the Eqs. (18.8),(18.14) and (18.15)) and emission probabilities (the Eqs. (18.17) and (18.19) are integrated over T_b). Then use the Eq. (18.21) to normalise calculated probabilities and obtain statistical weights for subprocesses. Select a subprocess according to the calculated statistical weights.

3. In the case of a transition update the number of excitons, if it is needed and proceed step (1). In the case of a fragment emission one should perform the next step.

4. Sample fragment kinetic energy according to the Eq. (18.17) or (18.19).

5. Sample fragment angles using the isotropic angular distribution in the exciton system rest frame. Calculate fragment momentum and perform boost to the nucleus rest frame.

6. Update characteristics of the residual nucleus according to the Eqs. (18.27) and proceed step (1).

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Chapter 19

Equilibrium evaporation model.

19.1 Reaction initial state.

The equilibrium evaporation model is based on the approach described in [1]. It is capable to predict final states photons, nucleons as well as nuclear fragments due to their evaporation from an excited nucleus with atomic number $A > 16$.

The initial information for simulation of the evaporation stage consists from the atomic mass number A, charge Z of residual (e. g. after kinetic stage) nucleus, its four momentum P_0, angular momentum $\vec{L}_0$ and excitation energy E^*.

The evaporation of photons, neutrons, protons, deutrons, thritiums, alphas, as well as the fission of heavy residual nuclei with $A > 65$ are taken into account as the competiting channels.

This model can be applied neither after kinetic stage of nuclear interaction or after another break-up process for an excited nucleus, e. g. the fission or the multifragmentation.

19.2 Simulation of fragment evaporation.

The evaporation of neutron, proton, deutron, thritium and alpha fragments are taken into account.

19.2.1 Evaporation threshold.

One should take into account the energy condition for fragment emission, i. e. the nucleus excitation energy should be higher than the reaction threshold:

$$T_b^{\max} = E^* - Q_b - V_b > 0. \tag{19.1}$$

Here $T_b^{\max}$ is the maximal kinetic energy carried by emitted fragment b. $Q_b = M(A,Z) - M(A_f,Z_f) - M_b$ is the fragment b binding energy. V_b is the Coulomb potential energy, i. e. the Coulomb barrier for fragment b. $M(A,Z)$ is the mass of the initial nucleus, $M(A_f,Z_f)$ is the mass of the nucleus after emission of fragment b and M_b is the fragment b mass (see the nuclear properties chapter). It should be noted that the expression (19.1) is only valid, when the recoil kinetic energy equals zero. Instead we can apply the condition:

$$T_b^{\max} = E_b^{\max} - M_b - V_b > 0, \tag{19.2}$$

where

$$E_b^{\max} = \frac{[M(A,Z)+E^*]^2 + M_b^2 - M^2(A_f,Z_f)}{2[M(A,Z)+E^*]}. \tag{19.3}$$

19.2.2 Coulomb barrier calculation.

The Coulomb barrier:

$$V_b = C_b \frac{Z_b Z_f}{R_f + R_b}, \tag{19.4}$$

where $C_b = 1.44$ MeVfm, Z_f and R_f are charge and radius of nucleus after fragment emission, Z_b and R_b are charge and radius of fragment. The value $V_b(E^*)$ varies with the excitation energy E^* by substitution [2]

$$V_b(E^*) = \frac{V_b}{1 + \sqrt{\frac{E^*}{2A}}}. \tag{19.5}$$

The radii of nuclei (in fm) are approximated by $R = r_C A^{1/3}$, where [1]

$$r_C = 2.173 \frac{1 + 0.006103 Z_b Z_f}{1 + 0.009443 Z_b Z_f}. \tag{19.6}$$

19.2.3 Fragment evaporation probability.

The statistical decay Weisskopf-Ewing theory [3] gives the probability to evaporate particle b in the energy interval $(T_b, T_b + dT_b)$ per unit of time:

$$W_b(T_b) = \sigma_b(T_b) \frac{(2s_b+1)m_b}{\pi^2\hbar^3} \frac{\rho_b(U_b - Q_b - T_b)}{\rho_c(U_c)} T_b, \tag{19.7}$$

where $\sigma_b(T_b)$ is the inverse (absorption of particle b) reaction cross section. Here s_b and m_b are particle spin and mass, respectively. ρ_c and ρ_b are level densities of the compound nucleus and the nucleus after particle evaporation, respectively. The energies U_b and U_c are defined as $U_b = E^* - \Delta_b$ and $U_c = E^* - \Delta_c$, where $\Delta_{b,c}$ are pairing energies Δ_{Pair} of the compound and residual nuclei, respectively. The pairing energy Δ_{Pair} (in MeV) is calculated according to

$$\Delta_{Pair} = \kappa \frac{12}{\sqrt{A}}, \tag{19.8}$$

were $\kappa = 0$, 1, and 2 for the odd-odd, the odd-even and the even-even nuclei, respectively.

We should note, that Eq. (19.7) is written for the case, when we have neglected angular momenta and parities of the compound and residual nuclei as well as the spin of the evaporated fragment (see below the extention of this equation).

19.2.4 Inverse reaction cross section.

The inverse reaction cross section is assumed to have the form [4]

$$\sigma_b(T_b) = (1 + C_b)(1 - k_b V_b/T_b)\pi R^2 \tag{19.9}$$

for charged charged fragments with $A_b \leq 4$ interaction and

$$\sigma_b(T_b) = \alpha(1 + \beta/T_b)\pi R^2 \tag{19.10}$$

for neutrons. Here, k_b is the barrier penetration coefficient (its tabulated values [4]) can be used. $R = r_0 A^{1/3}$ denotes the absorption radius, where $r_0 = 1.5$ fm, $\alpha = 0.76 + 2.2A^{-1/3}$ and $\beta = (2.12A^{-2/3} - 0.05)/(0.76 + 2.2A^{-1/3})$.

19.2.5 Level density.

The nuclear level density is approximated by the Fermi-gas approach [1]:

$$\rho(E^*) = C \exp{(2\sqrt{aE^*})}, \tag{19.11}$$

where C is a constant, which does not depend from nucleus properties and excitation energy E^*, a is the level density parameter, which was explained in the nuclear properties chapter. The system entropy is defined by $S = 2\sqrt{aE^*}$.

19.2.6 Total evaporation probability.

The total probability W_b or total partial width $\Gamma_b = \hbar W_b$ to evaporate particle b can be obtained from Eq. (19.7) by integration over T_b:

$$W_b = \int_{V_b}^{U-Q_b} W_b(T_b) dT_b. \tag{19.12}$$

Here the summation is carried out over all excited states of the fragment.

Integration in Eq. (19.12) for probability to emit fragment b can be performed analytically, if we will use Eq. (19.11) for level density and the

Eqs. (19.9)-(19.10) for inverse cross section. The probability to emit a charged particle:

$$W_b = \gamma_b A_b^{2/3} B \exp[-2\sqrt{aU}] \frac{(1+C_b)}{a_b^2} \{a_b T_b^{\max} [2\exp(2\sqrt{a_b T_b^{\max}}) + 1] - \\ -3\sqrt{a_b T_b^{\max}} \exp(2\sqrt{a_b T_b^{\max}}) - 3[1 - \exp(2\sqrt{a_b T_b^{\max}})]/2\}, \tag{19.13}$$

where T_b^{max} is defined by the equation (19.1). The following notations were introduced: $A_b = A - \Delta A_b$, $B = m_N r_0^2/(2\pi h^2)$, $\gamma_b = (2s_b + 1)m_b/m_N$. ΔA_b is the number of nucleons in b particle. m_b, m_N and s_b are mass of particle b, mass of nucleon and spin of particle b, respectively. a_b is level density parameter for nucleus after emission of fragment b. Similarly for the neutron evaporation probability we obtain the following equation:

$$W_n = \gamma_n A_n^{2/3} B \frac{\alpha}{2a_n^2} \exp[-2\sqrt{aE^*} + 2\sqrt{a_n T_n^{\max}}][4a_n T_n^{\max} + \\ +(2a_n\beta - 3)(\exp(-2\sqrt{a_n T_n^{\max}}) + 2\sqrt{a_n T_n^{\max}} - 1)]. \tag{19.14}$$

Using probabilities Eq. (19.13) and Eq. (19.14) we are able to sample type of the emitted fragment.

19.2.7 Kinetic energy of the emitted fragment.

The equation (19.7) can be used to sample kinetic energies of evaporated fragments. For example, keeping terms in Eq. (19.7), which depend from T_b and using the approximations for inverse cross section is given by Eq. (19.9) and for level densities are given by Eq. (19.11), we will obtain for the charged fragments:

$$W(x) = C_1 x \exp[2\sqrt{a(T_b^{\max} - x)}] = C_2 T_b \exp[2\sqrt{aE^*}], \tag{19.15}$$

where C_1 and C_2 do not depend from T_b, $x = T_b - V_b$. To generate values of x we can use the next procedure, changing the expression for $W(x)$ to have $W(x^{\max}) = 1$ ($x^{\max} = [(a_b + T_b^{\max} + 1/4)^{1/2} - 1/2]/a_b$):

1. Choose two random numbers ξ_1 and ξ_2 are distributed with equal probabilities between 0 and 1;
2. Find kinetic energy of particle b from $T_b = T_b^{\max}\xi_1 + V_b$, if the condition $\xi_2 \leq W(\xi_1 T_b^{\max})$ is fulfilled;
3. If the last condition is not fulfilled, proceed step (1).

19.2.8 Angular distribution and angular momenta of the emitted fragments.

We consider the angular distribution for emitted fragments as the isotropic distribution in spite of that the excited nucleus has an angular momentum.

We have no information about the residual nucleus polarisation (see below). The angular momenta of the evaporated particles are considered as classical vectors $\mathbf{l_i}$ and estimated in the sharp cut-off approximation (see the preequilibrium exciton model chapter).

19.2.9 Parameters of the residual nucleus.

After fragment emission we update parameter of decaying nucleus according to the equations:

$$\begin{array}{c} A_f = A - A_b; Z_f = Z - Z_b; P_f = P_0 - p_b; \\ E^*_f = \sqrt{E^2_f - \vec{P}^2_f} - M(A_f, Z_f); \vec{L}_f = \vec{L}_0 - \vec{l_b}. \end{array} \tag{19.16}$$

Here p_b is the evaporated fragment four momentum.

The spins of the emitted particles are not taken into account. Angular momenta of residual nuclei are calculated without taking into account the spin of initial target nucleus and of intermediate nuclei during emission of particles.

19.2.10 Counting of the excited nucleus angular momentum.

The angular momentum influence on evaporation and fission processes can be approximately taken into account. The angular momentum L dependence of the level density can be approximated [5] by

$$\rho(E^* - \Delta_{Pair}, L) = \rho(U, 0), \tag{19.17}$$

where $U = E^* - \Delta_{Pair} - E_R$ and E_R are the thermal and rotational energies of a nucleus, respectively. The fission barrier $B_{fis}(L)$ for a fissioning nucleus with the angular momentum L can also be approximated as follows

$$B_{fis}(L) = B_{fis}(0) - (E^{GS}_R - E^{SP}_R), \tag{19.18}$$

where E^{GS}_R and E^{SP}_R are nuclear rotational energies for the ground state and at the saddle-point, respectively. The rotational energies is calculated according to

$$E_R = \frac{L(L+1)}{2\theta}, \tag{19.19}$$

where θ is the moment of inertia of a nucleus. The moment of inertia for the compound and residual nuclei are calculated according to the rigid-body expression:

$$\theta = 0.4 m_N r_0^2 A^{5/3}, \tag{19.20}$$

where m_N is nucleon mass and $r_0 = 1.2$ fm. The moment of inertia of a nucleus at saddle-point J_{SP} is calculated in [6].

19.3 Fission probability calculation.

The fission decay channel (only for nuclei with $A > 65$) is taken into account as a competitor for fragment and photon evaporation channels.

19.3.1 Total fission probability.

The fission probability W_{fis} in the Bohr and Wheeler theory of fission [7] is proportional to the level density $\rho_{fis}(T)$ (the approximation by Eq. (19.11) is used) at the saddle point, i.e.

$$W_{fis} = \frac{1}{2\pi\hbar\rho_c(U_c)} \int_0^{U_f - B_{fis}} \rho_{fis}(U_f - B_{fis} - T)dT = \frac{1 + (C_f - 1)\exp(C_f)}{4\pi a_{fis} \exp(2\sqrt{a_c U_c})}, \tag{19.21}$$

where $U_f = E^* - \Delta_f$ and pairing energy (in MeV)

$$\Delta_f = \kappa \frac{14}{\sqrt{A}}. \tag{19.22}$$

In Eq. (19.21) B_{fis} is the fission barrier height (see nuclear properties chapter). The value

$$C_f = 2\sqrt{a_{fis}(U_f - B_{fis})} \tag{19.23}$$

and a_c, a_{fis} are the level density parameters of the compound and of the fission saddle point nuclei, respectively.

The value of the level density parameter is larger at the saddle point, when excitation energy is given by the initial excitation energy minus the fission barrier height, than in the ground state, i. e. $a_{fis} > a$. Thus, the values of $a_{fis} = 1.08a$ for $Z < 85$, $a_{fis} = 1.04a$ for $Z \geq 89$ and $a_f = a[1.04 + 0.01(89. - Z)]$ for $85 \leq Z < 89$ [1] can be used.

19.4 Simulation of the photon evaporation.

The photon evaporation channel is taken into account as a competitor for fragment evaporation and fission channels. For this case we consider only the giant dipole resonance photon evaporation.

19.4.1 Total probability of γ evaporation.

As the first approximation we assume that dipole $E1$-transitions is the main source of γ-quanta from highly-excited nuclei [8]. The probability to evaporate γ in the energy interval $(\epsilon_\gamma, \epsilon_\gamma + d\epsilon_\gamma)$ per unit of time is given by

$$W_\gamma(\epsilon_\gamma, L) = \frac{1}{\pi^2(\hbar c)^3} \sigma_\gamma(\epsilon_\gamma) \frac{\rho_\gamma(E^* - \epsilon_\gamma, L)}{\rho_c(U_c, L)} \epsilon_\gamma^2. \tag{19.24}$$

Here L is the nuclear angular momentum $\sigma_\gamma(\epsilon_\gamma)$ is the inverse (absorption of γ) reaction cross section. $\rho_\gamma(U-\epsilon_\gamma, L)$ and $\rho_c(U_c, L)$ are level densities of nucleus after and before γ evaporation, respectively. In Eq. (19.24) we assumed that $\rho_c(U_c, L+1) = \rho_c(U_c, L-1) = \rho_c(U_c, L)$ and $\rho_c(U_c, L) = \rho(U_c - E_{rot})$ and $\rho_\gamma(U-\epsilon_\gamma, L) = \rho(U-\epsilon_\gamma - E_{rot})$, where level density is defined by Eq. (19.11) and E_{rot} is defined by Eq. (19.19).

The photoabsorption reaction cross section is given by the expression

$$\sigma_\gamma(\epsilon_\gamma) = \frac{\sigma_0 \epsilon_\gamma^2 \Gamma_R^2}{(\epsilon_\gamma^2 - E_{GDP}^2)^2 + \Gamma_R^2 \epsilon_\gamma^2}, \tag{19.25}$$

where $\sigma_0 = 2.5A$ mbarn, $\Gamma_R = 0.3E_{GDP}$ and $E_{GDP} = 40.3A^{-1/5}$ MeV are empirical parameters of the giant dipole resonance [8]. The total radiation probability is

$$W_\gamma = \frac{3}{\pi^2(\hbar c)^3} \int_0^{E^*-E_{rot}} \sigma_\gamma(\epsilon_\gamma) \frac{\rho(E^* - E_{rot} - \epsilon_\gamma)}{\rho(U_c - E_{rot})} \epsilon_\gamma^2 d\epsilon_\gamma. \tag{19.26}$$

The integration can be performed numerically.

19.4.2 Energy of the evaporated photon.

The energy of γ-quantum can be sampled according to the Eq. (19.24) distribution.

19.4.3 Discrete photon evaporation.

The last step of the evaporation cascade consists of evaporation of photons with discrete energies. The competition between photons and fragments can be neglected at this step.

If we want to simulate the evaporation of photons with different multipolarities λ (e.g. electric and magnetic dipole $E1$ and $M1$ as well as the electric quadrupole $E2$ photons), we need more careful consideration of the angular momenta and parities for compound and residual nuclei. For this case we should rewrite the probability (see Eq. (19.7)) to evaporate particle b in the energy interval $(T_b, T_b + dT_b)$ per unit of time. The new probability is given by [1]

$$W_b(L_c, \pi_c, L_b, \pi_b, T_b) = \frac{1}{\hbar} \frac{\rho_b(U_b - Q_b - T_b, L_b, \pi_b)}{\rho_c(U_c, L_c, \pi_c)} \sum_{S=|L_b-s|}^{|L_b+s|} \sum_{l=|L_c-S|}^{|L_c+S|} T_l(T_b) \tag{19.27}$$

where L_c, π_c and L_b, π_b are the angular momenta and parities of the compound nucleus and residual nucleus, respectively and s is the fragment spin. The parity conservation should be taken into account in the summation over

l. The transmission coefficients of fragments $T_l(T_b)$ can be derived from the inverse absorption cross sections [1]

$$\frac{d\sigma_b(T_b)}{d\mathbf{l}} = \lambda^2 T_l(T_b)\delta(\mathbf{n},\mathbf{l}), \tag{19.28}$$

where λ is the de Broglie wavelength. The delta function takes into account the fact that the orbital momentum $\mathbf{l}$ is perpendicular to the direction $\mathbf{n}$ of the fragment motion.

Similarly for the photon evaporation:

$$W_\gamma^\lambda(L_c, L_\gamma, \epsilon_\gamma) = \frac{1}{\hbar}\frac{\rho(E^* - \epsilon_\gamma, L_\gamma, \pi_\gamma)}{\rho(U_c, L_c, \pi_c)} f_\lambda(\epsilon_\gamma) \tag{19.29}$$

where $f_\lambda(\epsilon_\gamma)$ are the strength functions, λ is the multipolarity of the γ-transition and ϵ_γ is the photon energy. The strength functions for the photon evaporation can be derived from the photoabsorption cross sections. This approach is similar to the considered before giant dipole resonance emission and requires the knowledge of the resonance parameters for nuclei. Another simplified approach to obtain the strength functions is based on the single particle transition estimates:

$$f_\lambda(\epsilon_\gamma) = \chi_\lambda \epsilon_\gamma^{2\lambda+1}, \tag{19.30}$$

where χ_λ are constants can be found in [9] or in [10].

The widths of different decays are

$$\Gamma_b(L_c) = \hbar \int_{V_b}^{U-Q_b} \sum_{L_b} W_b(L_c, L_b, T_b) dT_b \tag{19.31}$$

$$\Gamma_\gamma^\lambda(L_c) = \hbar \sum_{L_\gamma=|L_c-\lambda|}^{|L_c+\lambda|} \int_0^{E^*} \Gamma_\gamma^\lambda(L_c, L_\gamma, \epsilon_\gamma) d\epsilon_\gamma. \tag{19.32}$$

Using the so-called single-particle Weisskopf estimations of the strength functions (see Eq. (19.30)) one can estimate gamma-decay widths (in electronvolts):

$$\Gamma_\gamma(E1) = 0.07\epsilon_\gamma^3 A^{2/3}, \tag{19.33}$$

$$\Gamma_\gamma(M1) = 0.0021\epsilon_\gamma^3, \tag{19.34}$$

$$\Gamma_\gamma(E2) = 4.9 \times 10^{-8}\epsilon_\gamma^5 A^{4/3}. \tag{19.35}$$

From the above equations for the decay widths one can also estimate the corresponding lifetimes. From these equations we can see that $M1$ and $E2$ transitions are strongly suppressed as compared with the $E1$ transition. As we already discussed (see Eq. (19.17)) one can approximate level density by

$$\rho(U, L) = \frac{\sqrt{a}(2L+1)\hbar^3}{48\sqrt{2\theta^3}U^2} \exp(2\sqrt{aU}). \tag{19.36}$$

19.5 Anisotropic photon angular distribution.

The ability to observe an anisotropy in the angular distribution of the radiated photons depends on the ability to obtain a nonuniform population of the substates $\{L, M_i\}$ for excited nuclei with a given spin L, where M_i is its chosen projection.

19.5.1 Angular distributions of the dipole and quadrupole radiations.

The angular distribution of photons does not depend on whether the multipole (λ, μ) is electric or magnetic in nature (see, e.g. [11],[12]). Thus, if we consider only dipole and quadrupole radiations we obtain the next angular (θ) distributions:

1. For dipole

$$\omega_{1,0}(\theta) = \frac{3}{8\pi}\sin^2\theta, \tag{19.37}$$

$$\omega_{1,\pm1}(\theta) = \frac{3}{16\pi}(1+\cos^2\theta); \tag{19.38}$$

2. For quadrupole

$$\omega_{2,0}(\theta) = \frac{15}{8\pi}\sin^2\theta\cos^2\theta, \tag{19.39}$$

$$\omega_{2,\pm1}(\theta) = \frac{5}{16\pi}(1-3\cos^2\theta+4\cos^4\theta), \tag{19.40}$$

$$\omega_{2,\pm2}(\theta) = \frac{5}{16\pi}(1-\cos^4\theta). \tag{19.41}$$

19.6 MC procedure to simulate emission of fragments and dipole photons.

The equilibrium evaporation model algorithm consists from the repeating steps of binary break-ups of an excited nucleus:

1. Create an excited nucleus: assign atomic mass number A, electrical charge Z, fragment four vector P_0, fragment excitation energy E^* and fragment angular momentum $\vec{L}_0$;

2. Calculate the probabilities of break-up channels and sample a channel;

3. Sample evaporated fragment b kinetic energy at rest of the decaying nucleus (total energy for the emitted dipole photon);

4. Assuming isotropical evaporation of a fragment, sample its fly off angle at rest of decaying nucleus;

5. Boost the evaporated (no boost for the evaporated dipole photon) and residual fragment momenta into the observer frame.

6. Calculate residual fragment atomic mass number A_f, electrical charge Z_f, fragment four vector P_f, fragment excitation energy E_f^* and fragment angular momentum $\vec{L}_f$;

7. Repeat this procedure starting from the step (2) until no more fragments (the probabilities of break-up channels equal zeros) can be evaporated.

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Chapter 20

Symmetric and asymmetric fission model.

20.1 Reaction initial state.

The symmetric and asymmetric fission model is capable to predict final excited fragments as result of an excited nucleus symmetric or asymmetric fission. The fission process (only for nuclei with atomic number $A \geq 65$) is simulated according to the approach [1]. It is considered as a competitor for evaporation process, when nucleus transits from an excited state to the ground state. The competition with evaporation of neutrons, protons, deutrons, nuclei of thritium and helium as well as photon evaporation is taken into account.

The initial information for calculation of fission decay consists from the atomic mass number A, charge Z of residual (e. g. after kinetic stage of the nuclear reaction) nucleus, its four momentum P_0, angular momentum $\vec{L}_0$ and excitation energy E^*.

20.2 Nuclear fission cross section.

The probability P_n^{fis} that fission occurs at any step of evaporation chain with n evaporated fragments can be defined as follows

$$P_n^{fis} = 1 - P_n, \tag{20.1}$$

where P_n is the probability of the transition from an excited state to the ground state for the nucleus only by evaporation of n fragments. The probability P_n can be calculated using the equation:

$$P_n = \prod_{i=1}^{n} [1 - W_{fis}(E_i^*, A_i, Z_i)/W_{tot}(E_i^*, A_i, Z_i)], \tag{20.2}$$

where W_{fis} fission probability (per unit time) in the Bohr and Wheeler theory of fission [2]. It is assumed to be proportional to the level density $\rho_{fis}(T)$ at the saddle point:

$$W_{fis} = \frac{1}{2\pi\hbar\rho_c(U_c)} \int_0^{U_f - B_{fis}} \rho_{fis}(U_f - B_{fis} - T)dT, \tag{20.3}$$

where $U_f = E^* - \Delta_f$ and pairing energy (in MeV)

$$\Delta_f = \kappa \frac{14}{\sqrt{A}}. \tag{20.4}$$

In Eq. (20.3) B_{fis} is the fission barrier height (see the nuclear properties chapter). W_{tot} is the total decay probability (per unit time) of the excited nucleus:

$$W_{tot} = W_{fis} + \sum_{b=1}^{6} W_b. \tag{20.5}$$

W_b is the probability to evaporate fragment of type b. In the Weisskopf and Ewing theory of particle evaporation [3]:

$$W_b(T_b) = \sigma_b(T_b) \frac{(2s_b + 1)m_b}{\pi^2\hbar^3} \frac{\rho_b(U_b - Q_b - T_b)}{\rho_c(U_c)} T_b, \tag{20.6}$$

where $\sigma_b(T_b)$ is the inverse (absorption of particle b) reaction cross section, s_b and m_b are particle spin and mass, ρ_c and ρ_b are level densities of compound nucleus and nucleus after particle evaporation, respectively. The energies U_b and U_c are defined as $U_b = E^* - \Delta_b$ and $U_c = E^* - \Delta_c$, where $\Delta_{b,c}$ are pairing energies Δ_{Pair} of the compound and residual nuclei, respectively. The pairing energy Δ_{Pair} (in MeV) is calculated according to

$$\Delta_{Pair} = \kappa \frac{12}{\sqrt{A}}, \tag{20.7}$$

where $\kappa = 0$, 1, and 2 for the odd-odd, the odd-even and the even-even nuclei, respectively.

The Eq. (20.1) gives us a possibility to calculate numerically the so-called fisility of nucleus $P_{fis} = \sigma_{fis}/\sigma_{in}$ (see e.g. [1]), where σ_{in} is the inelastic nuclear reaction cross section and σ_{fis} is the fission:

$$\sigma_{fis} = \sigma_{in} P_{fis} = \sigma_{in} \frac{1}{N_{ch}} \sum_{n=1}^{N_{ch}} P_n^{fis}, \tag{20.8}$$

where N_{ch} is the number of fragment evaporation chains to be performed for the averaging.

As one can see from Eq. (20.3) the fission barrier height B_{fis} and the parameter of the level density of a nucleus a_{fis} at saddle point are the basic ingredients of the model, which are necessary for the calculation of fission cross section.

20.3 Fission process simulation.

20.3.1 Atomic number distribution of fission products.

As known from the experimental data [4] the mass distribution of fission products consists of the symmetric and the asymmetric components:

$$F(A_f) = F_{sym}(A_f) + \omega F_{asym}(A_f), \tag{20.9}$$

where $\omega(U, A, Z)$ defines relative contribution of each component and it depends from excitation energy U and A, Z of the fissioning nucleus. It was found in [5] that experimental data can be approximated with a good accuracy, if one takes

$$F_{sym}(A_f) = \exp\left[-\frac{(A_f - A_{sym})^2}{2\sigma_{sym}^2}\right] \tag{20.10}$$

and

$$\begin{aligned} F_{asym}(A_f) = \exp\left[-\tfrac{(A_f-A_2)^2}{2\sigma_2^2}\right] + \exp\left[-\tfrac{A_f-(A-A_2)^2}{2\sigma_2^2}\right] + \\ +C_{asym}\left\{\exp\left[-\tfrac{(A_f-A_1)^2}{2\sigma_1^2}\right] + \exp\left[-\tfrac{A_f-(A-A_1)^2}{2\sigma_2^2}\right]\right\}, \end{aligned} \tag{20.11}$$

where $A_{sym} = A/2$, A_1 and A_2 are the mean values and σ_{sim}^2, σ_1^2 and σ_2^2 are dispersions of the Gaussians, respectively. From the analysis of experimental data [5] the parameter $C_{asym} \approx 0.5$ and the next values for dispersions:

$$\sigma_{sym}^2 = \exp(0.00553U + 2.1386), \tag{20.12}$$

were determined (U in MeV).

$$2\sigma_1 = \sigma_2 = 5.6 \tag{20.13}$$

for $A > 235$ and

$$2\sigma_1 = \sigma_2 = 5.6 + 0.096(A - 235) \tag{20.14}$$

for $A \leq 235$ were found.

The weight $\omega(U, A, Z)$ was approximated as follows

$$\omega = \frac{\omega_a - F_{asym}(A_{sym})}{1 - \omega_a F_{sym}[(A_1 + A_2)/2]}. \tag{20.15}$$

The values of ω_a for nuclei with $96 \geq Z \geq 90$ were calculated from

$$\omega_a(U) = \exp(0.538U - 9.9564) \tag{20.16}$$

for $U \leq 16.25$ MeV,

$$\omega_a(U) = \exp(0.09197U - 2.7003) \tag{20.17}$$

for $U > 16.25$ MeV and

$$\omega_a(U) = \exp\left(0.09197U - 1.08808\right) \tag{20.18}$$

for $z = 89$. For nuclei with $Z \leq 88$ the authors of [5] constructed following approximation:

$$\omega_a(U) = \exp\left[0.3(227 - a)\right] \exp\left\{0.09197[U - (B_{fis} - 7.5)] - 1.08808\right\}, \tag{20.19}$$

where the height of fission barrier B_{fis} is taken in MeV. The corresponding factors occur in exponential functions are vanish at $A > 227$ and $U < B_{fis} - 7.5$.

20.3.2 Charge distribution of fission products.

At given mass of fragment A_f the experimental data [4] for the charge Z_f distribution of fragments are well approximated by the Gaussian with dispersion $\sigma_z^2 = 0.36$ and the average $< Z_f >$, which can be calculated from the expression:

$$< Z_f >= \frac{A_f}{A} Z + \Delta Z. \tag{20.20}$$

The parameter $\Delta Z = -0.45$ for $A_f \geq 134$, $\Delta Z = -0.45(A_f - A/2)/(134 - A/2)$ for $A - 134 < A_f < 134$ and $\Delta Z = 0.45$ for $A \leq A - 134$.

After sampling of fragment atomic numbers and fragment charges, we have to check that fragment ground state masses (see the nuclear properties chapter) do not exceed initial energy and calculate the maximal fragment kinetic energy according to

$$T^{max} < U + M(A, Z) - M_1(A_{f1}, Z_{f1}) - M_2(A_{f2}, Z_{f2}), \tag{20.21}$$

where U and $M(A, Z)$ are the excitation energy and mass of initial nucleus, $M_1(A_{f1}, Z_{f1})$, and $M_2(A_{f2}, Z_{f2})$ are masses of the first and second fragment, respectively.

20.3.3 Kinetic energy distribution of fission products.

We can use the empirically defined [6] dependence of the average kinetic energy $< T_{kin} >$ (in MeV) of fission fragments from the mass and the charge of a fissioning nucleus:

$$< T_{kin} >= 0.1178 Z^2/A^{1/3} + 5.8. \tag{20.22}$$

This energy is distributed differently for the cases of symmetric and asymmetric modes of fission. As it follows from the analysis of data [5] the average kinetic energy of fragments is higher for the symmetric mode, than that in the symmetric one by approximately 12.5 MeV. To approximate the average

numbers of kinetic energies $< T_{kin}^{sym}$ and $< T_{kin}^{asym} >$ for the symmetric and asymmetric modes of fission, respectively, the authors of [5] have suggested empirical expressions:

$$< T_{kin}^{sym} >=< T_{kin} > -12.5 W_{asim}, \tag{20.23}$$

$$< T_{kin}^{asym} >=< T_{kin} > +12.5 W_{sim}, \tag{20.24}$$

where

$$W_{sim} = \omega \int F_{sim}(A) dA / \int F(A) dA \tag{20.25}$$

and

$$W_{asim} = \int F_{asim}(A) dA / \int F(A) dA. \tag{20.26}$$

In the symmetric fission the experimental data for the ratio of the average kinetic energy of fission fragments $< T_{kin}(A_f) >$ to its maximum energy $< T_{kin}^{max} >$ as a function of the mass of a larger fragment A_{max} can be approximated by expressions:

$$< T_{kin}(A_f) > / < T_{kin}^{max} >= 1 - k[(A_f - A_{max})/A]^2 \tag{20.27}$$

for $A_{sim} \leq A_f \leq A_{max} + 10$ and

$$< T_{kin}(A_f) > / < T_{kin}^{max} >= 1 - k(10/A)^2 - 2(10/A)k(A_f - A_{max} - 10)/A \tag{20.28}$$

for $A_f > A_{max} + 10$, where $A_{max} = A_{sim}$ and $k = 5.32$. In the case of asymmetric fission fragments will have the maximal kinetic energy, if the larger fragment have the atomic number, which is equal $A_{max} = 134$. To approximate the experimental data for the asymmetric mode, we can also use the above equation with $k = 23.5$. For both modes of fission the distribution over kinetic energy of fragments T_{kin} is chosen Gaussian with their own average values $< T_{kin}(A_f) >=< T_{kin}^{sym}(A_f) >$ or $< T_{kin}(A_f) >=< T_{kin}^{asym}(A_f) >$ and dispersions σ_{kin}^2 equal 8^2 MeV or 10^2 MeV^2 for symmetric and asymmetric modes, respectively.

20.3.4 Calculation of the excitation energy of fission products.

The total excitation energy of fragments U_{frag} can be determined according to the equation:

$$U_{frag} = U + M(A, Z) - M_1(A_{f1}, Z_{f1}) - M_2(A_{f2}, Z_{f2}) - T_{kin}, \tag{20.29}$$

where U and $M(A, Z)$ are the excitation energy and mass of initial nucleus, T_{kin} is the fragments kinetic energy, $M_1(A_{f1}, Z_{f1})$, and $M_2(A_{f2}, Z_{f2})$ are masses of first and second fragment, respectively.

The value of excitation energy of fragment U_f determines the fragment temperature $T = \sqrt{U_f/a_f}$, where $a_f \sim A_f$ is the parameter of fragment level density. We assume that after disintegration fragments have the same temperature as the initial nucleus. Then the total excitation energy will be distributed between fragments in proportion to their mass numbers:

$$U_f = U_{frag}\frac{A_f}{A}. \tag{20.30}$$

20.3.5 Excited fragment momenta.

Assuming that fragment kinetic energy $T_f = P_f^2/(2(M(A_f, Z_f + U_f)$ we are able to calculate the absolute value of the fragment c.m. momentum

$$P_f = \frac{(M_1(A_{f1}, Z_{f1} + U_{f1})(M_2(A_{f2}, Z_{f2} + U_{f2})}{M_1(A_{f1}, Z_{f1}) + U_{f1} + M_2(A_{f2}, Z_{f2}) + U_{f2}}T_{kin}. \tag{20.31}$$

and its components, if we consider the fragment isotropic distribution.

20.4 MC procedure.

The Monte Carlo procedure to calculate characteristics of fission fragments can be outlined as follows:

- Select fission mode (symmetric or asymmetric). Sample atomic number A_f of a fission fragment according to experimentally defined distribution, which consists from the "symmetric" and "asymmetric" parts;
- For chosen A_f randomly in accordance with Gaussian distribution and the experimentally defined dispersion and average select the fragment charge Z_f;
- For chosen A_f, Z_f sample the kinetic energy of fragments according to the Gaussian distribution with experimentally defined average values and dispersions;
- Applying energy conservation and using fragment ground state masses calculate excitation energy of fragments and share it between fragments assuming that fragments have equal temperatures;
- Calculate absolute value of the c.m. fragment momentum (non-relativistic kinematics is used) and sample fragment fly off angles assuming isotropic angular distribution of fragments;

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Chapter 21

Multifragmentation model for light nuclei.

21.1 Multifragmentation of the light nuclei.

This model is based on the approach [1]. It can be used to predict final states as result of an excited light nucleus ($A \leq 16$) break-up (explosion).

For light nuclei the values of excitation energy per nucleon are often comparable with nucleon binding energy. Thus a light excited nucleus breaks into two or more fragments with the branching given by available phase space. To describe a process of nuclear disassembling the so-called Fermi break-up model is used [2], [3], [4].

The initial information for calculation of break-up stage consists from the atomic mass number A, charge Z and number of neutrons N of residual (e. g. after hadron kinetic stage or fission) nucleus and its excitation energy U. The total energy of nucleus in the rest system is $E = U + M(A, Z)$, where $M(A, Z)$ is the ground state mass of the nucleus (see the nuclear properties chapter).

21.1.1 Allowed channel.

The channel will be allowed for decay, if the total kinetic energy E_{kin} of all fragments of the given channel at the moment of break-up is positive. This energy can be calculated according to the equation:

$$E_{kin} = U + M(A, Z) - E_{Coulomb} - \sum_{b=1}^{n}(m_b + \epsilon_b), \qquad (21.1)$$

m_b and ϵ_b are masses and excitation energies of fragments, respectively, $E_{Coulomb}$ is the Coulomb barrier for the given channel. It is approximated

by

$$E_{Coulomb} = \frac{3}{5}\frac{e^2}{r_0}\left(1+\frac{V}{V_0}\right)^{-1/3}\left(\frac{Z^2}{A^{1/3}} - \sum_{b=1}^{n}\frac{Z^2}{A_b^{1/3}}\right), \qquad (21.2)$$

where V_0 is the volume of the system corresponding to the normal nuclear matter density, $r_0 = 1.3$ fm and $V = \kappa V_0$ is the decaying system volume ($\kappa = 1$ is usually used).

21.1.2 Multifragmentation probability.

The total probability per unit energy for nucleus to break-up into n components (nucleons, deutrons, tritons, alphas etc) in the final state is given by

$$W(E,n) = (V/\Omega)^{n-1}\rho_n(E), \qquad (21.3)$$

where $\rho_n(E)$ is the density of a number of final states, V is the volume of decaying system and $\Omega = (2\pi\hbar)^3$ is the normalisation volume and $\hbar = 0.197$ fm GeV. The density $\rho_n(E)$ can be defined as a product of three factors:

$$\rho_n(E) = M_n(E)S_nG_n. \qquad (21.4)$$

The first one is the phase space factor defined as

$$M_n = \int_{-\infty}^{+\infty}...\int_{-\infty}^{+\infty}\delta\left(\sum_{b=1}^{n}\mathbf{p_b}\right)\delta\left(E - \sum_{b=1}^{n}\sqrt{p^2+m_b^2}\right)\prod_{b=1}^{n}d^3p_b, \qquad (21.5)$$

where $\mathbf{p_b}$ are fragment momenta. The second one is the spin factor:

$$S_n = \prod_{b=1}^{n}(2s_b+1), \qquad (21.6)$$

which gives the number of states with different spin orientations. The last one is the permutation factor

$$G_n = \prod_{j=1}^{k}\frac{1}{n_j!}, \qquad (21.7)$$

which takes into account identity of components in the final state (n_j is the number of components of j- type particles and k is defined by $n = \sum_{j=1}^{k} n_j$). E.g. if in final state we have $n = 6$ particles and from them there are 2-alphas, 3-nucleons and 1-deutrons, then $G_6 = 1/(2!3!1!) = 1/12$.

For the non-relativistic case

$$M_n = \int_{-\infty}^{+\infty}...\int_{-\infty}^{+\infty}\delta\left(\sum_{b=1}^{n}\mathbf{p_b}\right)\delta\left(\sum_{b=1}^{n}\frac{p_b^2}{2m_b} - E_{kin}\right)\prod_{b=1}^{n}d^3p_b \qquad (21.8)$$

the integration in Eq. (21.5) can be evaluated analytically (see e. g. [5]) and the probability for a nucleus with energy E disassembling into n fragments with masses m_b ($b = 1, 2, 3, ..., n$) is

$$W(E_{kin}, n) = S_n G_n \left(\frac{V}{\Omega}\right)^{n-1} \left(\frac{1}{\sum_{b=1}^{n} m_b} \prod_{b=1}^{n} m_b\right)^{3/2} \frac{(2\pi)^{3(n-1)/2}}{\Gamma(3(n-1)/2)} E_{kin}^{3n/2-5/2}, \tag{21.9}$$

where $\Gamma(x)$ is the gamma function.

21.1.3 Fermi break-up model parameter.

The Fermi break-up model has only one free parameter V is the volume of decaying system, which can be calculated as follows:

$$V = 4\pi R^3/3 = 4\pi r_0^3 A/3, \tag{21.10}$$

where $r_0 = 1.3$ fm is usually used.

21.1.4 Fragment characteristics.

So far we considered only the formation of fragments in their ground and low-lying excited states, which are stable for nucleon emission. However, several unstable fragments with large lifetimes: ^5He, ^5Li, ^8Be, ^9B etc can also be considered [6]. Fragment characteristics A_b, Z_b, s_b and ϵ_b can be found in[6].

21.1.5 MC procedure.

1. We randomly (according to probability Eq. (21.9) and condition Eq. (21.1)) select decay channel.
2. Then for given channel we calculate kinematic quantities of each fragment according to the relativistic n-body phase space distribution (see Eq. (21.5)) using the Kopylov's methods[7] (see the resonance decay chapter).

We should note that outlined MC procedure, which is usually used[1], is not strictly consistent. The non-relativistic phase space is used for the channel probability calculation, but product momenta are sampled according to the relativistic phase space. However, it can be corrected since we are able to calculate relativistic phase space factor (Eq. (21.5)) numerically (see the Kopylov's method in the resonance decay model chapter).

We can take into account that the neutral fragments are not affected by the Coulomb field. When fragment move away to infinity its total kinetic energy can be approximated by the sum of its translational motion energy and the contribution from the Coulomb repulsion.

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Chapter 22

Multifragmentation model for heavy nuclei.

22.1 Multifragmentation process simulation.

The multifragmentation model is able to predict final states (nuclear fragments) as result of high excited nucleus break-up (explosion). The multifragmentation process simulation is based on the approach [1].

At high excitation energies $U/A > 5 - 7$ MeV the multifragmentation mechanism, when nuclear system can eventually breaks down into fragments, becomes dominant. Later on the excited primary fragments propagate independently in the mutual Coulomb field and undergo de-excitation. Detailed description of multifragmentation mechanism can be found in review [1].

The initial information for calculation of multifragmentation stage consists from the atomic mass number A, charge Z of residual (e. g. after kinetic stage of reaction) nucleus and its excitation energy U.

22.1.1 Multifragmentation probability.

The probability of a breakup channel b is given by the expression (in the so-called microcanonical approach [1], [2]):

$$W_b(U, A, Z) = \frac{1}{\sum_b \exp[S_b(U, A, Z)]} \exp[S_b(U, A, Z)], \qquad (22.1)$$

where $S_b(U, A, Z)$ is the entropy of a multifragment state of the corresponding breakup channel b. The channels $\{b\}$ can be parametrised by set of fragment multiplicities N_{A_f,Z_f} for fragments with atomic numbers A_f and charges Z_f. All partitions $\{b\}$ should satisfy constraints are derived from the total mass and charge:

$$\sum_f N_{A_f,Z_f} A_f = A \qquad (22.2)$$

and

$$\sum_f N_{A_f,Z_f} Z_f = Z. \tag{22.3}$$

It is assumed [2] that thermodynamic equilibrium is established in every channel, which can be characterised by the channel temperature T_b.

The channel temperature T_b is determined by the equation constraining the average energy $E_b(T_b, V)$ associated with partition b:

$$E_b(T_b, V) = U + E_{ground} = U + M(A, Z), \tag{22.4}$$

where V is the system volume, E_{ground} is the ground state (at $T_b = 0$) energy of system and $M(A, Z)$ is the mass of nucleus (see the nuclear properties chapter).

According to the conventional thermodynamics formulae the average energy of a partition b is expressed through the system free energy F_b as

$$E_b(T_b, V) = F_b(T_b, V) + T_b S_b(T_b, V). \tag{22.5}$$

Thus, if the free energy F_b of a partition b is known, we can find the channel temperature T_b from Eqs. (22.4) and (22.5), then the entropy $S_b = -dF_b/dT_b$ and hence, decay probability W_b defined by Eq. (22.1) can be calculated.

Calculation of the free energy is based on the liquid-drop description of individual fragments [2]. The free energy of a partition b can be split into several terms:

$$F_b(T_b, V) = \sum_f F_f(T_b, V) + E_C(V), \tag{22.6}$$

where $F_f(T_b, V)$ is the average energy of an individual fragment including the volume

$$F_f^V = [-E_0 - T_b^2/\epsilon(A_f)]A_f, \tag{22.7}$$

the surface

$$F_f^{Sur} = \beta_0[(T_c^2 - T_b^2)/(T_c^2 + T_b^2)]^{5/4} A_f^{2/3} = \beta(T_b) A_f^{2/3}, \tag{22.8}$$

the symmetry

$$F_f^{Sim} = \gamma(A_f - 2Z_f)^2/A_f, \tag{22.9}$$

the Coulomb

$$F_f^C = \frac{3}{5}\frac{Z_f^2 e^2}{r_0 A_f^{1/3}}[1 - (1 + \kappa_C)^{-1/3}] \tag{22.10}$$

and the translational

$$F_f^t = -T_b \left[\ln Z_{A_f,Z_f} - \ln (N_{A_f,Z_f}!)/N_{A_f,Z_f}\right] \tag{22.11}$$

terms. Using the Boltzmann gas approximation ($g_f = 1$ and translational energy $E_b^t = 3/2T_b$) the translation partition sum can be calculates as follows:

$$Z_{A_f,Z_f} = g_f V_f \int \frac{d^3 p_{A_f,Z_f}}{(2\pi\hbar)^4} \exp\left[-\frac{p^2_{A_f,Z_f}}{2m_{A_f,Z_f}T}\right] = g_f V_f \left(\frac{2m_{A_f,Z_f}}{2\pi\hbar^2}\right)^{3/2} \tag{22.12}$$

The expression for translational free energy can be approximated more, if we assume that $m_{A_f,Z_f} = m_N A_f$, where m_N is a nucleon mass and $N_{A_f,Z_f}! \approx (\frac{N_{A_f,Z_f}}{e})^N_{A_f,Z_f}$.

We can also introduce the thermal wavelength $\lambda_{T_b} = (2\pi\hbar^2/m_N T_b)^{1/2}$ and rewrite the Eq. (22.11):

$$F_f^t = -T_b \left(\ln \frac{V_f A_f^{3/2}}{\lambda^3_{T_b} N_{A_f,Z_f}} + 1\right). \tag{22.13}$$

The last term

$$E_C(V) = \frac{3}{5}\frac{Z^2 e^2}{R} \tag{22.14}$$

is the Coulomb energy of the uniformly charged sphere with charge Ze and the radius $R = (3V/4\pi)^{1/3} = r_0 A^{1/3}(1+\kappa_C)^{1/3}$, where $\kappa_C = 2$ [2].

Parameters $E_0 = 16$ MeV, $\beta_0 = 18$ MeV, $\gamma = 25$ MeV are the coefficients of the Bethe-Weizsacker formula at $T_b = 0$ (see the nuclear properties chapter). $g_f = (2S_f + 1)(2I_f + 1)$ is spin S_f and isospin I_f degeneracy factor for a fragment. Fragments with $A_f > 1$ are treated as the Boltzmann particles. $r_0 = 1.17$ fm and $T_c = 18$ MeV is the critical temperature, which corresponds to the liquid-gas phase transition. $\epsilon(A_f) = \epsilon_0[1 + 3/(A_f - 1)]$ is the inverse level density of the mass A_f fragment and $\epsilon_0 = 16$ MeV is considered as a variable model parameter, whose value depends on the fraction of energy transferred to the internal degrees of freedom of fragments [2]. The free volume $V_f = \kappa V = \kappa\frac{4}{3}\pi r_0^4 A$ available to the translational motion of a fragment, where $\kappa \approx 1$ and its dependence on the multiplicity of fragments can be taken from [2]:

$$\kappa = \left[1 + \frac{1.44}{r_0 A^{1/3}}(M^{1/3} - 1)\right]^3 - 1, \tag{22.15}$$

where $\kappa = 0$, if $M = 1$.

The light fragments with $A_f < 4$, which have no excited states, are considered as elementary particles characterised by the empirical masses M_f, radii R_f, binding energies B_f, spin degeneracy factors g_f of ground states, respectively. They contribute to the translation free energy and the Coulomb energy.

22.1.2 Direct simulation of the low multiplicity multifragment disintegration.

At comparatively low excitation energy (temperature) system will disintegrate into a small number of fragments $M \leq 4$ and number of channel is not huge. For such situation a direct (microcanonical) sorting of all decay channels can be performed. Then, using Eq. (22.1), the average multiplicity value $< M >$ can be found. To check that we really have the situation with the low excitation energy, the obtained value of $< M >$ is examined to obey the inequality $< M > \leq M_0$, where $M_0 = 3.3$ and $M_0 = 2.6$ for $A \sim 100$ and for $A \sim 200$, respectively [2]. If the discussed inequality is fulfilled, then the set of channels under consideration is believed to be suitable for a correct description of the break up. Then using calculated according Eq. (22.1) probabilities we can randomly select a specific channel with given values of A_f and Z_f.

22.1.3 Fragment multiplicity distribution.

The individual fragment multiplicities N_{A_f,Z_f} in the so-called macrocanonical ensemble [1] are distributed according to the Poisson distribution:

$$P(N_{A_f,Z_f}) = \exp\left(-\omega_{A_f,Z_f}\right)\frac{\omega_{A_f,Z_f}^{N_{A_f,Z_f}}}{N_{A_f,Z_f}!} \tag{22.16}$$

with mean value $< N_{A_f,Z_f} > = \omega_{A_f,Z_f}$ defined as

$$< N_{A_f,Z_f} > = g_f A_f^{3/2} \frac{V_f}{\lambda_{T_b}^3} \exp\left[\frac{1}{T_b}(F_f(T_b,V) - F_f^t(T_b,V) - \mu A_f - \nu Z_f)\right], \tag{22.17}$$

where μ and ν are chemical potentials. The chemical potentials can be found by substituting the Eq. (22.17) into the system of constraints:

$$\sum_f < N_{A_f,Z_f} > A_f = A \tag{22.18}$$

and

$$\sum_f < N_{A_f,Z_f} > Z_f = Z \tag{22.19}$$

and solving them by iteration.

22.1.4 Fragment atomic number distribution.

Fragment atomic numbers $A_f > 1$ are also distributed according to the Poisson distribution [1] (see Eq. (22.16)) with mean value $< N_{A_f} >$ defined

as

$$< N_{A_f} >= A_f^{3/2} \frac{V_f}{\lambda_{T_b}^3} \exp\left[\frac{1}{T_b}(F_f(T_b,V) - F_f^t(T_f,V) - \mu A_f - \nu < Z_f >)\right], \tag{22.20}$$

where calculating the internal free energy $F_f(T_b,V) - F_f^t(T_b,V)$ one has to substitute $Z_f \rightarrow < Z_f >$. The average charge $< Z_f >$ for fragment having atomic number A_f is given by

$$< Z_f(A_f) >= \frac{(4\gamma + \nu)A_f}{8\gamma + 2[1-(1+\kappa)^{-1/3}]A_f^{2/3}}. \tag{22.21}$$

22.1.5 Fragment charge distribution.

At given mass of fragment $A_f > 1$ the charge Z_f distribution of fragments are described by the Gaussian distribution:

$$P(Z_f(A_f)) \sim \exp\left[-\frac{(Z_f(A_f) - < Z_f(A_f) >)^2}{2(\sigma_{Z_f}(A_f))^2}\right] \tag{22.22}$$

with dispersion

$$\sigma_{Z_f(A_f)} = \sqrt{\frac{A_f T_b}{8\gamma + 2[1-(1+\kappa)^{-1/3}]A_f^{2/3}}} \approx \sqrt{\frac{A_f T_b}{8\gamma}}. \tag{22.23}$$

and the average charge $< Z_f(A_f) >$ defined by Eq. (22.21).

22.1.6 Fragment kinetic energy distribution.

It is assumed [2] that at the instant of nucleus break-up the kinetic energy of the fragment T_{kin}^f in the rest of nucleus obeys the Boltzmann distribution with given temperature T_b:

$$\frac{dP(T_{kin}^f)}{dT_{kin}^f} \sim \sqrt{T_{kin}^f} \exp\left(-T_{kin}^f/T_b\right). \tag{22.24}$$

Under assumption of thermodynamic equilibrium the fragment have isotropic velocities distribution in the rest frame of nucleus. The total kinetic energy of fragments should be equal $\frac{3}{2}MT_b$, where M is the fragment multiplicity. The total fragment momentum should be equal zero. These conditions are fulfilled by the proper choice of the momenta of two last fragments.

The initial conditions for the divergence of the fragment system are determined by random selection of fragment coordinates distributed with equal probabilities over the break-up volume $V_f = \kappa V$. It can be a sphere or prolongated ellipsoid. Then Newton's equations of motion can be solved for all fragments in the self-consistent time-dependent Coulomb field [2]. Thus the asymptotic energies of fragments determined as result of this procedure differ from the initial values by the Coulomb repulsion energy.

22.1.7 Calculation of the excitation energies of multifragmentation products.

The temperature T_b determines the average excitation energy of each fragment:

$$U_f(T_b) = E_f(T_b) - E_f(0) = \frac{T_b^2}{\epsilon_0} A_f + \left[\beta(T_b) - T_b \frac{d\beta(T_b)}{dT_b} - \beta_0\right] A_f^{2/3}, \quad (22.25)$$

where $E_f(T_b)$ is the average fragment energy at given temperature T_b and $\beta(T_b)$ is defined by the Eq. (22.8). There is no excitation for fragment with $A_f < 4$, for ^4He excitation energy can be taken as $U_{^4He} = 4T_b^2/\epsilon_o$.

22.1.8 MC procedure.

The Monte Carlo calculation of fragment characteristics can be outlined as follows:

1. Perform direct simulation of the low multiplicity disintegration using Eq. (22.1) and find average multiplicity value $< M >$. Examine that the found $< M >$ is really small, i.e. $< M > \leq M_0$. Using calculated according to the Eq. (22.1) probabilities, randomly select a specific channel with given values of A_f and Z_f. Then proceed to the step (5). If the obtained value of $< M > > M_0$, then proceed to the next step.

2. Sample atomic numbers of fragment A_f^i, where $1 \leq A_f^i \leq A$ according to the Poisson distribution Eq. (22.16) with the mean value $< N_{A_f} >$ defined by Eq. (22.20).

3. For chosen A_f^i randomly in accordance with the Gaussian distribution Eq. (22.22) with the dispersion defined by Eq. (22.23) and the average defined by Eq. (22.21) select fragment charge Z_f^i, where $0 \leq Z_f^i \leq Z$.

4. Repeat the sampling of A_f^i (step (2)) and Z_f^i (step (3)) i times until for all fragments their atomic numbers and charges will be defined. If the sum of nucleons and charge of all fragments exceed the values A and Z, then the procedure should be repeated starting from $i = 1$ at the step (2).

5. For chosen A_f, Z_f randomly according to the Boltzmann distribution Eq. (22.24) determine fragment kinetic energies T_{kin}^f at the instant of the nucleus break-up in the rest of decaying nucleus system. Then define fragment velocities and momenta under assumption of isotropic velocities distribution in the rest frame of decaying nucleus. Use the momenta of two last fragments to fulfil the energy-momentum constraints. By random selection of fragment coordinates distributed with equal probabilities over the break-up volume $V_f = \kappa V$ determine the

initial conditions for the divergence of the fragment system after break-up instant. Solve the Newton's equations of motion of all fragments in the self-consistent time-dependent Coulomb field to define the asymptotic energies of fragments.

6. Calculate excitation energies of fragments using the Eq. (22.25).

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Chapter 23

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