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PRIMORDIAL INFLATION[1]

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Abstract

A macroscopic universe may emerge naturally from a Planck cell fluctuation by unfolding through a stage of exponential expansion towards a homogeneous cosmological background. Such primordial inflation requires a large and presumably infinite degeneracy at the Planck scale, rooted in the unbounded negative gravitational energy stored in conformal classes. This complex Planck structure is consistent with a quantum tunneling description of the transition from the Planck scale to the inflationary era and implies, in the limit of vanishing Planck size, the Hartle-Hawking no-time boundary condition. On the other hand, string theory give credence to the holographic principle and the concomitant depletion of states at the Planck scale. The apparent incompatibility of primordial inflation with holography either invalidates one of these two notions or relegates the nature of the Planck size outside the realm of quantum physics, as we know it.

There are more things in heaven and earth, Horatio,
Than are dreamt of in your philosophy.
Shakespeare (Hamlet, Act I Scene V)

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1. INTRODUCTION

Primordial inflation is a mechanism whereby *a* universe emerges from a Planckian fluctuation of gravity and matter and is stabilized by an exponential expansion of the cosmological background[1].

The primordial inflationary scenario arose from an attempt to understand in scientific terms the problem posed by the birth of our universe and by the homogeneity of its huge cosmological background. The idea that our universe originated from an energy conserving quantum fluctuation was first proposed by Tryon [1]. We used related ideas to search for a mechanism which does not require a fine tuning of the initial conditions for the cosmological expansion [2, 3].

When extrapolated backwards in time the adiabatic expansion approaches one Planck time of the classical singularity, the presently observable universe still comprises about 10^{87} Planck cells. To avoid fine tuning, such a huge spatial extension requires the existence of a preadiabatic phase in which the cosmological background can develop causally from an initial Planckian cell fluctuation[2]. The basic ingredient which renders such an evolution possible is the fact that in general relativity the total Hamiltonian of *the* universe should be zero, a general feature which is expected to hold in any future development of gravity theory preserving invariance under time reparametrization. This feature renders possible the creation of positive matter energy out of the negative energy stored in conformally flat space-times, such as space-times describing homogeneous isotropic cosmological backgrounds. An expanding universe could then emerge, without cost of energy, from empty flat space-time. The requirement that this phenomenon be localized to a Planck cell implies that universe-like configurations can occur within a universe [3, 6].

Semi-classical considerations indicate that the cosmological expansion needed to generate matter energy in a self-consistent way is the exponential expansion of a de Sitter space-time [2]. Nevertheless, conventional classical general relativity cannot yield a satisfactory theory of a nascent universe.

[1]Talking about *a* universe may seem contradictory. The reason for this terminology is that one has often the prejudice that *the* universe, comprising everything, is endowed with a global arrow of time, but there is no theoretical indication that such an entity does exist. I shall call *a* universe (or a universe-like configuration) any space-time configuration with a well-defined arrow of time containing a large number of Planck cells, whether or not such structure resembles *our* universe.

[2]I am assuming here and in what follows the validity of classical general relativity up to the Planck scale.

Extending classical general relativity to the Euclidean section, it appears possible to interpret its birth as a tunneling process. In the limit of vanishing Planck size ($\hbar \to 0$) one recovers in this way the no-time Hartle-Hawking boundary condition [4]. More precisely the no-time boundary appears as the limit of a thermal state where the original Planck cell has infinite temperature[3]. Planck cells appear as very complex structures containing a huge and perhaps an infinite number of degrees of freedom.

Primordial inflation explains the large scale of a universe, the flatness and the homogeneity of the background. It also raises the possibility of creating the entropy of a universe during the inflationary phase itself as internal entropy. The latter would be released as radiation entropy during the turnover to the adiabatic era [3, 6]. Such scenario would be different from the reheating process envisaged in more mundane inflation mechanisms where the entropy is directly formed at turnover [8]. Whether this alternative would affect the seeding of large scale homogeneities as usually predicted by inflation remains to be seen, but is outside the scope of the present considerations.

The huge number of degrees of freedom needed at the Planck scale to accommodate primordial inflation is a feature corroborated by semi-classical considerations on black hole evaporation. But it is at odd with the holographic principle which states that the number of degrees of freedom within a given space volume is limited by the number of Planck cells of a surface bounding the volume [9]. The holographic principle appears to be encoded in the string and M-theory approach to quantum gravity and implies a depletion of quantum states at the Planck scale. The incompatibility of primordial inflation with holography seems to invalidate one of these two notions. Both could only survive if the complex structure of the Planck size would be outside the realm of quantum physics, as we know it. Recently, 't Hooft advocated that quantum physics gives a statistical description of an underlying deterministic and dissipative theory by coalescing fundamental states into equivalence classes which define the quantum states [10]. The merging of all Planckian degrees of freedom into a single class or into few classes would indeed reconcile, in the present context, holography with a complex structure of the Planck size. But no light is shed here on whether Planck size physics should be deterministic or not.

I present a critical review of the semi-classical approach to primordial

[3] As will be shown in Section 3, this result follows from the tunneling approach of reference [5]. The preheating of matter was anticipated in the references [7].

inflation in Section 2 and I discuss the tunneling approach in section 3. Conclusions are stated in section 4 where the nature of Planck cells resulting from these considerations are confronted with M-theory arguments.

2. THE SEMI-CLASSICAL APPROACH

The classical action for gravity and matter is

$$\begin{aligned} S &= S_{gravity} + S_{matter} \\ &= -\frac{1}{16\pi G}\int \sqrt{-g}\, R(g_{\mu\nu}) + \int \sqrt{-g}\, \{\mathcal{L}(\psi_i, g_{\mu\nu}) - \Lambda\} \end{aligned} \tag{1}$$

where ψ_i designate matter fields. The cosmological constant term Λ is included in the matter lagrangian density $\mathcal{L}$.

The invariance of the action Eq.(1) under time reparametrization leads to a constraint equation expressing the vanishing of the total Hamiltonian density. If, as is reasonable to assume for the whole universe, no boundary term contributes to the total energy, the total Hamiltonian H satisfies

$$H \equiv H_{gravity} + H_{matter} = 0 \ , \tag{2}$$

expressing the vanishing of the total energy. As the matter energy (assuming the net cosmological constant to be non negative) is positive definite the total gravitational energy must be negative if matter is present. To understand the origin of this negative energy let us perform the conformal transformation

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} \exp(-2\phi) \tag{3}$$

and rewrite the gravitational action $S_{gravity}$ as

$$\begin{aligned} S_{gravity} = &- \frac{1}{16\pi G}\int \sqrt{-\tilde{g}}\exp(2\phi) R(\tilde{g}_{\mu\nu}) \\ &- \frac{3}{8\pi G}\int \sqrt{-\tilde{g}}\exp(2\phi)\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \ . \end{aligned} \tag{4}$$

Conformal classes are defined by a reference metric $\tilde{g}_{\mu\nu}$ and the field ϕ. The "kinetic energy" of the ϕ-field is negative definite in the reference metric. The importance of this observation for cosmology is that all Robertson-Walker geometries describing homogeneous cosmological expansion are conformally flat. They can be described by a reference Minkowski space in which the constraint Eq.(2) follows from the the energy-momentum tensor in flat space-time deduced from Eqs.(4)

$$\tilde{T}^{gravity}_{\mu\nu} + \tilde{T}^{matter}_{\mu\nu} = 0 \ , \tag{5}$$

with

$$\begin{aligned}\tilde{T}^{gravity}_{\mu\nu} &= -\frac{3}{4\pi G}[\exp{(2\phi)}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\tilde{g}_{\mu\nu}\exp{(2\phi)}\tilde{g}^{\sigma\tau}\partial_\sigma\phi\partial_\tau\phi \\ &- \frac{1}{6}(\Delta_\mu\partial_\nu - \tilde{g}_{\mu\nu}\nabla^2)\exp{(2\phi)}] \ . \end{aligned} \tag{6}$$

Eqs.(5) and (6) give the usual description of the cosmological evolution of homogeneous matter in a Robertson-Walker metric. To make this explicit, let us write the Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t)d\Sigma^2 = a^2(\eta)[d\eta^2 - d\Sigma^2] \tag{7}$$

where $d\Sigma^2$ describes a reference sphere, hyperboloid or flat Euclidean 3-space. η is the conformal time defined by $dt = a(\eta)d\eta$.

Consider first the spatially flat case. Taking $\tilde{g}_{\mu\nu} = \eta_{\mu\nu} = (+1, -1, -1, -1)$ and choosing the conformal factor to be

$$\exp\phi(\eta) = a(\eta) \ , \tag{8}$$

one can write Eqs.(5) and (6) as :

$$-\frac{1}{2}\left(\frac{da(\eta)}{d\eta}\right)^2 + \frac{4\pi G}{3}a^4(\eta)\sigma = 0 \ , \quad d(\sigma a^3) = -pd(a^3) \ , \tag{9}$$

where $\sigma = T^t_t = a^{-4}\tilde{T}^{matter}_{\eta\eta}$ is the matter energy density in the comoving frame and p its pressure. Eq.(9) is the usual Einstein equation for the spatially flat universe written in a way which illustrates the negative "kinetic energy" carried by the conformal factor. Similarly, with different relations between a and $\exp\phi$, one recovers

$$-\frac{1}{2}\left(\frac{da(\eta)}{d\eta}\right)^2 \mp \frac{a^2(\eta)}{\rho^2} + \frac{4\pi G}{3}a^4(\eta)\sigma = 0 \tag{10}$$

with the minus (plus) sign for positive (negative) spatial curvature. ρ is the radius of the reference sphere or hyperboloid.

In absence of matter the only solution to Eqs.(5) and (6) is flat space-time if the vacuum energy is canceled by the Λ term in Eq.(1)[4]. Due to the negative sign in the gravitational energy tensor Eq.(6), one may envisage a energy conserving transition from empty flat space-time to a cooperative phenomenon whereby positive matter energy drives an expansion which in turn creates more matter energy. Fluctuations around the cosmological background will of course provoke a departure from conformally flat metrics but the expansion energy will remain a driving force encoded in the negative energy stored in conformal classes.

When matter is described by a scalar field with non vanishing expectation value, the cooperative process envisaged here is simply the mundane inflation mechanism : matter energy is produced at constant density as an effective cosmological constant resulting from some classical scalar field equation. If the original transition from zero to a finite effective cosmological constant could, in such models, originate from gravity-matter interactions, they would yield a primordial inflation.

I shall now review critically a model which explores, in a semi-classical approximation, the possibility that gravity would generate the effective cosmological constant from empty flat space [2, 6].

Consider a massive scalar field interacting only with gravity. The matter action is taken to be

$$S_{matter} = \int \sqrt{-g} \left\{ \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi - \frac{1}{2} (M - \frac{R}{6}) \psi^2 \right] - \Lambda \right\}. \tag{11}$$

The inclusion of the $R/6$ term restores, when $M \to 0$ classical conformal invariance ; it ensure that the number of massive particle produced by the cosmological expansion remains finite [11].

At some initial time $t = \eta = 0$ the geometry is taken to be empty flat Minkowskian space-time. Assume provisionally that after a time $t > t_0$, where t_0 is somewhat larger than the Planck time, space-time can be described by a non trivial conformally flat metric $g_{\mu\nu} = a(\eta)\eta_{\mu\nu}$. In the reference frame $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$, the normal modes of the rescaled scalar field $\tilde{\psi} = \psi\, a(\eta)$ acquire time-dependent frequencies $\omega_p = (p^2 + M^2 a^2(\eta))^{1/2}$. Expanding $\tilde{\psi}$ in creation and destruction operators $\alpha^+(p, 0), \alpha(p, 0)$ at time $t = \eta = 0$, one

[4]This cancellation is imposed here on phenomenological ground. Note that if the existence of a small value for the cosmological constant in our universe is confirmed, one should not perform a complete cancellation and the flat space-time solution would be replaced by a de Sitter space-time of large radius.

defines the Heisenberg state of the universe $|\Omega\rangle$ by $\alpha(p,0)|\Omega\rangle = 0$. As the universe expands, the time dependent normal modes get populated with a density $n_p(t) = \langle \alpha^+(p,t)\, \alpha(p,t)\rangle$. One looks for a self consistent solution for the gravity-matter system in the semi-classical limit where matter is treated quantum mechanically but gravity remains classical.

Trying, at sufficiently large time t, the de Sitter space-time solution given by

$$a(t) = \exp(t/\tau)\ , \tag{12}$$

or equivalently

$$a(\eta) = \tau/(\tau - \eta)\ , \tag{13}$$

one writes the proper energy density due to the created particles as

$$\sigma = a^{-4}(t) \int_0^\infty \frac{p^2 dp}{2\pi^2} n_p(t)\omega_p(t)\ . \tag{14}$$

Here, τ is the radius of the de Sitter space-time which is taken to be of order t_0. The zero point energy in Eq.(14) has been subtracted to ensure the vanishing of the cosmological constant in flat space-time.

One can show [2, 6] by computing $n_p(t)$ that σ given by Eq.(14) becomes independent of t after a time of order τ and thus that the created matter energy density is, at sufficiently large time t, constant in time. In addition, one can verify that $p = -\sigma$, in accordance with energy conservation for creation of a *constant* energy density. The integral Eq.(14) can be performed analytically to yield

$$\sigma = \frac{M^4}{64\pi^2}[\Psi(\frac{3}{2}+\nu) + \Psi(\frac{3}{2}-\nu) - 2\ln M\tau] \tag{15}$$

where Ψ is the digamma function and $\nu^2 = 1/4 - (M\tau)^2$ [12]. From the asymptotic form of the digamma functions one gets for $M\tau \gg 1$

$$\sigma \rightarrow \frac{M^2}{96\pi^2\tau^2}\ . \tag{16}$$

There is an additive correction to Eq.(15) revealed by neighboring non conformally flat metrics, namely the trace anomaly

$$\sigma_{anomaly} = \frac{1}{960\pi^2\tau^4}\ . \tag{17}$$

Clearly the anomaly contribution does not affect the asymptotic value of σ given in Eq.(16).

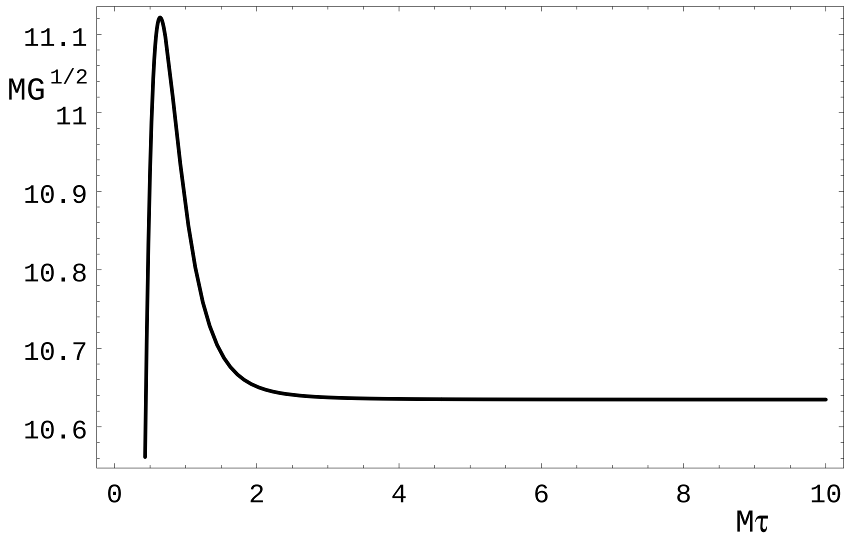

Fig.1 $M\sqrt{G}$ as a function of $M\tau$. The asymptotic value of M is $6\sqrt{\pi/G}$.

Equating the total contribution to σ due to quantum effects, as given by Eq.(15) and Eq.(17), to its value for the de Sitter solution of Einstein equation

$$\sigma_{deSitter} = \frac{3}{8\pi G\tau^2} \ , \tag{18}$$

one obtains a relation between τ and M :

$$G^{1/2}M = \left\{ \frac{(M\tau)^2}{24\pi}[\Psi(\frac{3}{2}+\nu) + \Psi(\frac{3}{2}-\nu) - 2\ln M\tau + \frac{1}{15(M\tau)^4}] \right\}^{-\frac{1}{2}} . \tag{19}$$

This equation was first solved numerically in reference [13]. The solution is depicted in Fig.1. One sees that for $M\tau > 1$ a self-consistent solution of the equations of motion exists only if M exceeds the critical value obtained by equating the asymptotic value Eq.(16) to the de Sitter solution Eq.(18). The critical mass is

$$M_{critical} = 6\sqrt{\frac{\pi}{G}} \ . \tag{20}$$

M remains very close to its critical value in the region $M\tau > 1$. In this region the Compton wavelength of the particle is small compared to the de Sitter radius and, as τ should be greater than $\sqrt{G}$, I shall consider only this region. The drop towards zero below $M\tau = 1$ is due to the trace anomaly which has a negligible effect for $M\tau > 1$.

It thus appears at first sight that an exponentially expanding universe, originating from some fluctuation in empty flat space-time, can be sustained through creation of particles of mass close to the critical mass. But there is a catch. The asymptotic value of σ in Eq.(16) yields a trace of the energy-momentum tensor proportional to the curvature and one could absorb it in a renormalization of the gravitational coupling constant. This would yield a renormalized matter density σ_{ren}

$$\sigma_{ren} = \frac{M^4}{64\pi^2}[\Psi(\frac{3}{2}+\nu) + \Psi(\frac{3}{2}-\nu) - 2\ln M\tau - \frac{2}{3(M\tau)^2} + \frac{1}{15(M\tau)^4}] \quad (21)$$

and a renormalized coupling constant G_{ren}

$$G_{ren} = \frac{G}{1 - GM^2/36\pi} \ . \quad (22)$$

These are well known results [14]. Clearly, the renormalization of the gravitational constant ensures that the subtraction in the renormalized energy density does not affect the solution, expressed in terms of the bare G, represented in Fig.1. But what goes wrong is the renormalization procedure for M greater than $M_{critical}$: the renormalized gravitational coupling becomes negative precisely at the lowest mass value for which a solution to the semi-classical equations exists.

The breakdown of the renormalization procedure indicates that new effects must be taken into account in the gravitational-matter system. This is not astonishing. The high value of the critical mass suggests that strong coupling effects appear in the system around this value. These could lead to the formation of black holes [3], or, if new matter degrees of freedom are introduced, to extended objects such as "strings" [15] . The relation between unrenormalized quantities depicted in Fig 1 might still describe approximately the formation of the massive objects but the scalar field should be viewed only as a phenomenological description of such objects [3]. An interesting possibility is that, in contradistinction with more mundane inflation scenario, these objects would carry internal entropy, (as black holes or strings do), which could bring them in (unstable?) thermodynamic equilibrium with the de Sitter background. The conversion of internal entropy, created exponentially during inflation, into radiation entropy at the turnover to the adiabatic era would then be the source of the cosmic background radiation[5].

Up to now, I have considered a spatially flat section of a de Sitter cosmological background emerging from flat space-time. To avoid fine tuning, the original fluctuation should be confined to a Planck size. As the inflation renders in a few Planck times the initial spatial curvature negligible, one is justified to consider simply spatially flat cosmological backgrounds. However the emergence of the universe should appear as a process localized on its Planck scale and not as a global process in an underlying flat background. To visualize how this could happen, choose a coordinate system in the flat background such that the time coordinate coincides with the conformal time η of the emerging universe. Namely define a local conformal time $(\eta, \mathbf{x})$ such that

$$ds^2 = a^2(\eta, \mathbf{x})(d\eta^2 - d\mathbf{x}^2) \tag{23}$$

where the $\mathbf{x}$ dependence of the scale factor is introduced to parametrize the transition region : $a = 1$ well outside the Planckian region $|\mathbf{x}| \gg \sqrt{G}$ and $a = a(\eta)$ for $|\mathbf{x}| \leq t_0$ where it describes at some initial time $\eta = 0$ the onset of a de Sitter universe of radius $\tau \simeq t_0$. Explicitly one has Eq.(13) for $\eta > 0$ and thus an exponential expansion in the proper time t.

Thus, in the "external" time η the creation process and the whole de Sitter period takes only a time of order τ. Although the universe has finite spatial extension of order τ in $|\mathbf{x}|$, an internal observer will contemplate an homogeneous cosmological background as long as his visible horizon is, in these coordinates, less than τ. The appearance of the inhomogeneous drop in scale at the edge of the universe would signal to the internal observer the end of his universe. It follows from the conformal flatness of the cosmological background that the whole history subsequent to the de Sitter period is delimited, in the conformal time η, by the light rays emitted from the edge and therefore is also of order τ. Hence the "external" observer would describe the rise and fall of our universe as a fluctuation on a scale τ comparable to

[5] The subtracted term in the renormalized energy density Eq.(21) includes the dominant contribution to the number of quanta created for $M\tau > 1$. Such subtraction can be done because the number of quanta of local field excitations is not an invariant in general relativity, but it becomes questionable in a phenomenological description.

the Planck scale. The reason "we" can exist during this seemingly short time span is the enormous dilation of the proper time t generated by the primordial inflation.

The possibility of generating a universe from a Planckian fluctuation in empty flat space raises the question of the significance of this reference space. Is it only a mathematical device or has it physical significance? If the transition from a Planckian event to the cooperative phenomenon does indeed occur, the answer must be that the reference space is physical because nothing could prevent a similar phenomenon to occur *within* our universe, as, on scales large compared to the Planck scale but small compared to cosmological scales, any background can be viewed as flat. This does not lead to a contradiction because universe-like configurations within our world would be only Planckian (or transplanckian -see below) fluctuations of our space-time. Universes within universes can therefore be generated in a large (infinite?) number of different ways and universe-like fluctuations may well be dominant Planckian effects. Note that there is no reason for two such baby-universes to have a common comoving time. It is with respect to their own comoving time that their original Planck size has a fixed scale and spatial extension. The comoving time of a baby-universe fixes locally in the mother universe the η variable in the metric Eq.(23). In particular two baby-universes may have opposite arrows of time.

The transition from a Planckian event to a universe-like configurations in an underlying space-time poses a new problem because, as mentioned above, the scale factor develops a huge gradient at its edge. We see from Eq.(6) that a spatial gradient of the scale factor gives rise to a negative energy density in $\tilde{T}_{\mu\nu}^{gravity}$ as does its time derivative hitherto considered. Local compensation by positive energy has to take place, possibly by forming new universe-like configurations. Note that these would seem to occur at much smaller scales than the Planck scale but such transplanckian effects are fictitious : the Planck scale is defined by the baby universe and is the same as before ; it is only its parametrization in the mother universe which makes it look different. However the generation of such throat-universe would only further steepen the gradient of the scale factor, thus giving birth to more and more new universes containing themselves potentially universe-like configurations. The space-time structure within a universe can thus be drastically altered at the Planck scale : universe-like configurations may form a "foam of universes" [3, 6]. Although the foam may appear as virtual, the underlying degrees of freedom should exist.

In view of the complexity of the structure due to the negative unbounded energy of the scale factor and to its necessary compensation by positive energy, the transition era can probably not be described at the classical or at the semi-classical level. If a cosmological background emerges out of a Planck size, $\hbar$ should enter the game in a fundamental way. An approximate quantum mechanical description of the creation process, taking into account the complex structure of its Planckian origin, should then be possible. Let us explore this approach.

3. THE TUNNELING INTERPRETATION

I shall assume that an approximate quantum description of the transition period can be obtained from a tunneling of the scale factor through a barrier from the original Planck scale to the macroscopic inflationary period. Although I cannot justify this procedure, there is an important consistency check : the tunneling of the scale factor must be consistent with a high degeneracy of states at the Planck side of the barrier and with an a priori complete lack of information about these states. Remarkably this will indeed be the case.

In order to describe a tunneling process, one must specify boundary conditions. These will be fixed from the requirement that the emerging de Sitter space-time has a well defined Hawking-Gibbons background temperature

$$\beta_{GH}^{-1} = 1/(2\pi\tau) \tag{24}$$

and an entropy $\mathcal{S}$ given, up to an integration constant, by the quarter of the area of the event horizon [16]. In our notations

$$\mathcal{S} = \frac{\pi\tau^2}{G} \ . \tag{25}$$

To understand how entropy and temperature determine the boundary condition of the tunneling process, I first review how entropy can be generated within a closed system in an energy eigenstate by a "tunneling of time" [5]. In such system, time evolution can only be defined by correlations between subsystems. Let us assume the existence of a subsystem for which the WKB limit is valid in certain regions of space ; this subsystem will be called the "clock". We shall see that correlations of subsystems to the clock define in these regions the time variable [17, 5] but that the tunneling of the clock can produce entropy. This mechanism will then be generalized to our problem where the total energy is zero from the constraint Eq.(2) and where the role of the clock will be played by the scale factor.

Consider a closed system where a non relativistic object with one very massive degree of freedom, x, representing the "clock", is correlated to the remaining "matter" part of the system by energy conservation only. No explicit interaction between clock and matter is imposed but the argument below is easily extended to the case of matter following adiabatically the clock. Labeling (unnormalized) matter eigenstate of energy ϵ_m by $|\chi_m\rangle$, each eigenstate $|\chi_m\rangle$ is correlated by quantum superposition to a clock state vector of energy $E - \epsilon_m$. One writes

$$\begin{aligned} H|\Psi\rangle &= (H_{clock} + H_{matter})|\Psi\rangle = E|\Psi\rangle, \\ H_{clock} &= -\frac{1}{2M}\frac{\partial^2}{\partial x^2} + U(x) \ , \\ H_{matter}|\chi_m\rangle &= \epsilon_m|\chi_m\rangle \ . \end{aligned} \tag{26}$$

Expanding $|\Psi\rangle$ in matter eigenstates

$$|\Psi\rangle = \sum_m \Phi_m(x)|\chi_m\rangle \ , \tag{27}$$

one gets

$$\left\{\frac{d^2}{dx^2} + 2M\left[E - \epsilon_m - U(x)\right]\right\}\Phi_m(x) = 0 \ . \tag{28}$$

Here $U(x)$ is some potential which vanishes as $x \to \pm\infty$. I shall take all the ϵ_m positive and assume

$$\epsilon_m \ll |E - U(x)| \ . \tag{29}$$

In the classically permitted regions, $E - U(x) > 0$, the WKB forward wave solution of Eq.(28) is

$$\Phi_m(x) = \frac{1}{\sqrt{p_m(E_m, x)}}\exp[iW(E_m)] \ . \tag{30}$$

W is the classical Legendre transform of the classical clock action

$$W(E_m, x) = S + E_m T = \int_{x_i}^{x} p_m(E_m, x')dx' \tag{31}$$

where T is the time recorded by the clock since a chosen initial position x_i. The clock energy E_m is equal to $E - \epsilon_m$ and its momentum $p_m(x, E_m)$ is $\sqrt{2M[E_m - U(x)]}$.

Expanding E_m to first order around $\epsilon_m = 0$ one gets

$$
\begin{aligned}
|\Psi\rangle &= \Sigma_m \frac{1}{\sqrt{p_m(x, E_m)}} \exp[iW(E_m, x)]|\chi_m\rangle \\
&\simeq \frac{1}{\sqrt{p(x, E)}} \exp(iW(E, x))\Sigma_m \exp[-i\frac{\partial W(E, x)}{\partial E}\epsilon_m]|\chi_m\rangle \\
&\simeq \frac{1}{\sqrt{p(x, E)}} \exp(iW(E, x)) \exp(-iTH_{matter})\Sigma_m|\chi_m\rangle
\end{aligned}
\tag{32}
$$

where from the classical action principle

$$
T(E, x) = \frac{\partial W(E, x)}{\partial E} \ . \tag{33}
$$

It follows from Eq.(32) that, under the prescribed conditions, any matter wave function $|\chi\rangle = \Sigma_m|\chi_m\rangle$ evolves according to the time dependent Schrodinger equation with the time recorded by the clock.

This result is valid in the WKB limit as long as backward waves generated from the second order equation Eq.(28) do not affect $|\Psi\rangle$ significantly. An extreme departure of this condition occurs if the clock has to tunnel through a barrier between two turning points $x = a$ and $x = b$, as shown in Fig.2 .

Let us call the regions $x < a$ and $x > b$ respectively the regions "before" and "after" tunneling.

Taking "before" tunneling forward waves only, backward wave will be generated "after" tunneling. In the limit of large clock mass, coherence "after" tunneling between forward and backward wave gets lost. The ratio N_m of the squared forward amplitudes "after" and "before" the tunneling of the clock with energy $E_m = E - \epsilon_m$ is the corresponding probability ratio for finding matter with energy ϵ_m. N_m is the inverse transmission coefficient through the barrier. For large barrier it is

$$
N_m = \exp[W_e(E_m)] \tag{34}
$$

where

$$
W_e(E_m) = \oint p_e(E_m, x)dx = \oint \sqrt{2M[U(x) - (E - \epsilon_m)]}dx \tag{35}
$$

is the "Euclidean" continuation of the action Eq.(31) defined by

$$W_e(E_m, U(x)) = W(-E_m, -U(x)) \ . \tag{36}$$

Taking into account the condition Eq.(29), one gets

$$N_m(E_m) = N_0 \exp[\frac{\partial W_e(E)}{\partial E}\epsilon_m] = N_0 e^{\theta\epsilon_m} \ , \tag{37}$$

where θ is the Euclidean time spanned by the clock in a round trip under the barrier, and

$$N_0 = \exp W_e(E) \ . \tag{38}$$

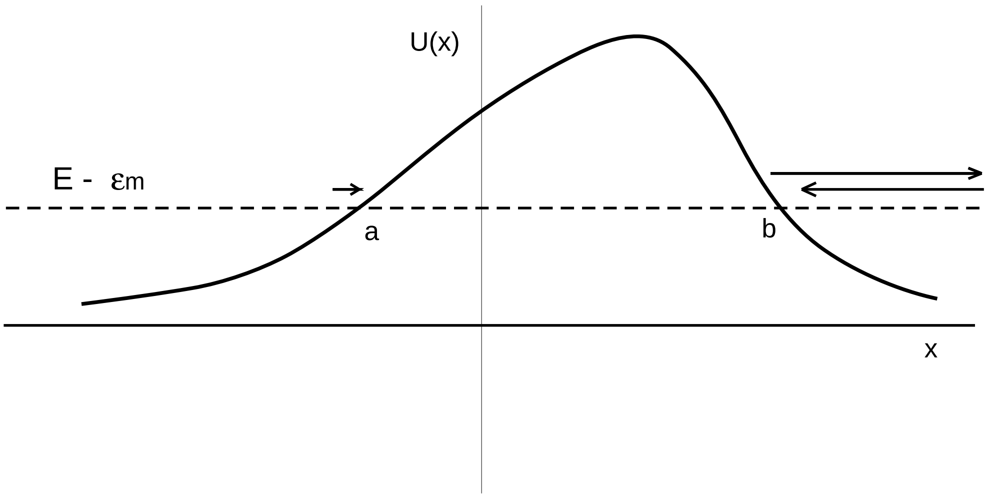

Fig.2 Tunneling of a clock of energy $E - \epsilon_m$.

The shift of sign in Eq.(36) makes the exponent in Eq.(37) positive. Thus the probability ratios N_m have a Boltsmann distribution with a *negative* temperature $\beta^{-1} = -\theta^{-1}$. The temperature is negative because higher energies for matter states mean lower energies for the the clock and hence higher inverse transmission coefficient N_m for the clock. Thus higher matter energies are favored by the tunneling.

To assess the thermodynamic significance of Eq.(37) we rewrite it more generally as

$$\delta W_e(E_m) = -\theta\delta E_m = \beta\delta E_m \ . \tag{39}$$

This equation is again the classical action principle but formulated for "Euclidean" closed trajectories. Consider matter in equilibrium with the clock "before" and "after" tunneling, respectively at temperatures β_a^{-1} and β_b^{-1} such that

$$\beta = \beta_b - \beta_a \ . \tag{40}$$

If $\mathcal{S}_b(E)$ and $\mathcal{S}_a(E)$ are the corresponding clock entropies, we see that Eq.(39) implies

$$W_e(E) = \mathcal{S}_b(E) - \mathcal{S}_a(E) \ . \tag{41}$$

$W_e(E)$ can be interpreted as the entropy gained by the clock from the backward wave generated by the tunneling process. In the particular case where matter is in equilibrium with the clock "after" tunneling at the temperature β^{-1}, the equilibrium temperature "before" tunneling must be infinite and the entropy of the clock becomes simply $W_e(E)$, up to an integration constant $\mathcal{S}_a$ independent of the clock energy.

Thus, this simple model indicates that a macroscopic clock may gain entropy by tunneling. The above analysis may seem rather academic because on the one hand the model does not contain dynamic elements to realize thermal equilibrium and infinite temperature and on the other hand the temperature β is negative. This is not the case. We see from Eq.(37) that the sign of β is related to the sign of the clock Hamiltonian. Changing indeed its sign and keeping the matter Hamiltonian positive definite would lead to $E_m = -E + \epsilon_m$ in the equations for W and W_e. Hence in Eqs.(37) and (39) θ should be replaced by $-\theta$.[6] Negative clock energies would lead to positive equilibrium temperature for matter. One might thus expect that in gravity where the "gravitational clock" has negative energy and where dynamics is contained in the many degrees of freedom, the tunneling of time should produce entropy and positive temperature. I now show that this is indeed the case.

In the cosmological context, I shall, as in previous sections, retain from gravity only the scale factor but I now allow it to be described quantum mechanically in classically forbidden regions. We shall then see that the gravitational clock can tunnel from a Planck size region to a de Sitter background of radius τ. As mentioned above, the boundary condition for the tunneling process should be consistent with the known entropy and thermal

[6]One may verify that the shift of sign does not affect the definition of time in the classically allowed region because for forward waves, the shift of sign in E_m is compensated by a shift of sign in the momentum which is then opposite to the velocity.

properties of the de Sitter background. This will determine on which side of the tunneling the wave function of the gravitational clock is small and its thermal properties on both sides.

The gravity Hamiltonian can be deduced, up to a multiplicative factor from Eq.(9) or more generally from Eq.(10). The mean matter density σ, that is the cosmological constant of the de Sitter space-time, is now included in the gravitational Hamiltonian As the open and flat spatial sections of the de Sitter space do not span the geodesically complete manifold, I shall describe it in terms of closed spatial sections. Eq.(10) yields

$$-\frac{1}{2}a'^2 - \frac{1}{2}\frac{a^2}{\rho^2} + \frac{4\pi G}{3}\Lambda a^4 = 0 \tag{42}$$

where the prime denotes differentiation with respect to the conformal time η and σ has been equated to the (dynamic) cosmological constant Λ of the de Sitter space-time. To obtain the Hamiltonian in the dimensionless conformal time $\tilde{\eta} = \eta/\rho$, the left hand side must multiplied by $(3\pi/2G)\rho^4$ to normalize correctly the (average) matter contribution in the comoving volume $2\pi^2\rho^3a^3$. Defining $\tilde{a}^2 = (3\pi/2G)a^2\rho^2$ one obtains

$$H_{grav} = -\frac{1}{2}p_{\tilde{\eta}}^2 - \frac{1}{2}(\tilde{a}^2 - \lambda\tilde{a}^4)\ ; \quad \lambda = \frac{2G}{3\pi\tau^2}\ , \tag{43}$$

where $p_{\tilde{\eta}}$ is the momentum conjugate to $\tilde{a}$.

In absence of quantum matter, $H_{grav} = 0$. $p_{\tilde{\eta}}^2$ is positive for $\tilde{a} > \lambda^{-1/2}$ and reaches zero at $\tilde{a} = 0$, and the latter point opens up into a finite neighborhood in presence of positive definite energy matter. As in the non relativistic model one has a potential barrier, shown in Fig.3, separating two turning points. The turning point at $\tilde{a} = 0$ can be identified with a Planck size wormhole and classical inflation sets up at the turning point $\tilde{a} = \lambda^{-1/2}$. Geometrically, the tunneling region is a half Euclidean 4-sphere of radius τ. Taking the radius of the reference 3-sphere in Eq.(7) to be τ, the turning point $\tilde{a} = \lambda^{-1/2}$ is the 3-sphere $a = 1$ (see Fig.4).

To get the entropy W_e and the temperature difference β^{-1} of the scale factor "clock" it is not necessary to include explicitly the quantum matter contribution. It suffices to evaluate the Euclidean action $W_e(E = 0)$ between the turning points and the Euclidean time spent under the barrier. Thus

$$W_e(0) = \oint p_{\tilde{\eta}}(\tilde{a})d\tilde{a} = 2\int_0^{\lambda^{-1/2}} z(1 - \lambda z^2)^{1/2}dz = \frac{2}{3\lambda} = \frac{\pi\tau^2}{G}\ . \tag{44}$$

This coincides with the horizon entropy of de Sitter space-time Eq.(25). Thus

$$\mathcal{S}_{deSitter} = W_e(0) + C \tag{45}$$

where C is an energy independent constant.

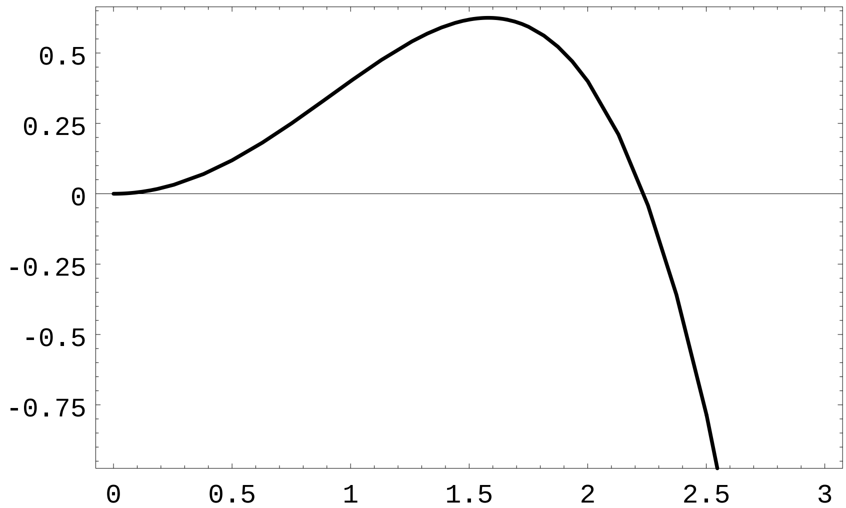

Fig.3 Potential $V(\tilde{a}) = (\tilde{a}^2 - \lambda\tilde{a}^4)/2$ as a function of $\tilde{a}$ for $\lambda = 0.2$

Consistency between Eq.(45) and Eq.(41) requires that the equilibrium temperature "before" tunneling be infinite. This can be checked more directly by evaluating the time $\theta = \beta$ spent under the barrier and showing that this time is equal to the inverse equilibrium temperature of the de Sitter space-time.

The latter is the (positive) Hawking-Gibbons background temperature Eq.(24) which is the global equilibrium temperature of de Sitter space-time with static test matter. This global temperature is given by the periodicity in the time t_s of the Euclidean continuation of the metric of a static de Sitter patch

$$ds^2 = (1 - \frac{r^2}{\tau^2})dt_s^2 + (1 - \frac{r^2}{\tau^2})^{-1}dr^2 + r^2 d\Omega^2 \ . \tag{46}$$

One easily verifies that the t_s-period is $2\pi\tau$.

Eq.(46) is the metric of the 4-sphere which can also be described by the Euclidean continuation of the coordinate system for comoving observers

$$\begin{aligned} ds^2 &= dt_c^2 + a^2(t_c) d\Sigma^2 \ , \\ a(t_c) &= \cos(t_c/\tau) \ , \end{aligned} \tag{47}$$

where $d\Sigma$ is the line element of a 3-sphere of radius τ. The tunneling region between $\tilde{a} = 0$ and $\tilde{a} = \lambda^{-1/2}$, shown in Fig.3, is depicted in Fig.4 as half the 4-sphere of radius τ laying between the wormhole at $a = 0$ and the equatorial 3-sphere $ABCD$ at $a = 1$.

This 3-sphere, which in the comoving system Eq.(47) is a slice of the 4-sphere at $t_c = 0$, is the union of two static patches : BCD at time $t_s = 0$ and BAD at time $t_s = \pi$. In the static coordinate system, the tunneling region is spanned by the motion of the patch BED from BCD to BAD. The period of the round trip is $\beta = 2\pi\tau$. Thus $\beta^{-1} = \beta_{GH}^{-1}$ which is the equilibrium temperature. This confirms that the equilibrium temperature "before" tunneling is infinite.

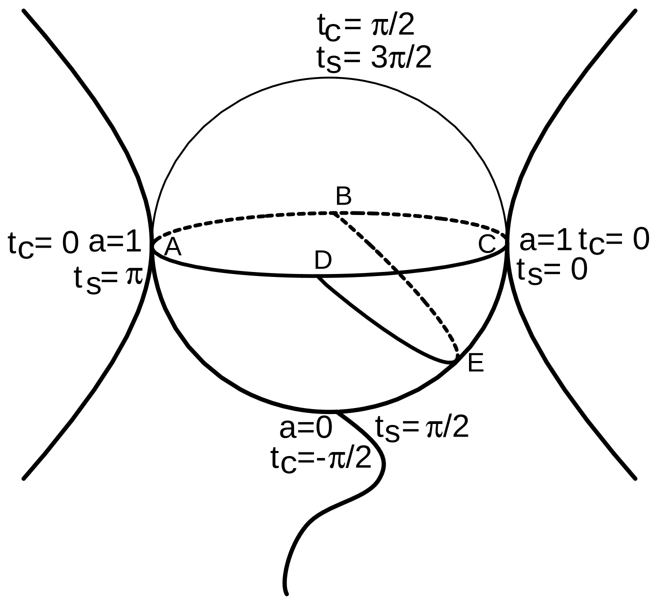

Fig.4 Tunneling through the half Euclidean sphere, parametrized by $\tilde{a}$ in Fig.3.

The preheating of matter at infinite temperature "before" tunneling is consistent with the qualitative semi-classical picture discussed in Section 2.

Namely, an infinite temperature, if realized dynamically, requires a large density of states at the Planckian origin and translates, in the scale factor approximation, complete ignorance about these states.

These results hinge upon the boundary conditions which must be chosen so that *the wave function for the scale factor is small "before" tunneling and exponentially large at the onset of the classical inflationary behavior.* In the limit of vanishing Planck size ($\sqrt{\hbar} \to 0$) such boundary conditions are equivalent to fixing the wave function to be zero at $a = 0$ in the Euclidean section. One thus recovers the no-time Hartle-Hawking boundary condition [4][7].

Finally we remark that when $\tau \to 0$ the entropy gained from tunneling disappears while the temperature of the mini-de Sitter goes to infinity, matching the wormhole temperature. The density of states there must be energy independent on the scale of matter excitations. If they could be counted as quantum mechanical states, they would provide an integration constant to the de Sitter entropy which may even be infinite. I now discuss this issue.

4. HOLOGRAPHY AND THE NATURE OF PLANCK CELLS

Semi-classical arguments on primordial inflation lead to a huge complexity of the Planck scale and this picture is consistent with the tunneling interpretation. The semi-classical approach to black hole leads to the same conclusion. Indeed, if one assumes, according to the original derivation [20], that the Hawking radiation carries no information about the infalling matter, one may increase indefinitely the information loss by sending more and more objects into the hole and letting it evaporate [21]. When finally the black hole evaporates up to the Planck size, it would leave an infinite or at least a very large[8] entropy. The black hole entropy would then contain, in addition to the horizon entropy $A/4$, a very large or perhaps infinite integration

[7] In Linde's chaotic inflation [18], which may be viewed as a realization of the foam, the boundary conditions in a tunneling interpretation are the opposite one [19]. Namely the wave function is exponentially small "after" tunneling. This appears motivated by considerations of creation probabilities but does not yield the thermal properties of the de Sitter space-time. I want to stress that here there is no global arrow of time for *the* universe and that the tunneling operates in the comoving time of the nascent universe. Such tunneling cannot be interpreted as a probability of creation.

[8] There might be some cut off on the matter that could be send into the hole due for instance to a finite size universe.

constant. The latter, like the integration constant needed in the de Sitter entropy, would count the degeneracy of Planckian states.

This is not what comes out of string theory. The most remarkable achievement of the superstrings and of M-theory considerations is the counting of quantum states for near extremal black holes. Not only is the value $A/4$ recovered but the integration constant is zero [22]. Also the Hawking radiation appears to contain the necessary information to ensure unitarity for the scattering matrix. Although no rigorous proof has been established for Schwarzschild black holes, the very fact that it seems possible to connect them adiabatically to near extremal ones, through performing reversible work in upper dimensions [23], indicates that these conclusions should not be altered in a fundamental way for Schwarzschild black holes. Namely the integration constant should remain zero or small. This supports the holographic principle which limits the number of quantum degrees of freedom in a volume L^3 by the number of Planckian cells on a surface of area L^2 [9]. Such drastic reduction in the number of states is expected in general relativity if, in accordance with the second principle of thermodynamics, the entropy within the volume is bounded by the area entropy of the largest black hole fitting the volume, *provided the integration constant in the entropy is not infinite or exceedingly large.*

In the string-M-theory approach to quantum gravity, T-duality and high temperature analysis [24] also suggest a depletion of states at the Planck scale. Even more direct evidence for holography [25], although in a peculiar local way and in cases which are not directly physically relevant, follows from the proposed correspondence between Anti de Sitter space-time and some Conformal Field Theory living on its boundary, correspondence which seems to be borne out for some limiting values of the parameters [26] .

We are confronted with a dilemma which raises fundamental questions. Possible alternatives are :

a) The holographic principle is true and the Planck size is essentially empty. Primordial inflation is incorrect. Also incorrect are all semi-classical considerations for black hole physics and in particular the original derivation of the Hawking radiation. At present, the objection to such conclusion is mainly of philosophical nature. The notion of space-time, as we use it, is perhaps not operative beyond the Planck scale. But why should physics disappear at a scale which might be the cradle of our universe. Such conclusion would carry a strong anthropomorphic connotation which hitherto did never improved our ken.

b) The holographic principle is wrong and the Planck scale has a huge or infinite degeneracy of quantum states. The string-M-theory considerations would be totally irrelevant for physics. But why should a tentative approach to quantum gravity, which led to a remarkable understanding of the entropy of at least some black holes in terms of a counting of quantum states, be completely void of physical significance.

c) The holographic principle is true and so is the complexity of a Planck cell, but usual quantum mechanics is not operative beyond the Planck scale. This could mean that the Planck energy scale marks the scale *below* which the quantum description of the world enters the physical description. The appearance of the Planck constant in physics would then have to be explained. Its origin should be in a scale, such as perhaps the original scale of a universe, hidden in gravity theory, or in some more fundamental framework at small distance scale[9].

[9]One shoud keep in mind that these conclusions rest on the assumption (see footnote 2) that classical general relativity remains valid up to the Planck scale. The existence of some intermediate fundamental scale could open different perspectives.

References

[1] E. P. Tryon, *Nature* **246** (1973) 396.

[2] R. Brout, F. Englert and E. Gunzig, *Ann. Phys.* **115** (1978) 78; *Gen. Rel. Grav.* **10** (1979) 1; R. Brout, F. Englert, and Ph. Spindel, *Phys. Rev. Lett.* **43** (1979) 417, 890; R. Brout, F. Englert, J-M Frere, E. Gunzig, P. Nardone, C. Truffin and Ph. Spindel, *Nucl. Phys.* **B170(FS1)** (1980) 228.

[3] A. Casher and F. Englert, *Physics Lett.* **104 B** (1981) 117.

[4] J.B. Hartle and S.W. Hawking, *Phys. Rev.* **D28** (1983) 2960.

[5] A. Casher and F. Englert, *Class. Quantum Grav.* **9** (1992) 2231.

[6] F. Englert, "Quantum Field Theory and Cosmology", in *Fundamental Interactions : Cargèse1981*, Eds M. Lévy, J.-L. Basdevant, D. Speiser, J. Weyers, M. Jacob, R. Gastmans, Plenum Press (1982).

[7] G. Horwitz and D. Weil, *Phys. Rev. Lett.* **48** (1982) 219; **48** (1982) 1136; R. Brout, G. Horwitz and D. Weil *Physics Lett.* **192** (1987) 318.

[8] For a review, see E.W. Kolb and M.S. Turner, "The Early Universe", Frontiers in Physics, *Addison-Wesley Publishing Company* (1990).

[9] G.'t Hooft, "Dimensional reduction in quantum gravity", in *Salam-festschrift, Word Scientific Series in 20th Century Physics*, **vol. 4**, Eds A. Ali, J. Ellis and S. Randjbar-Daemi (1993), gr-qc/9310023; L. Susskind, *J. Math. Phys.* **36** (1995) 6377, hep-th/9409089.

[10] G.'t Hooft, *Class. Quantum Grav.* **16** (1999) 3263, gr-qc/9903084.

[11] A. Grib, S. Mamayev and V. Mostepanenko, *Gen. Rel. Grav.* **7** (1976) 535.

[12] Ph. Spindel, *Physics Lett.* **107B** (1981) 361.

[13] Ph. Spindel and P. Vandepeutte, "Numerical Investigations in Cosmogenesis" in *Proceedings of the 26th Liège International Astrophysical Colloquium* (Université de Liège) (1986).

[14] J.S. Dowker and R. Critchley, *Phys. Rev.* **D12** (1976) 3224; Ph. Spindel, *Phys. Rev.* **D37** (1988) 2092.

[15] Y. Aharonov, F. Englert and J. Orloff, *Physics Lett.* **199B** (1987) 366.

[16] G.W. Gibbons and S.W. Hawking, *Phys. Rev.* **D15** (1977) 2738.

[17] T. Banks, *Nucl. Phys.* **B249** (1985) 332; R. Brout *Found. Phys.* **17** (1987) 603; F. Englert, *Physics Lett.* **228B** (1989) 111.

[18] A.D. Linde, *Physics Lett.* **129B** (1983) 177.

[19] A.D. Linde, "Quantum Cosmology and the Structure of Inflationary Universe" *PASCOS meeting, Baltimore, March 1995*, gr-qc/9508019.

[20] S. Hawking, *Commun. Math. Phys* **43** (1975) 199.

[21] Y. Aharonov, A. Casher and S. Nussinov, *Physics Lett.* **191B** (1987) 51.

[22] A. Strominger and C. Vafa, *Phys. Lett.* **B379** (1996) 99, hep-th/9601029; C. Callan and J. Maldacena, *Nucl. Phys.* **B472** (1996) 591, hep-th/9602043; G. Horowitz and A. Strominger, *Phys. Rev. Lett.* **77** (1996) 2368, hep-th/9602051; J. Maldacena and A. Strominger, *Phys. Rev. Lett.* **77** (1996) 428, hep-th/9603060; C.V. Johnson, R.R. Khuri and R.C. Myers, *Phys. Lett.* **378B** (1996) 78, hep-th/9603061.

[23] F. Englert and E. Rabinovici, *Physics Lett.* **426B** (1998) 269, hep-th/9801048; R. Argurio, F. Englert and L. Houart, *Physics Lett.* **426B** (1998) 275, hep-th/9801053.

[24] J.L.F. Barbon and E. Rabinovici, *Nucl.Phys.* **B545** (1999) 371, hep-th/9805143.

[25] E. Witten, *Adv. Theor. Math. Phys.* **2** (1998) 253, hep-th/9802150; E. Witten and L. Susskind, "The Holographic Bound in anti-de Sitter Space", hep-th/9805114.

[26] J. Maldacena, *Adv. Theor. Math. Phys.* **2** (1998) 231, hep-th/9711200.