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Charged macroscopic type-II strings and their networks

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Abstract: We write down charged macroscopic string solutions in type II string theories, compactified on tori, and present an explicit solution of the spinor Killing equations to show that they preserve 1/2 of the type II supersymmetries. The S-duality symmetry of the type IIB string theory in ten-dimensions is used to write down the $SL(2, \mathbb{Z})$ multiplets of such strings and the corresponding 1/2 supersymmetry conditions. Finally we present examples of planar string networks, using charged macroscopic (p, q)-strings. An interesting feature of some of these networks, which preserve 1/4 supersymmetry, is a required alignment among three parameters, namely the orientation of strings, a U(1) phase associated with the maximal compact subgroup of $SL(2, \mathbb{Z})$, and an (angular) parameter associated with a solution generating transformation, which is responsible for creating charges and currents on the strings.

Keywords: Superstrings and Heterotic Strings, String Duality, Superstring Vacua.

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Contents

1. Introduction

The study of macroscopic string solutions [1, 2, 3, 4] has been of importance in the last decade in several contexts, such as in black-hole physics [5], strong/weak duality applications [6, 3, 7, 8] etc. In the context of duality, they have been crucial in establishing several such symmetries of string theories. Prominent among these are the SL$(2, \mathbb{Z})$ duality [8] symmetries of the type-IIB string theory in ten dimensions and a string-string duality between the type IIA compactification on K3 and heterotic string compactification on T^4 [6, 3, 7]. The support for the later conjecture involved the construcation of certain BPS solutions carrying (1-form) gauge field charges. Such solutions for $K3$ compactified type-IIA theory were obtained using Charged Macrscopic String solutions of the heterotic strings with 1/2 supersymmetry [3] and then by mapping them to the type-II strings. In the former case, howerver, only a neutral string solution was needed, as type-IIB in ten dimensions does not have any 1-form gauge potential. A full duality multiplet of such neutral string solutions and the corresponding duality covaraint string tensions were also obtained in [8]. More recently, SL$(2,\mathbb{Z})$ as well as other U-duality multiplets of neutral string solutions have been used in constructing their networks with 1/4 [9], 1/8 [10] and lower supersymmetries in type-II theories. They are also expected to provide further confirmations of the duality conjectures.

The network solutions [9]–[22] of type-II strings have also found their applications elsewhere, namely in providing nonperturbative symmetry enhancements in orientifold models to show their matching with the F-theory predictions [19]. In addition, strings and networks which end on D3-branes have found a wide application in 4-dimensional gauge [20, 23, 21] and other world-volume theories [24].

The focus of attention in this paper are the Charged Macroscopic String solutions [2, 3] and their networks. As stated earlier, these solutions have been used earlier for constructing the soliton multiplets in type II string compactification on K3 in order to provide support for their duality conjecture with the heterotic theory on T^4. In this paper, howerver, we will concentrate on such solutions in type-II theories, when they are compactified on tori. As a result, a verification of supersymmetry requires an analysis of additional Killing equations than the ones which are present in the heterotic strings, namely one has to examine the supersymmetry conditions for the spinors arising from both the left and the right-moving sectors of the type-II theories. In this paper we perform this analysis explicitly for a class of such Charged Macroscopic String solutions which are analogous to the heterotic solutions presented in [2]. We also write down explicit supersymmetry conditions for several other class of examples in [3].

The Charged Macroscopic String solutions are generated from the neutral ones by a solution generating transformation and are in general parameterized by a group $O(d-1,1;d-1,1)$, arising out of one time and $d-1$ spatial translational isometries of the solution. These parameters also appear in the Charged Macroscopic String solutions. In particular, the solutions in [3] are characterized by two nontrivial $O(d-1,1;d-1,1)$ parameters α and β, which apply boost between the time direction and an internal direction in the left and the right-moving sectors respectively. The solutions of [2], which we use to explicitly show the 1/2 supersymmetric nature of these solutions in section 2 correspond to $\beta = 0$, but $\alpha \neq 0$. Our analysis then suggests that general solutions ($\alpha \neq 0, \beta \neq 0$) also preserve 1/2 supersymmetry.

Our results show that $O(d-1,1;d-1,1)$ transformations parameterized by α and β change the Killing equations in a nontrivial way. As a result, the supersymmetry conditions and the form of the Killing spinors is also modified. However both the supersymmetry conditions and the Killing spinors for the charged string can be generated from those for the neutral ones by lorentzian tranformations. The parameters of these Lorentz transformations turn out to be local, having a coordinate dependence on the transverse radius. The experience gained from this analysis (for $\beta = 0$) can in fact be used to write down the supersymmetry condition for other solutions characterized by parameters α and β. In view of our future application, in section 3 we confirm the 1/2 supersymmetry property of $\alpha = \beta$ and $\alpha = -\beta$ solutions by examining the consistency of the dilatino supersymmetry variation for the charged macroscopic string background. We also show that one of the above solutions, namely $\alpha = -\beta$, when decompactified to ten dimensions, is related to the neutral string solutions by a constant coordinate transformation. This is not surprising, as $O(d, d)$ group is known to contain a $\mathrm{GL}(d)$ subgroup of constant coordinate transformations. The other possibility, namely $\alpha = \beta$ that we have analyzed is an inequivalent solution even in ten-dimensional sense. This can be verified from the expression for the dilaton, which is now different from the one for the neutral string. However we like to point out that even $\alpha = -\beta$ solutions are in fact physically different in the compactified theory and represent genuine charged strings in $D \leq 9$.

We then use the $\mathrm{SL}(2, \mathbb{Z})$ duality symmetries of the type-IIB theories in ten dimensions to generate general (p, q)-charged macroscopic string solutions from the $(1, 0)$ or elementary-string solution discussed above. In particular we show that the supersymmetry conditions for both $\alpha = \pm\beta \neq 0$ soultions are of a form which allow the constructon of string networks preserving 1/4 supersymmetry. This is not surprising for the $\alpha = -\beta$ solution for the reason already stated in the last paragraph. As a result, the 1/4 supersymmetry of these networks already follows from that of the netutral planar string networks that exist in various dimensions. In this case, we find that the internal tori do not play any significant role and the string networks can be constructed by aligning the orinetation of the (p, q)-string, in a plane, with respect to a phase associated with the transformation of spinors under the $\mathrm{SL}(2, \mathbb{Z})$ duality symmetry transformation.

The charged string solutions with $\alpha = \beta$ turn out to be more interesting from the supersymmetry point of view for the construction of networks. We find that in this case a network construction, preserving certain supersymmetry, requires not only an alignment between the two angles discussed above, but in addition, one has to further align them with an angle coming from the soultion generating parameter. In our examples, in section 4.3, these strings carry not only the 2-form charges parameterized by integers (p, q) and moduli τ, but also by gauge charges characterized by a 2-dimensional unit vector $\hat{n}$. Physically, this alignment therefore implies a coupling between the SL(2) charges with that of the gauge charges and also a relationship between their conservation laws.

The outline of the paper is as following. In section 2 we write down the general charged string solution in arbitrary dimensions. Then to work out the supersymmetry, we restrict to a specific case, namely $\beta = 0$ and present the Killing spinors for this example. Although our analysis is performed specifically in 9-dimensions, we present the generalizations of the results to other lower dimensions as well. In section 3 of the paper, we write down explicit supersymmetry conditions for $\alpha = \beta \neq 0$ and $\alpha = -\beta \neq 0$. Once again we show that our background fields satisfy a nontrivial condition required for the consistency of these spinor equations. Again the derivations are given explicitly in 9-dimensions and then generalized to the lower ones. In section 4, following [8], we also write the SL$(2,\mathbb{Z})$ multiplets of the charged macroscopic strings and show the existence of network solutions for the examples worked out in section 3. This is done by demonstrating the existence of a unique spinor at aymptotic infinity, satisfying the supersymmetry conditions for arbitrary number of (p,q)-strings, provided the alignments we referred previously, also hold. Discussions and conclusions are presented in section 5.

2. Killing spinors for a charged macroscopic string in $D \leq 9$

2.1 Bosonic backgrounds

We start by writing down the bosonic backgrounds associated with the Charged Macroscopic strings in space-time dimensions D. They have been obtained from similar solutions for the heterotic strings [3], by turning off the sixteen gauge fields associated with the right-moving, bosonic sector. This is possible since this sector of the heterotic string is identical to the NS-NS sector of type-II theories in ten dimensions. The solution is given by,

$$\begin{aligned} ds^2 &= r^{D-4}\Delta^{-1}\Big[-(r^{D-4}+C)dt^2 + C(\cosh\alpha - \cosh\beta)dt dx^{D-1} + \\ &\qquad + (r^{D-4} + C\cosh\alpha\cosh\beta)(dx^{D-1})^2\Big] + (dr^2 + r^2 d\Omega^2_{D-3})\,, \end{aligned} \tag{2.1}$$

$$B_{(D-1)t} = \frac{C}{2\Delta}(\cosh\alpha + \cosh\beta)\left\{r^{D-4} + \frac{1}{2}C(1+\cosh\alpha\cosh\beta)\right\}, \tag{2.2}$$

$$e^{-\Phi} = \frac{\Delta^{1/2}}{r^{D-4}}\,, \tag{2.3}$$

$$A_t^{(a)} = \begin{cases} -\dfrac{n^{(a)}}{2\sqrt{2}\Delta} C\sinh\alpha\left\{r^{D-4}\cosh\beta + \dfrac{1}{2}C(\cosh\alpha + \cosh\beta)\right\} \\ \qquad\qquad\qquad\qquad \text{for } 1 \leq a \leq (10-D), \\ -\dfrac{p^{(a-10+D)}}{2\sqrt{2}\Delta} C\sinh\beta\left\{r^{D-4}\cosh\alpha + \dfrac{1}{2}C(\cosh\alpha + \cosh\beta)\right\} \\ \qquad\qquad\qquad\qquad \text{for } (10-D)+1 \leq a \leq (20-2D)\,, \end{cases} \tag{2.4}$$

$$A_{D-1}^{(a)} = \begin{cases} -\dfrac{n^{(a)}}{2\sqrt{2}\Delta} C\sinh\alpha\left\{r^{D-4} + \dfrac{1}{2}C\cosh\beta(\cosh\alpha + \cosh\beta)\right\} \\ \qquad\qquad\qquad\qquad \text{for } 1 \leq a \leq (10-D), \\ \dfrac{p^{(a-10+D)}}{2\sqrt{2}\Delta} C\sinh\beta\left\{r^{D-4} + \dfrac{1}{2}C\cosh\alpha(\cosh\alpha + \cosh\beta)\right\} \\ \qquad\qquad\qquad\qquad \text{for } (10-D)+1 \leq a \leq (20-2D), \end{cases} \tag{2.5}$$

$$M_D = I_{20-2D} + \begin{pmatrix} Pnn^T & Qnp^T \\ Qpn^T & Ppp^T \end{pmatrix}, \tag{2.6}$$

where

$$\Delta = r^{2(D-4)} + Cr^{D-4}(1+\cosh\alpha\cosh\beta) + \frac{C^2}{4}(\cosh\alpha+\cosh\beta)^2\,, \tag{2.7}$$

$$P = \frac{C^2}{2\Delta}\sinh^2\alpha\sinh^2\beta\,, \tag{2.8}$$

$$Q = -C\Delta^{-1}\sinh\alpha\sinh\beta\left\{r^{D-4} + \frac{1}{2}C(1+\cosh\alpha\cosh\beta)\right\}. \tag{2.9}$$

with $n^{(a)}$, $p^{(a)}$ being the components of $(10-D)$-dimensional unit vectors. A_μ's in eqs. (2.4) and (2.5) are the gauge fields appearing due to the Kaluza-Klein (KK) reductions of the ten dimensional metric and the 2-form antisymmetric tensor coming from the NS-NS sector. The matrix M_D parametrizes the moduli fields. The exact form of this parametrization depends on the form of the $O(10-D, 10-D)$ metric used. The above solution has been written for a diagonal metric of the form:

$$L_D = \begin{pmatrix} -I_{10-D} & \\ & I_{10-D} \end{pmatrix}. \tag{2.10}$$

Later on, while decompactifying these backgrounds, in order to check supersymmetry, we will use the notations and conventions in [25] which uses a different form of the metric, namely:

$$L = \begin{pmatrix} & I_{10-D} \\ I_{10-D} & \end{pmatrix}. \tag{2.11}$$

These two conventions are howerver related by:

$$L_D = \hat{P}L\hat{P}^T\,, \qquad M_D = \hat{P}M\hat{P}^T\,, \tag{2.12}$$

where

$$\hat{P} = \frac{1}{\sqrt{2}}\begin{pmatrix} -I_{10-D} & I_{10-D} \\ I_{10-D} & I_{10-D} \end{pmatrix}. \tag{2.13}$$

The gauge fields in two conventions are related as:

$$\begin{pmatrix} A^1_\mu \\ A^2_\mu \end{pmatrix} = \hat{P}\begin{pmatrix} \hat{A}^1_\mu \\ \hat{A}^2_\mu \end{pmatrix}, \tag{2.14}$$

with $A^{1,2}_\mu$'s in the above equation being $(10-D)$-dimensional columns consisting of the gauge fields A_μ's defined in (2.4-2.5), and coming from the left and the right-moving sectors.

In this section we now restrict ourselves to the $\beta = 0$ solutions. These solutions are analogous to the ones written for the hetrotic strings in [2] and are given by,

$$\begin{aligned}
ds^2 &= \frac{1}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}(-dt^2 + (dx^{D-1})^2) + \\
&\quad + \frac{\sinh^2\frac{\alpha}{2}(e^{-E}-1)}{(\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2})^2}(dt + dx^{D-1})^2 + \sum_{i=1}^{D-2} dx^i dx^i\,, \\
B_{(D-1)t} &= \frac{\cosh^2\frac{\alpha}{2}(e^{-E}-1)}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}\,, \\
A^{(1)}_{D-1} &= A^{(1)}_t = -\frac{1}{2\sqrt{2}}\frac{\sinh\alpha(e^{-E}-1)}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}\,, \\
\Phi &= -\ln\left(\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}\right), \qquad (2.15)
\end{aligned}$$

with e^{-E} being the Green function in the $D-2$ dimensional transverse space:

$$e^{-E} = \left(1 + \frac{C}{r^{D-2}}\right). \qquad (2.16)$$

and constant C determining the string tension.[1]

Now, in order to understand the type-II origin of various background fields and to verify the supersymmetry of these solutions, we decompactify the above solution back to ten dimensions. The decompactification exercise is done following a set of notations given in [25]. When restricted to the NS-NS sector of type-II theories, they can be written as:

$$\begin{aligned}
\hat{G}_{ab} &= G^{(10)}_{[a+(D-1),b+(D-1)]}\,, \\
\hat{B}_{ab} &= B^{(10)}_{[a+(D-1),b+(D-1)]}\,, \\
\hat{A}^{(a)}_{\bar{\mu}} &= \frac{1}{2}\hat{G}^{ab}G^{(10)}_{[b+(D-1),\bar{\mu}]}\,, \\
\hat{A}^{(a+(10-D))}_{\bar{\mu}} &= \frac{1}{2}B^{(10)}_{[a+(D-1),\bar{\mu}]} - \hat{B}_{ab}A^{(b)}_{\bar{\mu}}\,, \\
G_{\bar{\mu}\bar{\nu}} &= G^{(10)}_{\bar{\mu}\bar{\nu}} - G^{(10)}_{[(a+(D-1)),\bar{\mu}]}G^{(10)}_{[(b+(D-1)),\bar{\nu}]}\hat{G}^{ab}\,, \qquad (2.17) \\
B_{\bar{\mu}\bar{\nu}} &= B^{(10)}_{\bar{\mu}\bar{\nu}} - 4\hat{B}_{ab}A^{(a)}_{\bar{\mu}}A^{(b)}_{\bar{\nu}} - 2(A^{(a)}_{\bar{\mu}}A^{(a+(10-D))}_{\bar{\nu}} - A^{(a)}_{\bar{\nu}}A^{(a+(10-D))}_{\bar{\mu}})\,, \\
\Phi &= \Phi^{(10)} - \frac{1}{2}\ln\det\hat{G}\,, \qquad 1 \le a,b \le 10-D\,,\ 0 \le \bar{\mu},\bar{\nu} \le (D-1)\,.
\end{aligned}$$

We now start with a nine-dimensional ($D = 9$) solution in (2.15) and following the Kaluza-Klein (KK) compactification mechanism summarized above, write down

[1]There is an extra factor of $1/2\sqrt{2}$ appearing in (2.15) with respect to the one in [2]. This howerver has been taken care in [2] in the definitions of charges.

the solution directly in ten dimensions. We do this first for the $D = 9$ solution and later in section 2.4 generalize the results to $D < 9$. Only nonzero background fields are then given by

$$
\begin{aligned}
ds^2 &= \frac{1}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}(-dt^2 + (dx^8)^2) + \frac{\sinh^2\frac{\alpha}{2}(e^{-E}-1)}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}(dt+dx^8)^2 + \\
&\quad + \frac{\sinh\alpha(e^{-E}-1)}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}dx^9(dt+dx^8) + \sum_{i=1}^{7} dx^i dx^i + (dx^9)^2\,, \qquad (2.18)\\
B_{8t} &= \frac{\cosh^2\frac{\alpha}{2}(e^{-E}-1)}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}\,,\\
B_{9t} &= -\frac{\sinh\alpha}{2}\frac{(e^{-E}-1)}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}} = B_{98}\,. \qquad (2.19)
\end{aligned}
$$

The dilaton in ten dimensions remains same as the one in (2.15):

$$\phi^{(10)} = -\ln\left(\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}\right). \qquad (2.20)$$

Although it is already expected, we have also reconfirmed that many of the field equations in ten-dimensions are satisfied by the backgrounds in eqs. (2.18), (2.19) and (2.20).

We now study spinor Killing equatoins for type-IIB strings in ten dimensions and show that the solutions in (2.18)–(2.20) are consistent with 1/2 supersymmetry. We once again emphasize that 1/2 supersymmetry from the type-IIB string point of view is comparatively more nontrivial, than in the heterotic theory, due to the presence of extra equations to be satisfied by the background configuration. Later in section 2.3 we also find the corresponding Killing spinors.

2.2 Killing equations

The spinor Killing equations in ten dimensions, when restricted to NS-NS fields, follow from supersymmetry variations, (in string metric) [4, 27, 28]:

$$\delta\psi_M = \partial_M\eta + \frac{1}{4}\omega_M^{\hat{M}\hat{N}}\Gamma_{\hat{M}\hat{N}}\eta - \frac{1}{8}H_M^{\hat{M}\hat{N}}\Gamma_{\hat{M}\hat{N}}\eta^*\,, \qquad (2.21)$$

$$\delta\lambda = (\partial_M\phi^{(10)})\gamma^M\eta^* - \frac{1}{6}H_{MNP}\gamma^{MNP}\eta\,, \qquad (2.22)$$

where ψ_M is the ten-dimensional gravitino, λ the dilatino and $\eta \equiv (\epsilon_L + i\epsilon_R)$ are the supersymmetry parameters. $M = 0, \ldots, 9$ are the general coordinate indices in ten dimensions and $\hat{M}, \hat{N}$ are the Lorentz indices.

To analyze these equations for our nine-dimensional solution, we now denote the indices $(9, 0, 8)$ by greek indices μ. The corresponding Lorentz indices are denoted by $\hat{\mu}$ etc. The indices, transverse to the string are denoted by $m = 1, \ldots, 7$ and the corresponding Lorentz ones by $\hat{m}$'s etc. The ten-dimensional lorentzian metric for our purpose is taken to be of the form: $\eta_{\hat{M}\hat{N}} \equiv \mathrm{diag}(1, -1, 1, \ldots, 1)$ (with the first entry denoting the coordinate x^9), which implies: $\eta_{\hat{\mu}\hat{\nu}} = (1, -1, 1)$ and also $\eta_{\hat{m}\hat{n}} = \delta_{\hat{m}\hat{n}}$. Taking into account that the backgrounds depend only on transverse coordinates denoted by m's through radius r, the gravitino supersymmetry variation (2.21) can be written as:

$$\delta\psi_m = \partial_m \eta + \frac{1}{4}\omega_m^{\hat{\mu}\hat{\nu}}\Gamma_{\hat{\mu}\hat{\nu}}\eta - \frac{1}{8}H_m^{\hat{\mu}\hat{\nu}}\Gamma_{\hat{\mu}\hat{\nu}}\eta^*, \tag{2.23}$$

$$\delta\psi_\mu = \frac{1}{2}\omega_\mu^{\hat{\nu}\hat{m}}\Gamma_{\hat{\nu}\hat{m}}\eta - \frac{1}{4}H_\mu^{\hat{\nu}\hat{m}}\Gamma_{\hat{\nu}\hat{m}}\eta^* . \tag{2.24}$$

For the purpose of algebraic manipulations, we find it convenient to write these equations by introducing parameters:

$$\begin{aligned} g &= \frac{1}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}, \\ a &= \frac{\sinh^2\frac{\alpha}{2}(e^{-E}-1)}{\left(\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}\right)^2}, \\ b &= \frac{\sinh\alpha}{2}\frac{(e^{-E}-1)}{\cosh^2\frac{\alpha}{2}e^{-E} - sinh^2\frac{\alpha}{2}}, \end{aligned} \tag{2.25}$$

and 3×3 matrices $\mathcal{G}_{\mu\nu}$, $\mathcal{B}_{\mu\nu}$ and $\mathcal{E}_\mu^{\hat{\mu}}$, where the metric $\mathcal{G}$ and the antisymmetric tensor $\mathcal{B}$ can be read from the backgrounds in eqs.(2.18)-(2.20). $\mathcal{E}$ is the vielbein correponding to $\mathcal{G}$. In our case these 3×3 matrices can be written in terms of 2×2 matrices G, B and $\hat{E}$:

$$\mathcal{G} = \begin{pmatrix} 1 & \hat{b} \\ \hat{b}^T & G + \hat{b}^T\hat{b} \end{pmatrix} \qquad \mathcal{B} = \begin{pmatrix} 0 & -\hat{b} \\ \hat{b}^T & B \end{pmatrix}, \tag{2.26}$$

with $\hat{b} \equiv b(1,1)$, a 2-dimensional row-vector. The vielbein $\mathcal{E}$ is given by:

$$\mathcal{E} = \begin{pmatrix} 1 & 0 \\ \hat{b}^T & \hat{E} \end{pmatrix} \tag{2.27}$$

and satisfies $\mathcal{E}\eta\mathcal{E}^T = \mathcal{G}$, whereas $\hat{E}\hat{\eta}\hat{E}^T = G$, with $\hat{\eta}$ being a diagnoal 2×2 matrix: $\mathrm{diag}(-1, 1)$.

The 2×2 matrices G, B and $\hat{E}$ apprearing in eqs. (2.26), (2.27) have explicit forms:

$$\begin{aligned} G &\equiv \begin{pmatrix} -g+a & a \\ a & g+a \end{pmatrix}, \\ B &\equiv \begin{pmatrix} 0 & g-1 \\ 1-g & 0 \end{pmatrix}, \end{aligned} \tag{2.28}$$

and

$$\hat{E} \equiv \frac{1}{\sqrt{g-a}} \begin{pmatrix} g-a & 0 \\ -a & g \end{pmatrix} . \tag{2.29}$$

Note that G also represents the longitudinal part, or (0,8)-components, of the compactified metric in D-dimensions, as seen directly from eq. (2.15). Similarly B is the antisymmetric tensor in the compactified theory and $\hat{E}$ is the vielbein for the metric G. Using these notations we now start by simplifying the gravitino variation equation for the transverse coordinates, m, namely eq. (2.23).

The spin-connection matrix appearing in the r.h.s. of (2.23) in our case is given by, $\omega_m^{\hat{\mu}\hat{\nu}} = \frac{1}{2}(\mathcal{E}^T \mathcal{G}^{-1}\mathcal{E}_{,m} - \mathcal{E}^T_{,m}\mathcal{G}^{-1}\mathcal{E})^{\hat{\mu}\hat{\nu}}$ and has a form:

$$\omega_m^{\hat{\mu}\hat{\nu}} = \frac{1}{2} \begin{pmatrix} 0 & \frac{b_{,m}}{\sqrt{g-a}} & \frac{-b_{,m}}{\sqrt{g-a}} \\ \frac{-b_{,m}}{\sqrt{g-a}} & 0 & \frac{-g_{,m}}{g} + E_{,m} \\ \frac{b_{,m}}{\sqrt{g-a}} & \frac{g_{,m}}{g} - E_{,m} & 0 \end{pmatrix} . \tag{2.30}$$

Similarly $H_m^{\hat{\mu}\hat{\nu}} \equiv (\mathcal{E}^T \mathcal{G}^{-1}\mathcal{B}_{,m}\mathcal{G}^{-1}\mathcal{E})^{\hat{\mu}\hat{\nu}}$ is given by another antisymmetric matrix:

$$H_m^{\hat{\mu}\hat{\nu}} = \begin{pmatrix} 0 & \frac{b_{,m}}{\sqrt{g-a}} & \frac{-b_{,m}}{\sqrt{g-a}} \\ \frac{-b_{,m}}{\sqrt{g-a}} & 0 & \frac{-g_{,m}}{g} \\ \frac{b_{,m}}{\sqrt{g-a}} & \frac{g_{,m}}{g} & 0 \end{pmatrix} . \tag{2.31}$$

Equation (2.23) then implies for $\delta\psi_m = 0$:

$$\partial_m \epsilon_L + \frac{E_{,m}}{4}\Gamma_{\hat{0}\hat{8}}\epsilon_L = 0 , \tag{2.32}$$

$$\partial_m \epsilon_R + \frac{1}{4}\left[\left(-\frac{2g_{,m}}{g} + E_{,m}\right)\Gamma_{\hat{0}\hat{8}} + \frac{2b_{,m}}{\sqrt{g-a}}(\Gamma_{\hat{9}\hat{0}} - \Gamma_{\hat{9}\hat{8}})\right]\epsilon_R = 0 . \tag{2.33}$$

The variation of the gravitino components ψ_μ, eq. (2.24) can be rewritten as

$$\delta\psi_\mu \equiv \frac{1}{4}(\mathcal{G}^{,\hat{m}}\mathcal{G}^{-1}\mathcal{E})^{\hat{\nu}}_\mu \Gamma_{\hat{\nu}\hat{m}}\eta - \frac{1}{4}(\mathcal{B}^{,\hat{m}}\mathcal{G}^{-1}\mathcal{E})^{\hat{\nu}}_\mu \Gamma_{\hat{\nu}\hat{m}}\eta^* . \tag{2.34}$$

To simplify this further we write down the matrices appearing in the r.h.s. of this equation:

$$\mathcal{G}_{,m}\mathcal{G}^{-1}\mathcal{E} = \begin{pmatrix} 0 & \frac{-b_{,m}}{\sqrt{g-a}} & \frac{b_{,m}}{\sqrt{g-a}} \\ b_{,m} & \frac{g_{,m}-a_{,m}-bb_{,m}}{\sqrt{g-a}} & \sqrt{g-a}\frac{g_{,m}}{g} - \frac{g_{,m}-a_{,m}-bb_{,m}}{\sqrt{g-a}} \\ b_{,m} & -\frac{a_{,m}}{\sqrt{g-a}} - \frac{bb_{,m}}{\sqrt{g-a}} & \sqrt{g-a}\frac{g_{,m}}{g} + \frac{a_{,m}+bb_{,m}}{\sqrt{g-a}} \end{pmatrix} , \tag{2.35}$$

and

$$\mathcal{B}_{,m}\mathcal{G}^{-1}\mathcal{E} = \begin{pmatrix} 0 & \frac{b_{,m}}{\sqrt{g-a}} & \frac{-b_{,m}}{\sqrt{g-a}} \\ b_{,m} & \frac{bb_{,m}}{\sqrt{g-a}} & \sqrt{g-a}\frac{g_{,m}}{g} - \frac{bb_{,m}}{\sqrt{g-a}} \\ b_{,m} & \frac{g_{,m}}{\sqrt{g-a}} + \frac{bb_{,m}}{\sqrt{g-a}} & \sqrt{g-a}\frac{g_{,m}}{g} - \frac{g_{,m}}{\sqrt{g-a}} - \frac{bb_{,m}}{\sqrt{g-a}} \end{pmatrix} . \tag{2.36}$$

These can be used to show that six equations, $\delta\psi_\mu = 0$, following from the real and imaginary components of (2.34) reduce to only two independent ones with ϵ_L and ϵ_R satisfying the following conditions:

$$(\Gamma_{\hat{0}\hat{m}} - \Gamma_{\hat{8}\hat{m}})\,\epsilon_L = 0\,, \tag{2.37}$$

$$\left(\frac{2b_{,m}}{\sqrt{g-a}}\Gamma_{\hat{9}\hat{m}} + (2\frac{g_{,m}}{g} - E_{,m})\Gamma_{\hat{0}\hat{m}} + E_{,m}\Gamma_{\hat{8}\hat{m}}\right)\epsilon_R = 0\,. \tag{2.38}$$

Finally the Killing equations following from the variation of the dilatino can be written down in the notations introduced above as:

$$\delta\lambda = \partial_m\phi^{(10)}\gamma^m(\epsilon_L - i\epsilon_R) - \frac{1}{2}\left(\mathcal{E}^T\mathcal{G}^{-1}\mathcal{B}^{,\hat{m}}\mathcal{G}^{-1}\mathcal{E}\right)^{\hat{\beta}\hat{\gamma}}\Gamma_{\hat{m}\hat{\beta}\hat{\gamma}}(\epsilon_L + i\epsilon_R) = 0\,, \tag{2.39}$$

and using (2.31) gives:

$$(1 + \Gamma_{\hat{0}\hat{8}})\,\epsilon_L = 0\,, \tag{2.40}$$

$$\left(-\partial_m\phi^{(10)} + \frac{g_{,m}}{g}\Gamma_{\hat{0}\hat{8}} - \frac{b_{,m}}{\sqrt{g-a}}\Gamma_{\hat{9}\hat{0}} + \frac{b_{,m}}{\sqrt{g-a}}\Gamma_{\hat{9}\hat{8}}\right)\epsilon_R = 0\,. \tag{2.41}$$

The last two expressions can also be written in an alternative form, using (2.25):

$$(\epsilon_L - i\epsilon_R) = -[\Gamma_{\hat{0}\hat{8}} + tanh\frac{\alpha}{2}e^{E/2}(\Gamma_{\hat{9}\hat{0}} - \Gamma_{\hat{9}\hat{8}})](\epsilon_L + i\epsilon_R)\,, \tag{2.42}$$

which will be useful for discussions later on.

Eqs. (2.32), (2.37) and (2.40) therefore provide complete set of conditions that the Killing spinors ϵ_L have to satisfy. Similarly eqs. (2.33), (2.38) and (2.41) are the conditions to be satisfied by the Killing spinors ϵ_R. We also observe that the equations satisfied by ϵ_L are identical to the one for neutral strings. That is not surprising as the $O(d-1,1;d-1,1)$ transformation, used to generate solution (2.15) from neutral string solutions, act as identity in this sector.

The derivation of equations satisfied by the spinors also pass several consistency checks. First of these, as mentioned above, was the reduction of six equations in (2.24) into only two in (2.37) and (2.38). Moreover, the dilatino variation equations (2.40) and (2.41) are also equivalent to these. To show this, one simply has to multiply (2.38) by $(\Gamma_{\hat{0}} - \Gamma_{\hat{8}})$ from left. As a result, one gets a single independent constraint for ϵ_L, and similarly for ϵ_R. The equation involving a derivative on the spinors, (2.32) and (2.33) are also seen to be consistent with these constraint equations. We demonstrate this in section 2.3 by obtaining a solution for ϵ_L and ϵ_R satisying all the equations simultaneously. Moreover in section 2.3 we will also see that the final constraints, (2.38) or (2.41), satisfy certain consistency conditions on the eigen-values of operators appearing in these equations.

2.3 Killing spinors

We now present the solution of the Killing equations for spinors ϵ_L and ϵ_R. As already stated, ϵ_L satisfies the same condition as in the neutral case and corresponding solution is also identical:

$$\epsilon_L = e^{E/4}\epsilon_L^0 \,, \tag{2.43}$$

where ϵ_L^0 is a constant spinor satisfying,

$$(1 + \Gamma_{\hat{0}\hat{8}})\epsilon_L^0 = 0 \,. \tag{2.44}$$

The form of ϵ_R is more nontrivial. This is also obvious from the Killing equations (2.33), (2.38) and (2.41) that they satisfy. We already noticed that the two non-derivative equations (2.38) and (2.41) are in fact identical. As a result one finally has only two equations to solve, namely (2.33) and (2.38). Howerver before starting to solve these, we first show the self-consistency of (2.38) by writing it as:

$$\left(-1 + \left(\frac{2g_{,m}}{gE_{,m}} - 1\right)\Gamma_{\hat{0}\hat{8}} + \frac{2b_{,m}}{E_{,m}\sqrt{g-a}}\Gamma_{\hat{9}\hat{8}}\right)\epsilon_R = 0 \,, \tag{2.45}$$

and after substituting for g and b from equation (2.25) as:

$$\left(\frac{\cosh^2\frac{\alpha}{2}e^{-E} + \sinh^2\frac{\alpha}{2}}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}\Gamma_{\hat{0}\hat{8}} - \frac{2\sinh\frac{\alpha}{2}\cosh\frac{\alpha}{2}e^{-E/2}}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}\Gamma_{\hat{9}\hat{8}}\right)\epsilon_R = \epsilon_R \,. \tag{2.46}$$

A nontrivial check on our algebra in the previous sub-sections, as well as about 1/2 supersymmetry of our solution comes from the fact that the particular combination of matrices appearing in the l.h.s. of the above equation is idempotent, with only eigen-values ± 1, as required for the validity of the above equation. This can be checked by squaring the l.h.s. of (2.46).

The derivative equation (2.33) can also be simplified for our backgrounds using (2.25) and (2.41), and can be written as:

$$\partial_E \epsilon_R = \frac{1}{2}\frac{\cosh^2\frac{\alpha}{2}e^{-E}}{(\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2})}\epsilon_R - \frac{1}{4}\Gamma_{\hat{0}\hat{8}}\epsilon_R \,, \tag{2.47}$$

where we have now changed variable from $r \to E(r)$.

Now, to present an explicit solution of the Killing equations: (2.46) and (2.47) for ϵ_R we choose a basis for the ten dimensional Dirac (Γ) matrices as in [26]:

$$\Gamma_{\hat{0}} = i\sigma_2 \otimes I_{16} \,, \qquad \Gamma_{\hat{8}} = \sigma_1 \otimes I_{16} \,, \qquad \Gamma_{\hat{9}} = \sigma_3 \otimes I_{16} \,. \tag{2.48}$$

Also, we choose $\epsilon_R \equiv \hat{\epsilon}_R \otimes \chi_0$, with χ_0 an unconstrained sixteen-dimensional constant spinor and $\hat{\epsilon}_R$ is now a representation of Pauli-matrix algebra. Then the final equations to solve are:

$$\left[-I + \sigma_3 + \tanh\frac{\alpha}{2}e^{E/2}(\sigma_1 - i\sigma_2)\right]\hat{\epsilon}_R = 0 \tag{2.49}$$

and

$$\partial_E \hat{\epsilon}_R = \frac{1}{2}\frac{\cosh^2\frac{\alpha}{2}e^{-E}}{(\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2})}\hat{\epsilon}_R - \frac{1}{4}\sigma_3\hat{\epsilon}_R\,, \tag{2.50}$$

where we have now used eq. (2.41) instead of (2.38) or (2.46). The final solution for the Killing spinor is:

$$\hat{\epsilon}_R = \frac{1}{\sqrt{(\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2})}}\begin{pmatrix}\cosh\frac{\alpha}{2}e^{-E/4}\\ \sinh\frac{\alpha}{2}e^{E/4}\end{pmatrix}. \tag{2.51}$$

This Killing spinor reduces to the one for the neutral sting for $\alpha = 0$, for which we have

$$\hat{\epsilon}_R \to \hat{\epsilon}_R^N = e^{E/4}\begin{pmatrix}1\\0\end{pmatrix}, \tag{2.52}$$

and implies in our notations:

$$(1 - \Gamma_{\hat{0}\hat{8}})\epsilon_R^N = 0\,. \tag{2.53}$$

We have therefore explicitly solved for the Killing spinor and shown that a charged macroscopic string solution given in equation (2.15) is 1/2 supersymmetric. The 1/2 supersymmetry comes from the fact that half the components of $\hat{\epsilon}_R$ are related to the remaining ones as given in an explicit form in equation (2.51)

We now show that the supersymmetry conditions (2.46) and the Killing spinors (2.51) for the charged case are related to the neutral ones through a Lorentz boost. For this we parameterize the coefficients of $(\Gamma_{\hat{0}\hat{8}}, \Gamma_{\hat{9}\hat{8}})$ in equation (2.46) as $(\cosh\theta, -\sinh\theta)$ respectively and note that for ϵ_R in (2.51) satisfying this equation,

$$\epsilon_R^N = \left(\cosh\frac{\theta}{2} - \sinh\frac{\theta}{2}\Gamma_{\hat{9}\hat{0}}\right)\epsilon_R\,, \tag{2.54}$$

with

$$\cosh\frac{\theta}{2} = \frac{\cosh\frac{\alpha}{2}e^{-E/2}}{\sqrt{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}}\,, \qquad \sinh\frac{\theta}{2} = \frac{\sinh\frac{\alpha}{2}}{\sqrt{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}}\,, \tag{2.55}$$

reduces to the expression (2.52) and satisfies (2.53), which is also the condition satisfied by the Killing spinor for the neutral strings ($\alpha = \beta = 0$). Therefore the 1/2 supersymmetry condition for a charged macroscopic string, namely (2.46), is related to the one for neutral string by the action of a Lorentz boost on the spinor ϵ_R. This is expected, as the solution (2.15) for the charged macroscopic string is also generted from the neutral ones by a Lorentz boost in the right-moving sector. Howerver, we find it interesting to note that the action of this Lorentz transformation on the spinors is governed by a coordinate dependent parameter. Only in the $r \to \infty$ ($E \to 0$) limit, this parameter reduces to the one for a global Lorentz transformation. This is similar to the phase transformation of spinors induced by an $\mathrm{SL}(2,\mathbb{Z})$

S-duality transformation [29]. The transformation of the spinors are coordinate dependent under S-duality tranformations as well, although like $O(d-1,1;d-1,1)$ transformations, the SL(2)'s are themselves global.

2.4 $D < 9$ solutions

So far we have restricted ourselves to $D = 9$. Above analysis generalizes to the charged Macroscopic String solutions in $D < 9$ in a straightforward manner with only minor modifications. Using the KK procedure metioned in section 2.1, we can once again decompactify these solutions to ten dimensions. The resulting ten-dimensional metric now has a block-diagonal form:

$$\hat{G}^{(10)} = \begin{pmatrix} I_{9-D} & & \\ & \mathcal{G} & \\ & & I_{D-2} \end{pmatrix}, \tag{2.56}$$

with I_{9-D} representing an identity matrix for all the internal directions ranging from: $(x^{D+1}, \ldots, x^9)$ and I_{D-2} represents the tranverse space dimensions of the string in cartesian coordinates. Matrix $\mathcal{G}$ in eq. (2.56) is similar to the one in (2.26) and is now defined in a three dimensional space with coordinates (x^D, x^0, x^{D-1}), i.e. by replacing in eq. (2.26) the coordinates (x^9, x^8) by (x^D, x^{D-1}). Also, the explicit form of $\mathcal{G}$ is similar to the one in (2.26) except E is now a $D-2$ dimensional Green's function (2.16). Similarly, the antisymmetric tensor is represented by a matrix:

$$\hat{\mathcal{B}}^{(10)} = \begin{pmatrix} 0 & & \\ & \mathcal{B} & \\ & & 0 \end{pmatrix}. \tag{2.57}$$

The dilaton remains same as in the D-dimensional theory and is given by the same expression as in (2.20) with E modified as in (2.16).

Due to the block-diagonal form of the backgrounds that we have obtained, the supersymmetry analysis is exactly same as previously in this section. We have therefore shown the 1/2 supersymmetry of the $\beta = 0$ solution in dimensions $D \leq 9$. In next section, we also work out the supersymmetry of certain $\alpha \neq 0$, $\beta \neq 0$ solutions, in order to find a 1/4 supersymmetric network solution of charged macroscopic strings later in section 4.

3. Supersymmetry of $\alpha, \beta \neq 0$ Solutions

In this section we write down the 1/2 supersymmetry conditions for cases: $\alpha = -\beta$ and $\alpha = \beta$ in equations (2.1)–(2.9). Here we only write down the supersymmetry conditions which are the analogs of (2.42) given earlier. These conditions will be generalized to a maifestly SL(2, $\mathbb{Z}$)-covariant form later on. Although the full solution of the Killing equations can also be obtained as in the last section, we do not present them here.

3.1 $\alpha = -\beta \neq 0$ solutions

Once again we first discuss the solution in $D = 9$ and then generalize them to the lower dimensional cases. The solution in $D = 9$ is now characterized by a metric:

$$ds^2 = -\frac{1}{1 + C\cosh^2\alpha/r^5}dt^2 + \frac{1}{1 + C/r^5}(dx^8)^2 + \sum_{i=1}^{7} dx^i dx^i\,. \tag{3.1}$$

The only non-zero component of the antisymmetric tensor is of the form

$$B_{08} = -\frac{C\cosh\alpha}{2}\left[\frac{1}{(r^5 + C)} + \frac{1}{(r^5 + C\cosh^2\alpha)}\right]. \tag{3.2}$$

We also have a nontrivial modulus parametrizing the $O(1,1)$ matrix M_D in eq. (2.6):

$$\hat{G}_{99} \equiv \hat{g} = \frac{1 + C\cosh^2\alpha/r^5}{1 + C/r^5}\,. \tag{3.3}$$

The two gauge fields appearing in equations (2.17) and (2.14) for $D = 9$ are of the form:

$$\begin{aligned} \hat{A}^1_t &= \frac{C\sinh\alpha\cosh\alpha}{2(r^5 + C\cosh^2\alpha)}\,, & \hat{A}^1_8 &= 0\,, \\ \hat{A}^2_t &= 0\,, & \hat{A}^2_8 &= \frac{-C\sinh\alpha}{2(r^5 + C)}\,. \end{aligned} \tag{3.4}$$

The supersymmetry property of the above solution is obtained in the same manner as in section 2.1, after decompactifying the 9-dimensional backgrounds back to ten dimensions. The background fields in ten dimensions for $\alpha = -\beta$ case are now represented by 3×3 matrices analogous to the ones in (2.26):

$$\mathcal{G} = \begin{pmatrix} \frac{1+\frac{C\cosh^2\alpha}{r^5}}{1+\frac{C}{r^5}} & \frac{C}{r^5}\frac{\cosh\alpha\sinh\alpha}{(1+\frac{C}{r^5})} & 0 \\ \frac{C}{r^5}\frac{\cosh\alpha\sinh\alpha}{(1+\frac{C}{r^5})} & -\frac{[1-\frac{C\sinh^2\alpha}{r^5}]}{(1+\frac{C}{r^5})} & 0 \\ 0 & 0 & \frac{1}{(1+\frac{C}{r^5})} \end{pmatrix}, \tag{3.5}$$

$$\mathcal{B} = \begin{pmatrix} 0 & 0 & -\frac{C\sinh\alpha}{(r^5+c)} \\ 0 & 0 & -\frac{C\cosh\alpha}{(r^5+c)} \\ \frac{C\sinh\alpha}{(r^5+c)} & \frac{C\cosh\alpha}{(r^5+c)} & 0 \end{pmatrix}, \tag{3.6}$$

and

$$\phi^{(10)} = -\ln\left(1 + \frac{c}{r^5}\right). \tag{3.7}$$

The 1/2 supersymmetry conditions is now obtained from the dilatino variation (2.22), although other equations are expected to give the same answer as well. We once again need to compute the matrix $H_m^{\hat{\mu}\hat{\nu}}$, the analog of the one in eq. (2.31).

It now has a form:

$$H_m^{\hat\mu\hat\nu} = \frac{\partial_m[1+\frac{C}{r^5}]}{(1+\frac{C}{r^5})^2} \times \begin{pmatrix} 0 & 0 & -\frac{\sinh\alpha}{\sqrt{\hat g G_{88}}} \\ 0 & 0 & \frac{1}{\sqrt{G_{tt}G_{88}}}\frac{\cosh\alpha(1+\frac{C}{r^5})}{(1+\frac{C\cosh^2\alpha}{r^5})} \\ \frac{\sinh\alpha}{\sqrt{\hat g G_{88}}} & -\frac{1}{\sqrt{G_{tt}G_{88}}}\frac{\cosh\alpha(1+\frac{C}{r^5})}{(1+\frac{C\cosh^2\alpha}{r^5})} & 0 \end{pmatrix}. \tag{3.8}$$

Then after some algebra, the 1/2 supersymmetry condition is shown to be:

$$(\epsilon_L - i\epsilon_R) = \left[-\cosh\alpha\sqrt{\frac{1+\frac{C}{r^5}}{1+\frac{C\cosh^2\alpha}{r^5}}}\Gamma_{\hat 0\hat 8} + \frac{\sinh\alpha}{\sqrt{1+\frac{C\cosh^2\alpha}{r^5}}}\Gamma_{\hat 9\hat 8}\right](\epsilon_L + i\epsilon_R)\,. \tag{3.9}$$

Once again consistency of this equation is seen by observing that the matrix appearing in the r.h.s. of (3.9) is idempotent.

In the present case the 1/2 supersymmetry of the charged string, as well as that of the corresponding networks that will be discussed in section 4, can be argued in another way as well. As pointed out earlier, the solution generating transformations contain the group of constant coordinate transformations as a subgroup. One can show that $\alpha = -\beta$ solutions belong to this category. For this we note that the metric and antisymmetric tensors in the ten-dimensional theory, after decompactification, are related to the neutral string solutions as:

$$\mathcal{G} = \Lambda\mathcal{G}_0\Lambda^T\,, \qquad \mathcal{B} = \Lambda\mathcal{B}_0\Lambda^T\,, \tag{3.10}$$

where $\mathcal{G}_0$ and $\mathcal{B}_0$ are the ten-dimensional backgrounds for the netutral strings:

$$\mathcal{G}_0 = \begin{pmatrix} 1 & & \\ & -\frac{1}{(1+\frac{C}{r^5})} & \\ & & \frac{1}{(1+\frac{C}{r^5})} \end{pmatrix}, \tag{3.11}$$

$$\mathcal{B}_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{C}{(r^5+C)} \\ 0 & \frac{C}{(r^5+C)} & \end{pmatrix}, \tag{3.12}$$

and

$$\Lambda = \begin{pmatrix} \cosh\alpha & \sinh\alpha & 0 \\ \sinh\alpha & \cosh\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3.13}$$

We however like to point out that althought the two solutions are related by the above transformation, they are still physically different in the compactified theory. The generation of charged solutions through decompactification and constant coordinate transformations are known, including for many examples of black holes such as Reissner-Nordstrom from Schwarzschild etc. The transformations (3.10) in our case only points out that many of the classical properties, including supersymmetry are identical in two theories. In next subsection we will write down the 1/2 supersymmetry of the charged macrocopic strings for $\alpha = \beta$ case. These are inequivalent solutions with respect to the neutral ones even in ten dimensions.

The generalization of the supersymmetry condition (3.9) to $D < 9$ is once again straightforward and follows a similar path as in section 2.4. As long as the unit vectors $n^{(a)}$ and $p^{(a)}$ in eqs. (2.4), (2.5) are chosen to be along a single internal direction, say x^D, only modification in (3.9) comes in the power of r which is associated with the Green function in the tranverse directions, in addition to replacing $(\Gamma_{\hat{9}}, \Gamma_{\hat{8}}) \rightarrow (\Gamma_{\hat{D}}, \Gamma_{D\hat{-}1})$. A more interestring case is when we parameterize them by angular variables as $n^{(a)} = p^{(a)} \equiv (cos\omega, \sin\omega\cos\phi, \ldots)$, in $(10-D)$-dimensional internal space. Then $\Gamma_{\hat{9}}$ in eq. (3.9) is replaced by an orthogonal combination of Γ matrices in $(10-D)$ internal dimensions: $\Gamma_{\hat{9}} \rightarrow \Gamma_{\hat{n}}$. We will exploit this property in an eight-dimensional example in section 4.2 to show the existence of network type solutions.

3.2 $\alpha = \beta$ solutions

In this case the background metric and antisymmetric tensors are identical to the one in (3.1). The modulus field is now given by,

$$\hat{g} = \frac{1 + C/r^5}{1 + C\cosh^2\alpha/r^5}\,. \tag{3.14}$$

Finally the components of the gauge fields are now:

$$\begin{aligned} \hat{A}^1_t &= 0\,, & \hat{A}^1_8 &= \frac{C\sinh\alpha}{2(r^5 + C)}\,, \\ \hat{A}^2_t &= \frac{-C\sinh\alpha\cosh\alpha}{2(r^5 + C\cosh^2\alpha)}\,, & \hat{A}^2_8 &= 0\,. \end{aligned} \tag{3.15}$$

The ten-dimensional beackgrounds are now represented as:

$$\mathcal{G} = \begin{pmatrix} \hat{g} & 0 & \tilde{b} \\ 0 & -G_{tt} & 0 \\ \tilde{b} & 0 & G_{88} + \tilde{b}^2/\hat{g} \end{pmatrix}, \tag{3.16}$$

with G_{tt} and G_{88} as in (3.1), and $\tilde{b} = C\,\mathrm{sinh}\alpha/(r^5 + C\cosh^2\alpha)$. Antisymmetric tensor is represented as:

$$\mathcal{B} = \frac{C}{(r^5 + C\cosh^2\alpha)} \begin{pmatrix} 0 & -\sinh\alpha\cosh\alpha & 0 \\ \sinh\alpha\cosh\alpha & 0 & -\cosh\alpha \\ 0 & \cosh\alpha & 0 \end{pmatrix}, \tag{3.17}$$

and dilaton is given by the expression:

$$\phi^{(10)} = -\ln\left(1 + \frac{C\cosh^2\alpha}{r^5}\right). \tag{3.18}$$

The inequivalence of the charged solution with respect to the netutral ones can be seen by observing that the form of the dilaton in eq. (3.18) is now different from that in (3.7). A comparison of $\hat{g}$'s in (3.14) and (3.3) implies that $\alpha = \beta$ solutions are T-dual with respect to $\alpha = -\beta$ ones. Property of supercharges under T-duality has been studied in [30, 28]. We however obtain the 1/2 supersymmetry condition by directly using the background solutions.

The final form of the supersymmetry condition is now:

$$(\epsilon_L - i\epsilon_R) = -\left(\frac{1}{\cosh\alpha} \sqrt{\frac{1 + \frac{C\cosh^2\alpha}{r^5}}{1 + \frac{C}{r^5}}}\, \Gamma_{\hat{0}\hat{8}} + \tanh\alpha \sqrt{\frac{1}{1 + \frac{C}{r^5}}}\, \Gamma_{\hat{9}\hat{0}} \right) (\epsilon_L + i\epsilon_R), \quad (3.19)$$

and its self-consistency can again be checked by observing that the matrix in the r.h.s. of (3.19) is idempotent.

The extension of this result to $D < 9$ is again straight-forward. The final result is a replacement of $(\Gamma_{\hat{9}}, \Gamma_{\hat{8}})$ by $(\Gamma_{\hat{D}}, \Gamma_{\widehat{(D-1)}})$ respectively, for trivial unit vectors $n^{(a)}$ and $p^{(a)}$'s pointing only along x^D. At the same time, the power of r is modified in this equation appropriately to r^{D-4}. On the other hand, when $n^{(a)} = p^{(a)}$ represent a general rotated unit-vector in $(10 - D)$-dimensional internal space, the supersymmetry condition is also modified by replacing $\Gamma_{\hat{D}}$ by $\Gamma_{\hat{n}}$.

We end this section by implementing these changes for the case of $(\alpha = \beta)$ Charged Macroscopic Strings in $D = 8$, by defining unit vectors: $n^{(2)} = p^{(2)} = (\cos\omega, \sin\omega)$. Then 1/2 supersymmetry condition is:

$$\begin{aligned}(\epsilon_L - i\epsilon_R) = -&\left(\frac{1}{\cosh\alpha} \sqrt{\frac{1 + \frac{C\cosh^2\alpha}{r^4}}{1 + \frac{C}{r^4}}}\, \Gamma_{\hat{0}\hat{7}} + \tanh\alpha \sqrt{\frac{1}{1+\frac{C}{r^4}}} [cos\omega \Gamma_{\hat{9}\hat{0}} + sin\omega \Gamma_{\hat{8}\hat{0}}] \right) \times \\ &\times (\epsilon_L + i\epsilon_R)\,. \end{aligned} \quad (3.20)$$

4. SL$(2, \mathbb{Z})$-Multiplets and network Solutions

4.1 (p, q) charged macroscopic string solutions

The SL$(2, \mathbb{Z})$ multiplets of charged macroscopic strings and their supersymmetry properties can be written following [8, 9, 29]. The bosonic backgrounds for a general charged macroscopic string solution is generated in precisely the same manner as in [8] and can be written down using the ten-dimensional solutions that we introduced for our lower dimensional Charged Macroscopic Strings. First, the Einstein metric, defined in ten-dimensions:

$$G^E_{MN} = e^{-\phi^{(10)}/4} G^s_{MN}\,, \quad (4.1)$$

for our $(D = 9)$ examples of sections-2 and 3 take a form:

$$G^E = e^{-\phi^{(10)}/4} \begin{pmatrix} \mathcal{G} & \\ & I_7 \end{pmatrix}\,, \quad (4.2)$$

with $\mathcal{G}$ and $\phi^{(10)}$'s given for (i) $\beta = 0$ in eqs. (2.18), (2.26) and (2.20), (ii) $\alpha = -\beta$

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in eqs. (3.5) and (3.7) and (iii) $\alpha = \beta$ in eqs. (3.16) and (3.18) respectively. The Einstein metric defined by (4.2) is invariant under the $\mathrm{SL}(2,\mathbb{Z})$ transformation. Only modification in these are in the source terms in the Green function (2.16) to make it $\mathrm{SL}(2,\mathbb{Z})$ invariant [8]. Nonzero components of the antisymmetric tensor are given by 3×3 matrices:

$$(\mathcal{B})^{(i)} = (\mathcal{M}_0^{-1})_{ij} q_j \Delta_q{}^{-1/2} (\mathcal{B}) \,, \tag{4.3}$$

with $\Delta_q = q_i (\mathcal{M}_0^{-1})_{ij} q_j$. Components $(i = 1, 2)$ in the above equation correspond to the NS-NS and R-R sector fields and $(\mathcal{B})$ is a 3×3 matrix given in equations (2.26), (3.6) and (3.17) for cases (i), (ii) and (iii) listed above. The dilaton for the ten-dimensional extension of our (p, q)-string ($(p, q) \equiv (q_1, q_2)$ denoted above) solution is given by the same expression as in eq. (20) of [8], with A_q replaced by $e^{-\phi^{(10)}}$'s coming from equations (2.20), (3.7) and (3.18) in our three examples. We therefore have the $\mathrm{SL}(2,\mathbb{Z})$ covariant ten-dimesnional backgrounds for the ten-dimensional extension of our $D = 9$ Charged Macroscopic String solution. These can be compactified once again to $D = 9$. The compactification of type-II theories to lower dimensions has has been discussed in many papers [31, 32, 33] and we do not persue it here. The extension of the results to $D < 9$ solutions is straightforward as well. We now go on to discuss the supersymmetry properties of these generalized solutions.

4.2 Supersymmetry

The supersymmetry of a (p, q)-charged macrscopic $D \leq 9$ string solutions can be examined from the ten-dimensional point of view, with type-IIB Killing equations as obtained from the supersymmetry variations written in [27, 28]. It can be argued that the supersymmetry conditions that we have written in previous sections will be modified only by a phase factor for general (p, q)-strings. This becomes clear when one writes down the most general variation for the dilatino [28], in presence of both NS-NS and R-R backgrounds generated in section 4.1.

For our purpose, we however follow a path presented in [29] for the case of four-dimensional theories with $\mathrm{SL}(2,\mathbb{Z})$-duality symemtries. This argument has been applied to the case of type-IIB $\mathrm{SL}(2,\mathbb{Z})$-duality as well [9] and uses the fact that Killing spinors transform under $\mathrm{SL}(2,\mathbb{Z})$ by a phase. Explicitly for,

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \tag{4.4}$$

one has:

$$(\epsilon_L - i\epsilon_R) \rightarrow e^{\frac{i}{2}(c\tau + d)} (\epsilon_L - i\epsilon_R) \,. \tag{4.5}$$

In fact, as pointed out in [29], the transformation property of the spinors given in (4.5) holds for Killing spinors in general, including when they are explicitly dependent on coordinates, such as r in our case. We can now use the above tranformation to generate the supersymmetry condition for a general (p, q)-string starting from that for $(1, 0)$ ones.

We write down these supersymmetry conditions, only at asymptotic infinity, namely in the limit $r \to \infty$. This will be sufficient for our present purpose, following a line of study of string networks concentrating on the asymptotic properties of spinors [9, 10]. Although it is of importance to obtain the full supergravity solutions for the networks and examine complete supersymmetry properties, but we do not address the issue here.

We also note that the above procedure to generate the supersymmetry condition of a (p,q)-string, from $(1,0)$ ones, applies in Einstein frame whereas our supersymmetry conditions of sections 2 and 3 are written in the string frame. The translation among these frames involve redefinitions of fields written explicitly in Appendix of [28] and involve only dilaton-dependent scaling factors, when one restricts to the analysis of dilatino supersymmetry variation. However since the asymptotic values of the dilaton in all our examples in previous sections turn out to be independent of the parameter α with $\phi \to 0$ as $r \to \infty$, identical supersymmetry conditions hold in Einstein frame as well. They have explicit forms for $D = 9$ examples as:

$$\text{(i)}\ \ \alpha = -\beta : \qquad (\epsilon_L - i\epsilon_R) = e^{-i\Phi(p,q,\tau_0)} \left[-\cosh\alpha\Gamma_{\hat{0}\hat{8}} + \sinh\alpha\Gamma_{\hat{9}\hat{8}}\right](\epsilon_L + i\epsilon_R)\,, \tag{4.6}$$

$$\text{(ii)}\ \ \alpha = \beta : \qquad (\epsilon_L - i\epsilon_R) = -e^{-i\Phi(p,q,\tau_0)} \left(\frac{1}{\cosh\alpha}\Gamma_{\hat{0}\hat{8}} + \tanh\alpha\Gamma_{\hat{9}\hat{0}}\right)(\epsilon_L + i\epsilon_R)\,, \tag{4.7}$$

$$\text{(iii)}\ \ \beta = 0, \alpha \neq 0 : \ \ (\epsilon_L - i\epsilon_R) = -e^{-i\Phi(p,q,\tau_0)} \left[\Gamma_{\hat{0}\hat{8}} + \tanh\frac{\alpha}{2}(\Gamma_{\hat{9}\hat{0}} - \Gamma_{\hat{9}\hat{8}})\right](\epsilon_L + i\epsilon_R)\,, \tag{4.8}$$

with Φ denoting the phase associated with the complex parameter $p + q\tau_0$ and the subscript of τ denotes its asymptotic value. The value of the phase is once again given by the same expression, as for the neutral string, since the transformations that generate them from the charged $(1,0)$-string supersymmetry-condition is identical to the one for the neutral ones in [9].

4.3 Network solutions

To obtain the network solutions, we now start with case (i) above and find out if arbitrary number of (p,q)-strings can be arranged in a manner preserving some supersymmetry. For this we now generalize (4.6) further to accommodate arbitrary orientation of strings in spatial directions. In particular, for the string making an angle θ from x^8 axis in an $x^8 - x^7$ plane, the supersymmetry condition (4.6) modifies into:

$$(\epsilon_L - i\epsilon_R) = \exp(-i\Phi(p,q,\tau_0))\left[(-\cosh\alpha\Gamma_{\hat{0}} + \sinh\alpha\Gamma_{\hat{9}})(\cos\theta\Gamma_{\hat{8}} + \sin\theta\Gamma_{\hat{7}})\right](\epsilon_L + i\epsilon_R)\,. \tag{4.9}$$

The network solution with 1/4 supersymmetry is then found from the above equation by identifying the internal and space-time orientations of the strings, namely $\Phi = \theta$. Moreover since the above condition is solved by spinors satisfying the following conditions:

$$\begin{aligned} \epsilon_L &= -(\cosh\alpha\Gamma_{\hat{0}} - \sinh\alpha\Gamma_{\hat{9}})\Gamma_{\hat{8}}\epsilon_L\,, \\ \epsilon_R &= (\cosh\alpha\Gamma_{\hat{0}} - \sinh\alpha\Gamma_{\hat{9}})\Gamma_{\hat{8}}\epsilon_R\,, \end{aligned} \tag{4.10}$$

and

$$\epsilon_L = -(\cosh\alpha\Gamma_{\hat{0}} - \sinh\alpha\Gamma_{\hat{9}})\Gamma_{\hat{7}}\epsilon_R\,, \tag{4.11}$$

which are independent of the the orientation θ, we have the possibility of network solution by arranging arbitrarily large number of strings, provided charge conservations hold on every 3-string junctions.

Equations (4.10) and (4.11) are analogous to the supersymmetry conditions for the F and D-strings respectively in our case. We like to point out that for this example, the existence of a network solution is already gauranteed from its existence in the neutral case. This is because of our earlier observation that ($\alpha = -\beta$) charged solution is generated from neutral ones by a group of constant coordinate transformation. This property continues to hold even for a (p,q)-charged macroscopic string solution, as the group of constant coordinate transformations commutes with $\mathrm{SL}(2,\mathbb{Z})$. Above results can be generalized to the lower dimensional cases by making appropriate replacements already mentioned in section 3.1.

The network solution and its interpretations are more interesting in case (ii), namely for $\alpha = \beta$. First, as can be noticed from the supersymmetry condition, eq. (4.7), a solution like case (i) in $D = 9$ does not exist. This is because, only the first term in the bracket in the r.h.s. of eq. (4.7) can be modified, as in eq. (4.9), to include an orientation-dependence of the string through angle θ. The second term in the bracket, dependent on $\Gamma_{\hat{9}\hat{0}}$, namely the ones representing the internal and time coordinates, remains unchaged under any spatial rotation of string in $x^8 - x^7$ plane. As a result, solutions like the ones in eqs. (4.10, 4.11) do not work.

To obtain a network solution in this case, with a unique spinor satisfying the (p,q) string supersymmetry condition for their arbitrary orientations, one needs to go down to $D \leq 8$. This is done by introducing a parameter associated with rotation in internal space, in addition to the angle θ that the string now makes with x^7 axis in $x^7 - x^6$ spatial plane. The eight-dimensional supersymmetry conditions, employing internal rotations, was already given in eq. (3.20). A modification of this, for nonzero θ is given as:

$$\begin{aligned} (\epsilon_L - i\epsilon_R) = -e^{-i\Phi}\Bigg[&\frac{1}{\cosh\alpha}(\cos\theta\Gamma_{\hat{0}\hat{7}} + \sin\theta\Gamma_{\hat{0}\hat{6}}) + \\ &+ \tanh\alpha(\cos\omega\Gamma_{\hat{9}\hat{0}} + \sin\omega\Gamma_{\hat{8}\hat{0}})\Bigg](\epsilon_L + i\epsilon_R). \end{aligned} \tag{4.12}$$

To obtain θ-independent spinor-projections we now identify

$$\theta = \Phi = \omega \,. \tag{4.13}$$

This identification allows one to solve eq. (4.13) for ϵ's which are θ-independent and satisfy projection conditions:

$$-\left(\frac{1}{\cosh\alpha}\Gamma_{\hat{0}\hat{7}} + \tanh\alpha\Gamma_{\hat{9}\hat{0}}\right)\epsilon_L = \epsilon_L \,, \tag{4.14}$$

$$\left(\frac{1}{\cosh\alpha}\Gamma_{\hat{0}\hat{7}} + \tanh\alpha\Gamma_{\hat{9}\hat{0}}\right)\epsilon_R = \epsilon_R \,, \tag{4.15}$$

and

$$-\left(\frac{1}{\cosh\alpha}\Gamma_{\hat{0}\hat{6}} + \tanh\alpha\Gamma_{\hat{8}\hat{0}}\right)\epsilon_L = \epsilon_R \,. \tag{4.16}$$

The conditions (4.15) and (4.16) are again the analogs of the F-string and D-string supersymmetry conditions for the charged macroscopic ($D = 8$) strings considered here. The identifications (4.13) imply a coupling between the U(1) phase coming from S-dualtiy transformation to the one coming from the solution generating transformations. Physically this can be interpreted as implying a relationship between the gauge-charges with (p, q)-charges coming from 2-form fields. It will be interesting to analyze the precise implications of this relationship on the physical properties of the networks.

Finally we comment on the case (i) and other charged macroscopic string solutions. It is now evident that the condition (4.8) is of a form which does not lead to an obvious solution for an orientation independent projection condition. This can be related technically to the fact that in this case one has all three combination of $\Gamma_{\hat{\mu}\hat{\nu}}$ matrices (in $D = 9$) appearing in eq. (4.8), unlike in conditions (i) and (ii) where only two of the three combinations appeared, allowing above solutions. This is the property of other $\alpha \neq 0$, $\beta \neq 0$ solutions as well and may be related to the fact that a general left-right asymmetric solution generating transformation acts differently on ϵ_L and ϵ_R and is inconsistent with the conditions of having a network solution, as they require relationships like (4.11) and (4.16) between them.

5. Conclusions

In this paper we have obtained supersymmetry properties of the charged macrscopic strings. We have also shown the existence of a network solution of charged strings. Some of these are completely inquivalent with respect to the network of neutral string solutions.

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In the context of network construction, it should be pointed out that our exercise only shows the presence of a unique Killing spinor at asymptotic infinity in the presence of large number of (p, q) strings. We do not present the spinor at arbitrary space-time point. This however requires the knowledge of string network solutions for the full supergravity which is not completely understood even for neutral strings, although progress in this direction has been reported [22]. More precisely, we notice that the Killing spinor has a coordinate-dependence given by a covariant expression for the Green functions, leading to different spatial dependence for every (p, q)-string. It is hoped that the full Killing spinor of a supergravity solution for these networks will be given by a smooth funtion which will properly match on to every string in a network.

It will also be interesting to generalize these to non-planar networks and possibly to find the applications of such networks to four-dimensional gauge theories [20, 21]. Moreover, one can possibly also analyze the possibility of network solutions when strings are compactified on other manifolds like $K3$ etc. and be able to obtain a realization of various BPS states in string theories in this manner.

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