# Higgs limits in the MSSM: beyond the benchmark 

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## PRELIMINARY


#### Abstract

Up to now, the ALEPH limits on the MSSM neutral Higgs boson masses have been reported for specific sets of model parameters corresponding to the so-called minimal and maximal mixing configurations. An investigation of the robustness of these results is performed by means of a scan of the parameter space in which more than 30 million sets of $m_{0}, m_{1 / 2}, \mu, A_{t}, m_{\mathrm{A}}$ and $\tan \beta$ values are probed. In the low $\tan \beta$ regime, the $m_{\mathrm{h}}$ limit obtained in the cases of minimal and maximal mixing is found to remain valid for $99.99 \%$ of the parameter sets explored. In the case of large $\tan \beta$ and considering the limit on $m_{\mathrm{h}}+m_{\mathrm{A}}$, this fraction reduces to $99.9 \%$.


(ALEPH contribution to 1998 summer conferences)

## 1 Introduction

Except for one notable exception[1], all of the MSSM Higgs boson search results at LEP2 have up to now been presented for specific sets of model parameters corresponding to choices made at the time of the workshop on Physics at LEP2[2]. These sets are commonly referred to under the designations of "minimal" and "maximal mixing": a value of $1 \mathrm{TeV} / c^{2}$ is assumed for $M_{S}$, the quadratic average of the top squark masses, and the parameters controlling the stop mixing are chosen such that the impact on the Higgs sector is either minimal $\left(A_{t}=\mu=0\right)$ or maximal $\left(A_{t}=\sqrt{6} M_{S}, \mu=0\right)$ as far as the mass $m_{\mathrm{h}}$ of the lighter CP-even Higgs boson is concerned. In particular, those choices were made for the presentation of the latest ALEPH results at the April '98 LEPC[3].

The purpose of this note is to investigate how the ALEPH results on MSSM Higgs bosons are affected when the model parameters are allowed to vary. This study is conducted within the framework of a "semi-constrained" MSSM: Universal SUSY breaking masses $m_{0}$ and $m_{1 / 2}$ are assumed for all matter scalars and for the three gauginos at the GUT scale, respectively, but no such constraint is imposed in the Higgs sector and radiative breaking of the electroweak symmetry is not enforced. Therefore, the CP-odd Higgs boson mass $m_{\mathrm{A}}$ and the Higgs mixing supersymmetric mass $\mu$ remain as free parameters at the electroweak scale. The parameter set is further specified by the values of the ratio $\tan \beta$ of the vacuum expectation values of the two Higgs doublets and of the trilinear coupling $A_{t}$ which controls the stop mixing. The other trilinear couplings (e.g. $A_{b}$ or $A_{\tau}$ ) are assumed for simplicity to be equal to $A_{t}$ at the electroweak scale, but their rôle is insignificant in this study.

Once such a parameter set $\left\{m_{0}, m_{1 / 2}, \mu, \tan \beta, A_{t}, m_{\mathrm{A}}\right\}$ is specified, the masses and couplings of all sleptons, squarks, gauginos and Higgs bosons can be calculated, and hence all production cross-sections and decay branching ratios. Here, the slepton and squark masses are computed using the solutions of the renormalization group equations given in Ref. [4], ignoring the contributions of Yukawa interactions. The HZHA program[5] is used for all the calculations in the Higgs sector, with radiative corrections from Ref. [6].

The main lessons to be anticipated from such an exploration of the MSSM parameter space result from the observation of occurrences of parameter sets such that

- the cross-section of the Higgsstrahlung process $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{hZ}$ is vanishingly small for some low $m_{\mathrm{h}}$ value, but this is not compensated by a large cross section for the pair production reaction $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{hA}$ because of too large an $m_{\mathrm{A}}$ value;
Such situations, first pointed out in Ref. [7], are anomalous in the sense that a small value of the ZZh coupling normally goes together with h and A bosons close in mass. (This is the typical large $\tan \beta$ configuration.)
- or the Higgs boson decay patterns prevent the usual Higgs boson searches from being efficient.
This can be due, for instance, to a vanishing hbb coupling, or to large branching ratios into supersymmetric particles, visible or not, or into gg if the $h \rightarrow \tilde{\mathrm{t}} \overline{\tilde{t}}$ threshold is about to be reached.

A specificity of the study presented here, compared to those of Refs. [1] and [7], is the attempt made to quantify the level of fine-tuning which these anomalous configurations require.

## 2 Constraints on the parameter sets

To decide whether a given parameter set is excluded or not, the following theoretical and experimental constraints will be used, with $R$-parity conservation assumed throughout:

1. the lightest supersymmetric particle should be the lightest neutralino $\chi$, and no particle should be tachyonic.
These requirements are particularly relevant for stops and Higgs bosons in the case of large mixing, and for sneutrinos for moderate to large values of $\tan \beta$.
2. the chargino, sneutrino and stop masses should exceed their LEP1 limits;

For charginos and sneutrinos, the limits of 45 and $43 \mathrm{GeV} / c^{2}$, respectively, are deduced from the Z width measurement[8]. For stops, the excluded domain in the $\left\{m_{\tilde{\mathfrak{t}}}, m_{\chi}\right\}$ plane is taken from ALEPH [9], assuming the worst case of stops decoupling from the Z .
3. the partial width of the $\mathrm{Z} \rightarrow \mathrm{hA}$ decay should be smaller than 7 MeV ;

This is the upper limit for any non-standard contribution to the Z width within a supersymmetric model, i.e. assuming a light Higgs boson.
4. the squared ZZh coupling $\sin ^{2}(\beta-\alpha)$ should not exceed its upper limit as a function of $m_{\mathrm{h}}$, as determined by ALEPH at LEP1;
The published results[10] are based on searches for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{hZ} Z^{*}$ using data accumulated up to 1992. For $m_{\mathrm{h}}>30 \mathrm{GeV} / c^{2}$, these results have been updated for the purpose of the present analysis using the full LEP1 statistics. In the case of standard model like decays ( $\mathrm{h} \rightarrow$ hadrons, or $\mathrm{h} \rightarrow$ AA with $m_{\mathrm{A}}>2 m_{\mathrm{b}}$ ), the results from Ref. [11] have been used, restricted however to the $\mathrm{h} \nu \bar{\nu}$ channel. This is because b tagging was utilized in the $\mathrm{h} \ell^{+} \ell^{-}$channel while the LEP1 constraint on $\sin ^{2}(\beta-\alpha)$ is applied here independently of the $\mathrm{h} \rightarrow \mathrm{b} \overline{\mathrm{b}}$ branching ratio. Moreover, the dedicated search for $\mathrm{h} \nu \bar{\nu}$ with $\mathrm{h} \rightarrow \mathrm{AA}$ and $m_{\mathrm{A}}<2 m_{\mathrm{b}}$, also described in Ref. [10], has been applied to the full LEP1 statistics, leading to a single candidate event selected. The appropriate constraint on $\sin ^{2}(\beta-\alpha)$ resulting from these updates is applied taking into account the h decay branching ratios into standard model like final states, into pairs of low mass A bosons, or into supersymmetric final states for which a null efficiency is assumed at this stage.
5. the squared ZhA coupling $\cos ^{2}(\beta-\alpha)$ should not exceed its upper limit as a function of $m_{\mathrm{h}}$ and $m_{\mathrm{A}}$, as determined by ALEPH at LEP1;
The published results[10] are based on searches for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hA using data accumulated up to 1991. The search in the $\tau^{+} \tau^{-}$-hadrons final state was extended to
include the ALEPH data up to 1993 in Ref. [12]. These updated results are used in the present analysis for $m_{\mathrm{A}}>2 m_{\mathrm{b}}$, taking into account the values of the h and A decay branching ratios into $\tau^{+} \tau^{-}$and into hadrons. For $m_{\mathrm{A}}<2 m_{\mathrm{b}}$, the $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{hA}$ process leads to a three jet topology when $\mathrm{h} \rightarrow \mathrm{AA}$. To cope with this particular situation, a search for final states consisting of three jets, of which one reduces to a $\mu^{+} \mu^{-}$pair, was developed. This new analysis is described in Appendix A. Although low, the branching ratio of $\mathrm{A} \rightarrow \mu^{+} \mu^{-}$is never negligible when $m_{\mathrm{A}}<2 m_{\mathrm{b}}$; and since the resolution on the $\mu^{+} \mu^{-}$mass is such that only a very small $m_{\mathrm{A}}$ range would be affected by any single candidate, loose selection criteria can be used, leading to a high search efficiency. With no events selected in the data sample collected by ALEPH in 1994 and 1995, the results turn out to be sufficiently constraining whenever they are needed.
6. the value of $\sin ^{2}(\beta-\alpha)$ should not exceed its upper limit as a function of $m_{\mathrm{h}}$, as determined by ALEPH searches for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{hZ}$ at LEP2;
The results used in this analysis are the most recent ones[3]. Possible reductions of the sensitivity due to a $h$ decay branching ratio into $b \bar{b}$ lower than in the standard model are taken into account. It has been verified that the efficiencies of the searches in the $\mathrm{h} \ell^{+} \ell^{-}$and $\mathrm{h} \nu \bar{\nu}$ final states are unaffected if h decays to a pair of A bosons when $m_{\mathrm{A}}>2 m_{\mathrm{b}}$. In contrast, a reduction of the selection efficiency occurs in the $\mathrm{hq} \overline{\mathrm{q}}$ channel when $\mathrm{h} \rightarrow \mathrm{AA}$, due to the 6 -jet rather than 4 -jet structure of the final state. This is taken into account when relevant. A null efficiency is assumed in the case of $\mathrm{h} \rightarrow$ AA decays when $m_{\mathrm{A}}<2 m_{\mathrm{b}}$.
7. for a Higgs boson decaying invisibly, $\sin ^{2}(\beta-\alpha)$ should not exceed its upper limit as a function of $m_{\mathrm{h}}$, as determined by ALEPH searches at LEP1 [10] and LEP2[13];
The value of the branching ratio of $h \rightarrow \chi \chi$ is taken into account when this constraint is used.
8. the value of $\cos ^{2}(\beta-\alpha)$ should not exceed its upper limit as a function of $m_{\mathrm{h}}$ and $m_{\mathrm{A}}$, as determined by ALEPH searches for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hA at LEP2[3];
The values of the h and A decay branching ratios into $\tau^{+} \tau^{-}$and into $\mathrm{b} \overline{\mathrm{b}}$ are taken into account. The limits on $\cos ^{2}(\beta-\alpha)$ given in Ref. [3] apply for equal mass h and A bosons. The efficiency reduction which takes place in the case of unequal masses has been mapped as a function of $m_{\mathrm{h}}$ and $m_{\mathrm{A}}$ for the present analysis. Details are given in Appendix B.
9. the mass of the charged Higgs boson should not exceed its upper limit as a function of its decay branching ratio into $\tau \nu_{\tau}$, as determined by ALEPH searches for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ $\mathrm{H}^{+} \mathrm{H}^{-}$at LEP2[14];
At tree level, charged Higgs bosons are predicted to be heavier than the W boson, but radiative corrections can, although rarely, lead to significantly lower masses.
10. the masses of charginos, sleptons and stops should exceed their most recent ALEPH limits[15][16];

Care is taken to use these constraints strictly as applicable, in particular taking into account the sneutrino mass in the case of charginos, cascade decays in the case of sleptons, the mixing angle in the case of stops, and the mass difference with respect to the lightest neutralino in all cases.

In addition, it occasionnally happens that valuable constraints are obtained by the replacement of h by H , where H is the heavier CP-even neutral Higgs boson, with the appropriate coupling modifications.

The case of ultra-light Higgs bosons, i.e. $m_{\mathrm{h}}$ or $m_{\mathrm{A}}<2 m_{\mu}$, is not considered in the present analysis.

## 3 Scans of the MSSM parameter space: procedure

The limits placed by LEP2 in the Higgs sector are commonly presented as exclusion domains in the $\left\{m_{\mathrm{h}}, \tan \beta\right\}$ plane, for one or two of the conventional choices of mixing parameters. In contrast to these two-dimensional "benchmark cases", the problem addressed in this study is a six dimensional one. Therefore, choices have to be made as to the ranges investigated for the various parameters and as to the granularity with which each of these parameters is sampled. The scans performed to obtain the results presented below are constructed from the following samplings:

- nine values for $\tan \beta$ :

$$
\{1 / \sqrt{2}, 1, \sqrt{2}, 2,2 \sqrt{2}, 4,8,16,32\}
$$

- and for the dimensionful parameters $m_{0}, m_{1 / 2},|\mu|$ and $\left|A_{t}\right|$, with both signs of $\mu$ and $A_{t}$ considered:
- nine values for the "coarse logarithmic" scans:

$$
\begin{gathered}
\quad\left\{1 \text { and } 2000 / 2^{n}\right\} \mathrm{GeV} / c^{2} \text {, with } n=7 \text { to } 0 \\
\left(\text { i.e. } 1,15.6,31.3,62.5,125,250,500,1000,2000 \mathrm{GeV} / c^{2}\right)
\end{gathered}
$$

supplemented with a tenth value for $\left|A_{t}\right|: 4000 \mathrm{GeV} / c^{2}$;

- seventeen values for the "fine logarithmic" scans:

$$
\left\{1 \text { and } 2000 / 2^{n / 2}\right\} \mathrm{GeV} / c^{2}, \text { with } n=15 \text { to } 0
$$

$$
\text { (i.e. } 1,11.0,15.6,22.1,31.3, \ldots, 1000,1414,2000 \mathrm{GeV} / c^{2} \text { ) }
$$

supplemented with two more values for $\left|A_{t}\right|: 2000 \sqrt{2}$ and $4000 \mathrm{GeV} / c^{2}$;

- eleven values for the "linear" scans:

$$
\begin{gathered}
\{1 \text { and } n \times 200\} \mathrm{GeV} / c^{2} \text {, with } n=1 \text { to } 10 \\
\left(\text { i.e. } 1,200,400,600, \ldots, 1600,1800,2000 \mathrm{GeV} / c^{2}\right)
\end{gathered}
$$ supplemented with two more values for $\left|A_{t}\right|: 2200$ and $2400 \mathrm{GeV} / c^{2}$.

For each of those $\left\{\tan \beta, m_{0}, m_{1 / 2}, \mu, A_{t}\right\}$ sets, $m_{\mathrm{A}}$ is incremented using a still finer logarithmic sampling consisting of 35 values:

$$
\begin{gathered}
\left\{1 \text { and } 2000 / 2^{n / 4}\right\} \mathrm{GeV} / c^{2} \text {, with } n=33 \text { to } 0 \\
\left(\text { i.e. } 1,6.6,7.8,9.3,11.0, \ldots, 1414,1682,2000 \mathrm{GeV} / c^{2}\right) .
\end{gathered}
$$

The $m_{\mathrm{A}}$ scan is interrupted as soon as a value leading to an unexcluded situation is encountered, according to the criteria listed in Section 2. The interval separating the last excluded value and the first unexcluded one is then explored using a dichotomy technique. The procedure is stopped when the value of $m_{\mathrm{h}}$ changes by less than $100 \mathrm{MeV} / c^{2}$, and this $m_{\mathrm{h}}$ value is then taken as the CP-even Higgs boson mass limit for the $\left\{\tan \beta, m_{0}\right.$, $\left.m_{1 / 2}, \mu, A_{t}\right\}$ parameter set under consideration.

In order to gain some protection against gaps in the exclusion domain, dichotomies are also performed between successive values of $m_{\mathrm{A}}$ which are not excluded by at least one common constraint. Additional dichotomies are also performed between consecutive values of $m_{\mathrm{A}}$ with opposite signs of $\sin (\beta-\alpha)$, or of $\cos (\beta-\alpha)$, or of $\sin \alpha$. In such cases, the $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{hZ}$ cross section, or the $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hA cross section, or the $\mathrm{h} \rightarrow \mathrm{b} \overline{\mathrm{b}}$ branching ratio, respectively, is expected to vanish somewhere in between.

For practical reasons, the constraints from supersymmetric particle searches at LEP2 (criterion No. 10) are applied in an automatic way only for charginos, with some conservatism, while those from slepton and stop searches, and from chargino searches in a more refined fashion, are applied "by hand" to the few parameter sets for which they are actually useful.

Unless otherwise specified, a top quark mass of $175 \mathrm{GeV} / c^{2}$ is assumed in all scans.
Coarse logarithmic scans have been performed for the nine selected values of $\tan \beta$. Moreover, for two values of $\tan \beta$, namely $\sqrt{2}$ and 32 which are typical of the low and high $\tan \beta$ regimes, fine logarithmic scans have also been performed. Finally, for those two same $\tan \beta$ values, a number of additional scans were made:

- coarse logarithmic scans, but now sampling $m_{\mathrm{A}}$ linearly in steps of 5,50 and $250 \mathrm{GeV} / c^{2}$ up to 200,500 and $2000 \mathrm{GeV} / c^{2}$, respectively, still applying the same dichotomy procedure, in order to check the technique used to set the $m_{\mathrm{h}}$ limits;
- linear scans in order to investigate possible dependences of the results on the way the parameter space is sampled;
- coarse logarithmic scans for $m_{\mathrm{t}}=170$ and $180 \mathrm{GeV} / c^{2}$ to study the sensitivity of the results to the top quark mass.

In the coarse logarithmic scans, 26163 sets of $\left\{m_{0}, m_{1 / 2}, \mu, A_{t}\right\}$ values are explored, 352869 sets in the fine logarithmic scans and 63525 in the linear scans. (The sets with $\mu$ or $A_{t}=-1 \mathrm{GeV} / c^{2}$ give results essentially identical to those obtained with $\mu$ or $A_{t}=$ $1 \mathrm{GeV} / c^{2}$ and are therefore not counted as independent sets.) Altogether, the nine coarse logarithmic scans represent a sampling of the parameter space consisting of 8.2 million sets of $\left\{\tan \beta, m_{0}, m_{1 / 2}, \mu, A_{t}, m_{\mathrm{A}}\right\}$ values, and the two fine logarithmic scans each represent a sampling of 12.4 million sets of $\left\{m_{0}, m_{1 / 2}, \mu, A_{t}, m_{\mathrm{A}}\right\}$ values. (These counts do not include the additional $m_{\mathrm{A}}$ values tested in the dichotomy procedures.)

## 4 Scans of the MSSM parameter space: results

### 4.1 The low $\tan \beta$ regime

The low $\tan \beta$ regime is first addressed by the fine logarithmic scan for $\tan \beta=\sqrt{2}$. Out of the 352869 sets of $\left\{m_{0}, m_{1 / 2}, \mu, A_{t}\right\}$ values explored, 127994 are unphysical (criterion No. 1), 79254 are rejected by the LEP1 constraints on supersymmetric particles (criterion No. 2) and 58422 by the LEP2 limits on charginos (criterion No. 10). This leaves 87199 sets to be addressed by the Higgs searches at LEP1 and LEP2 (criteria No. 3 to 9).

Out of these 87199 sets, 45771 are excluded irrespective of the value of $m_{\mathrm{A}}$, which means that $\tan \beta=\sqrt{2}$ is excluded for such sets. The distribution of the upper edge of the physical domain, i.e. the largest possible $m_{\mathrm{h}}$ value, is shown for those sets in Fig. 1a. The "minimal mixing" configuration, with an upper edge value of $76 \mathrm{GeV} / c^{2}$, belongs to such excluded parameter sets.

The distribution of the $m_{\mathrm{h}}$ limit for the 41428 other sets is shown in Fig. 1b. In the vast majority of the cases, the limit is indistinguishable from the one obtained in the case of "maximal mixing" (Fig. 15 of Ref. [3]), i.e. $88 \mathrm{GeV} / c^{2}$.

There are however 62 sets for which a significantly lower limit is obtained, of which 34 are in fact eliminated by the stop, slepton or chargino searches at LEP2 (criterion No. 10). This leaves 28 sets for which the mass limit is actually degraded. This is to be compared to a total of about 225000 physically acceptable sets, or of about 87000 sets not excluded by supersymmetric particle searches. The proportion of sets for which the CP-even Higgs boson mass limit is degraded is therefore at the $(1 \div 3) 10^{-4}$ level.

The distribution of the $m_{\mathrm{h}}$ limit for the 28 remaining sets is shown in Fig. 1b (shaded histogram). These sets correspond to two main configurations:

- a rapid variation of $m_{\mathrm{h}}$ and $\sin ^{2}(\beta-\alpha)$ as a function of $m_{\mathrm{A}}$, usually at the edge of the physical domain (when the Higgs boson is about to become tachyonic);

In spite of occasionnally very low $m_{\mathrm{h}}$ values (less than $10 \mathrm{GeV} / c^{2}$ ), the production cross section via the Higgsstrahlung process is too low because $\sin ^{2}(\beta-\alpha)$ is very close to zero. An example of such a configuration is shown in Fig. 2a. The pair production process is useless either because $m_{\mathrm{A}}$ is so large that pair production is kinematically forbidden, or because the effectively three jet final state resulting from this process is not selected by the searches for hA at LEP2.

- a rapid variation of $\sin \alpha$, with $\sin \alpha$ passing through zero.

This leads to a vanishing branching ratio for $\mathrm{h} \rightarrow \mathrm{b} \overline{\mathrm{b}}$. An example is shown in Fig. 2b. In such a case, the searches at LEP2 involving b tagging are inefficient.

The additional scans do not reveal any new anomalous feature. As expected, the coarse logarithmic scan with linear sampling of $m_{\mathrm{A}}$ gives results identical to those of the standard coarse logarithmic scan. The linear scan leads to a similar fraction of pathological sets. The only noticeable effect of increasing (decreasing) the top quark mass is to reduce (increase) the fraction of parameter sets excluded irrespective of the value of $m_{\mathrm{A}}$, but the proportion of sets leading to a significantly reduced $m_{\mathrm{h}}$ limit is unaffected.

Although all of the exclusion criteria listed in the previous section turn out to be useful at some point, the main rôle is played in the end by the search for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{hZ}$ at LEP2 (criterion No. 6), as can be expected in this low $\tan \beta$ regime, leading to a CP-even Higgs boson mass limit of $88 \mathrm{GeV} / c^{2}$. The limit applies for $99.99 \%$ of the physically allowed parameter sets explored.

### 4.2 The high $\tan \beta$ regime

Similar investigations are made for the high $\tan \beta$ regime. In the fine logarithmic scan for $\tan \beta=32,352869$ sets of $\left\{m_{0}, m_{1 / 2}, \mu, A_{t}\right\}$ values are explored, of which 120223 are unphysical, 122713 are rejected by the LEP1 constraints on supersymmetric particles and 24712 by the LEP2 limits on charginos. This leaves 85221 sets to be addressed by the Higgs searches at LEP1 and LEP2. Out of those, only 536 are excluded irrespective of the value of $m_{\mathrm{A}}$, which is not unexpected since even the minimal mixing configuration is not excluded (Fig. 15 of Ref. [3]).

The distribution of the $m_{\mathrm{h}}$ limit for the 84685 other sets is shown in Fig. 3a. While a peak is clearly visible at $76 \mathrm{GeV} / c^{2}$, i.e. the value obtained in the benchmark cases, a broad tail is seen to extend down to $55 \mathrm{GeV} / c^{2}$. This comes from the fact that the main rôle is played in this high $\tan \beta$ regime by the search for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{hA}$ at LEP2, a search in which the kinematically relevant variable is $m_{\mathrm{h}}+m_{\mathrm{A}}$ rather than $m_{\mathrm{h}}$. Indeed, it can be seen in Fig. 3b that the peak in the limit, at $152 \mathrm{GeV} / c^{2}$, is much sharper when displayed using that variable.

There remain 232 sets for which a limit on $m_{\mathrm{h}}+m_{\mathrm{A}}$ lower than $144 \mathrm{GeV} / c^{2}$ is obtained, of which 50 are eliminated by the stop, slepton or chargino searches at LEP2. The characteristics of the 182 remaining sets are similar to those encountered for $\tan \beta=$ $\sqrt{2}$. This number of unexcluded sets is to be compared to a total of about 233000
physically acceptable sets, or of about 85000 sets not excluded by supersymmetric particle searches. The proportion of sets for which the $m_{\mathrm{h}}+m_{\mathrm{A}}$ limit is degraded is therefore at the $(1 \div 2) 10^{-3}$ level.

The conclusions drawn from the additional scans are the same as for $\tan \beta=\sqrt{2}$.

### 4.3 The general case

A summary of the results of the coarse logarithmic scans performed for the nine selected $\tan \beta$ values is displayed in Table 1.

As can be seen in Table 1 and in Fig. 4, the behaviour observed for $\tan \beta=1$ or 2 is very similar to the one detailed for $\tan \beta=\sqrt{2}$, and similarly for $\tan \beta=8$ and 16 compared to $\tan \beta=32$. The values $\tan \beta=2 \sqrt{2}$ and 4 correspond to the transition from the low to the high $\tan \beta$ regimes and share features of both, with at $\tan \beta=2 \sqrt{2}$ a small peak in the $m_{\mathrm{h}}$ limit developing around $65 \mathrm{GeV} / c^{2}$, in fact the reflection of an $m_{\mathrm{h}}+m_{\mathrm{A}}$ limit at $140 \mathrm{GeV} / c^{2}$, while at $\tan \beta=4$ most of the sets cluster at the $m_{\mathrm{h}}+m_{\mathrm{A}}$ limit, i.e. $144 \mathrm{GeV} / c^{2}$, but a few still correspond to an $m_{\mathrm{h}}$ limit of $87 \mathrm{GeV} / c^{2}$.

The case of $\tan \beta=1 / \sqrt{2}$ is quite different. Here, the apparent limit on $m_{\mathrm{h}}$ in excess of $60 \mathrm{GeV} / c^{2}$ simply corresponds to the lower edge of the physical region for small values of $m_{\mathrm{A}}$. The reason is that, in such a configuration, the search for the Higgsstrahlung process at LEP2 is inefficient in spite of large $\sin ^{2}(\beta-\alpha)$ values; this is because b tagging is involved while the $\mathrm{h} \rightarrow$ AA decay is dominant with $\mathrm{A} \rightarrow \mathrm{b} \overline{\mathrm{b}}$ kinematically forbidden. This problem does not arise for $\tan \beta>1$ because such low $m_{\mathrm{A}}$ values are now associated with $m_{\mathrm{h}}$ values small enough to be within the reach of searches at LEP1 which do not require $b$ tagging.

However, the region at low $m_{\mathrm{A}}$ corresponds to low charged Higgs boson masses. For $\tan \beta<1$, the dominant decay mode of charged higgs bosons is into cs. This configuration would show up indirectly as an apparent reduction of the t $\bar{t}$ production cross section in $\mathrm{p} \overline{\mathrm{p}}$ collisions. (In the measurement of this cross section, at least one leptonic W decay from $\mathrm{t} \rightarrow \mathrm{Wb}$ is required, while the $\mathrm{t} \rightarrow \mathrm{H}^{+} \mathrm{b}$ decay would dominantly lead to hadronic final states.) From the measurement of the $t \bar{t}$ production cross section at the Tevatron, it can be deduced that this particular region to which the LEP analyses are insensitive is indeed excluded [17].

## 5 Results in a less constrained MSSM

Among the model assumptions which have been made for this analysis, the one of a universal mass for all squarks and sleptons at GUT scale is the least compelling, in particular because the influence of the Yukawa interactions was neglected in the evolution of the masses down to the electroweak scale. To assess the impact of this assumption, coarse logarithmic scans have been repeated for $\tan \beta=\sqrt{2}$ and 32 , ignoring the constraints on sleptons and sneutrinos (Criteria No. 1, 2 and 10) and replacing the LEP2 limits
on charginos (Criterion No. 10), which depend on the sneutrino mass, by the constraint $m_{\chi^{ \pm}}>75 \mathrm{GeV} / c^{2}$, valid except in the case of chargino-sneutrino mass degeneracy[15]. Moreover, the $m_{0}$ parameter has been replaced in these scans by two independent soft supersymmetry breaking masses, $m_{Q}$ and $m_{U}$, appearing in the diagonal terms of the stop mass matrix. This leads to a total of 235467 sets of $\left\{m_{Q}, m_{U}, m_{1 / 2}, \mu, A_{t}\right\}$ values explored for each of the two $\tan \beta$ values.

The results are presented in Table 2. It can be seen that the fraction of sets with a limit significantly degraded compared to the benchmark case remains at or below the $10^{-3}$ level. Similar results are obtained with linear scans. The basic conclusions of this analysis therefore do not depend on the universality assumption for squark and slepton masses.

## 6 Conclusions

The main conclusion from this study is that the lower limit on the mass of the CP-even neutral Higgs boson is robust in the low $\tan \beta$ regime, i.e for $1<\tan \beta<\sim 2$. Irrespective of the way the MSSM parameter space is sampled, logarithmically or linearly, and at least for values of the dimensionful parameters not exceeding $2 \mathrm{TeV} / c^{2}$, the limit of $\sim 88 \mathrm{GeV} / c^{2}$ obtained in the benchmark case holds in $99.99 \%$ of the physically allowed domain. This means that, at the end of LEP2 and provided $150 \mathrm{pb}^{-1}$ are accumulated at 200 GeV , either the Higgs boson will have been discovered or the low $\tan \beta$ scenario will be disproven.

For larger $\tan \beta$ values, the conclusion is less strong. The safe exclusion is best expressed in terms of $m_{\mathrm{h}}+m_{\mathrm{A}}$, presently at the level of $140 \div 150 \mathrm{GeV} / c^{2}$.

While some modest improvement in the rejection of the parameter sets unexcluded for low $m_{\mathrm{h}}$ values could be obtained with dedicated searches, for instance for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{hA}$ with $m_{\mathrm{h}}<2 m_{\mathrm{b}}$, the picture which has emerged from this study would not be qualitatively changed: the fraction of those pathological parameter sets is already extremely low; this fraction will not be reduced to zero at LEP since sets have been identified with very low $m_{\mathrm{h}}$ values and with vanishing Higgsstrahlung cross sections, because of $\sin ^{2}(\beta-\alpha)=0$, and with forbidden pair production, because of $m_{\mathrm{h}}+m_{\mathrm{A}}>200 \mathrm{GeV} / c^{2}$ (an example is given in Fig. 5). Indirect constraints, such as those inferred from precision electroweak masurements or from rare B or $\tau$ decays, have not been used in this analysis. They might also restrict the fraction of pathological parameter sets. Finally, it may be remarked that those sets involve very large values of $|\mu|$ and $\left|A_{t}\right|$, and seem to be excluded by requirements on the absence of charge and colour breaking minima [18].

| $\tan \beta$ | test | physical | $\chi^{ \pm}, \tilde{\nu}, \tilde{\mathrm{t}}$ | excluded | limit > test | $\chi^{ \pm}, \tilde{\ell}, \tilde{\mathrm{t}}$ | limit < test |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1 / \sqrt{2}$ | 87 | 15096 | 9406 | 258 | 696 | 165 | 4571 |
| 1 | 87 | 16057 | 9796 | 3370 | 2882 | 5 | 4 |
| $\sqrt{2}$ | 87 | 16044 | 9745 | 2717 | 3570 | 6 | 6 |
| 2 | 87 | 16131 | 9638 | 1314 | 5155 | 11 | 13 |
| $2 \sqrt{2}$ | 80 | 17123 | 10443 | 275 | 6086 | 36 | 283 |
|  | $140^{*}$ |  |  |  | 6396 | 7 | 2 |
| 4 | 67 | 17258 | 10444 | 47 | 6490 | 82 | 195 |
|  | $140^{*}$ |  |  |  | 6674 | 53 | 40 |
| 8 | $140^{*}$ | 17016 | 10239 | 29 | 6694 | 18 | 36 |
| 16 | $142^{*}$ | 17002 | 10206 | 20 | 6760 | 6 | 10 |
| 32 | $144^{*}$ | 16711 | 10001 | 23 | 6681 | 0 | 6 |

Table 1: Results of the coarse logarithmic scans. The value of $\tan \beta$ is given in the first column. The second column indicates as "test" a mass value in $\mathrm{GeV} / c^{2}$ close to the limit obtained in the benchmark case of "maximal mixing" either for $m_{\mathrm{h}}$ (unstarred values) or for $m_{\mathrm{h}}+m_{\mathrm{A}}$ (starred values). The third column contains the number of physically acceptable parameter sets, out of a total of 26163 explored. The number of sets excluded by the sneutrino and stop mass limits at LEP1 and by the chargino searches at LEP1 and LEP2 is given in the fourth column. The fifth column contains the number of sets for which the whole physical domain is excluded by the searches for Higgs bosons. The number of sets for which the limit obtained for $m_{\mathrm{h}}$ or for $m_{\mathrm{h}}+m_{\mathrm{A}}$, as relevant, is equal to (or larger than) the test value is shown in the sixth column. Out of the remaining sets, a few, the number of which is given in the seventh column, are excluded by the searches for supersymmetric particles at LEP2 (including a more refined treatment of the chargino exclusion than in the fourth column). This leaves the number of sets indicated in the eighth column for which the limit is degraded with respect to the test value.

| $\tan \beta$ | test | physical | $\chi^{ \pm}, \tilde{\mathrm{t}}$ | excluded | limit > test | limit < test |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\sqrt{2}$ | 87 | 93224 | 77042 | 14715 | 1441 | 26 |
| 32 | $144 *$ | 89263 | 76275 | 88 | 12876 | 24 |

Table 2: Results of the coarse logarithmic scans performed without the universality assumption for scalar lepton and scalar quark masses. The definitions are the same as in Table 1 with the following adjustments: the number of parameter sets explored is 235467 instead of 26163 ; in the fourth column, the sneutrino mass limit is not used and the chargino mass limit is set at $75 \mathrm{GeV} / c^{2}$; the seventh column has been removed.

## Appendix A

Here, a search in the LEP1 data for final states resulting from the reaction $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{hA}$ followed by $\mathrm{h} \rightarrow \mathrm{AA}$ is described. It applies when one of the three final A bosons, with mass smaller than $2 m_{\mathrm{b}}$, decays into a $\mu^{+} \mu^{-}$pair. At least four good charged particle tracks are required. The event must be clustered into three jets, using the JADE algorithm with a $y_{\text {cut }}$ value of 0.01 . The maximum jet-jet angle must be smaller than $165^{\circ}$. The event is rejected if a jet contains only one charged particle. At least one jet $\left(j_{1}\right)$ must contain two oppositely charged and isolated muons. The isolation criterion requires that the energy in a cone of $30^{\circ}$ half-opening angle around the muon direction should be lower than 1 GeV , with the other muon removed. Finally the masses of the three jets should be compatible: $\left|\left(m_{j_{2}}+m_{j_{3}}\right) / 2-m_{j_{1}}\right|<2.5 \mathrm{GeV} / c^{2}$.

The efficiency of this search is typically $45 \%$ for $m_{\mathrm{A}}$ between 1 and $9 \mathrm{GeV} / c^{2}$ and $m_{\mathrm{h}}$ around $60 \mathrm{GeV} / c^{2}$. No events were found in the data collected in 1994 and 1995, which translates into a $95 \%$ C.L. upper limit of 1.2 MeV for the Z width into hA when the branching ratio of $\mathrm{A} \rightarrow \mu^{+} \mu^{-}$is $1.610^{-3}$.

## Appendix B

The search for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{hA}$ at LEP2 is described in Ref. [19]. An equal mass constraint is used in the analysis addressing the $\tau^{+} \tau^{-} \mathrm{b} \overline{\mathrm{b}}$ final state which can therefore not be used for mass differences above $\sim 10 \mathrm{GeV} / c^{2}$. In contrast, the equal mass constraint is used for the bbbb $\bar{b}$ final state only implicitely in the optimization procedure, but there is no explicit cut on the dijet mass difference. As a consequence, the analysis is expected to remain efficient even for substantial mass differences. This has been checked with samples of fully simulated signal events for various $m_{\mathrm{h}}$ and $m_{\mathrm{A}}$ values, using to interpolate between those points a fast simulation program incorporating a parametrization of the response of the b tagging algorithm. The resulting efficiency map is displayed in Fig. 6.

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## Figure captions

Figure 1: For $\tan \beta=\sqrt{2}$, (a): upper edge of the physical region for $m_{\mathrm{h}}$, for those parameter sets which are excluded irrespective of the value of $m_{\mathrm{A}}$, and (b): distribution of the lower limit on $m_{\mathrm{h}}$ for the other sets (in logarithmic scale). The shaded histogram corresponds to the parameter sets remaining once all constraints have been applied, and for which the limit is smaller than $87 \mathrm{GeV} / c^{2}$.

Figure 2: (a): Variation of $m_{\mathrm{h}}^{2}$ (right scale) and of $\sin ^{2}(\beta-\alpha)$ (left scale) as a function of $m_{\mathrm{A}}$ for $\tan \beta=\sqrt{2}, m_{0}=125 \mathrm{GeV} / c^{2}, m_{1 / 2}=353 \mathrm{GeV} / c^{2}, \mu=1414 \mathrm{GeV} / c^{2}$ and $A_{t}=4000 \mathrm{GeV} / c^{2}$. (b): Variation of $m_{\mathrm{h}}^{2}$ (right scale) and of $\sin \alpha$ (left scale) as a function of $m_{\mathrm{A}}$ for $\tan \beta=\sqrt{2}, m_{0}=125 \mathrm{GeV} / c^{2}, m_{1 / 2}=125 \mathrm{GeV} / c^{2}, \mu=1000 \mathrm{GeV} / c^{2}$ and $A_{t}=2000 \mathrm{GeV} / c^{2}$.

Figure 3: For $\tan \beta=32$, distributions of (a) the lower limit on $m_{\mathrm{h}}$ (in logarithmic scale) and of (b) the lower limit on $m_{\mathrm{h}}+m_{\mathrm{A}}$ (in doubly logarithmic scale). The shaded histograms correspond to the parameter sets remaining once all constraints have been applied, and for which the $m_{\mathrm{h}}+m_{\mathrm{A}}$ limit is smaller than $144 \mathrm{GeV} / c^{2}$.

Figure 4: For various values of $\tan \beta$, distributions in vertical linear (left) and vertical logarithmic (right) scales of (a) the lower limit on $m_{\mathrm{h}}$ in horizontal linear scale and of (b) the lower limit on $m_{\mathrm{h}}+m_{\mathrm{A}}$ in horizontal logarithmic scale.

Figure 5: Variation of $m_{\mathrm{h}}^{2}$ (right scale) and of $\sin ^{2}(\beta-\alpha)$ (left scale) as a function of $A_{t}$ for $m_{\mathrm{A}}=205 \mathrm{GeV} / c^{2}, \tan \beta=5, m_{0}=200 \mathrm{GeV} / c^{2}, m_{1 / 2}=125 \mathrm{GeV} / c^{2}$ and $\mu=1950 \mathrm{GeV} / c^{2}$.

Figure 6: Efficiency of the search for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{hA} \rightarrow \mathrm{b} \overline{\mathrm{b}} \mathrm{b} \overline{\mathrm{b}}$ at LEP2 as a function of $m_{\mathrm{h}}$ and $m_{\mathrm{A}}$.


Figure 1a


Figure 1b


Figure 2a


Figure 2b


Figure 3a


Figure 3b


Figure 4a (begin)


Figure 4a (cont.)


Figure 4a (end)


Figure 4b (begin)


Figure 4b (cont.)


Figure 4b (end)


Figure 5


Figure 6

