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Hard scattering in high-energy QCD[†]

Michelangelo L. Mangano[‡]

Theoretical Physics Division, CERN, 1211 Geneva 23, Switzerland
E-mail: `michelangelo.mangano@cern.ch`

Abstract

I review the recent results in the field of QCD at high energy presented to this Conference. In particular, I will concentrate on measurements of α_S from studies of event structures and jet rates, jet production in hadronic collisions, and heavy quark production.

1. INTRODUCTION

There is no doubt nowadays that QCD *is* the theory of strong interactions. Studies of QCD at high energy have left the phase of *testing* QCD, and have stepped into the phase of accurate measurements. A priori, the only fundamental quantity relevant to the high-energy regime of QCD that needs to be measured is α_S, as, in principle, everything else can be calculated from first principles. α_S is one of the fundamental "constants" of nature, and as such there is no limit to the accuracy with which we would like to know its value. The accurate knowledge of α_S, for example, is necessary to extract electroweak parameters from precision e^+e^- data, as well as to explore scenarios in which all forces of nature are unified at some high-energy scale. Pragmatically, one can therefore say that QCD studies at high energy mostly aim at assessing the accuracy of the approximations used by theorists to evaluate QCD cross sections. This accuracy forms the basis of our evaluation of the systematic errors entering the extraction of α_S from the data. Confidence in our practical understanding of QCD, on the other hand, has also an impact on our ability to predict cross-sections for processes which produce potential backgrounds to new physics, as well as to estimate the potential of experiments at future accelerators. An accurate understanding of high-energy QCD, furthermore, is necessary for an accurate determination of several other important parameters of the SM, such as the W mass (whose measurement is limited theoretically by the knowledge of structure functions in hadronic collisions, and of parton recombination effects in e^+e^-), and the top mass (where the properties of multi-gluon emission from initial and final states limit the theoretical accuracy of determinations from hadronic colliders). Upon closer scrutiny, one quickly realises that the ideal world in which α_S is the only ingredient needed to perform precise QCD predictions is far from reality. Since the observable final states are made out of mesons and baryons, and not of quarks and gluons, understanding of the interface between hard and soft QCD cannot be escaped even when we consider high-energy processes. This is particularly true when the initial states are given by hadrons, rather than by lepton pairs. As a result, ancillary information (such as parton densities and fragmentation functions) is needed for all practical cases. Deviations from the ideal factorisation regime, where all non-perturbative knowledge is dumped into phenomenological quantities, become also relevant when dealing with observables which have potentially large higher-twist (or power suppressed) contributions.

At the end of the day, one therefore concludes that future progress in QCD phenomenology will still benefit enormously from all possible sources of experimental information. It was then a great pleasure for me to deal with the almost 100 papers submitted to the QCD session of this Conference, number which testifies of the great interest which the field still raises, and of the great progress made by the experimental groups in developing new analysis tools and extracting useful information

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from their rich data samples. Given the large number of papers submitted, it will be difficult however to review in detail each contribution. I will try to keep a balance between the in-depth review of some critical issues, and the completeness of the report. As a result, I hope to provide the reader with a sort of hitchhiker's guide to the current open problems and to the available results, hoping that this will stimulate some to tackle the most interesting problems.

Since most submissions reviewed here have not appeared as yet as preprints, I have included references to the number of the submitted papers in the form (X_YYY), where X labels the session and YYY the paper number. The write-ups of most of these papers can be found from the Web pages of the experiments, listed in refs. [1]-[8]. To conclude this Introduction, I wish to thank the QCD physics coordinators of all experiments, for the prompt submission of draft papers, and for replying to my enquiries. I also thank S. Catani for several discussions related to the contents of this talk.

2. EXPERIMENTAL INPUTS AND THEORY TOOLS

The experimental papers submitted to the Conference cover all possible aspects of high-energy QCD phenomenology. Issues related to the structure of the proton and of the photon (parton densities, diffractive phenomena, small-x physics) are reviewed in the reports by Marage and Barreiro. The set of inputs I will cover in this talk is summarised here:

- Properties of final states (shapes, multiplicities, fragmentation functions, heavy-quark fractions). These have an important impact on perturbative QCD studies, as well as on the study of power corrections and non-perturbative physics, and on the extraction of $\alpha_S(M_Z)$
- Jet production in $p\bar{p}$, e^+e^- and ep collisions: these phenomena allow to explore the proton structure at the smallest possible distance scales, as well as to extract information on $\alpha_S(M_Z)$
- W/Z production
- Heavy quark production: gluon splitting to $c\bar{c}$ and $b\bar{b}$, pair production in $\gamma\gamma$, ep and $\gamma\gamma$ collisions.

Although several papers have been submitted, I will unfortunately be forced to leave out for lack of time and space topics which are not (or not yet) directly related to the regime of *hard* QCD: detection and study of resonances inside jets, Bose-Einstein correlations, Fermi-Dirac correlations, multi-particle production, etc. These subjects were recently reviewed by B. Webber [9] in his rapporteur presentation at the 1999 Lepton-Photon Symposium, and I refer the readers to his forthcoming contribution to the Proceedings for a good review of the topic.

The set of theoretical results presented at this conference represents only a minor sample of the most recent developments. A common thread among these contributions is the attempt to extend the range of applicability of perturbation theory (PT), by either increasing the order at which the perturbative calculations are performed, by resumming classes of potentially large (logarithmic) contributions appearing at all orders of PT, or by exploring the behaviour of PT in the infrared domain in the attempt to provide a phenomenological description of the hadronisation phase.

(*i*) Techniques introduced recently [10] for the evaluation of LO multi-particle amplitudes have been reviewed here by Draggiotis (1_652). (*ii*) The achievement of NNLO accuracy for jet observables in hadronic collisions represents today an open challenge for theorists. Several milestones need to be met before NNLO cross-sections become available. Among these milestones is the evaluation of the singular behaviour of amplitudes near multi-collinear and multi-soft poles. Analytical control over such behaviour allows to factorise the infrared and collinear singularities which appear at higher orders of PT, and is an important element in the construction of higher-order, parton-level event generators. Important progress has occurred in the recent past [11], as was discussed here by Uwer (1_129). (*iii*) Harlander (1_444) presented the results of a recent calculation [12] of the corrections of $\mathcal{O}(\alpha_s^3, m_Q^4)$ to $R_{\rm had}$, a quantity whose experimental measurement has reached high accuracy and for which higher-order mass corrections are non-negligible. (*iv*) Progress in the resummation of NLL threshold corrections, relevant for the production of heavy quarks, jets and prompt γ's at the edge of phase space [13]-[18], was reviewed by Kidonakis (1_164) [19]. Some applications in the case of top quark and prompt-γ production will be discussed in more detail later on. (*v*) Sterman (1_712) reviewed the status of the analytical understanding of power corrections [20], and (*vi*) explorations of the PT-non-PT transition region in QCD were illustrated by Eden (1_206).

3. SANITY CHECKS OF QCD

A large number of results submitted to the Conference do not find an obvious classification. To cope with this problem, I allowed myself to introduce a new category, namely that of *sanity checks* of QCD. I include in this category tests of QCD and QFT which do not have a direct impact on specific measurements (or not yet!). The lack of a direct impact could be due to several reasons:

- the measurements are not sufficiently accurate to improve our knowledge of some fundamental parameter (e.g. m_b)
- they are qualitative in nature (e.g. tests of colour coherence)
- they explore hard-wired fundamental features of QCD (e.g. the flavour independence of α_S, $N_c = 3, \dots$), and any deviation from the expected result would not signal a problem with QCD itself, but most likely the failure of some approximation used in the theoretical calculations, or, ultimately, the presence of unexpected new physics.

By no means classification under "sanity checks" should be taken as a negative judgement of the merits of a measurement. The tests discussed in the following testify in fact of the increased sophistication of experimental techniques. Furthermore, in several cases they set the groundwork for possible future applications, e.g.:

- background removal in searches for New Physics
- use of q-jet versus g-jet discrimination
- use of colour-coherence patterns to separate production of colour-singlet objects from multi-jet backgrounds

3.1. Evolution with $\sqrt{S}$

One of the most fundamental sanity checks of QCD is the behaviour of observables w.r.t. changes in the hardness of the process. The first preliminary measurements at the new energy frontier of LEP ($\sqrt{S} = 192 - 196$ GeV) have been submitted to this Conference (Aleph: (1_392); L3: (1_232); OPAL: (1_80)). There is no deviation from the expected evolution of track multiplicities, momentum fractions, and jet multiplicity rates.

3.2. Quark-mass effects

Improved tagging techniques have led in the recent past to a flourishing of QCD measurements based on the identification of heavy quarks inside the jets. This allows to test characteristic properties of Quantum Field Theory, such as the running of quark masses with energy or the screening of collinear singularities induced by the quark mass. In addition, as discussed later, the tagging of specific flavours inside the jets allows for example to isolate samples of high-purity gluon jets.

I will start from measurements of the b-quark mass at M_{Z^0}. These measurements use properties of b-tagged events such as the 3-jet rate or moments of some shape variable (e.g. $B_{W,2}$, the second moment of the wide-jet broadening). The b-quark mass m_b appears as a parameter in the theoretical evaluation of these quantities [21]-[23], and can be fitted from the the data. It is usually more convenient to work with a running $\overline{\text{MS}}$ mass, in which case the value one expects to extract from the fits to the data is the running b mass, evaluated at the scale M_Z. The results shown at this Conference are collected here (values in GeV):

(1_384) ALEPH:

$$m_b(M_Z) = \begin{matrix} 3.04 \pm 0.92 & \text{3-jet fraction} \\ 3.78 \pm 0.27 & B_{W_2} \end{matrix}$$

$$m_b^{\overline{\text{MS}}}(m_b) = \begin{matrix} 4.16 \pm 1.10 & \text{3-jet fraction} \\ 5.04 \pm 0.32 & B_{W_2} \end{matrix}$$

(1_223) DELPHI [24]:

$$m_b(M_Z) = 2.61 \pm .18_{st} \begin{matrix} +.45 \\ -.49 \end{matrix}_{frag} \pm .04_{tag} \pm .07_{th}$$

(1_449) A. Brandenburg et al. [25]:

$$m_b(M_Z) = 2.52 \pm .27_{st} \begin{matrix} +.33 \\ -.47 \end{matrix}_{sys} \begin{matrix} +.28 \\ -1.39 \end{matrix}_{had} \pm .48_{th}$$

These values are consistent with those obtained from the standard determination of $m_b^{\overline{\text{MS}}}(m_b) = 4.20(8)$, obtained from the application of NNLO PT and QCD sum rules to the $\Upsilon(1S)$ system [26]. The systematic uncertainties are however very large, and these measurements should therefore be taken just as overall consistency checks. For a comparative study of the measurements performed by Aleph and Delphi, see ref. [27].

The large value of the bottom quark mass is predicted by QCD to affect also other observables, such as for example the multiplicity of the final state. Soft-gluon emission is suppressed within a cone (the *dead* cone) of radius $2m_b/\sqrt{S}$ around the direction of the b quark, while it is identical to that originating from a light quark outside the dead cone. The difference between the multiplicities of heavy- and light-quark final states is then approximately proportional to the number of gluons emitted by the light quark inside the dead cone area, corrected by the average multiplicity produced during the heavy quark weak decay. When $\sqrt{S}$ increases, the size of the dead cone diminishes, but the density of emission from the light quark increases, and the two effects compensate each other, leaving a constant multiplicity. As a result, the difference of the average charged multiplicity of $b\bar{b}$ and light-quark events as a function of $\sqrt{S}$ is a constant [28]:

$$\delta_{bl} \equiv \langle n \rangle_{bb} - \langle n \rangle_{ll} \sim \text{const}\,.$$

This relation (valid asymptotically, and up to potentially large corrections of $\mathcal{O}(\sqrt{\alpha_S})$) is supported by the recent measurements by DELPHI (1_220), which used the large lever arm in energy available between LEP and LEP2. The results [29] are consistent with the theoretical expectations of a constant δ_{bl}, although the extracted value is slightly larger than anticipated, indicating large sub-dominant contributions.

$$\delta_{bl} = \begin{array}{ll} 2.96 \pm 0.20 & \sqrt{S} = 91.2\ GeV \\ 5.07 \pm 1.28_{stat} \pm 1.07_{syst} & \sqrt{S} = 183\ GeV \\ 3.97 \pm 0.83_{stat} \pm 0.68_{syst} & \sqrt{S} = 189\ GeV \end{array}$$

3.3. Flavour independence of α_S

These tests are performed by isolating samples of tagged heavy-flavour events, and extracting the value of α_S from standard event-shape and multi-jet analyses, using $\mathcal{O}(\alpha_S^2)$ QCD calculations including the heavy-quark mass effects [21]-[23], [30]. The following recent results were submitted to this Conference:

(1_25) OPAL [31]:

$$\frac{\alpha_S^c}{\alpha_S^f} = 0.997 \pm 0.050\,,\ \frac{\alpha_S^b}{\alpha_S^f} = 0.993 \pm 0.015$$

(1_223) DELPHI:

$$\frac{\alpha_S^b}{\alpha_S^f} = 1.005 \pm 0.012$$

The flavour independence of α_S is therefore tested at the 5% level for charm, and at the % level for bottom.

3.4. b couplings

Final states where the $b\bar{b}$ pair is accompanied by the emission of a hard gluon can be used to test the Lorentz structure of the $b\bar{b}g$ coupling, through the study of the spectrum and angular correlations of the 3 jets in the event. Limits on the anomalous chromomagnetic coupling κ defined by:

$$\Delta\mathcal{L} = \frac{\kappa}{4m_b} g_s\, \bar{b}\sigma_{\mu\nu} b\, G_{\mu\nu}$$

have been obtained by SLD (1_182): $-0.11 < \kappa < 0.08$ [32]. In a further study SLD (1_183) analysed correlations in the $b\bar{b}g$ decay of polarized Z^0. They found no evidence for T-odd CP-even or T-odd CP-odd asymmetries at a level of 5% [33].

3.5. Colour coherence

Quantum coherence is a fundamental property of radiation from multi-parton states. In the case of gluon emission, this is often referred to as colour-coherence, and manifests itself with non-trivial angular emission patterns for soft radiation [34]. Being a property of quantum theory, there is no doubt it should be there! As a result, it could become a very interesting tool to learn more about the underlying parton-level structure of a final state, for example to separate quark and gluon jets, or to identify events where jets come from colour-singlet sources (such as the decay of a neutral object) from generic QCD events. The following studies were presented at this Conference, providing further evidence that colour-coherence effects survive the parton-to-hadron transition, and can be detected with appropriate selection criteria:

(1_163e) DØ: Evidence of color coherence in W+jets events in $p\bar{p}$ at $\sqrt{s} = 1.8$ TeV [35].
(1_145) DELPHI: A test of QCD coherence and LPHD using symmetric 3-jet events.
(1_510) DELPHI: Testing of the New Parton Final State Reconstruction Method Using $Z^0 \to b\bar{b}g$ Mercedes Events.

3.6. C_A/C_F from N_{ch} and $F(z)$ in q/g jets

Heavy-flavour tagging techniques have also found a useful application in the study of fragmentation properties of light partons. The tagging of b quarks in a 3-jet event allows in fact to single out the gluon jet, and anti-tagged events provide samples of light-quarks. Using events with different 3-jet kinematics, it is possible to probe the jet properties at different scales, and test the scale evolution of observables such as the jet fragmentation function and the jet charged multiplicity N_{ch}. QCD provides detailed predictions for the evolution of these quantities and predicts clear differences between quark and gluon jets. These differences have been now measured with great accuracy, and have been turned into a measurement of the relative colour charge of gluons and quarks (C_A/C_F, equal to 9/4 in QCD). The following is a summary of these most recent results. A more complete account can be found in the contribution to these Proceedings by B. Gary [36].

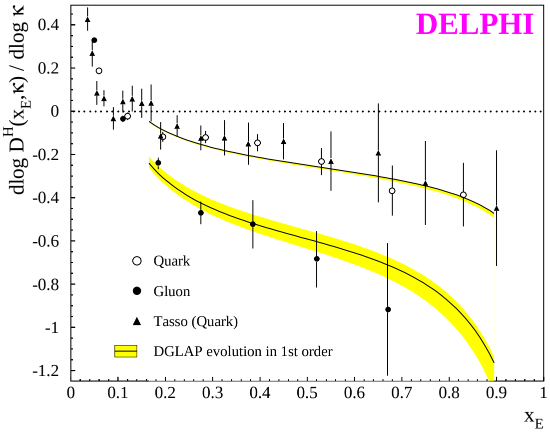

Figure 1. Scaling violations in the fragmentation functions $D(z)$ for quark and gluon jets, from DELPHI.

(1_571) DELPHI: Gluon fragmentation function and scaling violations in q/g jets (see fig. 1):

$$\frac{C_A}{C_F} = 2.23(9)(6)$$

(1_383) DELPHI: Scale Dependence of N_{ch} in q and g jets [37]:

$$\frac{C_A}{C_F} = 2.246 \pm 0.062_{stat} \pm 0.080_{sys} \pm 0.095_{th}$$

(1_24) OPAL: Experimental properties of gluon and quark jets from a point source [38]:

$$\frac{C_A}{C_F} = 2.29 \pm 0.09_{\rm stat} \pm 0.15_{\rm syst}$$

(See also OPAL (1_6), "A simultaneous measurement of α_S and QCD colour factors"). Interesting comparisons of the multiplicity of individual hadrons in quarks and gluon jets were presented by OPAL (1_4) and DELPHI (3_146). OPAL found that the observed ratio of η mesons in quark and in gluon jets agrees with what predicted by QCD Monte Carlo calculations, while the absolute number of η's is larger in the data than in the MC. Similar small discrepancies in the absolute prediction for kaons and protons in gluon jets have been observed by DELPHI. More details can be found in ref. [9].

4. $\alpha_S(M_Z)$ MEASUREMENTS

4.1. Event shapes in e^+e^- and ep

Event shapes are defined as infrared and collinear safe observables, characterising the structure of the final state. By construction, they should be calculable in PT, up to finite non-PT corrections suppressed by powers of the typical hard scale of the process. They are sensitive to properties of QCD radiation, and therefore allow the measurement of α_S. QCD predictions are available at $\mathcal{O}(\alpha_S^2)$ [39, 40, 41], which in the case of e^+e^- and of DIS corresponds to the NLO in PT. In most cases, these NLO calculations are improved by the resummation of next-to-leading logarithms (NLL) appearing at all orders of PT near the edge of phase-space [42]. Since shape variables are sensitive to non-perturbative power-suppressed (i.e. $\propto 1/Q$) effects, their study allows the analysis of the hadronisation phase.

Extractions of α_S in the past have used a description of non-PT corrections to event-shapes based on the hadronisation models of shower MC's (Herwig, Jetset). With the recent progress in the theoretical understanding of the structure of power corrections [43] the effect of hadronisation corrections can be described with simple analytical expressions, dependent on a small number of parameters. These parameters, in turn, can be interpreted [44] as moments of the strong coupling constant, averaged over the small Q^2 region. For most cases, what is required is the first moment of $\alpha_S(Q)$:

$$\alpha_0(\mu_0) = \frac{1}{\mu_0} \int_0^{\mu_0} {\rm d}q \; \alpha_S(q) \tag{1}$$

(in practical cases, one usually takes $\mu_0 = 2$ GeV). Early experimental tests of these ideas were presented in refs. [45]-[47], where evidence was found for values of $\alpha_0(2{\rm GeV}) \sim 0.5$.

It is common to use either *moments* of shape variables, or *distributions*. In the case of the first moments:

$$\langle \mathcal{F} \rangle = \langle \mathcal{F}_{PT} \rangle + \langle \mathcal{F}_{non-PT} \rangle \tag{2}$$

where $\langle \mathcal{F}_{PT} \rangle$ is obtained from the PT calculation, while the non-PT effects are shown to factorise into the contribution $\langle \mathcal{F}_{non-PT} \rangle = c_F \, \mathcal{P}$. Here

$$\mathcal{P} = \frac{4C_F}{\pi^2} \mathcal{M} \, \frac{\mu_0}{\sqrt{S}} \left[\alpha_0(\mu_0) - \alpha_S(\sqrt{s}) + \mathcal{O}(\alpha_S^2) \right] . \tag{3}$$

is a universal factor ($\mathcal{M} = 1.795$ is the so-called "Milan" factor [48], incorporating effects of 2-loop corrections), and c_F is an observable-dependent coefficient:

$$\begin{array}{lccccc} \mathcal{F} = & 1-T & M_H^2 & B_T & B_W & C \\ c_F = & 2 & 2 & 1 & 1/2 & 3\pi \end{array} \tag{4}$$

In the case of distributions, the effect of power corrections is to shift the value of the observable in

Q/GeV

the PT QCD prediction [49] (so long as the distance of the observable from its kinematic threshold value is larger than $\mathcal{O}(1/Q)$):

$$\frac{d\sigma}{d\mathcal{F}}(\mathcal{F}) = \frac{d\sigma^{PT}}{d\mathcal{F}}(\mathcal{F} - \mathcal{P}\, D_{\mathcal{F}}) \qquad (5)$$

$\mathcal{P}$ is the same as for the 1st moment, and $D_{\mathcal{F}}$ can be calculated for each observable. It can be a constant:

$$D_{\mathcal{F}} = \begin{cases} 2 & \mathcal{F} = 1 - T \\ 3\pi & \mathcal{F} = C \end{cases} \qquad (6)$$

or a function, as recently shown in the case of jet broadenings in ref. [50] ($\mathcal{F} = B_T,\, B_W$):

$$D_{\mathcal{F}} = \frac{1}{2}\log\frac{1}{\mathcal{F}} + B_{\mathcal{F}}(\mathcal{F}, \alpha_s(\mathcal{F}\sqrt{s})) \qquad (7)$$

4.2. Shape variables: results from QCD+MC fits

Let us begin with $\alpha_s(M_Z)$ determinations obtained from NLO+NLL QCD fits with non-PT effects described via MC programs. The following new measurements have been reported at this Conference:

(1_410) ALEPH (the third value, labelled by a (*), refers to the combination of LEP1+LEP2 results):

$$\begin{aligned} \alpha_s(189) &= 0.1119(15)_{stat}(11)_{sys}(30)_{th} \\ \alpha_s(M_Z) &= 0.1249(44) \\ \alpha_s^*(M_Z) &= 0.1216(39) \end{aligned}$$

(1_144) DELPHI (statistical errors only):

$$\begin{aligned} \alpha_s(189) &= 0.1116(24)_{stat} \\ \alpha_s(M_Z) &= 0.1246(30)_{stat} \end{aligned}$$

(1_279) L3 (the value labelled by a (*) refers to the combination of results in the range 30–189 GeV):

$$\begin{aligned} \alpha_s(189) &= 0.1101(18)_{exp}(56)_{th} \\ \alpha_s(M_Z) &= 0.1227(22)_{exp}(69)_{th} \\ \alpha_s^*(M_Z) &= 0.1220(62) \end{aligned}$$

(1_5) OPAL (the value labelled by a (*) uses data with $\sqrt{S}$ in the range 35–189 GeV, from JADE in addition to OPAL LEP1 and LEP2):

$$\begin{aligned} \alpha_s(189) &= 0.1085(15)_{stat}(27)_{sys}(20)_{had} \\ & \quad \left(\begin{smallmatrix} +22 \\ -3 \end{smallmatrix}\right)_{scale} \\ \alpha_s(M_Z) &= 0.1206 \left(\begin{smallmatrix} +54 \\ -46 \end{smallmatrix}\right) \\ \alpha_s^*(M_Z) &= 0.1199 \left(\begin{smallmatrix} +38 \\ -25 \end{smallmatrix}\right) \end{aligned}$$

An overall average of the above 189 GeV results, gives

$$\alpha_s(189) = 0.1105(4), \quad \alpha_s(M_Z) = 0.1232(5)$$

with the theoretical and systematic errors averaged among the various experiments.

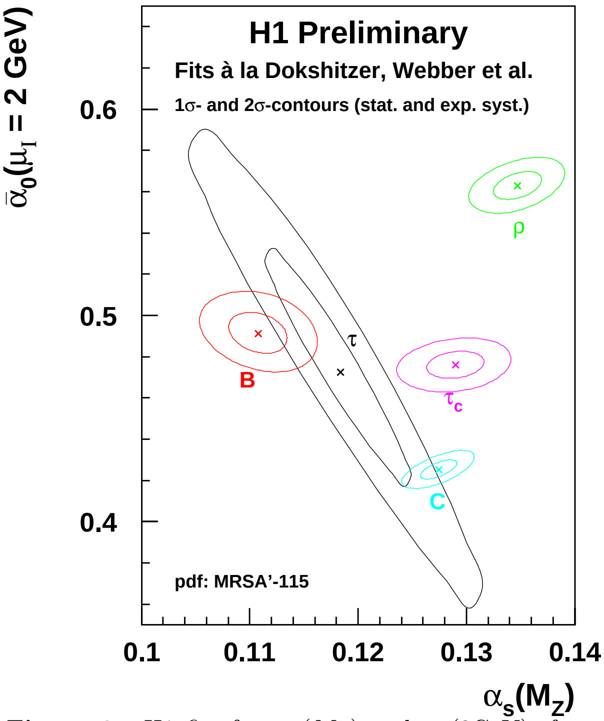

Figure 2. H1 fits for $\alpha_s(M_Z)$ and $\alpha_0(2\text{GeV})$, from different shape variables.

4.3. Shape variables: results from QCD+$\mathcal{O}(1/Q)$ fits

We review here the recent $\alpha_s(M_Z)$ determinations obtained from QCD fits, with non-PT effects described by analytic power corrections.

From 1st moments:

(1_144) DELPHI:

$$\begin{aligned} \alpha_0(2\,\text{GeV}) &= 0.5(1) \\ \alpha_s(189) &= 0.1102(23)_{stat}(18)_{sys}(24)_{th} \\ \alpha_s(M_Z) &= 0.1229(28)_{stat}(22)_{sys}(31)_{th} \end{aligned}$$

(1_157k) H1 (Fig. 2):

$$\begin{aligned} \alpha_0(2\,\text{GeV}) &= 0.50(5) \\ \alpha_s(M_Z) &= 0.12(1) \end{aligned}$$

From shape distributions:

(1_279) L3 (Fig. 3):

$$\begin{aligned} \alpha_0(2\,\text{GeV}) &= 0.490(46) \\ \alpha_s(M_Z) &= 0.1106(36)_{exp}(40)_{th} \end{aligned}$$

(1_113) Movilla Fernández et al, ref. [55], using data from JADE and LEP (the value labeled by a (*) excludes the B_W distribution from the fit, see

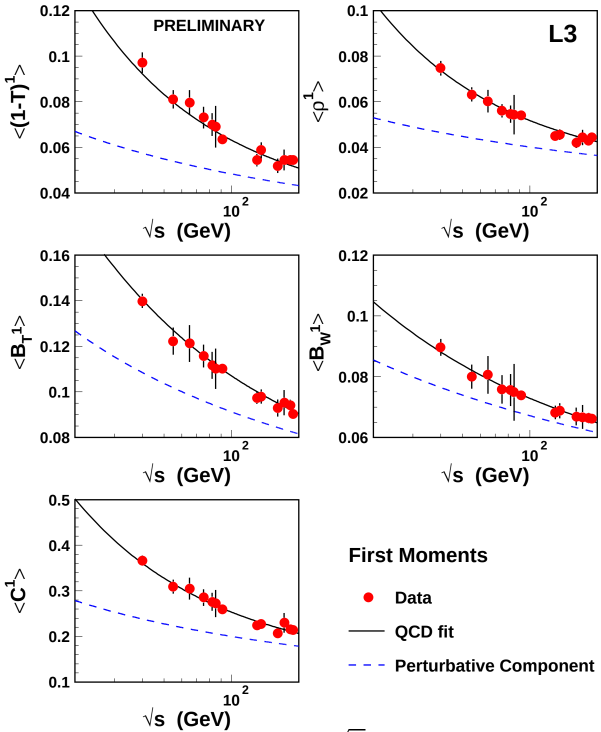

Figure 3. Evolution with $\sqrt{S}$ of the 1st moments of various shape variables, from L3. The dashed lines include only the PT contribution, the solid line is a fit including $\mathcal{O}(1/Q)$ effects.

Fig. 4):

$$\alpha_0(2\,\mathrm{GeV}) = 0.50\,^{+.9}_{-.6}$$
$$\alpha_S(M_Z) = 0.1068 \pm .0011_{stat}\,^{+.0033}_{-0.043}{}_{sys}\,^{+.0043}_{-0.029}{}_{th}$$
$$\alpha_s^*(M_Z) = 0.1141 \pm .0012_{stat}\,^{+.0034}_{-0.024}{}_{sys}\,^{+.0055}_{-0.041}{}_{th}$$

All of the above measurements show a good consistency in the extraction of $\alpha_S(M_Z)$ from QCD+hadronisation corrections, and from 1^{st} moments using analytic power corrections. Significant differences in $\alpha_S(M_Z)$ are vice-versa present when fitting shape distributions with power corrections. This is particularly true for the broadenings, for which the shift in the variable is given by the function in eq.(7). The inclusion of the variable shift, due to a subtle interplay between perturbative and non-PT effects, gives rise by itself to a large correction relative to the naive prescription of a constant shift, as shown in ref. [50] and as displayed in fig. 5. There are indications however that this correction is not sufficient, and that a bigger *squeezing* in the theoretical predictions is required. This is supported by the comparison with hadronisation corrections predicted by MC's [47]. I would conclude that we are moving in the right direction for a phenomenological understanding of power corrections, but more work is necessary before extractions of α_s can be improved further. It also remains to be evaluated whether the breakdown of factorisation for power corrections, shown in ref. [51] to occur at higher orders, will ultimately lead to an intrinsic limit in the theoretical accuracy attainable with this approach.

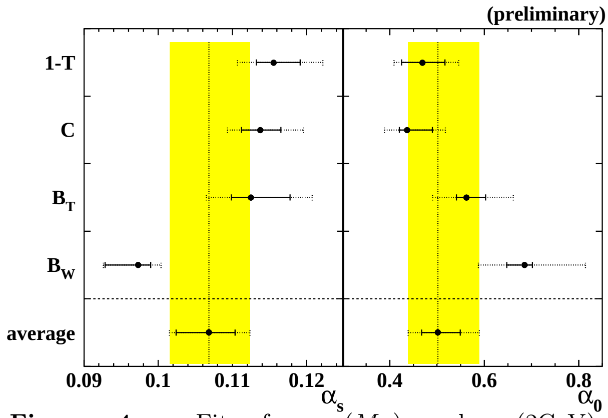

Figure 4. Fits for $\alpha_S(M_Z)$ and α_0(2GeV), from different shape variables in e^+e^- collisions (Movilla Fernández et al. [55]).

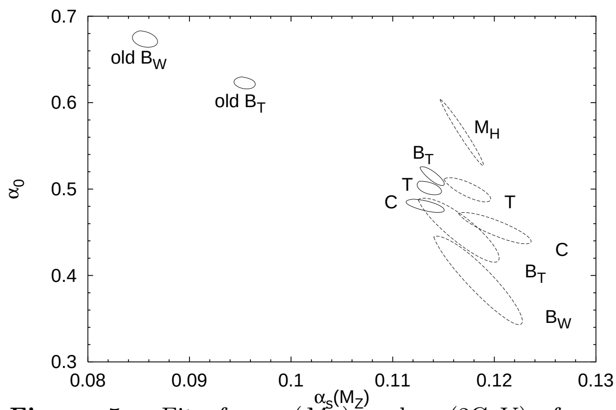

Figure 5. Fits for $\alpha_S(M_Z)$ and α_0(2GeV), from different shape variables in e^+e^- collisions. The dashed lines correspond to fit 1st moments, the solid lines to fits to distributions. Curves labeled by "old" refer to constant shifts in the broadening variable, while the others are obtained with the B-dependent shifts as in eq.(7). (Dokshitzer et al. [50]).

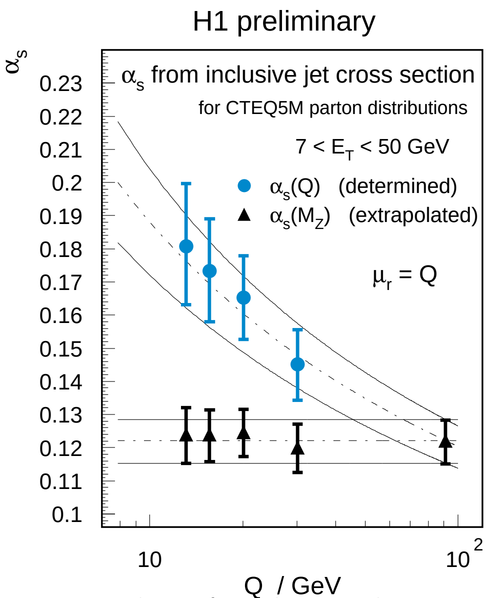

Figure 6. α_S evolution from the H1 inclusive jet rates.

4.4. Other $\alpha_S(M_Z)$ measurements

In addition to the above results on α_S obtained from precise shape-variable fits in e^+e^- collisions, other measurements of α_S have been reported at this Conference. The HERA measurements have been reviewed by T. Carli in his presentation [52]. I summarise here the main results:

(1_157) H1: Fit of the Inclusive Jet Rate $d^2\sigma/dE_TdQ^2$. The rates are compared to NLO QCD calculations [41, 53], and fit to the value of α_S. A recent comparison of various NLO calculations for jet production in DIS can be found in ref. [54]. (see fig. 6; the first value below corresponds to $\mu_R = E_T$, the second to $\mu_R = Q$):

$$\alpha_S(M_Z) = \begin{array}{l} 0.1181(30)_{exp}\begin{pmatrix}+39\\-46\end{pmatrix}_{th}\begin{pmatrix}+36\\-15\end{pmatrix}_{PDF} \\ 0.1221(34)_{exp}\begin{pmatrix}+54\\-59\end{pmatrix}_{th}\begin{pmatrix}+33\\-16\end{pmatrix}_{PDF}\end{array}$$

(1_157y) H1: Fit of the Dijet Rate $dn_{\rm dijet}/dy_2$ (the first value corresponds to use of the Durham algorithm, in the lab frame, the second to the $k_\perp^{DIS}$ algorithm in the Breit frame):

$$\alpha_S(M_Z) = \begin{array}{l} 0.1189\begin{pmatrix}+64\\-81\end{pmatrix}_{exp}\begin{pmatrix}59\\46\end{pmatrix}_{th}\begin{pmatrix}13\\55\end{pmatrix}_{PDF} \\ 0.1143\begin{pmatrix}+75\\-89\end{pmatrix}_{exp}\begin{pmatrix}74\\64\end{pmatrix}_{th}\begin{pmatrix}8\\54\end{pmatrix}_{PDF}\end{array}$$

(1_543) ZEUS: Fit of the Dijet fraction vs Q^2:

$$\alpha_S(M_Z) = 0.120(3)_{stat}\begin{pmatrix}+5\\-6\end{pmatrix}_{exp}\begin{pmatrix}+3\\-2\end{pmatrix}_{th}$$

(1_232) L3: QCD studies at 192 and 196 GeV:

$$\begin{array}{rcl}\alpha_S(192) &=& 0.1108(35)_{exp}(56)_{theory} \\ \alpha_S(M_Z) &=& 0.1220(15)_{exp}(60)_{theory}\end{array}$$

(1_80) OPAL: QCD studies at 192 and 196 GeV (the value at M_Z includes results in the range 30–192 GeV):

$$\begin{array}{rcl}\alpha_S(193) &=& 0.1025(38)_{stat}(54)_{syst} \\ \alpha_S(M_Z) &=& 0.1135(47)_{stat}(67)_{syst}\end{array}$$

(1_2) OPAL: QCD studies at 172-189 GeV (the value at M_Z includes results in the range 30–189 GeV):

$$\begin{array}{rcl}\alpha_S(187) &=& 0.106(1)_{stat}(4)_{syst} \\ \alpha_S(M_Z) &=& 0.117(5)\end{array}$$

(1_157y) DELPHI: $\mathcal{O}(\alpha_S^2)$ fits to oriented shape variables at 91.2 GeV:

$$\alpha_S(M_Z) = \begin{array}{ll} 0.1173(23) & \text{18 shape variables} \\ 0.1180(18) & \text{Jet cone E fraction[56]}\end{array}$$

In this analysis [57], the renormalisation scale is "experimentally optimised", i.e. it is chosen for each variable by searching for the best fit to the relative data distribution. It is extremely intriguing that the values of α_S extracted from each one of 18 variables are in extremely good agreement with one another (see fig. 7), leading to the very small errors quoted above. While this convergence is tantalising, it must be said that such an optimisation procedure has no theoretical basis, and I personally do not consider therefore the error estimate theoretically solid. Notice that there is a very large spread in the resulting values of optimised scales [57] which go from $x_\mu \equiv \mu^2/S = 3 \times 10^{-3}$ to $x_\mu = 7$. Altogether, I have no precise understanding of which bias could cause such an amazing convergence in the values of α_S. The result is extremely interesting, and certainly deserves theoretical attention, to try understand whether such a bias exists, or whether the procedure itself can be eventually justified. Until this understanding is achieved, I would not advocate taking the quoted error literally when this measurement is included in global averages.

4.5. $\alpha_S(M_Z)$ global averages

The latest pre-1999 compilation and average of results on α_S was given by S. Bethke [58]. Using measurements with $\Delta\alpha_S < 0.008$ only, he obtained: $\alpha_S(M_Z) = 0.119 \pm 0.004$. If the values from Lattice QCD, which tend to have some of the smallest absolute errors, are left out, then the average becomes: $\alpha_S(M_Z) = 0.120 \pm 0.005$.

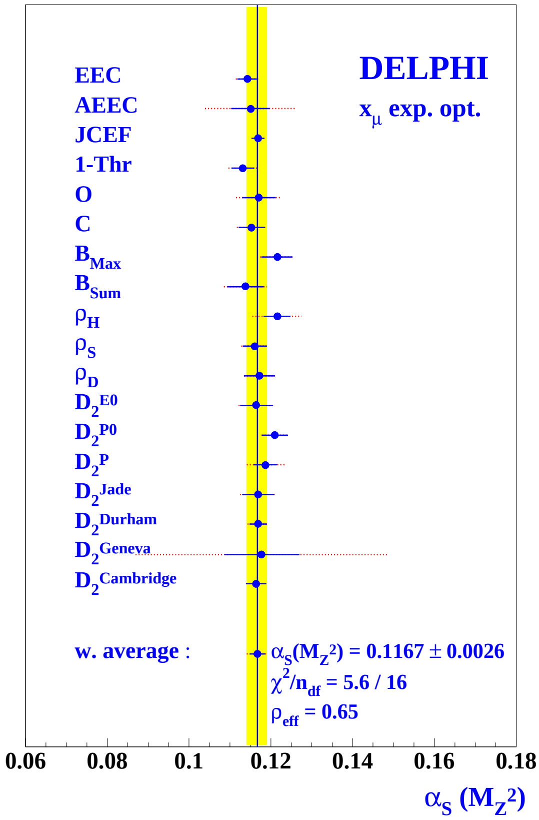

Figure 7. Values for $\alpha_S(M_Z)$ extracted by DELPHI by optimising the scale choice on a variable-by-variable basis.

I don't dare stealing from S.Bethke the pleasure to produce the new World Average for $\alpha_S(M_Z)$! Proper averaging of the new LEP results will require detailed knowledge of the correlation matrices for the various experiments, and will be done soon, I expect, by the QCD LEP Working Group. However, from the results submitted to this Conference I don't see indications that the most recent updates on the value of α_S will change significantly the central value and the determination of the error on $\alpha_S(M_Z)$. The most recent extractions of α_S from the fits to jet shapes at 189 GeV support a slightly larger value of $\alpha_S(M_Z)$ relative to the pre-EPS World average of 0.120 ± 0.005. So my best bet for the next world average is: $\alpha_S(M_Z) = 0.121 \pm 0.004$.

Progress in the analytic, phenomenological understanding of power corrections, needed to extract α_S from jet shapes, is remarkable. However, the current results should be taken in my view more as an indication that the direction of these theoretical developments is correct, rather than as a strong input for a reduction of the theoretical error on α_S.

Even more interestingly, they set the stage for future progress in the area of jet physics in hadronic collisions, where large statistics and huge lever arms in energy will lead to minuscule statistical uncertainties on α_S in the future years.

5. JET STRUCTURES IN $p\bar{p}$ and ep

Interesting new studies of the structure of jets in collisions involving hadrons in the initial state have been presented. All results confirm a good level of understanding of the theory of jet substructure, although it is still premature to employ these studies for accurate measurements of α_S.

(1_163d) DØ: Sub-jet multiplicity at $\sqrt{S} = 630$ and 1800 GeV [59]. Data at the different energies, where the relative q/g composition of jets are different for a fixed E_T, can be used to extract the average number of sub-jets contained in a quark and in a gluon jet. The result of the analysis gives:

$$\frac{\langle n_j - 1\rangle_g}{\langle n_j - 1\rangle_q} = 1.9 \pm 0.2$$

which is consistent with the LO QCD prediction of 9/4. A theoretical analyses of sub-jet multiplicities in hadronic collisions can be found in ref. [60].

(1_600) CDF: Jet fragmentation studies at the Tevatron [61]: the distribution of $\xi = log(1/x)$ is studied for jets over a wide range of energies, showing excellent agreement with MDLA both in terms of shapes, and in the evolution of the peak position with energy.

(1_530) ZEUS: Jet substructure in γp. This is a study of the sub-jet multiplicity as a function of the resolution parameter y_{cut}. The distribution of the average $\langle n_j\rangle$ is not consistent with the presence of a single type of partons (either quarks or gluons), but is well fitted by the appropriate composition of final state partons predicted by QCD, as shown in fig. 8.

(1_157x) H1: Jet substructure in DIS dijets. Figure 9 shows the fraction $\psi(r)$ of jet energy contained in a sub-cone of radius r inside the jet. As for the ZEUS measurement presented above, the data are consistent with the right composition of quark and gluon jets expected from QCD.

6. JETS AT THE TEVATRON

At the Tevatron, jets up to 450 GeV transverse momentum have been observed [62, 63]. These data can be used for many interesting purposes:

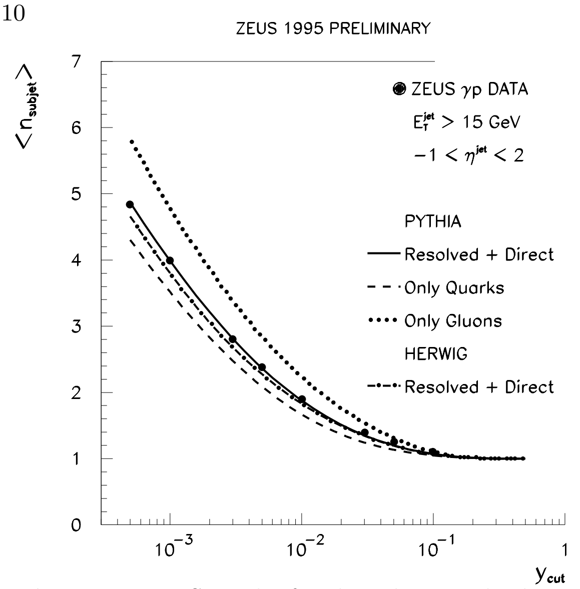

Figure 8. ZEUS results for the sub-jet multiplicity.

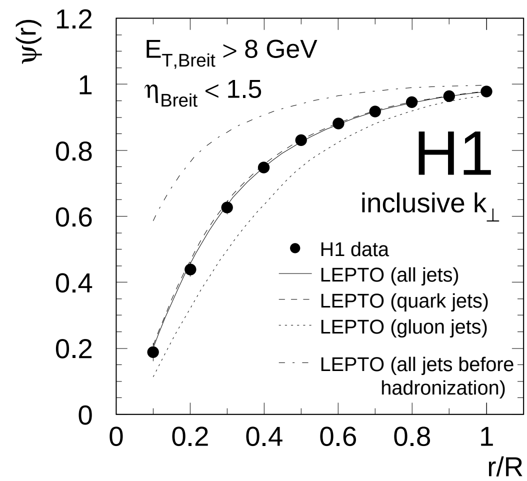

Figure 9. H1 distribution for the energy flow inside a jet.

- Tests of QCD: calculations are available up to NLO [64].
- Extract information on the partonic densities, $f_{q,g}(x, Q^2)$ at large Q^2.
- Look for deviations from QCD (e.g. resonances in the dijet mass spectrum), explore quark structure at small distances.

The results of the most recent analyses of the data from the full Run I of the Tevatron have been submitted to this Conference [65] (DØ: (1_163c); CDF: (1_593) and (1_594)).

The studied range of transverse energies corresponds to values of $x \gtrsim 0.5$, at $Q^2 \simeq 160,000$ GeV^2. This is a domain not accessible to DIS experiments. The current agreement between

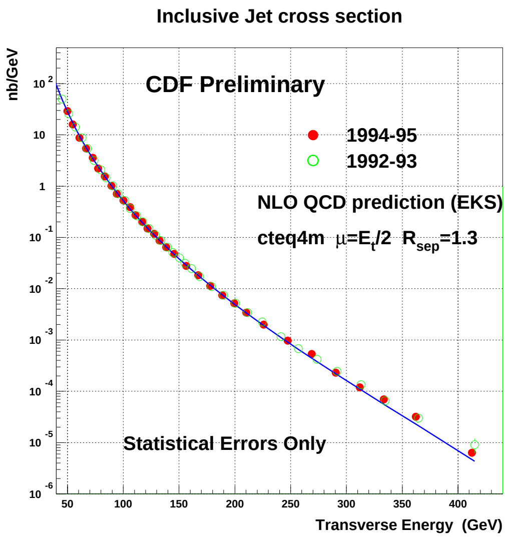

Figure 10. Inclusive jet transverse energy (E_T) distribution as measured by CDF, compared to the absolute NLO QCD calculation.

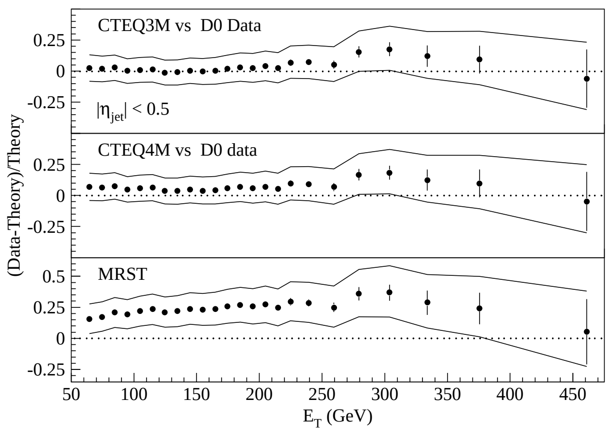

Figure 11. Deviations of QCD predictions from DØ jet data for various sets of PDFs.

theory and data is at the level of 30 % over 8 orders of magnitude of cross-section, from $E_T \sim 20$ to $E_T \sim 450$ GeV (see fig. 10) In spite of the general good agreement, a large dependence on the chosen set of parton densities [66, 67] is present, as shown in fig. 11. The presence of this uncertainty limits the use of high-E_T jet data to set constraints on possible new physics.

An important question is therefore the following: to which extent do independent measurements of parton densities constrain the knowledge of PDFs at large x, and what is the residual uncertainty on the jet E_T distributions?

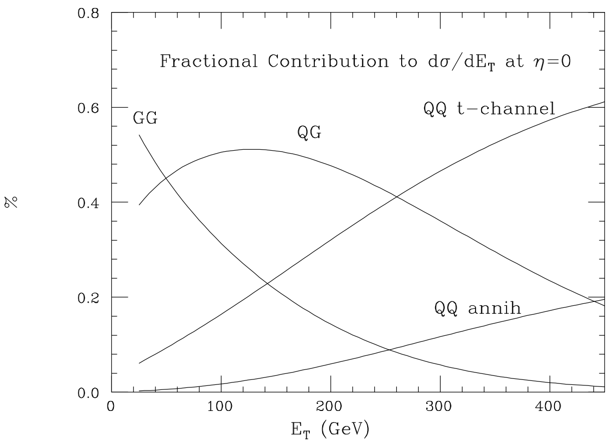

Figure 12. Contributions from different initial states to the jet cross section at $\sqrt{s} = 1.8$ TeV

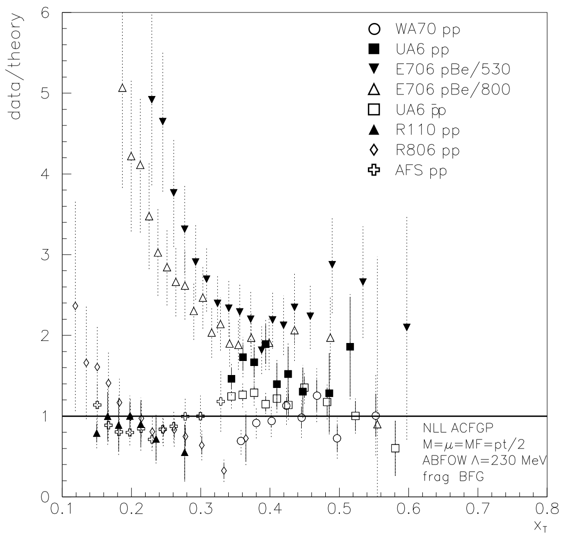

Figure 13. Relative deviations between NLO QCD and prompt photon data, as a function of $x_T = 2p_T/\sqrt{S}$, for various fixed target experiments.

To address this issue, let us first show what is the relative contribution of different initial state partons to the jet cross section. This is plotted in fig. 12, where some standard PDF set (CTEQ4M [67] in this case) was chosen. At the largest energies accessible to today's Tevatron data, 80% of the jets are produced by collisions involving only initial state quarks. The remaining 20% comes from processes where at least one gluon was present in the initial state.

Quark densities at large x are constrained by DIS data to within few percent, leading to an overall uncertainty on the high-E_T jet rate of at most 5%. What is the uncertainty on the remaining 20% coming from gluon-induced processes? How are we guaranteed that the gluons are known to better than a factor of 2, limiting the overall uncertainty to 20-30%?

The only independent constraint on $f_g(x, Q^2)$ comes from fixed-target production of prompt photons. This process is induced at LO by two mechanisms, $q\bar{q}\to g\gamma$ and $qg\to q\gamma$. In pN collisions $g(x) \gg \bar{q}(x)$, and therefore $d\sigma/dE_T(qg\to q\gamma) \gg d\sigma/dE_T(q\bar{q}\to g\gamma)$ Data from FNAL and CERN fixed target experiments can therefore be used to extract $f_g(x, Q^2)$ at large x. Unfortunately, a comparison [68] of data and NLO theory shows discrepancies at small E_T, as well as inconsistencies between the various experiments, as shown in fig. 13 (1_635). The authors of ref. [68] suggest that the large scale uncertainty in the theoretical calculations can be sufficient to explain away the differences observed at small E_T between data and theory, in particular in view of the apparent inconsistency between some of the experimental results. As a possible additional explanation for these discrepancies, the presence of a large non-perturbative contribution from the intrinsic k_T of partons inside the nucleon has been suggested [69, 70]. This could also explain the differences between the x_T distributions of the various experiments in fig. 13, since different experiments run at different energies and are subject to k_T effects in a different way. k_T effects give rise to power-like corrections to the spectrum of order k_T/p_T, with possibly very large coefficients due to to the steepness of the spectrum itself. The effect of the intrinsic k_T is to smear the p_T distribution, as shown in fig. 14. Inclusion of these effects, however, has a big impact also on the rate at large E_T (i.e. $x \sim 0.6$). Due to the large size of the effects, and to their intrinsic non-perturbative nature (they cannot be understood from first principles, and need to be described by ad hoc models), it is hard to trust the theoretical predictions obtained in this way, and to claim that prompt photons provide a reliable way of extracting the gluon content of the proton at large x. Recent theoretical improvements, such as the resummation of large-x_T logarithms [13, 16, 17, 18], should help understanding the large-x problem, but more work is necessary to achieve a satisfactory picture of the data. In conclusion, the issue of the large-x behaviour of $f_g(x)$ is still an open problem.

Concerning the possible excess observed by CDF in its highest E_T jet data [62], additional input

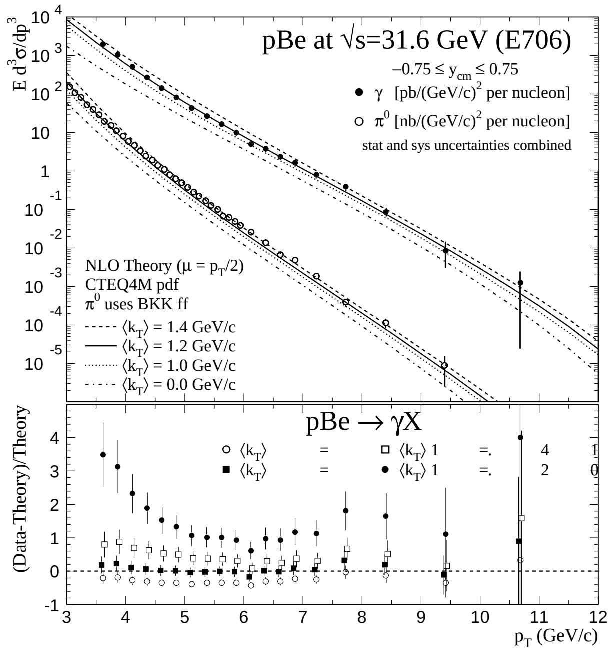

Figure 14. Comparison of E706 data [69] with NLO QCD, before and after inclusion of an intrinsic-k_T.

will be available with the data from the upcoming run of the Tevatron (due to start in the late 2000), thanks to an increased energy ($\sqrt{S}\to 2$ TeV, 10% increase). Should the excess be due to a problem with the gluon density at large x, a discrepancy similar to the one observed at 1.8 TeV will appear at jet E_T values larger by 10%. If the excess is instead due to really new phenomena, one expects the excess to appear at the same value of E_T as seen in the data at 1.8 TeV. Time will tell!

6.1. Cross-section ratios at 630/1800 GeV

It is expected that a large fraction of the theoretical and experimental systematics will cancel when taking the ratio:

$$R(x_T = \frac{2E_T}{\sqrt{S}}) = \frac{[E_T^3\, d\sigma/dE_T]_{\sqrt{S}=630}}{[E_T^3\, d\sigma/dE_T]_{\sqrt{S}=1800}} \qquad (8)$$

The measurement of $R(x_T)$ can therefore provide a useful additional tool to explore the physics of high-energy jets. In the exact scaling limit $R(x_T) = 1$. Deviations from 1 arise from scaling violations in α_S and in the parton densities. The NLO theoretical uncertainty on this ratio is better than 10%. CDF and DØ observe however serious deviations from theory at $x_T \lesssim 0.15$ ($E_T^{630} \lesssim 50$ GeV), as can be seen in fig. 15. What's more, the pattern of deviations

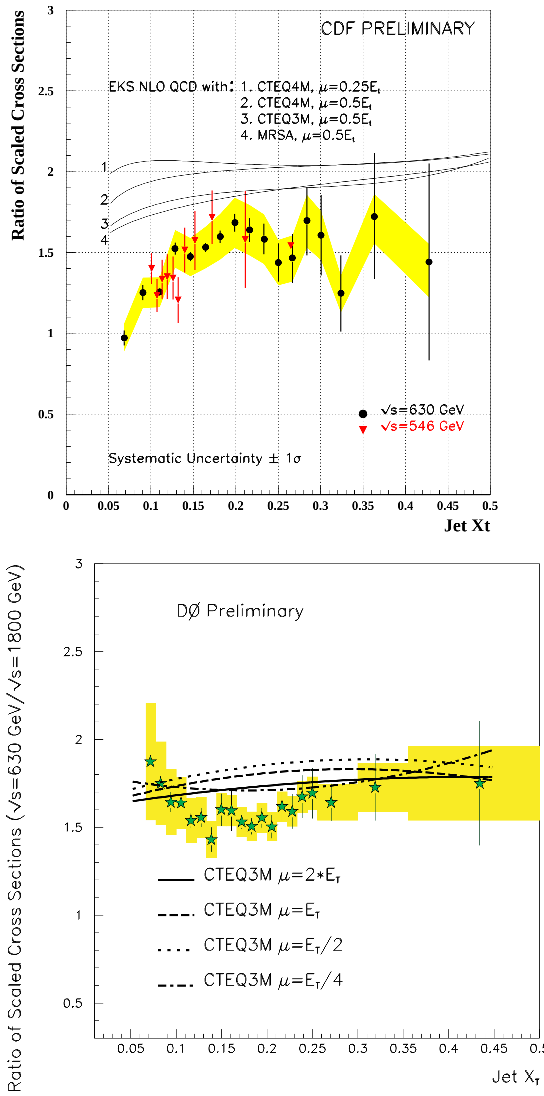

Figure 15. Cross-section ratios from CDF (upper) and DØ (lower).

is inconsistent between the two experiments. I feel that this is a clear indication of the contamination of the PT results by power-suppressed corrections, as will be shown in the next section.

6.2. A pedestrian's evaluation of x_T ratios and power corrections

To study the theoretical systematics of the x_T distributions, it is useful to consider a simplified treatment, which however contains all relevant ingredients. Let us approximate the inclusive jet rate with the value of the differential cross-section at $y = 0$ for both jets. In this case, at LO, one gets:

$$R(x_T) \ = \ \Sigma(x_T,\ 630\ \mathrm{GeV})/\Sigma(x_T,\ 1800\ \mathrm{GeV}) \quad (9)$$

with

$$\Sigma(x_T, \sqrt{S}) \ = \ \alpha_S^2(\mu)\ F^2(x_T, \mu), \quad \mu = x_T\sqrt{S}/2\ . \quad (10)$$

Here

$$F(x) = G(x) + \frac{4}{9} \sum_{q,\bar{q}} \left[Q(x) + \bar{Q}(x) \right] \quad (11)$$

is the so-called effective structure function for dijet production [71]. Power-suppressed corrections can be included via a factor:

$$\Sigma(x_T, \sqrt{S}) \rightarrow \Sigma(x_T, \sqrt{S}) \times \left(1 \ + \ \frac{A}{E_T} \right)\ , \quad (12)$$

What is the possible origin of A, and what is its right order of magnitude? Here is a partial list of sources:

- Energy lost outside the jet cone ($A < 0$)
- Energy from the underlying event inside the jet cone ($A > 0$)
- Intrinsic k_T effects ($A > 0$)

PT contributions to the energy gain/loss can be evaluated and removed. However this can be done at LO only, since they are effects of $\mathcal{O}(\alpha_S^3)$ in PT. Some energy shifts induced by non-PT effects can be extracted from the data and corrected for. This is the case, for example, of the energy deposited in the cone by the Minimum Bias component of the underlying event. Correcting for the above effects may leave us with an A of arbitrary sign, depending on whether one under- or over-corrects.

In addition, however, there is also a class of non-PT effects which are out of solid theoretical control, and which cannot be measured in a direct way. This is the case of parton recombinations with the beam fragments and with nearby jets.

The scale for all these effects is $\Lambda \sim \mathcal{O}(1\ \mathrm{GeV})$. Assuming a $1/E_T^n$ fall-off of the cross-section, one gets $A \sim n\Lambda$. Values of $A \sim 5$ GeV should therefore not be surprising. For $A \sim \pm 5$ GeV the effects are large, and can be consistent with the deviations observed by CDF and DØ, as can be seen in fig. 16. Notice that at $x_T \sim 0.05$ all scaling violations are due to the running of α_S, since this is an approximate fixed point for the evolution of the partonic luminosity $F^2(x_T)$. This is a solid result, independent of the PDF set chosen, since in this range of x_T structure functions are known with great accuracy. As a result, we don't expect that an anomaly in the 630/1800 ratio can be explained by playing with PDF's.

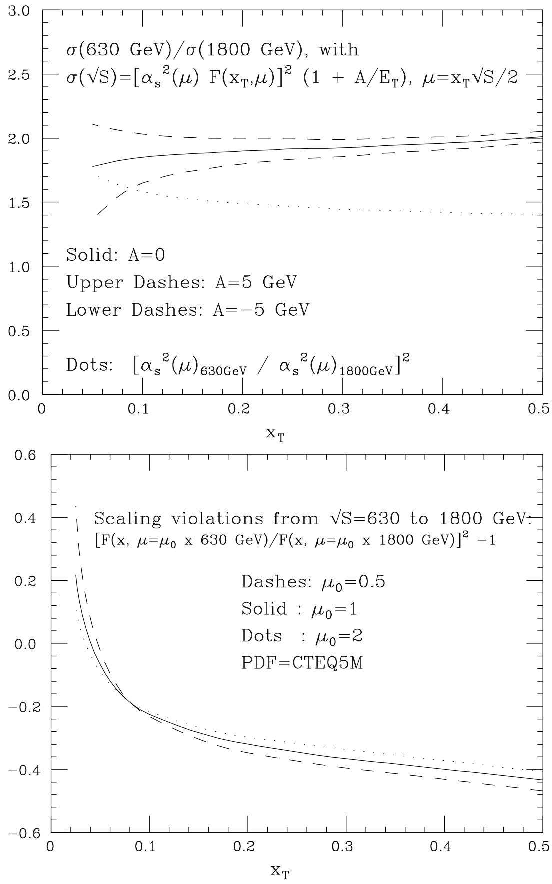

Figure 16. Upper: effect of power corrections to the scaling violations in the x_T ratios; the dotted line represents the sole effect due to the running of $\alpha_S(E_T)$. Lower: contribution to x_T scaling violations due to the parton luminosities.

As the results above show, the case is compelling for an explanation in terms of (acceptably sized) power-like corrections. For previous studies of power-suppressed effects in the jet cross-sections and ratios, see e.g. ref. [72], as well as work in progress by Huston et al. It is possible to fit the CDF data on x_T ratios using the exact NLO jet cross-section (CTEQ3M, $\mu = E_T/2$), assuming a universal and E_T-independent shift in the jet energy. The results of the fit are given in fig. 17. As can be seen, a shift of -2.8 GeV relative to the NLO parton-level jet E_T provides a good fit to the CDF data. Notice that the effect induced by such a shift is large even at large x_T. Is the size of such a shift acceptable? One can estimate the amount of energy lost through the hadronisation phase using the Herwig MC. Predictions for the quantity

$$E_{T,jet}^{\mathrm{hadron-level}} - E_{T,jet}^{\mathrm{parton-level}} \quad (13)$$

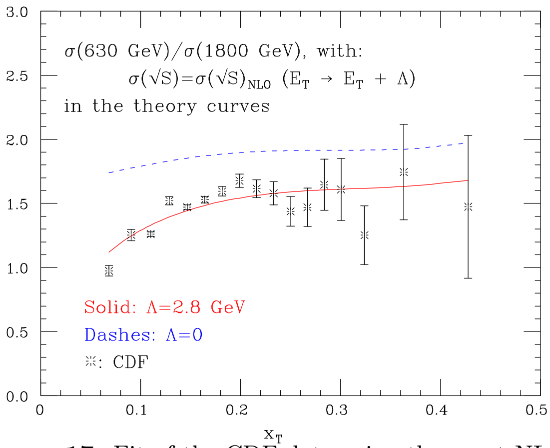

Figure 17. Fit of the CDF data using the exact NLO jet cross-section (CTEQ3M, $\mu = E_T/2$), assuming an E_T-independent shift Λ in the jet energy.

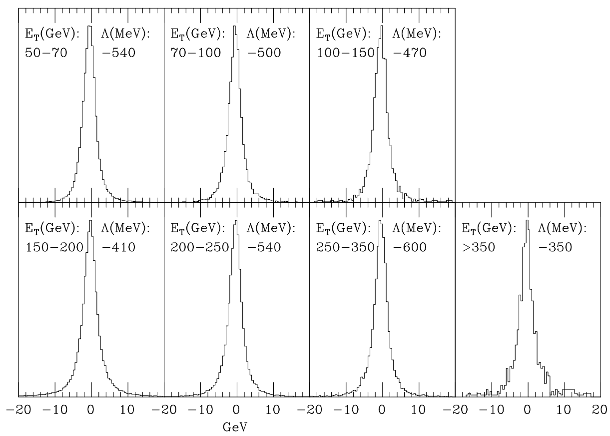

Figure 18. Hadronisation corrections to the jet energy, for different ranges of E_T. The values of Λ indicated corresponds to the average of the distributions (in MeV).

are shown in fig. 18, where I plot the distribution of the above quantity for jets in several bins of E_T. The corrections are of the order of 500 MeV, and are remarkably independent of the value of jet E_T, in the range $50 < E_T < 500$ GeV. This is by itself a non-obvious result, since in this range the jet compositions in terms quarks and gluons varies a lot. The 500 MeV are a non-negligible fraction of what is needed to explain the discrepancies between CDF/D∅ data and NLO theory, confirming the importance of such phenomena. It would be very interesting to try put on a firmer theoretical standing the analysis of power corrections to jet spectra in hadronic collisions, and in particular to explore the relation between the jet energy correction, and the correction to other possible observables. Uncovering some universality relation similar to those found for e^+e^- and DIS observables would open the way to a new set of interesting measurements in hadronic collisions. A first study of the effect of power corrections on jet-shape observables in hadronic collisions was performed by Seymour in ref. [73]

6.3. Conclusions on jet production at the Tevatron

There is no evidence in my view for departures from QCD in the inclusive jet data. Generally good agreement with QCD was also found in the studies of multi-jet final states presented by CDF ((1_595): Two-jet Differential Cross Section from CDF; (1_596): The fully corrected dijet invariant mass distribution from CDF) and by D∅ ((1_163a): The triple differential dijet cross section at D∅).

Current discrepancies (E_T spectrum at CDF, x_T ratios 630/1800 at both CDF and D∅) are within theoretical and experimental uncertainties once proper account is taken of:

- true uncertainties on the extraction of the gluon density
- power corrections
- limitations of the cone algorithm (see ref. [73] for a discussion)

In view of this, it is premature in my view to use jets for accurate measurements, such as the extraction of $\alpha_S(Q^2)$. However, better use can be made in the future of the large statistics, high E_T reach, and powerful control of the experimental systematics, if progress on the theory side can achieve:

- firmer understanding of the intrinsic k_T effects in fixed-target γ production
- NLL resummations for jet shape variables
- control (even at the phenomenological level) of the power corrections

New ideas are needed for observables which can help disentangling the various components of the theoretical uncertainties.

7. MULTI-JET PHENOMENA IN e^+e^- and *ep*

Several contributions were submitted with studies of multi-jet processes. In some cases, new NLO calculations have recently become available for these observables. Given the large powers of α_S involved, however, the scale dependence of the results is still large, and it is premature to use these data for a better measurement of α_S. In all cases, however, the agreement between data and theory is satisfactory:

(1_544) ZEUS: Three-jet distributions ($M_{3j} > 50$ GeV) in γp and LO QCD.
(1_553) ZEUS: NLO tests of high-mass dijet cross-sections in γp, $47 < M_{jj} < 140$ GeV [74].
(1_531) ZEUS: Dijet cross-sections in DIS.
(1_540) ZEUS: Dijet X-sections in γp.
(1_386) ALEPH: NLO tests for 4-jet observables in Z^0 decays. This measurement could be important to definitely rule out the existence of light gluinos, whose presence would affect these distributions[75]. The comparison of data and theory (for a recent review, and extensive references, see [76]) is shown in fig. 19 for the specific example of the T_{min} (Thrust minor) variable. No good overall fit to the data can be obtained, even at the price of changing the renormalisation scale over a wide range. No conclusion on the issue of a light gluino can however be drawn from these data either.

8. GAUGE BOSON PRODUCTION in $p\bar{p}$ and ep

Production of gauge bosons (photons, W and Z) is an extremely useful tool to probe several aspects of QCD. First of all the clear experimental signatures allow for very efficient trigger strategies, and make it possible to probe regions of phase-space were subtle QCD effects become evident. This is the case, for example, of the p_T distributions of W and Z bosons, which can be probed in hadronic collisions down to small values of p_T, in a domain where perturbative resummation and intrinsic k_T effects are dominant [77]. Large amounts of theoretical work have been done recently [78, 79]. The data (CDF (1_601) and DØ (1_71d) [80]-[82]) confirm the need of resummation of large logarithms of $M_{W,Z}/p_T$, as well as the presence of an additional intrinsic k_T, of the order of 2 GeV. we all look forward to the quality and statistics of future data from the Tevatron, which will allow stringent tests of the different theoretical approaches [79].

The presence of a small intrinsic k_T contribution is also advocated in the case of prompt photon production, similarly to the fixed-target case discussed above. These results were covered in the following contributions:
(1_599) CDF: Measurement of the Isolated Photon Cross Section [83].
(1_598) CDF: Diphoton Production [83].
(1_531) ZEUS: Prompt Photon Processes in Photoproduction [84].

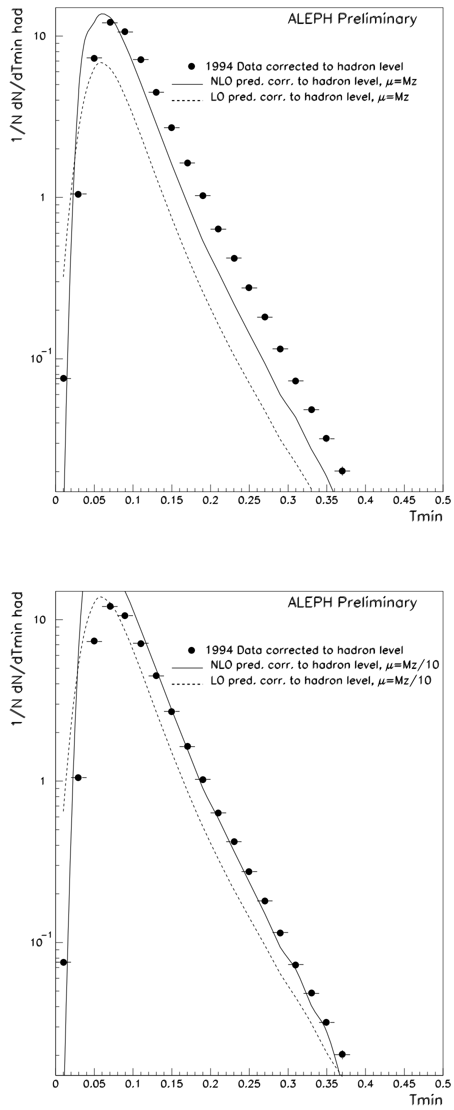

Figure 19. ALEPH study of the thrust-minor distribution in 4-jet Z^0 decays. Comparison with NLO QCD for different values of μ_R.

9. PRODUCTION OF HEAVY QUARKS

9.1. HVQ's in $\gamma\gamma$ collisions at LEP2

Beautiful results have appeared, including the first measurements of $b\bar{b}$ production by L3. Here is a brief summary:
(1_265) L3: $c\bar{c}$ and $b\bar{b}$ production in $\gamma\gamma$ at 91-189 GeV (see fig. 20)
(1_275) L3 [85]: D^* production and p_T spectra in $\gamma\gamma$ at 183-189 GeV

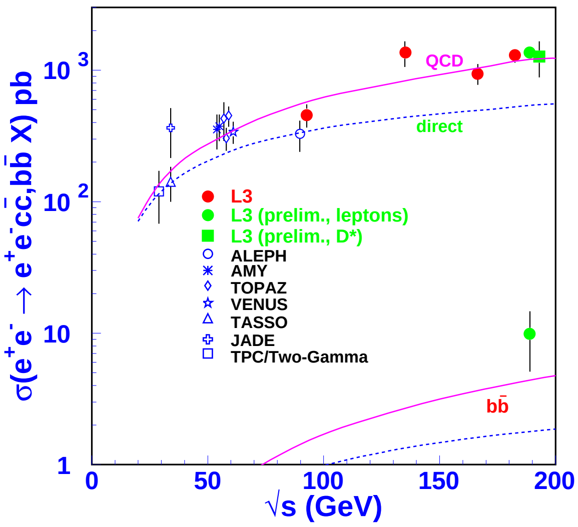

Figure 20. L3 measurements of the heavy quark production rates in $\gamma\gamma$ collisions.

(1_23) OPAL: D^* production and p_T spectra in $\gamma\gamma$ at 183-189 GeV

All papers share the same conclusion: the agreement with QCD is very good, provided the resolved component of the γ is included. The accuracy of the measurements is however not yet sufficient to uniquely disentangle the gluon density of the photon.

9.2. $g\to c\bar{c}$, $b\bar{b}$ splitting fractions in Z^0 decays

In addition to providing an interesting playground for studies of PT QCD [86, 87], the production of heavy quark pairs via gluon splitting during a jet evolution provides also an important contribution to the accurate measurement of important electroweak properties of heavy quarks, such as R_b [88]. Studies of the gluon-splitting fraction have therefore been an important subject for research by the LEP collaborations. An important thing to keep in mind however is that these fractions should not to be used as *universal* gluon-splitting probabilities, as they strongly reflect the spectrum of gluons in $Z\to$ jets. Several new results from the latest analyses of LEP1 data have been shown in Tampere:

(1_9, 1_10) OPAL [89]:

$$\langle n_{Z^0}(g\to c\bar{c})\rangle \cdot 10^2 = 3.20(21)(38)$$
$$\langle n_{Z^0}(g\to b\bar{b})\rangle \cdot 10^3 = 2.15(43)(80)$$

(1_281) L3:

$$\langle n_{Z^0}(g\to c\bar{c})\rangle \cdot 10^2 = [2.45(35)(45) - 3.74(n_{b\bar{b}} - 0.26)]$$

(1_226) DELPHI [90]:

$$\langle n_{Z^0}(g\to b\bar{b})\rangle \cdot 10^3 = 3.3 \pm 1.0 \pm 0.7$$

(1_184) SLD [91]:

$$\langle n_{Z^0}(g\to b\bar{b})\rangle \cdot 10^3 = 3.07(71)(66)$$

These numbers are to be compared with the most recent QCD determinations [87] (NLL, $\alpha_S = 0.120$):

$$n_{Z^0}(g\to Q\bar{Q})\rangle = \begin{array}{ll} (m_c = 1.2) & (m_c = 1.5) \\ 2.3\% & 1.7\% \\ (m_b = 4.5) & (m_b = 4.75) \\ 0.27\% & 0.24\% \end{array}$$

The QCD predictions are on the low side of the experimental results. One should notice however that the experimental detection efficiencies are very small, $\mathcal{O}$(few %), and therefore require a large theoretical extrapolation to extract the full rate. In ref. [87] it was pointed out that agreement between resummed QCD and shower MC's (used for the experimental analyses) is rather marginal, and it is therefore likely that the true systematics are larger than quoted.

9.3. c, b quark production at HERA

Measurements of charm photo-production at HERA have been available for some time already [92, 93]. Updates of these measurements have been presented at this Conference (ZEUS [94], (1_525) and (1_528)), including studies of $D_s^\pm$ production. The agreement between data and massive NLO QCD calculations [95, 96] is generally good, compatibly with the theoretical uncertainties due to the choice of the charm quark mass, and of the renormalisation/factorisation scales. H1 finds good agreement for all kinematical variables considered, while ZEUS (which however has a looser cut in Q^2, and therefore a less strict definition of photo-production), finds some slight excess in the forward region and for $p_T > 3$ GeV. As an example of the quality of the comparisons, the ZEUS p_T spectrum for D^* mesons is shown in Fig. 21. Theoretical estimates including the resummation of large-p_T logarithms, done in a framework where the charm is treated as a massless parton, up to mass thresholds built into the evolution of structure and fragmentation functions, have recently been performed [97, 98]. Attempts to describe the

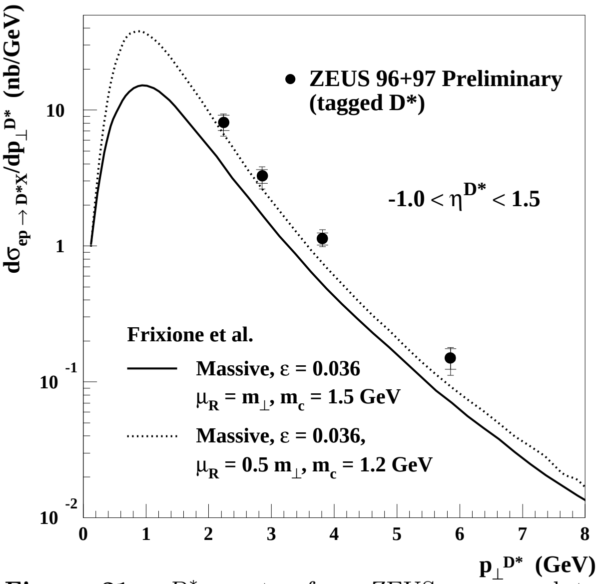

Figure 21. D^* spectra from ZEUS, compared to NLO QCD calculations for different values of the input parameters. ϵ corresponds to the parameter of the Peterson fragmentation function.

current data using this approach, however, would not seem justified in my view in the range of p_T accessible to the experiments, as corrections of $\mathcal{O}(m^2/(p_T^2+m^2))$ are not negligible. By definition, if the massive and the massless calculation were to differ in the region $p_T <$ few$\times m_c$, one should trust the massive result. It will be therefore interesting to see the results of calculations where the massive cross-sections are matched at high p_T to the resummed expressions, similarly to what was done in refs. [99, 100] in the case of hadro-production. For a review of the theoretical status, see e.g. ref. [101].

Studies have also been presented on bottom photoproduction at HERA [103]. The results here are more puzzling than in the case of charm. Both H1 [102] (5_157v) and ZEUS (1_498) tag b events using semileptonic decays. Charm and fake-lepton backgrounds are separated using the transverse momentum distribution of the lepton relative to the jet direction. The same selection and cuts applied to the data are used on samples of MC events generated using LO matrix elements for the $b\bar{b}$ photoproduction. The comparison of observed and expected rates is reported here:

(5_157v) H1 single lepton (LO=Aroma MC):

$$\sigma^{vis}_{b\bar{b}}(\mathrm{nb}) = \begin{array}{ll} 0.93 \pm 0.08_{stat}\,{}^{+0.21}_{-0.12}{}_{syst} & \text{H1 Data} \\ 0.19 & \text{LO} \end{array}$$

(5_157v) H1 dimuons (LO=Aroma MC):

$$\sigma^{vis}_{b\bar{b}}(\mathrm{pb}) = \begin{array}{ll} 55 \pm 30_{stat} \pm 7_{syst} & \text{H1 Data} \\ 17 & \text{LO} \end{array}$$

(1_498) ZEUS single lepton (LO: Herwig MC):

$$\sigma^{vis}_{b\bar{b}}(\mathrm{pb}) = \begin{array}{ll} 39 \pm 11_{stat}\,{}^{+0.23}_{-16}{}_{syst} & \text{ZEUS Data} \\ 10 & \text{LO} \end{array}$$

The results indicate an excess of data relative to the LO QCD expectations by a factor of approximately 4. In my view it is very hard to accept this result, given the excellent agreement observed in the case of charm production. Bottom production at HERA is expected to be more reliably estimated than charm production [95], due to the larger bottom mass. It will be interesting to see how these studies evolve once more statistics will have become available. More accurate studies of the bottom quark distributions will be necessary to validate the MC tools used by H1 and ZEUS to estimate the detection acceptances and efficiencies.

9.4. Bottom quark production at the Tevatron

The prediction of bottom cross-sections in hadronic collisions is a sore point for perturbative QCD. NLO calculations have been available for several years now for the total cross sections [104], for single-inclusive distributions [105] and for correlations [106]. As pointed out in the original papers [104], the inclusion of NLO corrections increases the rates by factors of order 2, and leaves a large scale dependence (of order 2, and more if renormalisation and factorisation scales are varied independently). As a result, any comparison with data (for a recent complete review, see ref. [107]) will at best be qualitative, and certainly will not provide a compelling test of the theory. The current comparison with NLO QCD of single-inclusive rates, as measured by CDF (1_37) and D∅, is summarised in Fig. 22 (differential B-meson p_T spectra from CDF and integrated b-quark p_T spectra from D∅ [108]).

Within the theoretical uncertainties, the agreement with data is acceptable. The comparison indicates that smaller values of the renormalisation and factorisation scales are favoured. Indeed, if one were to push the scale down to values of the order of $\sqrt{m_b^2+p_T^2}/4$, the theory curve would exactly overlap the data. In spite of the large uncertainty in the prediction of the absolute rates, the NLO predictions for the shapes of the $b\bar{b}$ correlations are better defined. Evidence was given in the past [107], and confirmed recently in [108], that NLO QCD provides a good description of the shape

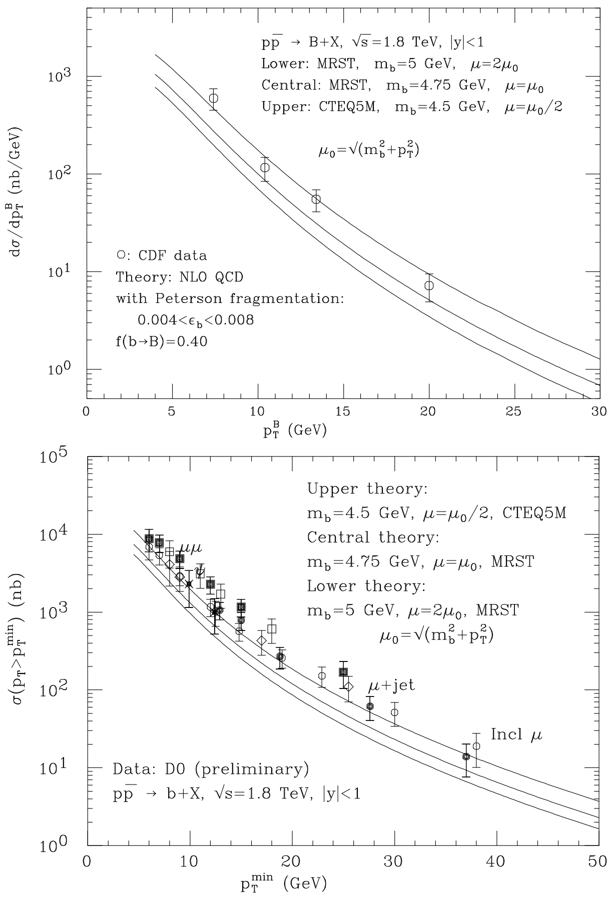

Figure 22. Left: differential B-meson p_T spectra from CDF, (1_37)). Right: integrated b-quark p_T spectra from DØ [108].

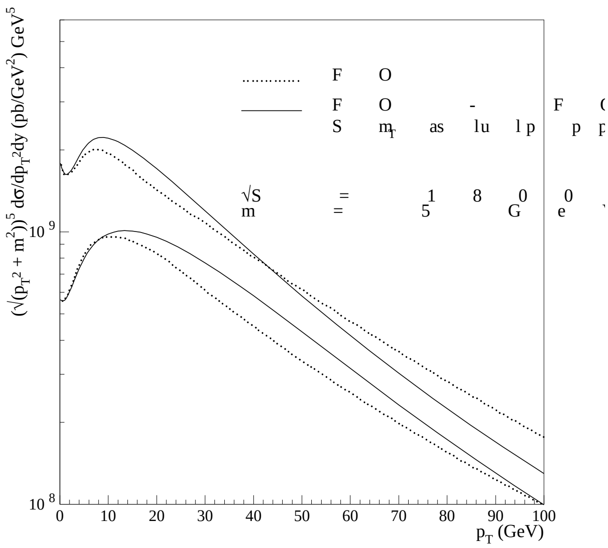

Figure 23. Improvement in the scale dependence of the bottom quark p_T spectrum after inclusion of large-p_T resummation contributions (Cacciari et al., [100]. The bands correspond to changes of μ in the range $m_T/2 < \mu < 2m_T$, with $m_T^2 = m^2 + p_T^2$ (dotted lines: NLO; solid lines: NLO+resummed).

of azimuthal $b\bar{b}$ correlations. It was shown by CDF in this conference, (1_123), that the theory provides also a good description of the $b\bar{b}$ rapidity correlations [109]. All of these observations make therefore rather intriguing the anomaly observed by DØ in the inclusive forward production of b quarks [110]. In this paper, DØ reports a factor of 2 excess in the production of forward b's, relative to what expected by extrapolating the rate measured in the central rapidity region. Possible mechanisms have been proposed to increase the expected rates for forward production of B mesons (e.g. a harder non-perturbative fragmentation function [111]), but none of them can explain the large effect observed by DØ.

Progress in theory took place in the recent past. This includes studies of resummation of the large logarithms of p_T/m_b which appear at any order of PT [99, 100]. The accuracy of these calculations is now full $\mathcal{O}(\alpha_S^3)$ plus the resummation of NL logarithms arising in the fragmentation function of the heavy quark [112]. The results (see fig. 23) show an improved scale dependence at large p_T, as expected, and an increase in rate in the region of $10 \gtrsim p_T \lesssim 40$ GeV, where most of the data from CDF and DØ are sitting. Unfortunately the resummation of this class of logarithms does not provide reliable information in the region of $p_T \lesssim 20$ GeV, since in this region mass corrections have been shown to be large [100]. The large K-factor shown in fig. 23 at small p_T cannot be used, therefore, to improve the agreement between data and theory.

On the front of the experimental inputs, I should recall here the recent accurate experimental studies of B-meson spectra at LEP and SLD [114] (1_182). These will provide the necessary ingredients for accurate extractions of the non-perturbative component of the fragmentation functions, as discussed recently for example in ref. [113].

9.5. Top quark production at the Tevatron

Theoretical predictions for $t\bar{t}$ production at the Tevatron are expected to be rather robust, given the large value of the top mass and the correspondingly small value of the coupling, $\alpha_S(m_{top})$, appearing in the QCD perturbative expansion. The next-to-leading-log (NLL) resummation of Sudakov

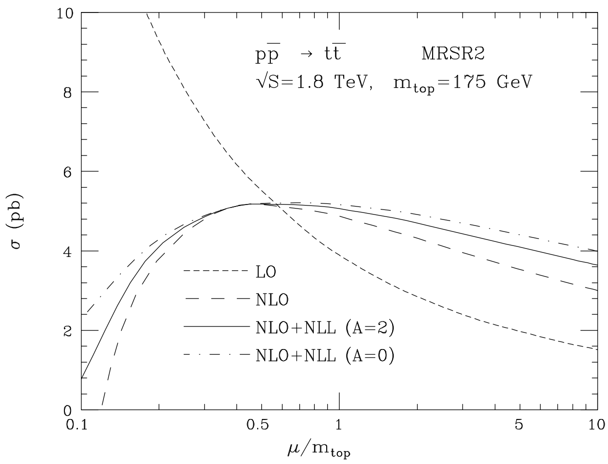

Figure 24. Scale dependence of $\sigma_{t\bar{t}}$ at the Tevatron (1.8 TeV), for various degrees of accuracy in the QCD calculation.

threshold effects has been carried out in the past year [13, 14] Results indicate a good reduction in scale uncertainty, to the level of $\pm5\%$, as shown in Fig. 24 [14]

In addition to the scale-variation uncertainty, a $\pm7\%$ variation in the theoretical predictions is present due to the choice of PDF's (σ's in pb):

PDF	$\mu = m_t/2$	$\mu = m_t$	$\mu = 2m_t$
MRST	5.04	4.92	4.57
MRST$g\uparrow$	5.22	5.09	4.72
MRST$g\downarrow$	4.90	4.79	4.45
MRST$\alpha_S\downarrow$	4.84	4.74	4.42
MRST$\alpha_S\uparrow$	5.20	5.07	4.68
CTEQ5M	5.41	5.30	4.91
CTEQ5HJ	5.61	5.50	5.10

(MRST: [66]; CTEQ5: [67]; NLO+NLL results from [14], using the prescription for the inverse Mellin transform introduced in [115])

At this Conference a new determination of the $t\bar{t}$ cross-section measured by CDF was presented by Ptohos (5_455). The new value is approximately 1 standard deviation lower than the previous one, and in much better agreement with the QCD predictions. The overall cross-section averages from CDF and DØ (in this last case rescaled to m_{top} = 175 GeV) are shown in Table 9.5, and compared to various theoretical results appeared in the literature. Now that the CDF number has come down a bit, the average of the experimental determinations (5.9 ± 1.3 pb at 175 GeV) is within less than one standard deviation from the QCD NLO+NLL resummed result of (5.0 ± 0.6 pb) ([14], with scale and PDF uncertainties added linearly). It is interesting to notice that both CDF and DØ [116] report significantly lower values for $\sigma(t\bar{t})$ in the single-lepton plus jets channels than in the all-jet or dilepton ones. These lower values are in closer agreement with QCD than the overall average. It is clearly premature to draw any conclusion on this small discrepancy between the determinations obtained using the various channels. Several studies of kinematical properties of top final states have been presented by CDF [119] and DØ. All results are in good agreement with the predictions from NLO QCD [120].

Table 1. $t\bar{t}$ cross-sections, in pb, for m_{top} = 175 GeV. Upper rows: CDF: (5_455); DØ: [116]. Lower rows: BCMN: [115]; BC: [117]; K: [118]

CDF	DØ	
6.5 ± 1.5	5.4 ± 1.5	
BCMN	BC	K
5.0 ± 0.6	$5.57^{+0.07}_{-0.42}$	7

10. CONCLUSIONS

After over 25 years since its discovery, QCD is still a very rich an exciting field, with progress both in the experimental techniques and in the theoretical understanding. Measurements are becoming more and more sophisticated, and the challenge for theorists is becoming harder and harder. The accuracy in the extraction of α_S is reaching its limits. New theoretical developments will be necessary to take full advantage of the future ep and $p\bar{p}$ (as well as LHC) data.

A consistent phenomenological picture of the impact of the hadronisation phase on the structure of final states is emerging. Tests in e^+e^- collisions are becoming very compelling, and the universality of the description of power corrections has been tested even in ep collisions, to a level of accuracy which is consistent with the current expectations. At this time, however, the uncertainties in the determination of α_S performed using the analytic description of hadronisation effects are not smaller than those found using the Monte Carlo modelling. The largest source of theoretical uncertainties in the extraction of α_S still remains the renormalisation-scale dependence of the results. New approaches, such as the experimentally optimisation of the scale pursued by DELPHI, will have to await more solid theoretical justification before their potential can be fully exploited.

Application of these ideas to hadronic collisions will require more work. The new frontier is the evaluation of NNLO cross-sections and NLL resummations, and applications to the study of jet shapes. A recent calculation of 3-jet production at $\mathcal{O}(\alpha_S^4)$ [121] could be used for a first evaluation of jet shapes at NLO. Attention should go to the use of appropriate jet algorithms, and to the identification of appropriate observables. Extraction of the gluon density of the proton from photon and jet data has also reached the limit of theoretical accuracy. Progress on the above points will be necessary before further improvements can be achieved.

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