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THE LONGITUDINAL HIGH–FREQUENCY IMPEDANCE OF A PERIODIC ACCELERATING STRUCTURE*

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Abstract

In many future collider and FEL designs intense, short bunches are accelerated in a linear accelerator. For example, in parts of the Linac Coherent Light Source (LCLS) a bunch with a peak current of 3.4 kA and an rms length of 30 microns will be accelerated in the SLAC linac. In such machines, in order to predict the beam quality at the end of acceleration it is essential to know the short range wakefields or, equivalently, the high frequency impedance of the accelerating structure. R. Gluckstern[1] has derived the longitudinal, high-frequency impedance of a periodic structure, a solution which is valid for a structure with a small gap-to-period ratio. We use his approach to derive a more general result, one that is not limited to small gaps. In addition, we compare our results with numerical results obtained using a field matching computer program.

1 INTRODUCTION

Let us consider the infinitely periodic, cylindrically symmetric structure depicted in Fig. 1. R. Gluckstern has derived the high frequency behavior of the impedance of such a structure, to order $(kg)^{-1/2}$ relative to the leading term, with k the wave number and g the gap, as[1]

$$Z_L(k) = \frac{iZ_0}{\pi k a^2}\left[1 + (1+i)\frac{\alpha L}{a}\sqrt{\frac{\pi}{kg}}\right]^{-1} \quad , \qquad (1)$$

with $\alpha = 1$. Note that Z_L is the average impedance per unit length (averaged locally over frequency to give a smooth function, and averaged over a distance in the structure large compared to the period L), $Z_0 = 120\pi\Omega$, and a is the iris radius. Gluckstern's result was meant to be valid for $g/L \ll 1$. In this report, following Gluckstern's method, we will show that Eq. 1 is still valid in the general case g/L not small, but with α a function of g/L.

Other authors have investigated the high frequency behavior of the same structure. Their results agree in the leading order term ($iZ_0/\pi k a^2$), but not in the constant α in the higher order term. E. Keil, describing the so-called Sessler–Vaynstein optical resonator model of high frequency impedance, obtains a constant $\alpha = 0.67$[2] and S. Heifets and S. Kheifets give $\alpha = 8/\pi \approx 2.55$[3]. G. Stupakov, considering the limiting case of a structure with infinitesimally thin irises (*i.e.* $g/L = 1$), finds that, in this case, $\alpha \approx 0.46$[4].

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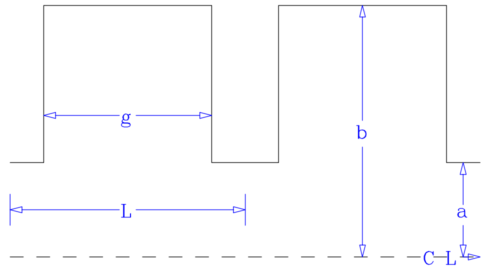

Figure 1: Two cells of the geometry under consideration.

2 ANALYTICAL STUDY

Let us begin by briefly summarizing Gluckstern's method: He divides the geometry of Fig. 1 into two regions, the pipe region ($r \leq a$) and the cavity region $r \geq a$. The fields are expanded in terms of Bessel functions in the pipe region and in term of the cavity eigenfunctions in the cavity region with the (perfectly conducting) metallic boundary condition on the iris surface $r = a$. He obtains a relation (Eq. 2.14 in the first of Ref. [1]) between the azimuthal magnetic field and the axial electric field along $r = a$. Then, by matching the fields in the pipe and cavity regions along $r = a$, he obtains an integral equation for the axial electric field along $r = a$ (normalized by $Z_0 I_0/ka^2 e^{-ikz}$, with I_0 the arbitrary driving current), $F(z)$ (we follow his notation):

$$\int_0^g dz' \left[\widehat{K}_c(z,z') + \sum_{m=-\infty}^{\infty} \widehat{K}_p(mL+z'-z)\right] F(z') = -i. \qquad (2)$$

The kernels in this equation are those of the cavity region $\widehat{K}_c$ and of the pipe region $\widehat{K}_p$, with the $\widehat{K}_c$ term and the $m = 0$ term involving $\widehat{K}_p$ giving the contribution to the impedance of the cell that includes the point z, and the $m \neq 0$ terms giving the contribution of the other cells. Once we know $F(z)$ the impedance is simply given by

$$Z_L(k) = \frac{Z_0}{ka^2 L}\int_0^g dz F(z) \quad . \qquad (3)$$

For our purposes we do not need to know the details of the kernels $\widehat{K}_c$ and $\widehat{K}_p$, but only their high frequency behavior. Gluckstern found that the high frequency behavior of the cavity kernel $\widehat{K}_c$ is independent of the details of the cavity shape, and is given by

$$\widehat{K}_c(z,z') = -\frac{(1+i)\sqrt{\pi}}{a\sqrt{k(z-z')}}\Theta(z-z') \quad , \qquad (4)$$

where $\Theta(z)$ is the step function (0 for $z < 0$ and 1 for $z \geq 0$). The kernel in the pipe region $\widehat{K}_p$ is given by

$$\begin{aligned} \widehat{K}_p(z) &= -\frac{2\pi i}{a} e^{ikz} \sum_{s=1}^{\infty} \frac{1}{b_s} e^{ib_s|z|/a} \quad , \\ b_s &= (k^2a^2 - j_s^2)^{1/2} \quad , \qquad [\Im\mathrm{m}(b_s) \geq 0] \, , \end{aligned} \tag{5}$$

with j_s the s-th zero of the Bessel function J_0. Gluckstern shows that, at high frequencies, the $m = 0$ pipe kernel, $\widehat{K}_p(z' - z)$, is equal to the cavity kernel $\widehat{K}_c(z, z')$, as given in Eq. 4. As for the $m \neq 0$ terms in $\widehat{K}_p$, Gluckstern points out, that those with $m > 0$ oscillate rapidly at high k, and therefore do not contribute to the average impedance. The sum over negative m gives

$$\begin{aligned} K_s(v) &\equiv \sum_{m=-\infty}^{-1} \widehat{K}_p(mK + z' - z) \\ &= \frac{2\pi i}{a} \sum_{s=1}^{\infty} \frac{1}{b_s} \frac{e^{i(\theta-\phi_s)v}}{1 - e^{i(\theta-\phi_s)v}} \quad , \end{aligned} \tag{6}$$

with $v = (z' - z)/L$, $\theta = kL$, $\phi_s = b_s L/a$. Noting that up to order $k^{-1/2}$, $(\theta - \phi_s)$ can be replaced by $j_s^2 L/2ka^2$, this becomes

$$\begin{aligned} K_s(v) = -\frac{4\pi}{L} \sum_{s=1}^{\infty} \Bigg[\frac{1}{j_s^2} - \\ \frac{iL}{2ka^2} \left(\frac{e^{ij_s^2 vL/2ka^2}}{1 - e^{ij_s^2 L/2ka^2}} + \frac{2ka^2}{ij_s^2 L} \right) \Bigg] \, . \end{aligned} \tag{7}$$

The first term in square brackets can be summed: $\sum 1/j_s^2 = 1/4$; the second term can be approximated by an integral: $\sum_{s=1}^{\infty} f(j_s) \approx (1/\pi) \int_0^{\infty} f(x)dx$, for $k \to \infty$. Thus, we obtain

$$K_s(v) = -\frac{\pi}{L} \left[1 + \sqrt{\frac{2L}{\pi k a^2}} e^{\pi i/4} G_0(v) \right] \quad , \tag{8}$$

with

$$\begin{aligned} G_0(v) &= \frac{2}{\sqrt{\pi}} \int_0^{\infty} dx \left[\frac{e^{x^2 v}}{e^{x^2} - 1} - \frac{1}{x^2} \right] \\ &= \sum_{n=1}^{\infty} \left[\frac{1}{\sqrt{n - v}} - \frac{1}{\sqrt{n}} \right] + \zeta(\tfrac{1}{2}) \quad , \end{aligned} \tag{9}$$

where ζ is the Riemann zeta function ($\zeta(1/2) = G_0(0) = -1.460$). Note that it is the second term in the brackets of Eq. 7 that Gluckstern has let go to zero, which will account for the difference in his final result and ours.

The original integral equation, Eq. 2, thus becomes

$$\begin{aligned} \frac{(1-i)}{\sqrt{\pi k L}} \int_0^{g/L} dv' \bar{G}(v' - v) F(v') \\ - \frac{ia}{L} \int_0^{g/L} dv' F(v') = \frac{a}{\pi L} \, , \end{aligned} \tag{10}$$

with

$$\bar{G}(v) = \frac{2\Theta(-v)}{\sqrt{-v}} + G_0(v) \quad . \tag{11}$$

This equation can be rewritten in the form

$$\int_0^{g/L} dv' \bar{G}(v' - v) F_0(v') = 1 \quad , \tag{12}$$

with

$$F_0(v) = F(v)/\mathcal{A} \quad , \tag{13}$$

$$\mathcal{A} = \frac{\sqrt{\pi k L}}{(1-i)} \left[\frac{a}{\pi L} + \frac{ia}{L} \int_0^{g/L} dv \, F(v) \right] \quad . \tag{14}$$

Note that Eq. 12, with $F_0(v)$ the unknown, contains only one parameter $\gamma \equiv g/L$. The equation for the impedance, Eq. 3, becomes

$$Z_L(k) = \frac{Z_0}{ka^2} \mathcal{A} \int_0^{\gamma} dv \, F_0(v) \quad . \tag{15}$$

Finally, the solution to this equation is an impedance of the form Eq. 1, with

$$\alpha(\gamma) = \frac{\sqrt{\gamma}}{\pi} \left[\int_0^{\gamma} dv \, F_0(v) \right]^{-1} \quad , \tag{16}$$

a result that can be verified by substitution.

We find that for small γ, $\alpha = 1 + G_0(0)/\pi\sqrt{\gamma} + O(\gamma^{3/2})$, and for $\gamma = 1$, $\alpha \equiv \alpha_1 = -G_0(0)/\pi \approx 0.4648$. The numerical solution of Eq. 12 gives α in general (see Fig. 2). A polynomial fit in $\sqrt{\gamma}$

$$\alpha(\gamma) = 1 - \alpha_1\sqrt{\gamma} - (1 - 2\alpha_1)\gamma \quad , \tag{17}$$

given by the dashes in Fig. 2, agrees to within 1.5% with the numerical result. Note that $\alpha(0)$ is in agreement with the result of Gluckstern, and $\alpha(1)$ with that of Stupakov.

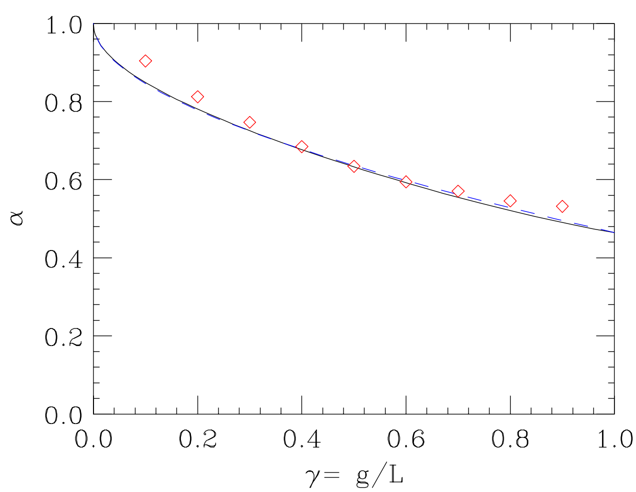

Figure 2: The coefficient $\alpha(\gamma)$. The dashed curve gives the analytical fit, Eq. 17. The plotting symbols are numerical results discussed in the next section.

3 NUMERICAL COMPARISON

To confirm these results numerically we have used a field matching computer program working in the frequency domain[5] (see also Ref. [6]). For the geometry of Fig. 1 and for a given k, this program matches the tangential fields at $r = a$, and then performs normalizing integrals. The result is an infinite dimensional matrix equation that is truncated and inverted to obtain $Z_L(k)$. In order to better study the asymptotic behavior, the program calculates the impedance along a path slightly shifted off the real k axis, which has the effect of averaging and smoothing out the many narrow impedance spikes otherwise found at high frequencies. (Note that to obtain the short–range wakefield from this impedance, after performing the inverse Fourier transform, the result must also be multiplied by the factor $\exp[\mathrm{Im}(k)s]$, with s the distance between driving and test particles.) As example geometry we consider that of a typical cell of the NLC accelerating structure known as the damped, detuned structure (DDS)[7]. One simplification in our model, however, is that the irises are not rounded, unlike those in the real structure. The dimensions are $a = 4.924$ mm, $g = 6.89$ mm, and $L = 8.75$ mm (note that for the average, high–frequency impedance neither the cavity radius b, nor the coupling manifolds that couple through slots at $r = b$ in the DDS, play a role).

The numerical results, giving the real (R_L) and imaginary (X_L) parts of the impedance Z_L, when $\mathrm{Im}(k) = 0.5$ mm^{-1}, are given in Fig. 1 (the solid curves). We note that this impedance is indeed a relatively smooth function of $\mathrm{Re}(k)$ (on the real axis, R_L would be a collection of many infinitesimally–narrow spikes). We should point out, however, that with this method, to get good convergence in the solution at high frequencies, the size of the matrix that needs to be solved becomes very large: at $k_R = 200\,\mathrm{mm}^{-1}$ its size is $\sim 600 \times 600$. In Fig. 3 the dashed curves give the analytical result, Eq. 1 with $\alpha = 0.52$. We see that agreement is good for frequencies $ka \gtrsim 1.5$.

The impedance was calculated for several values of g/L with the field matching program, keeping the other dimensions fixed. The results, for fixed $\mathrm{Im}(k)$, become bumpier as g/L decreases, due to the shorter reflection time $2g/c$ of a wave between irises; and for the calculations we let $\mathrm{Im}(k) \sim 2/g$. Then to obtain α, the numerically obtained $\mathrm{Re}(1/Z_L)$ was fit to the form $f(k) + \alpha g(k)$ (with f and g known functions taken from Eq. 1). The results of the fit are given in Fig. 2 (the plotting symbols). The agreement is relatively good, though, due to residual bumpiness in the impedance curve, the accuracy of the fit to the numerical result is limited to $\sim 10\%$.

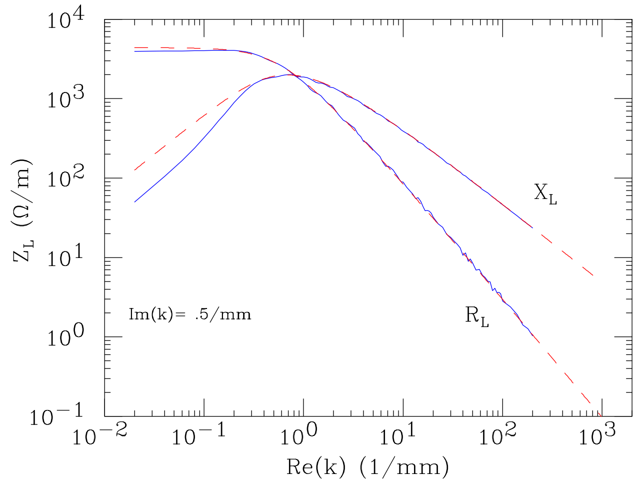

Figure 3: The real (R_L) and imaginary (X_L) parts of the impedance for the dimensions of the average NLC cell, as obtained by field matching (solid lines). The dashes represent Eq. 1 with $\alpha = 0.52$.

4 CONCLUSIONS

We have extended R. Gluckstern's analytical result for the high–frequency, longitudinal impedance of a periodic accelerating structure, a result valid for small gaps, to one valid for all gap to period ratios. We have, in addition, performed numerical calculations and obtained results that confirm the analytical result to an accuracy of $\sim 10\%$.

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