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Some Thermodynamical Aspects of String Theory[1]

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1. Introduction

The possible phases of gauge theory have been uncovered and studied even in the absence of an exact solution of such theories. Their low energy physics can be classified according to the charges of the dyons which condense (or not). In particular the Standard Model utilizes three of the possible phases of gauge systems. The weak, colored and electromagnetic interactions correspond to the condensation of electric, magnetic and no condensation respectively. A similar structure is yet to be uncovered in detail in theories which contain gravity. Several pieces of information correlating the phase structure of the worldsheet theory with that of the target space theory are known. Theories which are in the topological phase on

[1]From contributions by Eliezer Rabinovici to "The Many Faces of the Superworld": Yuri Golfand Memorial Volume, World Scientific (Singapore) 1999, and the EnglertFest, Université Libre de Bruxelles, 24-27 march 1999. Based on a series of works with J. Barbon, I. Kogan and S. Abel [1, 2, 3].

the worldsheet lead to target space theories which are also topological [4, 5]. Theories which describe perturbatively strings moving in a background of the form $(AdS)_{p+2} \times \mathcal{N}_{8-p}$ (AdS stands for an Anti-de Sitter spacetime and $\mathcal{N}$ for some appropriate compact manifold) turn out to be well described by a $p+1$ dimensional target space theory [5] which is just a field theory living on the boundary of the manifold [6]. There are quite a few examples of this behavior. One is also familiar with the fact that perturbatively a string moving on a background of the form $R_{3,1} \times \mathcal{C}_6$ ($R_{3,1}$ is for example four dimensional Minkowski space and $\mathcal{C}_6$ is an appropriate compact manifold) does not seem to be described by a regular field theory in target space but rather by a theory with string like excitations which possesses a very high degree of symmetry. There are indications that a system intermediate in some sense between field theory and string theory may also exist [7]. These are several possible phases of string theory. Here I will focus on some questions raised by the field theoretical description of string theory on $(AdS)_{p+2} \times \mathcal{N}_{8-p}$.

2. Should 4=10 be read in English or in Hebrew?

Another title for this section could be 'Extensivity vs. Holography'. It had been suggested that in theories of gravity residing in D spacetime dimensions the number of degrees of freedom (somehow suitably defined) should reflect a $D-1$ structure [8]. In string theory examples the reduction of degrees of freedom may seem even more drastic, for example the propagation of a string on $AdS_5 \times \mathcal{N}_5$ is described by an $N=4$ SUSY $SU(N)$ Yang-Mills theory living in 4 spacetime dimensions. This relation can be approximately described using a very smooth appropriate supergravity background when:

$$N \gg (g_{YM}^2 N)^{1/4} \gg 1. \tag{2.1}$$

N is the number of colors and g_{YM} is the Yang-Mills gauge coupling.

We first reflect on the equation $4 = 10$ in Hebrew, that is we study if it is possible and if it is true that what seems like a ten dimensional theory actually exhibits four dimensional behavior. For this purpose the effective spacetime dimensionality of the system is associated with the temperature dependence of the entropy of the system at temperatures smaller than string scale temperatures and larger than Kaluza-Klein temperatures. For a temperature obeying:

$$m_{KK} \ll T \ll m_s, \qquad\qquad VT^{D-1} \gg 1 \tag{2.2}$$

where V is the spatial volume of the system, the entropy is expected to behave as:

$$S \sim VT^{D-1} \tag{2.3}$$

from which one can read off the effective dimension D of the system. We will discuss two successive terms in the perturbation theory around the supergravity background. To leading order in $1/N^2$ common wisdom expects that the full quantum calculation of the entropy of the strongly coupled gauge theory would be of the form

$$S = N^2 \, f(g_{YM}^2 \, N) \, VT^3 \tag{2.4}$$

where f is an appropriate function of the Yang-Mills coupling. On the supergravity side there are at least two classical backgrounds whose bulk geometry has the same behavior on the four dimensional boundary on which the Yang-Mills theory lives. One could imagine that one needs to sum over all such bulk geometries which have the same boundary [9, 10]. It will turn out that the failure to do so will not be consistent with the duality conjecture. For the case at hand [9, 10], one such background which exists for all values of the temperature is that of the AdS_5 at finite temperature. The other background is that of a Schwarzschild-AdS black hole. Both have a $S^3 \times S^1$ as a boundary. The temperature above which the black hole is formed is proportional to the curvature energy scale $1/b$, of AdS space. The thermodynamical analysis on the supergravity side indicates that the black hole configuration starts to dominate for temperatures not much higher than that above which it may be formed. For those temperatures for which the AdS space dominates, the entropy vanishes to order N^2. For high enough temperatures, the entropy is indeed of the expected form (2.4). In this regime the classical supergravity calculation confirms holography and reproduces the features of the 4 dimensional gauge theory. The low temperature result is interpreted as reflecting a finite size effect on the gauge theory side. For large N the cooled YM theory is thus supposed to pass to a phase in which the entropy is of order 1. An even more severe test to the holography idea comes about to next order in $1/N$. On the gauge theory side no qualitative changes of the formula (2.4) is expected. On the other hand on the supergravity side for temperatures (2.2) one may expect the full ten dimensionality of the system to be rediscovered.

Indeed, had the $AdS_5 \times \mathcal{N}_5$ been the only contributing configuration, D as appearing in (2.3) would have been 9, invalidating the holography property. More precisely, as shown in Figure 1, the temperature in this background depends on a radial coordinate r, while the KK gap $1/b$ is constant along r.

For large values of r, the temperature is red-shifted to very low values. For a temperature T_0 (the temperature at $r = 0$) larger than $1/b$ there would be essentially two regions in the radial direction: for small r the temperature would be hot enough to probe the full

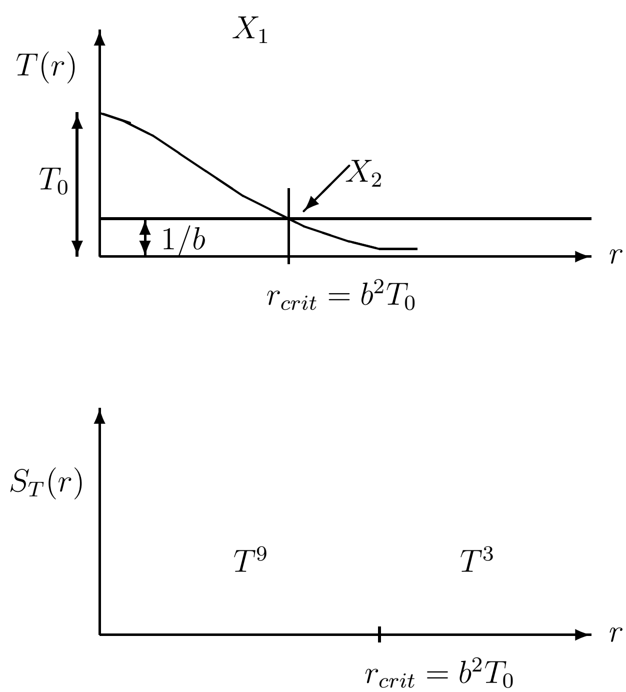

Figure 1: The radial dependence of the effective temperature of the AdS manifold X_1 and the black hole X_2. The bottom part shows the radial variation of the temperature dependence of the entropy in the X_1. $1/b$ is the KK mass threshold.

10 dimensional structure of the system; for r above a certain critical radius the temperature would be too cool to excite the KK modes and the naive effective dimension of the system is 5. In fact, the red-shift in radial directions of AdS is so strong that the true effective dimension actually drops to 4. The critical radius r_c is proportional to the scale set by the AdS curvature b and given by:

$$r_c = b^2 T_0. \tag{2.5}$$

This apparent violation of holography is overturned by the emergence of the second bulk supergravity configuration, namely that of the black hole. Precisely for those temperatures at which holography is at risk, the Euclidean black hole dominates the functional integral, it hides beyond its horizon exactly that region in r which would have revealed the ten dimensional nature of the system. The region in r which actually exists for the black hole configuration is cold, preserving the holographic four dimensional nature of the entropy. This is possible because the location of the horizon of the black hole is correlated to the radius of the compact manifold $\mathcal{N}$ (which dictates the KK transition temperature) in the appropriate manner. Several lessons emerge: first it is essential to sum over bulk geometries with different topologies in order to enforce holography, and second the formation of black holes is essential for the same purpose. One may also attempt to draw a lesson directly for string theory, that is, given an initial perturbative background, non perturbative effects in string theory would cause all backgrounds with the same boundary (and perhaps other data) to contribute as well. We have employed the methods used to derive the above results also for other string backgrounds with non-constant negative curvature. In these cases, not only is the temperature red-shifted for large values of r, but the KK mass gap itself narrows for large values of r. These two effects are competing. It turns out that Dirichlet p-branes continue to obey holography as long as p is smaller than 5. $p = 5$ is a marginal case, and for $p > 5$ an extensive dual field theory would not reproduce the supergravity results. In this analysis it was important that the black hole configuration dominated the AdS configuration.

As we turn to inspect the relation $4 = 10$ in English, more respect will be paid to the 'losing' configurations. The more precise question is the following: suppose one could fully diagonalize the strongly coupled SUSY YM Hamiltonian on a finite spatial volume of radius R. In that case one could plot the density of states as a function of the energy. Are there energy bands for which the behavior of the density of states would be different than that expected for a 4 dimensional theory? In particular would there be a finite band for which the 4 dimensional system would exhibit a 10 dimensional behavior? It was conjectured [11] that this is indeed the case. The conjectured behavior is shown in Figure 2.

The curve contains four sections: at the high energy end the entropy $S(E)$ is proportional to $N^{1/2}\,(RE)^{3/4}$. This is the conventional behavior of a 4 dimensional system with N^2 fields. However for other values of E there are conjectured to be bands for which $S(E)$ is proportional to $(RE)^{9/10}$ reflecting a ten dimensional behavior for a finite range of energy, bands for which $S(E) \sim (g_{YM}^2 N)^{-1/4}\,RE$ reflecting the usual perturbative Hagedorn spectrum of strings and finally a finite size band for which $S(E) \sim N^{-2/7}\,(RE)^{8/7}$ reflecting the temporary formation of 10 dimensional Schwarzschild black holes. Can one find any hint of this microcanonical behavior in the canonical analysis that we have performed on the supergravity side? We believe that the answer is yes, and that the evidence can be obtained by following the losing configu-

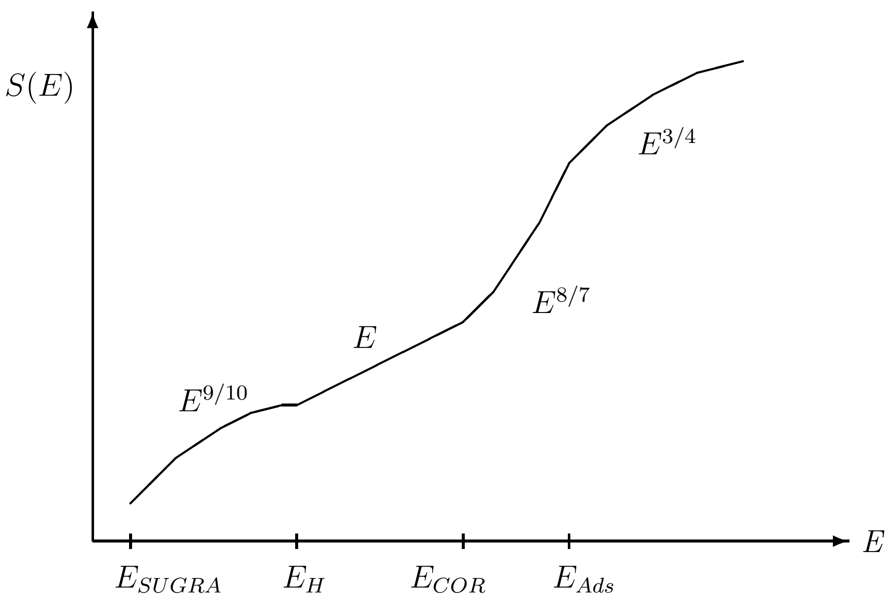

Figure 2: The conjectured energy dependence of the entropy of $D = 4$, maximally supersymmetric Yang-Mills on a sphere of radius R. The following definitions are used: $E_H \equiv \frac{1}{R}(g_s N)^{5/2}$, $E_{COR} \equiv \frac{1}{R}N^2(g_s N)^{-7/2}$, $E_{Ads} \equiv \frac{1}{R}N^2$.

ration. The 10 dimensional behavior is hinted by the non-leading AdS_5 contribution to the entropy which we have discussed earlier. Indeed, this contribution comes from small values of r which are short distance effects in the supergravity theory and thus long distance effects in the gauge theory exactly as expected for finite size effects. The ten dimensional black holes can be traced to black hole configuration with negative specific heat which may form in AdS space above a certain temperature. These objects will be localized or delocalized on $\mathcal{N}_5$ according to the temperature. It is only the stringy Hagedorn regime which cannot be traced as easily on the supergravity side. Actually some attempts to expose it have failed [1, 2] leading to a conjecture of a 'Hagedorn censorship'. However without a full stringy treatment this problem is still open. In conclusion we have learned that indeed the relation $4 = 10$ should be read in Hebrew, but as far as finite size effects are concerned, it is just as interesting to view it in English. Imagine that we ourselves are collecting data in some energy band misleading ourselves into believing that the dimension of our space is larger than it really is...

3. String thermodynamics in D-brane backgrounds

The spectrum of hadrons in the dual model was given by the formula

$$S(E) = cE^a e^{\beta_H E} \tag{3.6}$$

where a and β_H are determined by the theory. This system has a limiting temperature, the Hagedorn temperature $1/\beta_H$. After being amused by encountering a limiting temperature, physicists suggested that the limit was rather on our knowledge and that its emergence reflected the existence of a phase transition. At temperatures around $1/\beta_H$ the system is much better described by underlying constituents of the hadrons, quarks and gluons [12]. When the system is expressed in terms of its constituents the temperature can be raised indefinitely leading to the liberation of the confined quarks and gluons. Many string theories have a similar density of states, the difference between the various theories is reflected by the values of the parameters a and β_H. Over the years the thermodynamics of open and closed, bosonic, supersymmetric and heterotic string theories was studied in some detail (see [13, 14] among others), sometimes in a hope to uncover the existence of constituents of strings, the stuff the strings are made of.

In ten non-compact dimensions, open strings indeed exhibit a limiting temperature. Closed strings on the other hand do not exhibit such a behavior. It is indeed tempting to consider the possibility that the Hagedorn threshold may be crossed in that case. However microcanonical studies of the system were necessary due to the large fluctuations exhibited near the Hagedorn temperature. In certain circumstances a non-extensive behavior emerged driven by the formation of a single very large string. It was conjectured that such a large (effectively tensionless?) string signaled a genuine phase transition. The system also seemed to exhibit a negative specific heat. On the other hand in many cases where the system was regularized by embedding it in a finite volume, it turned out that one cannot surpass the Hagedorn and eventually the energy pumped into the system was distributed more evenly among the string modes, winding modes restored in some cases a positive specific heat. We reexamined all these issues in the presence of D-branes. The details appear in [3]. The flavor of the results can be reproduced by simple random walk arguments.

In addition to providing a nice physical interpretation and checks of the calculations, this point of view leads to some possible generalizations beyond toroidal backgrounds.

For example consider the single-string distribution function $\omega(\varepsilon)$ for closed strings in D large space-time dimensions. The energy ε of the string is proportional to the length of the random walk. The number of walks with a fixed starting point and a given length ε grows exponentially as $\exp(\beta_c\,\varepsilon)$. Since the walk must be closed, this overcounts by a factor of the volume of the walk, which we shall denote by $V(\text{walk}) = W$. Finally, there is a factor of V_{D-1} from the translational zero mode, and a factor of $1/\varepsilon$ because any point in the closed string can be a starting point. The final result is

$$\omega(\varepsilon)_{\text{closed}} \sim V_{D-1} \cdot \frac{1}{\varepsilon} \cdot \frac{e^{\beta_c\,\varepsilon}}{W}. \tag{3.7}$$

Now, the volume of the walk is proportional to $\varepsilon^{(D-1)/2}$ if it is well-contained in the volume ($R \gg \sqrt{\varepsilon}$), or roughly V_{D-1} if it is space-filling ($R \ll \sqrt{\varepsilon}$). One has the known result

$$\omega(\varepsilon)_{\text{closed}} \sim V_{D-1}\,\frac{e^{\beta_c\,\varepsilon}}{\varepsilon^{(D+1)/2}} \tag{3.8}$$

in D effectively non-compact space-time dimensions, and

$$\omega(\varepsilon)_{\text{closed}} \sim \frac{e^{\beta_c\,\varepsilon}}{\varepsilon} \tag{3.9}$$

in an effectively compact space.

We can generalize this analysis to open strings in the presence of branes for a general $(\mathrm{D}p, \mathrm{D}q)$ sector by a slight modification of the combinatorics. The leading exponential degeneracy of a random walk of length ε with a fixed starting point in say the Dp-brane is the same as for closed strings: $\exp(\beta_c\,\varepsilon)$. Fixing also the end-point at a *particular* point of the Dq-brane requires the factor $1/W$ to cancel the overcounting, just as in the closed string case. Now, both end-points move freely in the part of each brane occupied by the walk. This gives a further degeneracy factor

$$(W_{NN}\,W_{ND})\cdot(W_{NN}\,W_{DN}) \tag{3.10}$$

from the positions of the end-points. N and D refer to Neumann and Dirichlet boundary conditions. Finally, the overall translation of the walk in the excluded NN volume gives a factor V_{NN}/W_{NN}. The final result is:

$$\omega(\varepsilon)_{\rm open} \sim \frac{V_{NN}}{W_{NN}}\cdot W_{NN+ND}\cdot W_{NN+DN}\cdot\frac{1}{W}\cdot\exp\left(\beta_c\,\varepsilon\right) \sim \frac{V_{NN}}{W_{DD}}\;\exp\left(\beta_c\,\varepsilon\right). \tag{3.11}$$

Thus, we find that the density of states is only sensitive to the effective volume of the random walk in DD directions. If the walk is well-contained in DD directions ($R_{DD} \gg \sqrt{\varepsilon}$), we find $W_{DD} \sim \varepsilon^{d_{DD}/2}$ and

$$\omega(\varepsilon)_{\rm open} \sim \frac{V_{NN}}{\varepsilon^{d_{DD}/2}}\;\exp\left(\beta_c\,\varepsilon\right). \tag{3.12}$$

On the other hand, if it is space-filling in DD directions ($R_{DD} \ll \sqrt{\varepsilon}$), the DD-volume of the walk is just $W_{DD} \sim V_{DD}$ and we find

$$\omega(\varepsilon)_{\rm open} \sim \frac{V_{NN}}{V_{DD}}\;\exp\left(\beta_c\,\varepsilon\right). \tag{3.13}$$

The random walk picture gives a geometric rationale for the similarity between non-compact closed-string and open-string densities of states. It is related to the fact that the random walk must ‘close on itself’ in some effective co-dimension (the full space for closed strings and the DD space for open strings). Canonically, open strings attached to Dp-branes in infinite transverse space for $p < 5$ thus have the non-limiting characteristics of closed strings in ten dimensions. These formulas are very useful to determine the canonical and microcanonical behavior of the system in various environments, depending on the values of compactification moduli.

I wish to note here one speculative feature which emerges out of the analysis. It turns out that in a system containing a collection of D-branes, Dp-branes with $p \geq 5$ attract energy from their neighboring branes. One possible result of being an energy sink could be the melting of these branes, leaving the arena free for Dp-branes with $p < 5$. One can find various counter-arguments to this scenario in [3]. Nevertheless, we find this Darwinistic concept worthy of further investigation.

4. Phases in Gravity

We end this contribution with an impressionistic discussion of bulk and boundary phase diagrams in string theory at moderately small coupling. I believe such diagrams will come to play as an important a role as those drawn some years ago for gauge theories.

4.1. Bulk Phase Diagram

A supergravity gas in ten dimensions has entropy

$$S(E)_{\rm sgr} \sim V^{1/10}\, E^{9/10} \tag{4.14}$$

and can be matched to a bulk black hole with entropy ($\alpha' \sim \ell_s^2 = 1$ throughout this section)

$$S(E)_{\rm bh} \sim E\, (g_s^2\, E)^{1/7}. \tag{4.15}$$

The coexistence line $S_{\rm sgr} \sim S_{\rm bh}$ gives a black hole in equilibrium with radiation in a finite volume, with energy of order

$$E({\rm sgr} \leftrightarrow {\rm bh}) \sim \frac{1}{g_s^2}\, (g_s^2\, V)^{7/17}, \tag{4.16}$$

and microcanonical temperature

$$T({\rm sgr} \leftrightarrow {\rm bh}) \sim \left(\frac{1}{g_s^2\, V}\right)^{1/17}. \tag{4.17}$$

Since the black-hole-dominated region has negative specific heat, this temperature is maximal in the vicinity of the transition. This configuration is microcanonically stable in finite volume, in a range of energies between the matching point and the Jeans bound.

The graviton gas can also be matched to a gas of long closed strings. The coexistence curve $S_{\rm sgr} \sim S_{\rm Hag}$ at temperatures $T \sim O(1)$ in string units, is independent of the string coupling and is given by the Hagedorn energy density:

$$E({\rm sgr} \leftrightarrow {\rm Hag}) \sim V. \tag{4.18}$$

This Hagedorn phase can be exited at high energy or large coupling through the correspondence curve $S_{\rm Hag} \sim S_{\rm bh}$:

$$E({\rm bh} \leftrightarrow {\rm Hag}) \sim \frac{1}{g_s^2}, \tag{4.19}$$

into a black-hole dominated phase at lower temperatures. The resulting phase diagram for the bulk or closed-string sector is depicted in Figure 3.

An interesting feature of the phase diagram is the existence of a triple point at the intersection of the phase boundaries of the massless supergravity gas, Hagedorn, and black-hole-dominated regimes. This point lies at Hagedorn energy density $E_c \sim V$, string scale temperatures $T \sim O(1)$, and considerably weak coupling $g_s \sim 1/\sqrt{V}$ and, somewhat optimistically, we would like to interpret its existence as evidence for completeness of this phase structure. Namely, we are not missing any major set of degrees of freedom. According to this picture, the Hagedorn phase goes into a black-hole-dominated phase at large energy or coupling, well within the Jeans

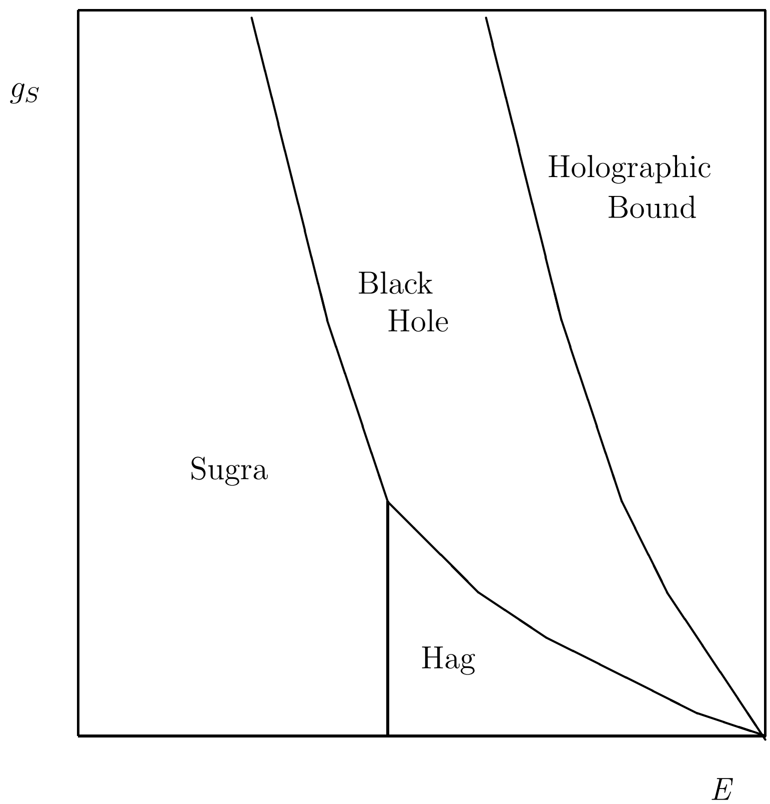

Figure 3: An impressionistic bulk phase diagram. Only the region $g_s < 1$ is represented in this picture. The triple point separating the supergravity gas, black hole, and Hagedorn-dominated regimes is located at $g_s \sim 1/\sqrt{V}$, and $E \sim V$. The rightmost region is excluded by the holographic bound.

or holographic bound:

$$E < E_{\rm Hol} \sim \frac{V^{7/9}}{g_s^2}, \tag{4.20}$$

provided we are at weak string coupling $g_s < 1$. We see that the Hagedorn regime has no thermodynamic limit whatsoever. If we scale the total energy E linearly with the volume, we run into the black-hole phase, which ends when the horizon crushes the walls of the box (i.e. the black hole fills the box). Moreover, if the string coupling is larger that $1/\sqrt{V}$, we miss the Hagedorn phase altogether, as the supergravity gas goes into the black-hole-dominated phase directly. In this case, the system has a sub-stringy maximum temperature

$$T_{\rm max} \sim T({\rm sgr} \leftrightarrow {\rm bh}) < 1. \tag{4.21}$$

4.2. World-Volume Phase Diagram

Similar remarks apply to the open-string sector in the vicinity of the D-branes. In Figure 4 the world-volume phase diagram is presented for small string coupling. Here, the details of the correspondence principle depend on the excitation energy of the D-brane, i.e. in the geometric picture, we must distinguish between the near-extremal ($r_0 \ll r_Q$) and non-extremal or Schwarzschild ($r_0 \gg r_Q$) regimes.

Before proceeding further, it is important to notice that Dp-branes with $p > 6$ cannot be considered as well-defined asymptotic states in weakly-coupled string theory. The massless fields specifying the closed-string vacuum, including the dilaton, grow with transverse distance to the D-brane. As a consequence, introducing a $p > 6$ Dp-brane in a given perturbative background inevitably results in a non-perturbative modification of the vacuum itself. Thus,

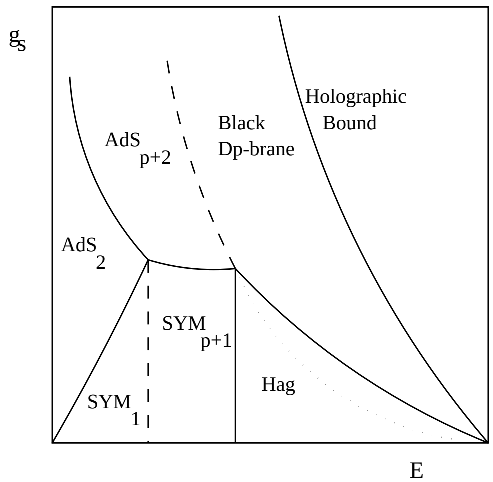

Figure 4: An impressionistic world-volume phase diagram for small string coupling.

consistency with the requirement of weak string coupling throughout the system means that such branes are never far from orientifold boundaries, and should be better considered as part of the specification of the background geometry. In the following, we shall restrict to $p < 7$, unless specified otherwise.

The matching of the near-extremal ($r_0 \ll r_Q$) black-brane entropy or Anti- de Sitter-type (AdS) throats:

$$S(E)_{\mathrm{AdS}_{p+2}} \sim N^{1/2}\,(V_{\|})^{\frac{5-p}{2(7-p)}}\,g_s^{\frac{p-3}{2(7-p)}}\,E^{\frac{9-p}{2(7-p)}} \tag{4.22}$$

to a weakly-coupled Yang–Mills gas on the world-volume:

$$S(E)_{\mathrm{SYM}_{p+1}} \sim N^{\frac{2}{p+1}}\,(V_{\|})^{\frac{1}{p+1}}\,E^{\frac{p}{p+1}}, \tag{4.23}$$

is the content of the generalized SYM/AdS correspondence [6], and was studied in detail in [15, 2, 16] (we call these manifolds AdS although, properly speaking, they are only conformal to $AdS_{p+2} \times \mathbf{S}^{8-p}$).

There are interesting finite-size effects at low temperatures, $T \lesssim 1/R_{\|}$, in the form of large N phase transitions of the gauge theory. For $p = 3$ and spherical topology of the brane world-volume, the gravitational counterpart is the Hawking–Page transition [9, 10] between the AdS black-hole geometry and the AdS vacuum geometry (intermediate metastable phases can be found [11]). For our case ($p < 7$ and toroidal topology of the branes) the finite-size effects setting in at the energy threshold $E \lesssim N^2/R_{\|}$ are associated to the transition to zero-mode dynamics in the Yang–Mills language and to finite-volume localization [17] in the black-hole language. At sufficiently low temperatures one must use a T-dual description of the throat, resulting in an effective geometry of 'smeared' D0-branes. When these D0-branes localize as in [17] the description involves an AdS-type throat with $p = 0$, which we denote by AdS_2. In this case of toroidal topology, there is no regime of *vacuum* AdS dominance, provided N is large enough [2, 18]. We refer the reader to [2, 16, 18] for a detailed discussion of such low-temperature phenomena, as well as for an extension of the phase diagram to large values of the string coupling, beyond the 't Hooft limit discussed here.

At temperatures $T > 1/R_{\|}$ these finite-size effects can be neglected, and the SYM/AdS transition is determined by the matching of (4.22) and (4.23). The transition temperature,

$$T(\text{SYM}_{p+1} \leftrightarrow \text{AdS}_{p+2}) \sim (g_s\, N)^{\frac{1}{3-p}}, \tag{4.24}$$

is smaller than the Hagedorn temperature as long as stringy energy densities are not reached in the world-volume.

At this point, it should be noted that the interpretation of the AdS throats as SYM dynamics at large 't Hooft coupling (the standard AdS/SYM correspondence) is problematic for $p = 5, 6$. For

$p = 5$, the AdS regime has a density of states typical of a string theory, with renormalized tension $T_{\rm eff} = 1/\alpha' \, g_s \, N$. For $p = 6$ the qualitative features of the thermodynamics of the near-extremal and Schwarzschild regimes are essentially the same, so that the boundary $r_0 \sim r_Q$ does not mark a significant change in behaviour. The holography properties required to interpret the AdS physics *only* in terms of gauge-theory dynamics seem to break down for these cases [19, 1, 20, 2, 21]. However, the SYM/AdS correspondence line in the sense of [22] can always be defined, independently of whether there is a candidate microscopic interpretation for the entropy (4.22) in the AdS regime.

At stringy energy densities $E \sim N^2 \, V_\|$, the SYM/AdS correspondence line joins the open-string Hagedorn regime. The transition from a Yang–Mills gas on the world-volume to a Hagedorn regime of open strings ($S_{\rm SYM} \sim S_{\rm Hag}$) occurs at the energy

$$E({\rm SYM}_{p+1} \leftrightarrow {\rm Hag}) \sim N^2 \, V_\|. \qquad (4.25)$$

This line joins the SYM/AdS correspondence curve at a *triple* point (see Figure 4), the other phase boundary being the correspondence curve between the long open strings in the Hagedorn phase, and the non-extremal black-brane phase. Black Dp-branes in the Schwarzschild regime ($r_0 \gg r_Q$) have entropy:

$$S(E)_{{\rm B}p} \sim E \left(\frac{g_s^2 E}{V_\|} \right)^{\frac{1}{7-p}}. \qquad (4.26)$$

and match the world-volume Hagedorn phase along the curve:

$$E({\rm Hag} \leftrightarrow {\rm B}p) \sim \frac{V_\|}{g_s^2}. \qquad (4.27)$$

Notice that the boundary line separating the near-extremal (AdS) and Schwarzschild (Bp) regimes of the black branes, given by $r_0 \sim$

r_Q, or

$$E(\mathrm{AdS}_{p+2} \leftrightarrow \mathrm{B}p) \sim N\,\frac{V_\|}{g_s}, \tag{4.28}$$

also joins the triple point located at $E \sim N^2\, V_\|$ and $g_s\, N \sim 1$. The temperature along this line is

$$T(\mathrm{AdS}_{p+2} \leftrightarrow \mathrm{B}p) \sim \left(\frac{1}{g_s\, N}\right)^{\frac{1}{7-p}}. \tag{4.29}$$

This temperature is locally maximal for small energy variations if $p < 5$.

All these phases lie well within the holographic bound, defined by the condition that the horizon of the black brane saturates the available transverse volume:

$$E < E_{\mathrm{Hol}} \sim \frac{V_\|}{g_s^2} \cdot (V_\perp)^{\frac{7-p}{9-p}}. \tag{4.30}$$

In all of the above, supersymmetry was the *éminence grise*. In its absence none of the above calculations could have been consistently done.

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