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Black Hole Thermodynamics and Two-Dimensional Dilaton Gravity Theory

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Abstract

We relate various black hole solutions in the near-horizon region to black hole solutions in two-dimensional dilaton gravity theories in order to argue that thermodynamics of black holes in $D \geq 4$ can be effectively described by thermodynamics of black holes in two-dimensional dilaton gravity theories. We show that the Bekenstein-Hawking entropies of single-charged dilatonic black holes and dilatonic p -branes in arbitrary spacetime dimensions and with an arbitrary dilaton coupling parameter are exactly reproduced by the Bekenstein-Hawking entropy of the two-dimensional black hole in the associated two-dimensional dilaton gravity model. We comment that thermodynamics of non-extreme stringy four-dimensional black hole with four charges and five-dimensional black hole with three charges may be effectively described by thermodynamics of the black hole solutions with constant dilaton field in two-dimensional dilaton gravity theories.

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1 Introduction

The recent development in string duality and the observation [1] that D-branes can carry charges of form potentials in the RR sector of string theories enabled us to address fundamental issues in quantum gravity such as statistical interpretation of black hole entropy within the framework of string theory. By applying the D-brane counting technique, pioneered in Ref. [2], it has been successful in reproducing the Bekenstein-Hawking entropies of black hole solutions in string theories. Generally, the D-brane counting technique can be applied to the BPS and the near-BPS cases and few other special cases such as non-BPS extreme rotating black hole in four dimensions.

Holographic principle [3, 4, 5] also has contributed to the microscopic interpretation of the Bekenstein-Hawking entropy. Brown and Henneaux [6] showed that the asymptotic symmetry group of AdS_3 space is the conformal group in two dimensional spacetime at the boundary. By using this idea, Carlip [7, 8] and Strominger [9] were able to exactly reproduce the Bekenstein-Hawking entropy of the three-dimensional constant curvature black hole of Banãdos, Teitelboim and Zanelli (BTZ) [10] by counting microscopic degrees of freedom of the conformal field theory at the boundary. An important observation [11] by Hyun that (uplifted) black hole solutions in string theory can be put in the form of the product of the BTZ black hole and a sphere through a series of U -duality transformations enabled the microscopic counting of non-extreme stringy black holes in four and five dimensions by applying the result of Carlip and Strominger (for example, Ref. [12]).

As pointed out in Ref. [11], higher-dimensional black hole solutions in string theories can also be related to two-dimensional black hole solutions through U -duality transformations. The four- and the five-dimensional non-extreme black holes in string theory are related [17, 18, 19, 20] to two-dimensional black holes in the Jackiw-Teitelboim (JT) model [21, 22] and the Callan-Giddings-Harvey-Strominger (CGHS) model [23] and to the two-dimensional charged black hole of McGuigan, Nappi and Yost [24]. Recently, progress has been made [14, 15, 16] in reproducing the Bekenstein-Hawking entropies of the two-dimensional black holes by using the Cardy's formula [13] for the boundary conformal theory. Furthermore, it is argued in Refs. [25, 26] that the Bekenstein-Hawking entropy of the generic black holes in arbitrary dimensional pure gravity theories can be reproduced by the microscopic calculation based on the conformal theory associated with the two-dimensional subset of the spacetime. So, it seems that black holes in two-dimensional gravity theories, which were originally studied as unrealistic toy models for quantum theory of gravity, have closer connection with realistic black holes in $D \geq 4$ than it was originally thought.

It is the purpose of this paper to relate various black hole solutions in $D \geq 4$ to black hole solutions in two-dimensional dilaton gravity theories. In section 2, we

relate a single-charged dilatonic black hole solution in $D \geq 4$ with an arbitrary dilaton coupling parameter to a black hole solution in a two-dimensional dilaton gravity theory. We find that the Bekenstein-Hawking entropies of these two black holes are the same in the large charge limit or the near-extremal limit. In section 3, we argue that the thermodynamics of a dilatonic p -brane in arbitrary spacetime dimensions and with an arbitrary dilaton couple parameter can also be described by a black hole solution in a two-dimensional dilaton gravity theory. In section 4, we comment that thermodynamics of the non-extreme four-dimensional stringy black hole with four charges and the non-extreme five-dimensional stringy black hole with three charges may be more naturally described by the black holes with constant dilaton field in two-dimensional dilaton gravity theories.

2 Single-Charged Dilatonic Black Holes

In this section, we consider single-charged dilatonic black hole solutions with an arbitrary dilaton coupling parameter a and in arbitrary spacetime dimensions D . The result in this section can be applied not only to all the single-charged dilatonic black holes but also to multi-charged black holes in string theories, because in the near-horizon limit the actions for the multi-charged black hole solutions in string theories can be reduced (with the guidance of the explicit solutions in the near-horizon limit) to the actions for the two-dimensional dilaton gravity with possibly different forms of the dilaton potential term from the one obtained in this section. The exceptional case is the multi-charged stringy black hole solutions with regular BPS limit, i.e., the four-dimensional black hole with four charges and the five-dimensional black hole with three charges. In this special case, all the scalars of the solution become constant in the near-horizon limit and therefore one cannot relate such black hole solutions to black hole solutions with non-trivial dilaton field in two-dimensional dilaton gravity models². This exceptional case will be separately discussed in the last section.

The corresponding Einstein-frame action is

$$S_E = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-G^E} \left[\mathcal{R}_{G^E} - \frac{4}{D-2} (\partial\phi)^2 - \frac{1}{4} e^{2a\phi} F_2^2 \right], \quad (1)$$

²On the other hand, the near-horizon region solutions for this case can be related to the BTZ black hole solution when the solutions are uplifted to one higher spacetime dimensions, after the U -duality transformations when necessary. (Note, however, that in such embeddings of black hole solutions the harmonic function for the charge associated with the gravitational wave is allowed to not take the near-horizon form and therefore some of scalars are not constant.) The action for the BTZ black hole solution can be reduced to a two-dimensional dilaton gravity model action upon dimensional reduction. But in this paper, we shall ignore the higher-dimensional origin of black hole solution, just considering the Einstein-frame metric itself. See the second paragraph of the last section for more comments.

where κ_D is the D -dimensional Einstein gravitational constant and F_2 is the field strength of the $U(1)$ gauge potential $A^{(1)} = A_M dx^M$ ($M = 0, 1, \dots, D-1$). The non-extreme black hole solution to the field equations of this action is given by

$$\begin{aligned} ds_E^2 &= -H^{-\frac{4(D-3)}{(D-2)\Delta}} f dt^2 + H^{\frac{4}{(D-2)\Delta}} \left[f^{-1} dr^2 + r^2 d\Omega_{D-2}^2 \right], \\ e^\phi &= H^{\frac{(D-2)a}{2\Delta}}, \quad A_t = -H^{-1}, \end{aligned} \quad (2)$$

where

$$\begin{aligned} H &= 1 + \frac{m \sinh^2 \alpha}{r^{D-3}}, \quad f = 1 - \frac{m}{r^{D-3}}, \\ \Delta &= \frac{(D-2)a^2}{2} + \frac{2(D-3)}{D-2}. \end{aligned} \quad (3)$$

The Bekenstein-Hawking entropy S_{BH} of the dilatonic black hole solution (2) is determined by the surface area A_H of the event horizon (located at $r = r_H = m^{\frac{1}{D-3}}$):

$$S_{BH} = \frac{A_H}{4G_D} = \frac{m^{\frac{D-2}{D-3}} V_{S^{D-2}} \cosh^{\frac{4}{\Delta}} \alpha}{4G_D}, \quad (4)$$

where $V_{S^{D-2}} = 2\pi^{\frac{D-2}{2}} / \Gamma(\frac{D-1}{2})$ is the volume of S^{D-2} with the unit radius.

One can also think of the dilatonic black hole in D dimensions as being magnetically charged under the $(D-2)$ -form field strength F_{D-2} . The corresponding Einstein-frame action is

$$\tilde{S}_E = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-G^E} \left[\mathcal{R}_{G^E} - \frac{4}{D-2} (\partial\phi)^2 - \frac{1}{2 \cdot (D-2)!} e^{-2a\phi} F_{D-2}^2 \right]. \quad (5)$$

In terms of this ‘‘dual’’ field parametrization, the dilatonic black hole solution takes the following form:

$$\begin{aligned} ds_E^2 &= -H^{-\frac{4(D-3)}{(D-2)\Delta}} f dt^2 + H^{\frac{4}{(D-2)\Delta}} \left[f^{-1} dr^2 + r^2 d\Omega_{D-2}^2 \right], \\ e^\phi &= H^{\frac{(D-2)a}{2\Delta}}, \quad F_{D-2} = \star(dH \wedge dt). \end{aligned} \quad (6)$$

In the so-called dual-frame [27, 28, 29, 30], the spacetime of the BPS dilatonic black hole in the near-horizon region takes the $\text{AdS}_2 \times S^{D-2}$ form. The following dual-frame action is related to (5) through the Weyl rescaling transformation $G_{MN}^E = e^{-\frac{2a}{D-3}\phi} G_{MN}^d$:

$$\tilde{S}_d = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-G^d} e^{\delta\phi} \left[\mathcal{R}_{G^d} + \gamma (\partial\phi)^2 - \frac{1}{2 \cdot (D-2)!} F_{D-2}^2 \right], \quad (7)$$

where the parameters in the action are defined as

$$\delta \equiv -\frac{D-2}{D-3} a, \quad \gamma \equiv \frac{D-1}{D-2} \delta^2 - \frac{4}{D-2}. \quad (8)$$

In the dual-frame, the dilatonic black hole solution takes the following form:

$$dx_d^2 = -H^{\frac{2\Delta-4(D-3)}{(D-3)\Delta}} f dt^2 + H^{\frac{2}{D-3}} \left[f^{-1} dr^2 + r^2 d\Omega_{D-2}^2 \right], \quad (9)$$

where the dilaton and the $(D-2)$ -form field strength take the same forms as in Eq. (6). In the near-horizon region, the metric (9) is approximated to

$$ds_d^2 \approx - \left(\frac{\hat{\mu}}{r} \right)^{2-\frac{4(D-3)}{\Delta}} \left(1 - \frac{m}{r^{D-3}} \right) dt^2 + \left(\frac{\hat{\mu}}{r} \right)^2 \left(1 - \frac{m}{r^{D-3}} \right)^{-1} dr^2 + \hat{\mu}^2 d\Omega_{D-2}^2, \quad (10)$$

where $\hat{\mu} \equiv (m \sinh^2 \alpha)^{1/(D-3)}$, and the dilaton and the non-zero component of the $U(1)$ gauge field are approximated to

$$e^\phi \approx \left(\frac{\hat{\mu}}{r} \right)^{\frac{(D-2)(D-3)a}{2\Delta}}, \quad A_t \approx \left(\frac{r}{\hat{\mu}} \right)^{D-3}. \quad (11)$$

By compactifying the dual-frame action (7) on S^{D-2} with the radius $\hat{\mu}$, one obtains the following two-dimensional effective action:

$$S = \frac{1}{2\kappa_2^2} \int d^2x \sqrt{-g} e^{\delta\phi} \left[\mathcal{R}_g + \gamma(\partial\phi)^2 + \Lambda \right], \quad (12)$$

where the κ_2^2 is the two-dimensional gravitational constant and the cosmological constant Λ is given by

$$\Lambda = \frac{D-3}{2\hat{\mu}^2} \left[2(D-2) - \frac{4(D-3)}{\Delta} \right]. \quad (13)$$

To bring the action (12) to the standard form of the two-dimensional dilaton gravity action, one redefines the dilaton as $\Phi = e^{\delta\phi}$ and then applies the Weyl rescaling of the metric $g_{\mu\nu} = \Phi^{-\frac{\gamma}{\delta^2}} e^{\frac{\Phi}{2}} \tilde{g}_{\mu\nu}$. The resulting action has the following form [31]:

$$S = \frac{1}{2\kappa_2^2} \int d^2x \sqrt{-\tilde{g}} \left[\Phi \mathcal{R}_{\tilde{g}} + \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi + \Lambda \Phi^{1-\frac{\gamma}{\delta^2}} e^{\frac{\Phi}{2}} \right]. \quad (14)$$

The kinetic term for the dilaton Φ can be removed by applying one more Weyl rescaling $\tilde{g}_{\mu\nu} = e^{-\frac{\Phi}{2}} \bar{g}_{\mu\nu}$, resulting in the following action [31]:

$$S = \frac{1}{2\kappa_2^2} \int d^2x \sqrt{-\bar{g}} \left[\Phi \mathcal{R}_{\bar{g}} + \Phi^{1-\frac{\gamma}{\delta^2}} \Lambda \right]. \quad (15)$$

Particularly interesting special cases of this action are the JT model for the $(D, a) = (4, 1/\sqrt{3})$ case and the CGHS model for the $(D, a) = (4, 1)$ case.

The field equations of the action (15) are

$$\mathcal{R}_{\bar{g}} + \left(1 - \frac{\gamma}{\delta^2} \right) \Phi^{-\frac{\gamma}{\delta^2}} \Lambda = 0,$$

$$\nabla_\mu \nabla_\nu \Phi - \bar{g}_{\mu\nu} \frac{1}{2} \Phi^{1-\frac{\gamma}{\delta^2}} \Lambda = 0. \quad (16)$$

In the Schwarzschild gauge, the general time-dependent solution to these field equations takes the following form ³:

$$\begin{aligned} ds^2 &= - \left[\frac{\delta^2}{2\delta^2 - \gamma} \left(\frac{x}{\ell} \right)^{2-\frac{\gamma}{\delta^2}} - 2\ell M \right] d\tau^2 + \left[\frac{\delta^2}{2\delta^2 - \gamma} \left(\frac{x}{\ell} \right)^{2-\frac{\gamma}{\delta^2}} - 2\ell M \right]^{-1} dx^2, \\ \Phi &= \frac{x}{\ell}, \end{aligned} \quad (17)$$

where $\ell \equiv 1/\sqrt{\Lambda}$ and the diffeomorphism invariant parameter M [33] defined in the following is the mass of the solution:

$$M = -\frac{1}{2\ell} \left[(\nabla\Phi)^2 \ell^2 + \frac{\delta^2}{2\delta^2 - \gamma} \Phi^{2-\frac{\gamma}{\delta^2}} \right]. \quad (18)$$

The thermodynamic properties of the solution (17) is determined by the behaviour of the solution at the event horizon. The event horizon is located at the root of $\bar{g}_{\tau\tau}(\Phi_H) = 0$, namely at

$$\Phi_H = \left[2\ell M \left(2 - \frac{\gamma}{\delta^2} \right) \right]^{\frac{\delta^2}{2\delta^2 - \gamma}}. \quad (19)$$

At the event horizon, the Killing vector $k^\mu = \ell \eta^{\mu\nu} \Phi_{,\nu}$ is null due to the definition (18) of the mass M :

$$|k|^2|_{\Phi_H} = -\ell^2 |\nabla\Phi|^2|_{\Phi_H} = 2\ell M - \frac{\delta^2}{2\delta^2 - \gamma} \Phi^{2-\frac{\gamma}{\delta^2}} \Big|_{\Phi_H} = \bar{g}_{\tau\tau}(\Phi_H) = 0. \quad (20)$$

The surface gravity κ , which determines the Hawking temperature $T_H = \frac{\kappa}{2\pi}$ and is defined by $\kappa^2 = -\frac{1}{2} \nabla^\mu k^\nu \nabla_\mu k_\nu|_{\Phi_H}$, is given by

$$\kappa = \frac{1}{2\ell} \Phi_H^{1-\frac{\gamma}{\delta^2}} = \frac{1}{2\ell} \left[2\ell M \left(2 - \frac{\gamma}{\delta^2} \right) \right]^{\frac{\delta^2 - \gamma}{2\delta^2 - \gamma}}. \quad (21)$$

Given the above expressions for the mass M and the surface gravity κ of the solution, one can see by using the first law of the thermodynamics that the Bekenstein-Hawking entropy is

$$S_{BH} = \frac{2\pi}{\kappa_2^2} \Phi_H = \frac{2\pi}{\kappa_2^2} \left[2\ell M \left(2 - \frac{\gamma}{\delta^2} \right) \right]^{\frac{\delta^2}{2\delta^2 - \gamma}}. \quad (22)$$

We now show that the Bekenstein-Hawking entropy (22) of the two-dimensional black hole compactified from the dilatonic black hole in $D \geq 4$ is the same as the

³The most general solution in two-dimensional dilaton gravity theory with general dilaton potential is previously constructed in Ref. [32].

Bekenstein-Hawking entropy (4) of the original dilatonic black hole in $D \geq 4$. One can bring the two-dimensional part $g_{\mu\nu} = G_{\mu\nu}^d$ ($\mu, \nu = t, r$) of the near-horizon metric (10) to the form (17) of the solution of the 2-dimensional dilaton gravity theory with the action (15) by applying the Weyl rescaling $g_{\mu\nu} = \Phi^{-\frac{\gamma}{\delta^2}} \bar{g}_{\mu\nu} = e^{-\frac{\gamma}{\delta}} \bar{g}_{\mu\nu}$ and then redefining the coordinates in the following way:

$$t = \frac{|\delta|}{\sqrt{2\delta^2 - \gamma}} \tau, \quad r = \hat{\mu} \left(\frac{x}{\ell} \right)^{\frac{2\Delta}{(D-2)^2 a^2}}. \quad (23)$$

The resulting metric $\bar{g}_{\mu\nu}$ has the form (17) with the mass M given by

$$M = \frac{\delta^2}{2\ell(2\delta^2 - \gamma) \sinh^2 \alpha}. \quad (24)$$

By plugging the expression (24) for the mass M into the expression (22) for the entropy of the two-dimensional solution (17), making use of the following relation:

$$\kappa_2^2 = \frac{\kappa_D^2}{\hat{\mu}^{D-2} V_{S^{D-2}}} = \frac{\kappa_D^2}{m^{\frac{D-2}{D-3}} \sinh^{\frac{2(D-2)}{D-3}} \alpha V_{S^{D-2}}}, \quad (25)$$

one obtains the following expression for the entropy:

$$S_{BH} = \frac{2\pi}{\kappa_2^2} \left(\frac{1}{\sinh^2 \alpha} \right)^{\frac{\delta^2}{2\delta^2 - \gamma}} = \frac{2\pi}{\kappa_D^2} m^{\frac{D-2}{D-3}} V_{S^{D-2}} \sinh^{\frac{4}{D}} \alpha. \quad (26)$$

Note, the Einstein gravitational constant κ_D is related to the Newton constant G_D as $\kappa_D^2 = 8\pi G_D$ in the unit $c = 1$. So, in the limit of large α (i.e. the large charge limit or the near-extremal limit), in which $\sinh \delta \approx \cosh \delta$, the Bekenstein-Hawking entropy (26) of the two-dimensional solution (17) with (24) becomes exactly same as the Bekenstein-Hawking entropy (4) of the D -dimensional dilatonic black hole (2). Therefore, thermodynamics of the D -dimensional dilatonic black hole solution (2) can be effectively described by thermodynamics of the 2-dimensional black hole solution (17).

3 Dilatonic p -Branes

When all the longitudinal directions are compactified on a compact manifold, a p -brane in D spacetime dimensions reduces to a dilatonic black hole in $D - p$ spacetime dimensions. So, by using the result of the previous section, one can see that thermodynamics of dilatonic p -branes can also be effectively described by black holes in two-dimensional dilaton gravity models. Also, thermodynamics of the delocalized intersecting brane solutions can be described by thermodynamics of black holes in two-dimensional dilaton

gravity theories, since all the delocalized intersecting brane solutions reduce to multi-charged black holes after all the longitudinal and the relative transverse directions are compactified. In the following, we relate a dilatonic p -brane in D spacetime dimensions to a dilatonic black hole in $D - p$ spacetime dimensions.

The Einstein-frame action for the D -dimensional dilatonic p -brane with an arbitrary dilaton coupling parameter b is given by

$$S_E^p = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-G^E} \left[\mathcal{R}_{G^E} - \frac{4}{D-2} (\partial\phi)^2 - \frac{1}{2 \cdot (p+2)!} e^{2b\phi} F_{p+2}^2 \right], \quad (27)$$

where F_{p+2} is the field strength of the $(p+1)$ -form potential $A^{(p+1)} = A_{M_1 \dots M_{p+1}} dx^{M_1} \wedge \dots \wedge dx^{M_{p+1}}$ ($M_1, \dots, M_{p+1} = 0, 1, \dots, D-1$). The non-extreme dilatonic p -brane solution to the field equations of this action has the following form:

$$\begin{aligned} ds_E^2 &= H_p^{-\frac{4(D-p-3)}{(D-2)\Delta_p}} \left[-f_p dt^2 + dx_1^2 + \dots + dx_p^2 \right] + H_p^{\frac{4(p+1)}{(D-2)\Delta_p}} \left[f_p^{-1} dr^2 + r^2 d\Omega_{D-p-2}^2 \right], \\ e^{\hat{\phi}} &= H_p^{\frac{(D-2)b}{2\Delta_p}}, \quad A_{tx_1 \dots x_p} = -H_p^{-1}, \end{aligned} \quad (28)$$

where

$$\begin{aligned} H_p &= 1 + \frac{m \sinh^2 \alpha_p}{r^{D-p-3}}, \quad f_p = 1 - \frac{m}{r^{D-p-3}}, \\ \Delta_p &= \frac{(D-2)b^2}{2} + \frac{2(p+1)(D-p-3)}{D-2}. \end{aligned} \quad (29)$$

The Bekenstein-Hawking entropy S_{BH} of the dilatonic p -brane solution (28) is determined by the surface area A_H of the horizon (located at $r = r_H = m^{\frac{1}{D-p-3}}$):

$$S_{BH} = \frac{A_H}{4G_D} = \frac{m^{\frac{D-p-2}{D-p-3}} V_{SD-p-2} \cosh^{\frac{4(p+1)(D-p-2)}{(D-2)\Delta_p}} \alpha_p}{4G_D}. \quad (30)$$

Since the p -brane solution (28) does not depend on the longitudinal coordinates x_i ($i = 1, \dots, p$), i.e., has the isometry along these directions, one can compactify the solution along the longitudinal directions on T^p to obtain a black hole solution in $D - p$ spacetime dimensions. Such dimensional reduction of the dilatonic p -brane solution (28) leads to the dilatonic black hole solution of the form (2) in $D - p$ spacetime dimensions (i.e., D in the solution (2) is replaced by $D - p$) with the dilaton coupling parameter a given by

$$a = \sqrt{\frac{D-p}{D-p-2} b^2 + \frac{4(D-p-3)^2 p}{(D-2)(D-p-2)^2}}. \quad (31)$$

The Bekenstein-Hawking entropy of such black hole solution obtained from the dilatonic p -brane through the dimensional reduction procedure has the following form:

$$S_{BH} = \frac{A_H}{4G_{D-p}} = \frac{m^{\frac{D-p-2}{D-p-3}} V_{S^{D-p-2}} \cosh^{\frac{4}{\Delta}} \alpha_p}{4G_{D-p}}, \quad (32)$$

where Δ is given by Eq. (3) with D replaced by $D-p$ and a given by Eq. (31). By using the fact that the value of Δ_p does not change under the dimensional reduction (so, Δ_p in Eq. (30) and Δ in Eq. (32) are the same) and the following relation between the D -dimensional Newton constant G_D and the $(D-p)$ -dimensional Newton constant G_{D-p} :

$$G_{D-p} = \frac{G_D}{V_{T^p}} = \frac{G_D}{\cosh^{\frac{4p(D-p-3)}{(D-2)\Delta_p}} \alpha_p}, \quad (33)$$

where V_{T^p} is the volume of T^p , on which the dilatonic p -brane is compactified, one can see that the entropy (32) of the dimensionally reduced black hole in $(D-p)$ -dimensions is the same as the entropy (30) of the dilatonic p -brane (28) in D -dimensions. In the previous section, we have shown that the Bekenstein-Hawking entropy of the dilatonic black hole is the same as the Bekenstein-Hawking entropy of the corresponding two-dimensional solution in the limit of large charge or the near-extremal limit. So, the Bekenstein-Hawking entropy (30) of the dilatonic p -brane (28) in D dimensions has to be the same as the Bekenstein-Hawking entropy of a black hole solution (17) of the associated two-dimensional dilaton gravity theory in the large charge limit or the near-extremal limit.

4 Non-Extreme Black Holes with Regular BPS Limit

In this section, we consider black holes in string theories with regular BPS limits. Such black holes are four-dimensional black hole with four charges and five-dimensional black hole with three charges. The previous related works (e.g. Refs. [17, 18, 19]) relate such black holes to two-dimensional charged black hole solution of McGuigan, Nappi and Yost [24], which has non-trivial dilaton field as well as $U(1)$ gauge field. In such works, black hole solutions which contain a charge associated with the gravitational wave are considered or U -duality transformations are applied to obtain solutions with the gravitational wave charge. Then, one takes the limit in which only the harmonic functions associated with other charges take the near-horizon limit form, while the harmonic function associated with the gravitational wave (and the harmonic function of the fundamental string with the charge assumed to be equal to the gravitational wave charge) does not take the near-horizon form. In this limit, the string-frame spacetime metric is put into the form of the direct product of the McGuigan, Nappi

and Yost black hole and a sphere after the coordinate transformation. Then, the Bekenstein-Hawking entropy of the two-dimensional black hole becomes exactly same as the Bekenstein-Hawking entropy of the $D = 4, 5$ black holes.

When one counts the microscopic degrees of freedom associated with the black hole entropy by making use of two-dimensional model or three-dimensional model of the BTZ black hole [10], one considers conformal theory at the spacetime boundary, which is associated with the gravity theory only and therefore does not have anything to do with string theories (although it is shown [34] that the BTZ black hole solution can be embedded as a solution of string theory). We also note that the form of the Einstein-frame spacetime metric, which gives rise to the Bekenstein-Hawking entropy formula, is insensitive to different ways of embedding black holes as the higher-dimensional intersecting branes in string theories or M-theory. So, it seems to be unnatural to uplift the black hole solutions to ten dimensions and apply series of U -duality transformations to make the black hole solutions carry the charge associated *specifically* with the gravitational wave of string theory for the purpose of relating the near-horizon limit spacetime metric to the BTZ black hole solution (in spacetime in one higher dimensions) or to the two-dimensional charged black hole solution of McGuigan, Nappi and Yost. Also, it seems to be unnatural to let only part of harmonic functions take near-horizon limit forms (by applying the series of U -duality transformations and the so-called shift transformation), while that associated with the gravitational wave (and fundamental string) not taking near-horizon limit form, in order to relate the $D = 4, 5$ black hole solutions to the BTZ black hole solution (and to the two-dimensional charge black hole of McGuigan, Nappi and Yost), when we take notice of the fact that all the harmonic functions of the black hole solutions in the Einstein-frame are actually on the equal footing (i.e., the Einstein-frame metric is symmetric under the permutations of harmonic functions). Such unnaturalness becomes pronounced for the particular case of the Reissner-Nordstrom black holes, i.e, the case of equal constituent charges. First, when all the charges are equal, all the scalars of the solutions are constant, but equal charge limit of the above mentioned near-horizon limit solutions does not lead to constant scalar fields since some of harmonic functions have near-horizon form and some do not. Second, it is unnatural to let some of harmonic functions not take near-horizon forms, when all the charges have the same magnitude.

So, in this section, we consider only the generic Einstein-frame spacetime metric for black hole solutions, disregarding higher-dimensional origin of constituent charges and taking all the harmonic functions on the equal footing. We will therefore not uplift the $D = 4, 5$ black holes to higher dimensions and we will let all the harmonic functions associated with the constituent charges take the near-horizon limit forms. Perhaps, our description of non-extreme black hole with regular BPS limit in this section may not lead to the correct description of black hole thermodynamics. But it seems to be

more natural from the perspective of the Einstein-frame form of the spacetime metric of black hole solutions. The generic property of stringy black holes with regular BPS limit is that in the near-horizon limit all the scalar fields (including dilaton) of the black hole solutions become constant and the spacetime metric takes the $\text{AdS}_2 \times S^n$ form. Therefore, it seems that the solution of the associated two-dimensional model should have constant dilaton field.

The most general action for the two-dimensional dilaton gravity, which depends at most on two derivatives of the fields, can be transformed to the following form [35, 36]:

$$S = \frac{1}{2\kappa_2^2} \int d^2x \sqrt{-g} [\phi \mathcal{R}_g + V(\phi)]. \quad (34)$$

The field equations of this action has a solution with constant dilaton $\phi = \phi_0$, provided that the potential $V(\phi)$ satisfies the following conditions [37]:

$$V(\phi_0) = 0, \quad \left. \frac{dV(\phi)}{d\phi} \right|_{\phi_0} \neq 0. \quad (35)$$

Then, in the conformal gauge with the following spacetime metric:

$$g_{\mu\nu} dx^\mu dx^\nu = e^{2\rho} (-dt^2 + dx^2), \quad (36)$$

the following field equations of the action (34) for a static configuration:

$$\begin{aligned} \frac{d^2\rho}{dx^2} + \frac{1}{2} e^{2\rho} \frac{dV}{d\phi} &= 0, \\ \frac{d^2\phi}{dx^2} - e^{2\rho} V &= 0, \end{aligned} \quad (37)$$

lead to the solution with constant spacetime curvature, i.e. $\phi = \phi_0$ and $\mathcal{R}_g = 2e^{-2\rho} \frac{d^2\rho}{dx^2} = -V'(\phi_0)$. This is in accordance with the fact that the near-horizon region spacetimes of the four-dimensional black hole with four charges and the five-dimensional black hole with three charges contain the AdS_2 space. The following spacetime metric solution [37] to the field equations (37) in the Schwarzschild gauge is obtained by redefining the spatial coordinate through the relation $dy = e^{2\rho} dx$:

$$ds_2^2 = - \left(\frac{\mathcal{R}_0}{2} y^2 - k \right) dt^2 + \left(\frac{\mathcal{R}_0}{2} y^2 - k \right)^{-1} dy^2, \quad (38)$$

where $\mathcal{R}_0 = -V'(\phi_0)$ is the Ricci scalar of the metric and k is an integration constant.

In the following subsections, we bring the near-horizon region metrics of the $D = 4, 5$ black holes to the form of the solution (38). Then, the study of thermodynamics of non-extreme black holes in $D = 4, 5$ with regular BPS limit reduces to the study of two-dimensional black holes with constant dilaton field.

4.1 Four-dimensional black hole

The generic form of the Einstein-frame metric of the four-dimensional black hole solution with four charges is

$$g_{\mu\nu}^E dx^\mu dx^\nu = -\frac{1}{\sqrt{H_1 H_2 H_3 H_4}} f dt^2 + \sqrt{H_1 H_2 H_3 H_4} [f^{-1} dr^2 + r^2 d\Omega_2^2], \quad (39)$$

where $f = 1 - \frac{m}{r}$ and $H_i = 1 + \frac{m \sinh^2 \alpha_i}{r}$. Here, the $U(1)$ charges $Q_i \sim m \sinh 2\alpha_i$ can have any higher-dimensional origin. Namely, the metric (39) can be the metric of a heterotic black hole solution [38, 39, 40] with the Kaluza-Klein $U(1)$ electric and magnetic charges and the NS-NS 2-form $U(1)$ electric and magnetic charges or the metric of a type-IIB black hole solution compactified from intersecting D3-branes. Regardless of various higher-dimensional origins as intersecting branes, the Einstein-frame metric for all the four-dimensional stringy black hole with four charges have the form (39).

In the near-horizon region, in which $H_i \approx \frac{m \sinh^2 \alpha_i}{r}$ ($i = 1, \dots, 4$), the metric (39) is approximated to

$$g_{\mu\nu}^E dx^\mu dx^\nu \approx -\frac{r^2}{m^2 \prod_{i=1}^4 \sinh \alpha_i} \left(1 - \frac{m}{r}\right) dt^2 + \frac{m^2 \prod_{i=1}^4 \sinh \alpha_i}{r^2} \left(1 - \frac{m}{r}\right)^{-1} dr^2 + m^2 \prod_{i=1}^4 \sinh \alpha_i d\Omega_2^2. \quad (40)$$

The two-dimensional part of this near-horizon region metric can be put into the following suggestive form of the constant dilaton solution (38) of the two-dimensional dilaton gravity by redefining the spatial coordinate as $y = r - \frac{m}{2}$:

$$ds_2^2 \approx -\left(\frac{y^2}{m^2 \prod_{i=1}^4 \sinh \alpha_i} - \frac{1}{4 \prod_{i=1}^4 \sinh \alpha_i}\right) dt^2 + \left(\frac{y^2}{m^2 \prod_{i=1}^4 \sinh \alpha_i} - \frac{1}{4 \prod_{i=1}^4 \sinh \alpha_i}\right)^{-1} dy^2. \quad (41)$$

This corresponds to two-dimensional solution with constant dilaton and constant space-time curvature $\mathcal{R}_g = -V'(\phi_0) = 2/(m^2 \prod_{i=1}^4 \sinh \alpha_i)$.

4.2 Five-dimensional black hole

The generic form of the Einstein-frame metric of the five-dimensional black hole solution in string theory with three charges, regardless of the higher-dimensional origins of charges, is as follows:

$$g_{\mu\nu}^E dx^\mu dx^\nu = -\frac{1}{(H_1 H_2 H_3)^{\frac{2}{3}}} f dt^2 + (H_1 H_2 H_3)^{\frac{1}{3}} [f^{-1} dr^2 + r^2 d\Omega_3^2], \quad (42)$$

where $f = 1 - \frac{m}{r^2}$ and $H_i = 1 + \frac{m \sinh^2 \alpha_i}{r^2}$. In the near-horizon region, in which $H_i \approx \frac{m \sinh^2 \alpha_i}{r^2}$ ($i = 1, 2, 3$), the metric (42) is approximated to

$$g_{\mu\nu}^E dx^\mu dx^\nu \approx -\frac{r^4}{m^2 \prod_{i=1}^3 \sinh^{4/3} \alpha_i} \left(1 - \frac{m}{r^2}\right) dt^2 + \frac{m \prod_{i=1}^3 \sinh^{2/3} \alpha_i}{r^2} \left(1 - \frac{m}{r^2}\right)^{-1} dr^2 + m \prod_{i=1}^3 \sinh^{2/3} \alpha_i d\Omega_3^2. \quad (43)$$

The two-dimensional part of the metric (43) can be put into the following suggestive form of the two-dimensional constant dilaton solution (38) by redefining the spatial coordinate as $y = \frac{1}{2m^{1/2} \prod_{i=1}^3 \sinh^{1/3} \alpha_i} (r^2 - \frac{m}{2})$:

$$ds_2^2 \approx -\left(\frac{4y^2}{m \prod_{i=1}^3 \sinh^{2/3} \alpha_i} - \frac{1}{4 \prod_{i=1}^3 \sinh^{4/3} \alpha_i}\right) dt^2 + \left(\frac{4y^2}{m \prod_{i=1}^3 \sinh^{2/3} \alpha_i} - \frac{1}{4 \prod_{i=1}^3 \sinh^{4/3} \alpha_i}\right)^{-1} dy^2. \quad (44)$$

This corresponds to the two-dimensional solution with the constant dilaton and the constant spacetime curvature $\mathcal{R}_g = -V'(\phi_0) = 8/(m \prod_{i=1}^3 \sinh^{\frac{2}{3}} \alpha_i)$.

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