

QCD factorization for exclusive, non-leptonic B decays

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Abstract

Exclusive, non-leptonic, two-body decays of B mesons simplify greatly in the heavy quark limit. In this talk I discuss the factorized structure that holds in this limit and some of its consequences: (a) naive factorization is recovered in a certain limit; (b) ‘non-factorizable’ effects are hard and can be calculated; (c) strong interaction phases vanish in the heavy quark limit (and can be calculated as well). As an illustration, I compute the penguin contribution to the decay $B_d \rightarrow \pi^+ \pi^-$ and its effect on the determination of $\sin 2\alpha$.

1. Introduction

Measuring asymmetries in exclusive decays of B vs. \bar{B} mesons is the primary task of four experimental facilities soon in operation. While a non-zero asymmetry establishes CP violation unambiguously, the interpretation of such a measurement in terms of fundamental theory parameters is mostly rather difficult. Hence many strategies that use different measurements in an attempt to eliminate or minimize theoretical input on strong interaction dynamics, essentially trading theoretical uncertainties for experimental ones. It would be clearly useful to have some theoretical guidance as well.

In this talk I describe the approach put forward in [1]. The result is that most non-leptonic, two-body B decays can be treated as hard processes in QCD and become simple in the heavy quark limit. The naive factorization approximation follows as the leading term in a systematic expansion. A particularly interesting consequence is that soft final state interactions vanish in the heavy quark limit. Strong phases are therefore calculable, if the form factors and light-cone distribution amplitudes of mesons are known. A simple factorized (in a sense to be explained below) expression holds only for the leading term in an expansion in Λ_{QCD}/m_b . Whether the heavy quark limit is an adequate approximation for B mesons is quite a different matter, which I do not address in this talk. For this reason, all numbers that may follow should be considered as illustrative of what can be done, once we are convinced that the heavy quark limit is good.

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2. The factorization formula

The basic objects are the matrix elements $\langle M_1 M_2 | \mathcal{O}_i | B \rangle$, where M_1 and M_2 denote the two final state mesons and \mathcal{O}_i an operator in the weak effective Hamiltonian. Such matrix elements contain both short-distance (‘hard’) effects, related to the large scale m_b , and long-distance (‘soft’, ‘collinear’) effects. The idea is to factorize (and compute) hard contributions and to parameterize soft and collinear ones, hoping that this leads to some simplification. I take the heavy quark limit, i.e. relative corrections of order Λ_{QCD}/m_b are neglected. Hard and infrared contributions will be separated by looking at Feynman diagrams, a method familiar from other hard QCD processes (hadron-hadron collisions, fragmentation, jets, hard diffraction ...). It is then assumed that factorization holds non-perturbatively. This assumption that soft effects that may not be visible from Feynman diagrams do not destroy the power counting is always implicit in factorization ‘theorems’ and I do not know of any example, where it would go wrong – but it is an assumption.

Before applying these considerations to the case at hand, note that two light final state mesons carry energy and momentum $M_B/2$. If all the mesons’ constituents have large momentum, the meson is appropriately described by light-cone distribution amplitudes. There is a finite probability for asymmetric partonic fluctuations in which a subset of partons carries almost all momentum. For example, for a pion with momentum E there is a probability of order $1/E^2$ for a $q\bar{q}$ component, in which one of the quarks carries momentum of order Λ_{QCD} . This probability can be estimated from the endpoint behaviour of the asymptotic

distribution amplitude, since the distribution amplitude approaches the asymptotic one in the high energy limit. Such endpoint contributions are not appropriately described by overlap integrals of light-cone distribution amplitudes. If unsuppressed, they require introducing more general non-perturbative parameters.

The main results are summarized as follows. Let M_1 be the meson that absorbs the light spectator quark from the B meson. M_1 can be a light meson ($M_{M_1}/m_b \rightarrow 0$ in the heavy quark limit) or a heavy meson ($M_{M_1}/m_b \rightarrow \text{finite}$). The second meson M_2 is required to be light. Then:

(1) ‘Non-factorizable’ contributions, i.e. contributions that do not belong to the $B \rightarrow M_1$ form factor or the decay constant f_{M_2} of M_2 , are dominated by hard gluon exchange. This effect can be calculated. Since hard gluons transfer large momentum to M_2 , one obtains a convolution with the light-cone distribution amplitude of M_2 rather than f_{M_2} as in naive factorization. ‘Non-factorizable’ soft exchange is suppressed, because the $q\bar{q}$ pair that forms M_2 is produced as a small colour dipole [2]. The cancellation of soft gluons is not enough to arrive at a factorization formula. It is important that ‘non-factorizable’, collinear gluons cancel as well.

(2) The $B \rightarrow M_1$ form factor is dominated by soft gluon exchange for both heavy and light M_1 by power counting. The reason for this is that the B meson contains a soft spectator quark. If M_1 is light, the endpoint suppression of the light-cone distribution amplitude of M_1 is not sufficient to render the soft contribution power suppressed. In fact, the hard gluon correction is suppressed by one power of α_s relative to the soft one. If M_2 is heavy, the light-cone distribution amplitude favours the absorption of a soft quark and the hard contribution is suppressed even further. This discussion refers to counting powers. It ignores the possibility that resummation of Sudakov logarithms suppresses the soft contribution beyond naive power counting. This possibility deserves further investigation, even though it appears unrealistic for realistic B mesons.

(3) ‘Non-factorizable’, hard gluon exchange between M_2 and the B meson spectator quark is a leading power effect. Because the gluon is hard, the interaction is local in transverse distance and can be described by the convolution of three light-cone distribution amplitudes. If the ‘non-factorizable’, hard gluon exchange occurs between M_2 and the other quarks of B and M_1 , the spectator quark can be a distance $1/\Lambda_{\text{QCD}}$ away. This implies that in this case we must keep the $B \rightarrow M_1$ form factor.

(4) Annihilation topologies and higher Fock states of M_2 give contributions suppressed by powers of Λ_{QCD}/m_b .

The observations collected above are expressed by the factorization formula

$$\begin{aligned} \langle M_1(p') M_2(q) | \mathcal{O}_i | B(p) \rangle = & \\ & F_{B \rightarrow M_1}(0) \int_0^1 dx T_i^I(x) \Phi_{M_2}(x) \\ & + \int_0^1 d\xi dx dy T_i^{II}(\xi, x, y) \Phi_B(\xi) \Phi_{M_1}(y) \Phi_{M_2}(x), \end{aligned} \quad (1)$$

where the last line accounts for the hard interaction with the spectator quark in the B meson and the equality sign is valid up to corrections of order Λ_{QCD}/m_b . T_i denote hard scattering functions that depend on longitudinal momentum fractions and Φ label the light-cone distribution amplitudes. See Fig. 1 for a graphical illustration of (1). If M_1 is heavy, the last line is suppressed and should be dropped. In this case an equation like (1) has already been used in [3]. If M_2 is a heavy, onium-like meson, factorization still holds. If, on the other hand, M_2 is a heavy-light meson, factorization does not occur.

The validity of (1) has been demonstrated by an explicit 1-loop calculation [1]. General arguments support factorization to all orders in perturbation theory. For a heavy-light final state (such as $D\pi$), an explicit demonstration of the cancellation of soft and collinear divergences at two loops exists [4].

One of the consequences of (1) is that in those cases where it applies, it also justifies naive factorization as the limit in which corrections of order α_s and Λ_{QCD}/m_b are neglected. The underlying physical picture is that the meson M_1 is produced in an asymmetric configuration, in which one quark carries almost all of the meson’s momentum, while the other meson M_2 is produced in a symmetric $q\bar{q}$ configuration which leaves the decay region with little probability for interactions.

The possibility to compute systematically logarithmic corrections to naive factorization solves the problem of scheme-dependence often discussed in the literature in the context of naive factorization. The hard scattering kernels require ultraviolet subtractions related to the renormalization of the operator \mathcal{O}_i . If performed

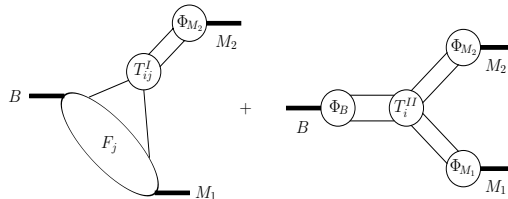


Figure 1. Graphical representation of (1).

consistently, these subtractions compensate the scheme- and scale-dependence of the Wilson coefficients in the weak effective Hamiltonian.

Another consequence of (1) is that strong final state interactions are computable. The imaginary part of the amplitude is generated by the imaginary parts of the hard scattering amplitudes only. As all other soft effects, soft final state interactions are also suppressed by a power of Λ_{QCD}/m_b and so soft strong phases disappear in the heavy quark limit. Since this statement may be a point of controversy [5], it requires further discussion.

3. Final state interactions

When discussing final state interactions, we may choose to use a partonic or hadronic language. The partonic language is justified by the dominance of hard rescattering in the heavy quark limit. In this limit the number of physical intermediate states is arbitrarily large. We may then argue on the grounds of parton-hadron duality that their average is described well enough (say, up to Λ_{QCD}/m_b corrections) by a partonic calculation. This is the picture implied by (1). The hadronic language is in principle exact. However, the large number of intermediate states makes it almost impossible to observe systematic cancellations, which usually occur in an inclusive sum of intermediate states.

To be specific, consider the decay of a B meson into two pions. Unitarity implies that

$$\text{Im } \mathcal{A}_{B \rightarrow \pi\pi} \sim \sum_n \mathcal{A}_{B \rightarrow n} \mathcal{A}_{n \rightarrow \pi\pi}^*. \quad (2)$$

The elastic rescattering contribution is related to the $\pi\pi$ scattering amplitude, which exhibits Regge behaviour in the high-energy ($m_b \rightarrow \infty$) limit. Hence the soft, elastic rescattering phase increases slowly in the heavy quark limit [5]. On general grounds, it is rather improbable that elastic rescattering gives an appropriate description in the heavy quark limit. This expectation is also borne out in the framework of Regge behaviour, see [5], where the importance of inelastic rescattering is emphasized. However, the approach pursued in [5] leaves open the possibility of soft rescattering phases that do not vanish in the heavy quark limit, as well as the possibility of systematic cancellations, for which the Regge language does not provide an appropriate theoretical framework.

Eq. (1) implies that such systematic cancellations do occur in the sum over all intermediate states n . It is worth recalling that such cancellations are not uncommon for hard processes. Consider the example of $e^+e^- \rightarrow \text{hadrons}$ at large energy q . While the production of any hadronic final

state occurs on a time scale of order $1/\Lambda_{\text{QCD}}$ (and would lead to infrared divergences if we attempted to describe it in perturbation theory), the inclusive cross section given by the sum over all hadronic final states is described very well by a $q\bar{q}$ pair that lives over a short time scale of order $1/q$. In close analogy, while each particular hadronic intermediate state n in (2) cannot be described partonically, the sum over all intermediate states is accurately represented by a $q\bar{q}$ fluctuation of small transverse size of order $1/m_b$, which therefore interacts little with its environment. Note that precisely because the $q\bar{q}$ pair is small, the physical picture of rescattering is very different from elastic $\pi\pi$ scattering – hence the Regge picture is difficult to justify in the heavy quark limit. Technically, 2-gluon exchange (plus ladder graphs) between a compact $q\bar{q}$ pair with energy of order m_b and transverse size of order $1/m_b$ and the other pion does not lead to large logarithms and hence no possibility to construct the pomeron. (Notice the difference with elastic vector meson production through a virtual photon, which also involves a compact $q\bar{q}$ pair. However, in this case one considers $s \gg Q^2$ and this implies that the $q\bar{q}$ fluctuation is born long before it hits the proton. Hence the possibility of pomeron exchange.)

It follows from (1) that the leading strong interaction phase is of order α_s in the heavy quark limit. The same statement holds for rescattering in general. For instance, according to the duality argument, a penguin contraction with a charm loop represents the sum over all intermediate states of the form $D\bar{D}$, $J/\Psi\rho$, etc. that rescatter into two pions.

As is clear from the discussion, parton-hadron duality is crucial for the validity of (1) beyond perturbative factorization. Proving quantitatively to what accuracy we can expect duality to hold is a yet unsolved problem in QCD. Short of a solution, it is worth noting that the same (often implicit) assumption is fundamental to many successful QCD predictions in jet physics, hadron-hadron physics and heavy quark decays.

4. CP asymmetry in $B_d \rightarrow \pi^+\pi^-$ decay

I now use the factorization formula to compute the time-dependent, mixing-induced asymmetry in $B_d \rightarrow \pi^+\pi^-$ decay. For reasons discussed below I do not consider this result final. However, it illustrates the predictivity of a systematic approach to calculate non-leptonic decay amplitudes.

The complete expressions for the decay amplitudes into two pions of any charge are given in [1] including α_s corrections to naive factorization. The relevant Feynman diagrams are shown in Fig. 2.

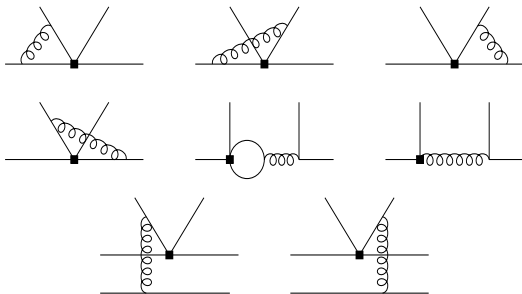


Figure 2. Order α_s corrections to the hard scattering kernels T_i^I (first two rows) and T_i^{II} (last row). In the case of T_i^I , the spectator quark does not participate in the hard interaction and is not drawn. The two lines directed upwards represent the two quarks that make up M_2 .

In the following I use the same input parameters as in [1]. The time-dependent asymmetry can be expressed as

$$A(t) = -S \cdot \sin(\Delta M_{B_d} t) + C \cdot \cos(\Delta M_{B_d} t). \quad (3)$$

In the absence of a penguin contribution (defined as the contribution to the amplitude which does not carry the weak phase γ in standard phase conventions) $S = \sin 2\alpha$ (where α refers to one of the angles of the CKM unitarity triangle) and $C = 0$. Fig. 3 shows S as a function of $\sin 2\alpha$ with the amplitudes computed according to (1). The central of the solid lines refers to the heavy quark limit including α_s corrections to naive factorization and including the power-suppressed term $a_6 r_\chi$ (for notation see [1]) that is usually also kept in naive factorization. The other two solid lines correspond to dropping this term or multiplying it by a factor of 2. This exercise shows that formally power-suppressed terms can be non-negligible (see below), but it also shows that a measurement of S can be

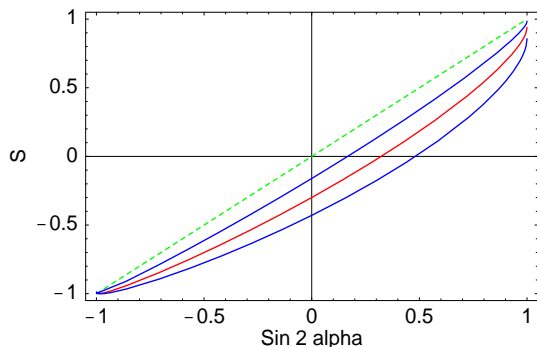


Figure 3. Coefficient of $-\sin(\Delta M_{B_d} t)$ vs. $\sin 2\alpha$. $\sin 2\beta = 0.7$ has been assumed. See text for explanation.

converted into a range for $\sin 2\alpha$ which may already provide a very useful constraint on CP violation.

5. Outlook

Much work remains to be done on the theoretical and phenomenological side. On the theoretical side, the proof of factorization for a final state of two light mesons has to be completed. Then, to define the heavy quark limit completely, one must control all logarithms of m_b . While this is straightforward for the first term on the right hand side of (1), this is less trivial for the hard scattering term, since $\xi \sim \Lambda_{\text{QCD}}/m_b$. This means that one must control logarithms of ξ that may appear in higher orders. To do this, one has to define the appropriate B meson wave function, since $\Phi_B(\xi)$ as it stands still depends on m_b . Work on these issues is in progress.

Power corrections are an important issue, as m_b is not particularly large. There exist ‘chirally enhanced’ corrections that involve the formally power suppressed, but numerically large parameter $r_\chi = 2m_\pi^2/(m_b(m_u + m_d))$. All terms involving such chiral enhancements can be identified, but they involve non-factorizable soft gluons. The size of these terms has to be estimated to arrive at a realistic phenomenology.

If this can be done, we expect promising constraints and predictions for a large number of non-leptonic, two-body final states.

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