Domain walls in supersymmetric QCD

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Abstract

We consider domain walls that appear in supersymmetric $SU(N)$ with one massive flavour. In particular, for $N \geq 3$ we explicitly construct the elementary domain wall that interpolates between two contiguous vacua. We show that these solutions are BPS saturated for any value of the mass of the matter fields. We also comment on their large N limit and their relevance for supersymmetric gluodynamics.

The study of supersymmetric gauge theories that are in the strong coupling regime has been intensified recently by the realization that some of their constructions could admit exact solutions. In particular, the issue of domain walls in SU(N) supersymmetric gluodynamics, the theory of gluons and gluinos, is one of the most exciting ones. These walls arise because this theory has an axial U(1) symmetry broken by the anomaly to a discrete Z_{2N} chiral symmetry. Due to non-perturbative effects gluino condensates ($\langle\lambda\lambda\rangle$) form, breaking the symmetry further down to Z_2. This leaves us with a set of N different vacua labelled by

$$\langle \mathrm{Tr}\lambda\lambda\rangle = \Lambda^3 e^{2\pi i k/N} \quad k = 0, 1, ..., N-1 \ , \tag{1}$$

where Λ is the condensation scale, and, as indicated above, a set of domain walls interpolating between them. If we assume that they are BPS saturated, the energy density of these walls is exactly calculable and given by [1, 2, 3]

$$\epsilon = \frac{N}{8\pi^2} |\langle \mathrm{Tr}\lambda\lambda\rangle_{\infty} - \langle \mathrm{Tr}\lambda\lambda\rangle_{-\infty}| \ , \tag{2}$$

In fact, it has been shown in Ref. [4] that, in the large N limit, these domain walls are BPS states. On the other hand, these solutions preserving half of the supersymmetry would play an important role in the D-brane description of $N = 1$ SQCD [5].

In order to get to study pure gluodynamics, it is convenient to add matter fields to the theory and analyze the limit where these extra fields (usually taken to be pairs of chiral superfields transforming as $(N, \bar{N})$ under the colour group) become very heavy. In the strong coupling regime, we should expect the formation of squark condensates. These models were considered in Refs [6, 7], for the case of $(N-1)$ flavours. Their analysis of the vaccuum structure led to the conclusion that the existence of BPS saturated domain walls was restricted to values for the mass m of the squark fields below a certain critical one. It therefore looked like it would be impossible to recover pure gluodynamics by taking the limit $m \to \infty$, and this is precisely the main issue we will address during this talk.

In order to do that let us consider supersymmetric QCD with $SU(N)$ gauge group and one couple of chiral superfields $(Q, \bar{Q})$ transforming as $N, \bar{N}$. Non-perturbative effects become relevant at the scale Λ, where condensates form. The gaugino and squark colourless condensates are described by the following composite fields

$$\begin{aligned} S &= \frac{3}{32\pi^2} \mathrm{Tr} W^2 \ , \\ M &= Q\bar{Q} \ , \end{aligned} \tag{3}$$

where W^2 is the composite chiral superfield whose lowest component is $\lambda\lambda$. In this regime, the relevant degrees of freedom are described by a Wess-Zumino model, as shown in Ref. [8]. Its effective Langrangian is given by

$$\mathcal{L} = \frac{1}{4} \int d^4\theta \, \mathcal{K} + \frac{1}{2} \left[\int d^2\theta \, \mathcal{W} + \mathrm{h.c.} \right] \ , \tag{4}$$

where $\mathcal{K}$ is the Kähler potential and $\mathcal{W}$ is the superpotential

$$\mathcal{W} = \frac{2}{3} S \ln \frac{S^{N-1} M}{\Lambda^{3N-1} e^{N-1}} - \frac{1}{2} m M \ , \tag{5}$$

with m the mass for the matter superfields. This superpotential has N extrema labeled by the different phases of the gaugino condensate. At the minimum we have the gaugino condensate fixed to

$$S_*^N = \frac{3}{4} m \Lambda^{3N-1} \ . \tag{6}$$

The matter condensate is aligned with respect to the former and given by

$$M_* = \frac{1}{m}\frac{4}{3}S_* \ . \tag{7}$$

We want to study domain wall configurations that interpolate between the different minima. Here a technical problem appears: the superpotential has several branches associated with its logarithmic piece [9]. In the pure SUSY gluodynamics limit described by Veneziano and Yankielowicz [8] this is a severe problem, since any configuration connecting two vacua has to cross this branch. This is not necessarily the case when we include other fields, given that the variation in the phase of the gaugino condensate can be partially compensated by these new fields. In this case, this will be done by matter fields.

Let $(S, M)_a$ be a particular vacuum. We can continuously deform it into another vacuum, $(S, M)_b$. For this path in the configuration space, we define δ, w such that

$$\begin{aligned} S|_b &= e^{i\delta} S|_a \ , \\ M|_b &= e^{i(\delta+2\pi w)} M|_a \ . \end{aligned} \tag{8}$$

Notice that Eq. (7) implies that w must be some integer number. On the other hand, one necessary condition to avoid crossing the logarithmic branch along this path is

$$(N-1)\delta + (\delta + 2\pi w) = 0 \ . \tag{9}$$

Since we are interested in configurations interpolating between the vacua i and $i-1$, we will fix $\delta = -\frac{2\pi}{N}$ and then $w = 1$.

If we assume that there is a BPS domain wall connecting these two vacua, it will be described by the following differential equations

$$\begin{aligned} \mathcal{K}_{S\bar{S}} \partial_z \bar{S} &= e^{i\gamma} \frac{\partial \mathcal{W}}{\partial S} \ , \\ \mathcal{K}_{M\bar{M}} \partial_z \bar{M}_i^i &= e^{i\gamma} \frac{\partial \mathcal{W}}{\partial M_i^i} \ , \end{aligned} \tag{10}$$

where $\mathcal{K}_{\phi\bar{\phi}}$ is the induced metric from the Kähler potential $\mathcal{K}$, and γ is given by

$$\gamma = -\frac{1}{2}(\delta + \pi) = \frac{\pi}{N} - \frac{\pi}{2} \ . \tag{11}$$

The configuration is described by four real functions

$$\begin{aligned} M(z) &= |M_*| \, \rho(z) e^{i\alpha(z)} \ , \\ S(z) &= |S_*| \, R(z) e^{i\beta(z)} \ . \end{aligned} \tag{12}$$

Notice that we have defined $\rho(z), R(z)$ in such a way that $\rho(\pm\infty) = R(\pm\infty) = 1$. On the other hand, α varies from 0 to $2\pi(1-1/N)$ and β from 0 to $-2\pi/N$. A consistent ansatz under reflection $z \to -z$ is given by: $\rho(z) = \rho(-z)$, $R(z) = R(-z)$, $\beta(z) = -2\pi/N - \beta(-z)$ and $\alpha(z) = 2\pi(1-1/N) - \alpha(-z)$. These relations fix the boundary conditions at $z = 0$. Eqs (10) imply the following BPS constraint

$$Im\left[e^{i\gamma}\mathcal{W}(S, M)\right] = const \ . \tag{13}$$

In Refs [6, 7] a similar analysis was presented for $N_f = N - 1$ couples of matter fields. In these papers it was shown that these domain walls are BPS states only for squark masses lower than some critical value, m_*, that depends on N and the Kähler potential. The existence of this bound is related to the presence of two different BPS domain wall solutions for small enough values of m, which became identical at the critical value.

Here we have worked in detail a different case: $N = 3$, $N_f = 1$, using the same Kähler potential, i.e. $\mathcal{K} = (S\bar{S})^{1/3} + (M\bar{M})^{1/2}$. We have found that the equations can be solved for *all* values of the squark mass, and we have checked that the logarithmic branch is never crossed. The profiles for R are shown in Fig. 1 for several values of m (given in units of Λ), focusing on their central region. The spatial coordinate z is expressed in units of $\tilde{\Lambda}^{-1}$, where $\tilde{\Lambda} = \Lambda(\frac{3m}{4\Lambda})^{1/3N}$ is the effective QCD scale that arises in the large m limit.

In our case there is only one BPS solution for every value of m. This can be understood analyzing both the large and small m limit, as explained in detail in [10]. These limits depend on the number of flavours.

Let us consider the limit $m \to \infty$, that is expected to describe pure gluodynamics. From Fig. 1 we see that there is a well defined gaugino condensate profile in that limit. In fact, the following constraints apply in the asymptotic regions

$$\begin{aligned} \rho(z) e^{i\alpha(z)} &= R(z) e^{i\beta(z)} \ \ (z << -1/m) \ , \\ \rho(z) e^{i\alpha(z)} &= R(z) e^{i(\beta(z) - 2\pi)} \ \ (z >> 1/m) \ . \end{aligned} \tag{14}$$

Therefore, using Eq. (14) in this large m limit we can get rid of α and ρ in the BPS equations. Also the BPS constraint involves only the gaugino condensate and can be written as

$$Im\left\{e^{i[\gamma+\beta(z)]} R(z) \left[\ln\left(R(z) e^{i\tilde{\beta}(z)}\right) - 1\right]\right\} = const \ , \tag{15}$$

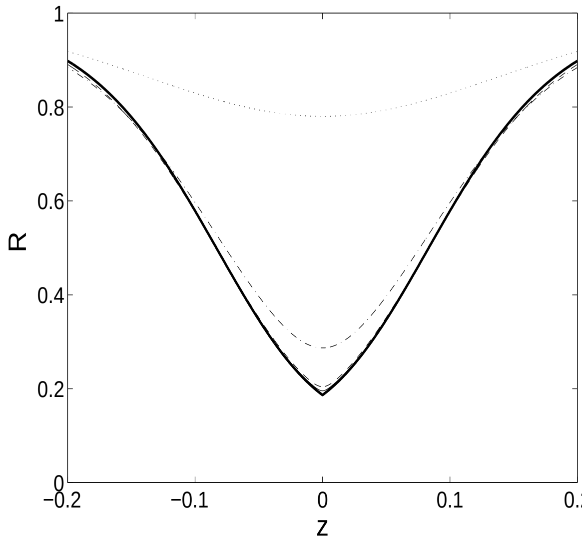

Figure 1.
$R(z)$ as defined in Eqs (12) versus z (in units of $\tilde{\Lambda}^{-1}$), for $m = 2$ (dotted line), 20 (dash-dotted), 100 (dashed), 200 (solid). The thick solid line corresponds to the $m \to \infty$ solution given by Eq. (16).

where $\tilde{\beta}(z) = \beta(z)$ for $z < 0$ and $\tilde{\beta}(z) = \beta(z) + 2\pi/N$ for $z > 0$. This constraint allows us to express β as a function of R, and we end up with the following BPS equation for $R(z)$

$$\partial_z R(z) = 6N(R(z))^{4/3}\tilde{\Lambda}\left\{\cos(\gamma + \beta([R(z)]))\ \ln R(z) - \sin(\gamma + \beta([R(z)]))\ \tilde{\beta}[R(z)]\right\} \ , \qquad (16)$$

Notice that, in the large m limit, the profile has two branches which are smoothly connected for finite values of m. In this limit, the domain wall has two typical scales. One is associated to the gaugino condensate, while the other one is associated to the additional field that compensates the phase changes of the gaugino condensate, in our case the matter field‖.

Let us now analyze the large N limit of this asymptotic configuration [11]. In order to do that we expand the normalized modulus

$$R(z) = 1 - r(z)/N + \mathcal{O}(1/N^2) \ , \qquad (17)$$

and we rewrite $\beta(z) = b(z)\frac{2\pi}{N}$. Then Eq. (15) implies

$$b(z) = -r(z) \qquad (18)$$

for the left branch, i.e. for $z < 0$. If we want to analyze the structure of the domain wall, we have to include properly the dependence of S on N. One expects [4, 5]

$$S \equiv \langle \mathrm{Tr}\lambda_\alpha\lambda^\alpha\rangle \sim \mathcal{O}(N) \ . \qquad (19)$$

Then, we have to replace Λ^3 by $N\Lambda^3$ in the above equations. This implies that the energy of the previous elementary domain wall, as given by Eq. (2), scales like $\epsilon \sim \mathcal{O}(N)$. If we want to study the width, i.e. how the energy is spread along the domain wall, we need some information about the N dependence of kinetic terms of both gaugino and regulator -matter- fields.

For example, if we assume that the gaugino kinetic term term is $\mathcal{O}(N^a)$, then $r(z)$ is given by

$$\partial_{|z|} r(|z|) = -k^2\tilde{\Lambda}N^{1-a} r(|z|) \ , \qquad (20)$$

where the precise value of the positive constant k^2 depends on the particular Kähler potential. Under this assumption, the scale associated with the gaugino field variations is $\mathcal{O}(1/(\tilde{\Lambda}N^{1-a}))$.

In summary, it is possible to build BPS domain walls in SQCD with one flavour for any value of the mass of the matter fields. This allows us to study the limit where the theory approaches pure supersymmetric gluodynamics.

‖ See [4] for other possibilities.

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