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Fluctuations from dissipation in a hot non-Abelian plasma

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Abstract

We consider a transport equation of the Boltzmann-Langevin type for non-Abelian plasmas close to equilibrium to derive the spectral functions of the underlying microscopic fluctuations from the entropy. The correlator of the stochastic source is obtained from the dissipative processes in the plasma. This approach, based on classical transport theory, exploits the well-known link between a linearized collision integral, the entropy and the spectral functions. Applied to the ultra-soft modes of a hot non-Abelian (classical or quantum) plasma, the resulting spectral functions agree with earlier findings obtained from the microscopic theory. As a by-product, it follows that Bödeker's effective theory is consistent with the fluctuation-dissipation theorem.

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It has been recognized that dynamical properties of (non-perturbative) quasi-particle excitations in non-Abelian plasmas can be described very efficiently by means of effective transport equations. A prominent recent example is given by Bödeker's effective classical theory for the ultra-soft modes in a hot non-Abelian plasma close to equilibrium [1], which corresponds to a transport equation of the Boltzmann-Langevin type. In [2], a general procedure has been presented, based on classical coloured point particles, to obtain effective transport equations from the microscopic theory after integrating-out the fluctuations about the mean values, and taking the Gibbs ensemble average in phase space. On the one-loop level, the same mean field equations of [2] have been obtained recently within a many-particle world line formalism [3] (see also [4]). The collision integral and the source of stochastic noise of [1] have been obtained from [2] to leading order in a weak coupling expansion, and at logarithmic accuracy. It was also realized that the dynamical equations are the same for the classical and the quantum plasma, changing only in the value of the Debye mass [2]. Other approaches to obtain the collision term of [1] have been reported as well [5–7].

In the present Letter we consider, based on classical transport theory, a generic Boltzmann-Langevin equation for the one-particle distribution function $f(x,p,Q)$, given as

$$p^\mu\left(\frac{\partial}{\partial x^\mu} - g f^{abc} A^b_\mu Q^c \frac{\partial}{\partial Q^a} - g Q_a F^a_{\mu\nu}\frac{\partial}{\partial p_\nu}\right) f(x,p,Q) = C[f](x,p,Q) + \zeta(x,p,Q)\ . \qquad (1)$$

Here, the variables Q describe the non-Abelian colour charges. The transport equation contains an effective collision term $C[f]$ and an associated source for stochastic noise. The SU(N) gauge fields appearing in the above equation are self-consistent, that is, generated by the same particles of the plasma. The Yang-Mills equation are

$$\left(D_\mu F^{\mu\nu}\right)_a = J^\mu_a(x) = g \sum_{\substack{\text{helicities}\\ \text{species}}} \int dP\, dQ\, Q_a\, p^\mu\, f(x,p,Q)\ , \qquad (2)$$

where the momemtum measure reads $dP = d^4p 2\Theta(p_0)\delta(p^2-m^2)$, and the colour measure dQ was defined in [2]. We work in natural units $c=\hbar=k_B=1$, unless otherwise specified. From now on we will omit the sum over different species of particles and helicities. In the collisionless limit $C=\zeta=0$, the above set of transport equation reduces to those introduced by Heinz [8]. In the general case however, the r.h.s. of (1) does not vanish due to effective interactions (collisions) in the plasma, resulting in the term $C[f]$. In writing (1), we have already made the assumption that the one-particle distribution function f is a fluctuating quantity. This is quite natural having in mind that f describes a 'coarse-grained' microscopic distribution function for coloured point particles, and justifies the presence of the stochastic source ζ in the transport equation. For non-charged particles, a similar kinetic equation has already been considered in [9] (see also [10], where stochastic noise is introduced to a Schwinger-Dyson approach).

Given the stochastic dynamical equation (1), the question raises as to what can be said on general grounds about the spectral functions of f and ζ. Here, we shall assume that the dissipative processes are known close to equilibrium, but no further information is given regarding the underlying fluctuations. This way of proceeding is complementary to [2], where the r.h.s. of (1) has been obtained from correlators of the microscopic statistical fluctuations. We then show that the spectral function of the fluctuations and the noise correlator close to equilibrium can be obtained from the knowledge of the entropy of the plasma, and from the dissipative term in the effective transport equation. This gives a well-defined prescription as to how the correct source for noise can be identified without the detailed knowledge of the underlying microscopic dynamics responsible for the dissipation. The basic idea behind this approach relies on the essence of the fluctuation-dissipation theorem (FDT). While this theorem is more general, here we will only discuss the close to equilibrium situations. According to the FDT if a fluctuating system remains close to equilibrium, then the dissipative process occurring in it are known. Vice versa, if one knows the dissipative process in the system, one can describe the fluctuations without an explicit knowledge of the microscopic structure or processes in the system. The cornerstone of our approach is the entropy of the fluctuating system, which serves to identify the thermodynamical forces, and leads to the spectral function for the deviations from the non-interacting equilibrium.

Before entering into the discussion of plasmas, we will illustrate this way of proceeding by reviewing the simplest setting of classical linear dissipative systems [11]. A generalization to the more complex case of non-Abelian plasmas will then become a natural step to perform. We consider a classical homogeneous system described by a set of variables x_i, where i is a discrete index running from 1 to n. These variables are normalized in such a way that their mean values at equilibrium vanish. The entropy of the system is a function of the quantities x_i, $S(x_i)$. If the system is at equilibrium, the entropy reaches its maximum, and thus $(\partial S/\partial x_i)_{\rm eq} = 0$, $\forall i$. If the system is taken slightly away from equilibrium, then one can expand the difference $\Delta S = S - S_{\rm eq}$, where $S_{\rm eq}$ is the entropy at equilibrium, in powers of x_i. If we expand up to quadratic order, then

$$\Delta S = \frac{1}{2}\left(\frac{\partial^2 S}{\partial x_i \partial x_j}\right)_{\rm eq} x^i x^j \equiv -\frac{1}{2}\beta_{ij} x^i x^j \ . \tag{3}$$

The matrix β_{ij} is symmetric and positive-definite, since the entropy reaches a maximum at equilibrium. The thermodynamic forces F_i are defined as the gradients of ΔS

$$F_i = -\frac{\partial \Delta S}{\partial x_i} \ . \tag{4}$$

For a system close to equilibrium the thermodynamic forces are linear functions of x_i, $F_i = \beta_{ij} x^j$. If the system is at equilibrium, the thermodynamic forces vanish. In more general situations the variables x_i will evolve in time. The time evolution of these variables is given as functions of the thermodynamical forces. In a close to equilibrium case one can expect that the evolution is linear in the forces

$$\frac{dx^i}{dt} = -\gamma^{ij} F_j + \zeta^i \ , \tag{5}$$

which, in turn, can be expressed as

$$\frac{dx^i}{dt} = -\lambda^{ij} x_j + \zeta^i \ , \tag{6}$$

The first term in the r.h.s. of the above equation describes the mean regression of the system towards equilibrium, while the second term is the source for stochastic noise. The quantities γ^{ij} are known as the kinetic coefficients, and it is not difficult to check that $\gamma_{ij} = \lambda_{ik}\beta^{-1}_{kj}$. From the value of the coefficients β_{ij} one can deduce the equal time correlator

$$\langle x_i(t) x_j(t) \rangle = \beta^{-1}_{ij} \ , \tag{7}$$

which is used to obtain Einstein's law

$$\Big\langle x^i(t) F_j(t) \Big\rangle = \delta^i_j \ . \tag{8}$$

After taking the time derivative of (8), assuming that the noise is white and Gaussian

$$\Big\langle \zeta^i(t) \zeta^j(t') \Big\rangle = \nu^{ij} \delta(t - t') \ , \tag{9}$$

we find that the strength of the noise self-correlator ν is determined by the dissipative process

$$\nu^{ij} = \gamma^{ij} + \gamma^{ji} \ , \tag{10}$$

which is the FDT relation we have been aiming at.

We now come back to the case of a non-Abelian plasma and generalize the above discussion to the case of our concern. We will consider the non-Abelian plasma as a linear dissipative system, assuming that we know the collision term in the transport equation. In order to adopt the previous reasoning, we have to identify the dissipative term in the transport equation, and to express it as a function of the thermodynamical force obtained from the entropy. The deviation from the equilibrium distribution is given here by

$$\Delta f(x, p, Q) = f(x, p, Q) - f_{\rm eq}(p_0) \ , \tag{11}$$

and replaces the variables x_i discussed above. The deviation Δf goes as $\mathcal{O}(g)$ to leading order in a small gauge coupling expansion. The entropy flux density for classical plasmas is given as

$$S_\mu(x) = - \int dP dQ \, p_\mu \, f(x, p, Q) \Big(\ln \left(f(x, p, Q) h^3 \right) - 1 \Big) \ , \tag{12}$$

where h is an arbitrary constant such that $f(x, p, Q)h^3$ is dimensionless. The $\mu = 0$ component of (12) gives the entropy density of the system. The entropy itself is then obtained as $S = \int d^3x \, S_0(x)$.

We shall now assume that the deviation from the equilibrium distribution is small, $\Delta f \ll f_{\rm eq}$, which can always be arranged for at small gauge coupling $g \ll 1$. We then obtain ΔS just by expanding the expression of the entropy density in powers of Δf up to quadratic order. It is important to take into account that we will consider situations where the small deviations from equilibrium are such that both the particle number and the energy flux remain constant, thus

$$\int dP dQ \, \Phi(p) \, \Delta f(x, p, Q) = 0 \ , \quad \text{for} \quad \Phi(p) = p_0, \, p_0 p_\mu \, . \tag{13}$$

Under those assumptions, one reaches to

$$\Delta S_0(x) = -\int dP dQ\, p_0 \frac{(\Delta f(x,p,Q))^2}{f_{\rm eq}(p_0)}$$
$$= -\int d^3p\, dQ \frac{(\Delta f(x,\mathbf{p},Q))^2}{f_{\rm eq}(\omega_p)} \;, \qquad (14)$$

where in the last equality we have taken into account the mass-shell condition, with $p_0 = \omega_p = \sqrt{\mathbf{p}^2 + m^2}$. Without loss of generality, we will consider from now on the case of massless particles, so $\omega_p = p = |\mathbf{p}|$.

The thermodynamic force associated to Δf is defined from the entropy as

$$F(x,\mathbf{p},Q) = -\frac{\delta \Delta S}{\delta \Delta f(x,\mathbf{p},Q)} = 2\,\frac{\Delta f(x,\mathbf{p},Q)}{f_{\rm eq}(p)} \qquad (15)$$

We now linearize the transport equation (1) and express the collision integral close to equilibrium in terms of the thermodynamical force. Dividing (1) by p_0 and imposing the mass-shell constraint, we find

$$v^\mu D_\mu \Delta f - g v^\mu Q_a F^a_{\mu 0} \frac{df_{\rm eq}}{dp} = C[\Delta f](x,\mathbf{p},Q) + \zeta(x,\mathbf{p},Q) \;, \qquad (16)$$

where $v^\mu = p^\mu/p_0 = (1,\mathbf{v})$, with $\mathbf{v}^2 = 1$. We also introduced the shorthand $D_\mu \Delta f \equiv (\partial_\mu - g f^{abc} A_{\mu,b} Q_c \partial^Q_a)\Delta f$ [2]. It is understood that the collision integral has been linearized, and we write it as

$$C[\Delta f](t,\mathbf{x},\mathbf{p},Q) = \int d^3x' d^3p'\, dQ'\, K(\mathbf{x},\mathbf{p},Q;\mathbf{x}',\mathbf{p}',Q')\Delta f(t,\mathbf{x}',\mathbf{p}',Q') \,, \qquad (17)$$

with $t \equiv x_0$. For simplicity, we take the collision integral local in time, but unrestricted elsewise. According to the FDT, the source of stochastic noise has to obey

$$\langle \zeta(x,\mathbf{p},Q)\zeta(x',\mathbf{p}',Q')\rangle = -\left(\frac{\delta C[F](x,\mathbf{p},Q)}{\delta F(x',\mathbf{p}',Q')} + \frac{\delta C[F](x',\mathbf{p}',Q')}{\delta F(x,\mathbf{p},Q)}\right) \qquad (18)$$

in full analogy to (10). With the knowledge of the thermodynamical force (15) and the linearized collision term (17) we arrive at

$$\langle \zeta(x,\mathbf{p},Q)\zeta(x',\mathbf{p}',Q')\rangle = -\left(\tfrac{1}{2} f_{\rm eq}(p) K(\mathbf{x},\mathbf{p},Q;\mathbf{x}',\mathbf{p}',Q') + \text{sym.}\right)\delta(t-t') \;. \qquad (19)$$

Here, symmetrisation in $(\mathbf{x},\mathbf{p},Q) \leftrightarrow (\mathbf{x}',\mathbf{p}',Q')$ is understood.

Notice that we can derive the equal time correlator for the deviations from the equilibrium distribution simply from the knowledge of the entropy and the thermodynamical force, exploiting Einstein's law in full analogy to the corresponding relation (7). Using (15) we find

$$\langle \Delta f(x,\mathbf{p},Q)\Delta f(x',\mathbf{p}',Q')\rangle_{t=t'} = f_{\rm eq}(p)\delta^{(3)}(\mathbf{x}-\mathbf{x}')\delta^{(3)}(\mathbf{p}-\mathbf{p}')\delta(Q-Q') \;. \qquad (20)$$

If the fluctuations Δf have vanishing mean value, then (20) reproduces the well-known result that the correlator of fluctuations at equilibrium is given by the equilibrium distribution function. In order to make contact with the results of [2], we go a step further and consider the case where Δf has a non-vanishing mean value to leading order in the gauge coupling. Splitting $\Delta f = g\bar{f}^{(1)} + \delta f$ into a deviation of the mean part $\langle \Delta f \rangle = g\bar{f}^{(1)}$ and a fluctuating part $\langle \delta f \rangle = 0$ and using (20), we obtain the equal time correlator for the fluctuations δf as

$$\begin{aligned} \langle \delta f(x, \mathbf{p}, Q) \delta f(x', \mathbf{p}', Q') \rangle_{t=t'} &= f_{\rm eq}(p) \delta^{(3)}(\mathbf{x} - \mathbf{x}') \delta^{(3)}(\mathbf{p} - \mathbf{p}') \delta(Q - Q') \\ &\quad - g^2 \bar{f}^{(1)}(x, \mathbf{p}, Q) \bar{f}^{(1)}(x', \mathbf{p}', Q') \Big|_{t=t'} \ . \end{aligned} \tag{21}$$

This result agrees with the correlator obtained in [2] from the Gibbs ensemble average as defined in phase space in the limit where two-particle correlations are small and given by products of one-particle correlators.

Up to now we have dealt with purely classical plasmas. On the same footing, we can consider the soft and ultra-soft modes in a hot quantum plasma. These can be treated classically as their occupation numbers are large. The sole effect from their quantum nature reduces to the different statistics, Bose-Einstein or Fermi-Dirac as opposed to Maxwell-Boltzmann. The corresponding quantum FDT reduces to an effective classical one [11,12].

Some few changes are necessary to study hot quantum plasmas. As in [2], we change the normalisation of f by a factor of $(2\pi\hbar)^3$ to obtain the standard normalisation for the (dimensionless) quantum distribution function. Thus, the momentum measure is also modified by the same factor, $dP = d^4p 2\Theta(p_0)\delta(p^2)/(2\pi\hbar)^3$ for massless particles, and $\hbar = 1$. To check the FDT relation in this case one needs to start with the correct expression for the entropy for a quantum plasma. The entropy flux density, as a function of $f(x, p, Q)$, is given by

$$S_\mu(x) = -\int dP dQ \, p_\mu \Big(f \ln f \mp (1 \pm f) \ln (1 \pm f) \Big) \ , \tag{22}$$

where the upper/lower sign applies for bosons/fermions. From the above expression of the entropy one can compute ΔS, and proceed exactly as in the classical case, expanding the entropy up to quadratic order in the deviations from equilibrium. Thus, we obtain the noise correlator

$$\langle \zeta(x, \mathbf{p}, Q) \zeta(x', \mathbf{p}', Q') \rangle = -(2\pi)^3 \Big(\tfrac{1}{2} f_{\rm eq}(p)(1 \pm f_{\rm eq}(p)) K(\mathbf{x}, \mathbf{p}, Q; \mathbf{x}', \mathbf{p}', Q') + \text{sym.} \Big) \delta(t - t') \ . \tag{23}$$

Again, the spectral functions of the deviations from equilibrium are directly deduced from the entropy. As a result, we find

$$\langle \Delta f(x, \mathbf{p}, Q) \Delta f(x', \mathbf{p}', Q') \rangle_{t=t'} = (2\pi)^3 f_{\rm eq}(p)(1 \pm f_{\rm eq}(p)) \delta^{(3)}(\mathbf{x} - \mathbf{x}') \delta^{(3)}(\mathbf{p} - \mathbf{p}') \delta(Q - Q') \ . \tag{24}$$

Expanding $\Delta f = g\bar{f}^{(1)} + \delta f$ as above, we obtain the equal time correlator for δf, which agrees with the findings of [2] in the case where two-particle distribution functions can be expressed as products of one-particle distributions.

With the knowledge of the above spectral functions for the fluctuations in a classical or quantum plasma one can derive further spectral distributions for different physical quantities. In particular, we can find the correlations of the self-consistent gauge field fluctuations once the basic correlators as given above are known. This is how those spectral functions were deduced in [2].

As a particular example of the above, we consider the dynamical equations for the ultra-soft modes with $p \ll m_D$, where m_D is the Debye mass in a non-Abelian plasma close to equilibrium. The linearized collision integral has been obtained to leading logarithmic accuracy by several different approaches [1,2,5–7]. They all employ an IR cut-off of the order of gm_D for the elsewise unscreened magnetic modes.

We will first concentrate on the classical plasma, for particles carrying two helicities. It is most efficient to write the transport equation not in terms of the full one-particle distribution function, but in terms of the current density

$$\mathcal{J}_a^\rho(x,\mathbf{v}) = 8\pi\, g\, v^\rho \int dp\, dQ\, p^2\, Q_a\, \Delta f(x,\mathbf{p},Q)\,. \tag{25}$$

(Notice that $f_{\rm eq}$ gives no contribution to the current.) The current of (2) follows after integrating over the angles of $\mathbf{v}$, $J_a^\mu(x) = \int \frac{d\Omega}{4\pi} \mathcal{J}_a^\rho(x,\mathbf{v})$ [2]. Expressed in terms of (25), the linearized Boltzmann-Langevin equation (16) becomes

$$[v^\mu D_\mu, \mathcal{J}^\rho](x,\mathbf{v}) = -m_D^2 v^\rho v_\mu F^{\mu 0}(x) + v^\rho C[\mathcal{J}^0](x,\mathbf{v}) + \zeta^\rho(x,\mathbf{v})\,. \tag{26}$$

where m_D is the Debye mass [2]

$$m_D^2 = -8\pi g^2 C_2 \int dp\, p^2\, \frac{df_{\rm eq}}{dp} \tag{27}$$

and the constant C_2 depends on the representation of the coloured particles

$$\int dQ\, Q_a Q_b = C_2 \delta_{ab}\ . \tag{28}$$

The linearized collision integral is related to (17) by

$$C[\mathcal{J}_a^0](x,\mathbf{v}) = 8\pi\, g \int d^3x'\, d\Omega_{\mathbf{v}'}\, dp\, dp'\, dQ\, dQ'\, p^2 p'^2\, Q_a\, K(\mathbf{x},\mathbf{p},Q;\mathbf{x}',\mathbf{p}',Q')\, \Delta f(t,\mathbf{x}',\mathbf{p}',Q') \tag{29}$$

and has been obtained explicitly [1,2,5–7] as

$$C[\mathcal{J}_a^0](x,\mathbf{v}) = -\gamma \int \frac{d\Omega_{\mathbf{v}'}}{4\pi}\, \mathcal{I}(\mathbf{v},\mathbf{v}') \mathcal{J}_a^0(x,\mathbf{v}') \tag{30}$$

where the kernel reads

$$\mathcal{I}(\mathbf{v},\mathbf{v}') = \delta^{(2)}(\mathbf{v}-\mathbf{v}') - \frac{4}{\pi} \frac{(\mathbf{v}\cdot\mathbf{v}')^2}{\sqrt{1-(\mathbf{v}\cdot\mathbf{v}')^2}} \tag{31}$$

and $\gamma = g^2 NT \ln(1/g)/4\pi$. Comparing (29) with (30) we learn that only the part of the kernel K which is symmetric under $(\mathbf{x}, \mathbf{p}, Q) \leftrightarrow (\mathbf{x}', \mathbf{p}', Q')$ contributes in the present case. This part can be expressed as

$$K(\mathbf{x}, \mathbf{p}, Q; \mathbf{x}', \mathbf{p}', Q') = -\gamma \frac{\mathcal{I}(\mathbf{v}, \mathbf{v}')}{4\pi p^2} \delta(p - p')\delta(Q - Q')\delta^{(3)}(\mathbf{x} - \mathbf{x}') \,. \tag{32}$$

According to our findings above, the self-correlator of the stochastic source for the classical plasma obeys

$$\begin{aligned} \langle \zeta_a^\mu(x, v)\, \zeta_b^\nu(y, v') \rangle &= 2^2 (4\pi)^2\, g^2 \int dp\, dp'\, dQ\, dQ'\, p^2 p'^2\, Q_a Q_b'\, v^\mu v'^\nu\, \langle \zeta(x, \mathbf{p}, Q)\, \zeta(y, \mathbf{p}', Q') \rangle \\ &= 2\, \gamma\, T\, m_D^2\, v^\mu v'^\nu\, \mathcal{I}(\mathbf{v}, \mathbf{v}')\, \delta_{ab}\, \delta^{(4)}(x - y) \ . \end{aligned} \tag{33}$$

The factor of 2^2 accounts for the helicities of the particles. In order to obtain (33), we have made use of (19), (27) to (30), and of the relation $f_{\rm eq} = -T\, df_{\rm eq}/dp$ for the Maxwell-Boltzmann distribution.

The quantum plasma can be treated in exactly the same way. To confirm (33), we only need to take into account the change of normalization as commented above, and the relation $f_{\rm eq}(1 \pm f_{\rm eq}) = -T\, df_{\rm eq}/dp$ for the Bose-Einstein and Fermi-Dirac distributions, respectively.

We thus found that the correlator (33) is in full agreement with the result of [1,2] for both the classical or the quantum plasma. While this correlator has been obtained in [1,2] from the corresponding microscopic theory, here, it follows solely from the FDT. This way, it is established that the effective Boltzmann-Langevin equation found in [1] is indeed fully consistent with the fluctuation-dissipation theorem. More generally, the important observation is that the spectral functions as derived here from the entropy and the FDT do agree with those obtained in [2] from a microscopic phase space average. This guarantees that the formalism of [2] is consistent with the FDT.

In the above discussion we have considered the stochastic noise as Gaussian and Markovian. These characteristics can be understood from the formalism in [2] as a consequence of the small coupling expansion to leading logarithmic accuracy. More precisely, the noise follows to be Gaussian due to the second moment approximation, valid for small couplings, which allows to neglect higher order correlators beyond quadratic ones. The Markovian character of the noise follows because the ultra-soft modes are well separated from the soft ones, and suppressed in the collision integral at leading logarithmic order. This way, the collision term and the correlator of stochastic noise are both local in x-space. Going beyond the leading logarithmic approximation, we expect from the explicit computation in [2] that the coupling of the soft and the ultra-soft modes makes the collision term non-local in coordinate space. This non-trivial memory kernel should also result in a non-Markovian, but still Gaussian, source for stochastic noise.

The present line of reasoning can in principle be extended to other approaches. Using the phenomenological derivation of (30) from [5], the same arguments as above justify the presence of a noise source with (33) in the corresponding Boltzmann equation [5–7]. It might also be fruitful to follow a similar line based on the entropy within a quantum field theoretical language. An interesting proposal to include self-consistently the noise within a

Schwinger-Dyson approach has been made recently in [10]. Along these lines, it might be feasible to derive the source for stochastic noise directly from the quantum field theory [6].

While we have concentrated the discussion on plasmas close to equilibrium, it is known that a fluctuation-dissipation theorem can be formulated as well for (stationary and stable) systems out-of-equilibrium [12]. It can also be extended to take non-linear effects into account [13]. Both the out of equilibrium situations and non-linear effects can be treated, in principle, with the general formalism presented in [2].

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