Dirac Relation and Renormalization Group Equations for Electric and Magnetic Fine Structure Constants

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Abstract

The quantum field theory describing electric and magnetic charges and revealing a dual symmetry was developed in the Zwanziger formalism. The renormalization group (RG) equations for both fine structure constants – electric α and magnetic $\tilde{\alpha}$ – were obtained. It was shown that the Dirac relation is valid for the renormalized α and $\tilde{\alpha}$ at the arbitrary scale, but these RG equations can be considered perturbatively only in the small region: $0.25 \overset{<}{\sim} \alpha, \tilde{\alpha} \overset{<}{\sim} 1$ with $\tilde{\alpha}$ given by the Dirac relation: $\alpha\tilde{\alpha} = \frac{1}{4}$.

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1 Introduction

The existence of the renormalization group (RG) in the quantum field theory was discovered by E.C.G.Stueckelberg and A.Peterman [1]. RG techniques were successfully developed by Gell–Mann and Low [2] in their investigation of the effective charge behavior. They first noticed that the derivative $d\log\alpha(p)/dt$ is only a function of the effective fine structure constant:

$$\alpha(p) = \frac{e^2(p)}{4\pi} \tag{1}$$

where $e(p)$ is the effective charge, p is a 4–momentum and

$$t = \log\frac{p^2}{M^2} \tag{2}$$

with M as a momentum cut–off.

In gauge theories without monopoles the Gell–Mann–Low RG equation has the following form:

$$\frac{d\log\alpha(p)}{dt} = \beta(\alpha(p)) \tag{3}$$

where the function $\beta(\alpha)$ depends on the Lagrangian describing the theory.

At sufficiently small charge ($\alpha < 1$) the β–function is given by a series over $\alpha/4\pi$:

$$\beta(\alpha) = \beta_2(\frac{\alpha}{4\pi}) + \beta_4(\frac{\alpha}{4\pi})^2 + ... \tag{4}$$

The first two terms of this series were calculated in QED a long time ago [3,4]. The following result was obtained in the framework of the perturbation theory (in the one– and two–loop approximations):

a)

$$\beta_2 = \frac{4}{3}, \qquad \beta_4 = 4 \qquad - \qquad \text{for fermion (electron) loops} \tag{5}$$

and

b)

$$\beta_2 = \frac{1}{3}, \qquad \beta_4 = 1 \qquad - \qquad \text{for scalar particle loops.} \tag{6}$$

This result means that for both cases a) and b) the β–function can be represented by the following series:

$$\beta(\alpha) = \beta_2(\frac{\alpha}{4\pi})(1 + 3\frac{\alpha}{4\pi} + ...) \tag{7}$$

and we are able to use the one–loop approximation up to $\alpha \sim 1$ (with accuracy $\approx 25\%$ for $\alpha \approx 1$).

In the present paper we consider the Abelian quantum field theory when both electrically and magnetically charged particles with charges e and g, respectively, present in the theory which we call below QEMD ("quantum electromagnetodynamics") following the terminology used in Ref.[5]. In their review [5], M.Blagojevic and P.Senjanovic described the various formulations [6-15] of the Abelian quantum field theories containing two coupling constants (electric and magnetic charges) connected via the quantization condition. Several topics were treated there: Dirac's and Schwinger's quantum mechanics of the monopole, connection with non-Abelian monopoles, a supersymmetric generalization of the theory and other aspects.

The aim of this paper is to investigate in QEMD the corresponding RG equations for electric (α) and magnetic ($\tilde{\alpha}$) fine structure constants in accordance with the Dirac relation for the minimal charges:

$$eg = 2\pi, \tag{8}$$

or

$$\alpha\tilde{\alpha} = \frac{1}{4} \tag{9}$$

where

$$\tilde{\alpha} = \frac{g^2}{4\pi}. \tag{10}$$

Below we consider QEMD in the Zwanziger formalism [9].

2 The Zwanziger formalism for the Abelian gauge theory with electric and magnetic charges

A version of the local field theory for the Abelian gauge fields interacting with electrically and magnetically charged particles is represented by the Zwanziger formalism [9,10] (see also [16]) which considers two potentials $A_\mu(x)$ and $B_\mu(x)$ describing one physical photon with two physical degrees of freedom.

In this theory the total field system of the gauge, electrically- (Ψ) and magnetically-charged (Φ) fields is described by the partition function which has the following form in Euclidean space:

$$Z = \int [DA][DB][D\Phi][D\bar{\Phi}][D\Psi][D\bar{\Psi}]e^{-S} \tag{11}$$

where

$$S = S_{Zw}(A, B) + S_{gf} + S_e + S_m. \tag{12}$$

The Zwanziger action $S_{Zw}(A, B)$ is given by:

$$S_{Zw}(A, B) = \int d^4x \left[\frac{1}{2}(n \cdot [\partial \wedge A])^2 + \frac{1}{2}(n \cdot [\partial \wedge B])^2 + \right.$$

$$\left. +\frac{i}{2}(n \cdot [\partial \wedge A])(n \cdot [\partial \wedge B]^*) - \frac{i}{2}(n \cdot [\partial \wedge B])(n \cdot [\partial \wedge A]^*)\right] \tag{13}$$

where we have used the following designations:

$$[A \wedge B]_{\mu\nu} = A_\mu B_\nu - A_\nu B_\mu, \quad (n \cdot [A \wedge B])_\mu = n_\nu (A \wedge B)_{\nu\mu},$$

$$G^*_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}G_{\lambda\rho}, \tag{14}$$

with n^μ representing the direction of the frozen Dirac string.

S_{gf} in Eq.(12) is the gauge–fixing action and the actions S_e and S_m:

$$S_{e,m} = \int d^4x L_{e,m}(x) \tag{15}$$

describe the matter fields with the electric and magnetic charges, respectively.

Here we have a number of possibilities. The electrically- and magnetically-charged fields can be described by the following Lagrangian expressions (given in Minkowski space):

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a)

$$L_e = \bar{\Psi}\gamma_\mu(iD_\mu - \mu_e)\Psi, \tag{16}$$

$$L_m = \bar{\Phi}\gamma_\mu(i\tilde{D}_\mu - \mu_m)\Phi \tag{17}$$

if they are fermions (electrons or monopoles, respectively). In Eqs.(16), (17)

$$D_\mu = \partial_\mu - ieA_\mu \tag{18}$$

and

$$\tilde{D}_\mu = \partial_\mu - igB_\mu \tag{19}$$

are the covariant derivatives.

b)

$$L_e = \frac{1}{2}[|D_\mu\Psi|^2 - \mu_e{}^2|\Psi|^2], \tag{20}$$

$$L_m = \frac{1}{2}[|\tilde{D}_\mu\Phi|^2 - \mu_m{}^2|\Phi|^2] \tag{21}$$

if the electrically- and magnetically-charged particles are the Klein–Gordon complex scalars. But for the Higgs scalars with electric and magnetic charges we have the following Lagrangians:

c)

$$L_e = \frac{1}{2}|D_\mu\Psi|^2 - U(|\Psi|), \tag{22}$$

$$L_m = \frac{1}{2}|\tilde{D}_\mu\Psi|^2 - U(|\Phi|) \tag{23}$$

where

$$U(|\Psi|) = \frac{1}{2}\mu_e^2|\Psi|^2 + \frac{\lambda_e}{4}|\Psi|^4 \tag{24}$$

and

$$U(|\Phi|) = \frac{1}{2}\mu_m^2|\Phi|^2 + \frac{\lambda_m}{4}|\Phi|^4 \tag{25}$$

are the Higgs potentials for the electrically- and magnetically-charged fields, respectively. The complex scalar fields:

$$\Phi = \phi + i\chi_1 \qquad \text{and} \qquad \Psi = \psi + i\chi_2 \tag{26}$$

contain Higgs (ϕ, ψ) and Goldstone (χ_1, χ_2) boson fields.

Below we shall consider the gauge-fixing action S_{gf} chosen in Ref.[16]:

$$S_{gf} = \int d^4x[\frac{M_A^2}{2}(n \cdot A)^2 + \frac{M_B^2}{2}(n \cdot B)^2] \tag{27}$$

which has no ghosts.

3 Dual Symmetry and Charge Quantization Conditions

In the last years gauge theories essentially operate with the fundamental idea of duality (see, for example, reviews [8] and references there).

Duality is a symmetry appearing in free electromagnetism as invariance of the free Maxwell equations:

$$\nabla \cdot \vec{\mathbf{B}} = \mathbf{0}, \qquad \nabla \times \vec{\mathbf{E}} = -\partial_0 \vec{\mathbf{B}}, \tag{28}$$

$$\nabla \cdot \vec{\mathbf{E}} = \mathbf{0}, \qquad \nabla \times \vec{\mathbf{B}} = \partial_0 \vec{\mathbf{E}}, \tag{29}$$

under the interchange of electric and magnetic fields:

$$\vec{\mathbf{E}} \to \vec{\mathbf{B}}, \qquad \vec{\mathbf{B}} \to -\vec{\mathbf{E}}. \tag{30}$$

Letting

$$F = \partial \wedge A = -(\partial \wedge B)^*, \tag{31}$$

$$F^* = \partial \wedge B = (\partial \wedge A)^*, \tag{32}$$

it is easy to see that the following equations:

$$\partial_\lambda F_{\lambda\mu} = 0, \tag{33}$$

which together with the Bianchi identity:

$$\partial_\lambda F^*_{\lambda\mu} = 0 \tag{34}$$

are equivalent to Eqs.(28), show invariance under the Hodge star operation on the field tensor:

$$F^*_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma} \tag{35}$$

(here $F^{**} = -F$).

This Hodge star duality applied to the free Zwanziger Lagrangian (13) leads to its invariance under the following duality transformations:

$$F \leftrightarrow F^*, \qquad (\partial \wedge A) \leftrightarrow (\partial \wedge B), \qquad (\partial \wedge A)^* \leftrightarrow -(\partial \wedge B)^*. \tag{36}$$

Introducing the interacting Maxwell equations:

$$\partial_\lambda F_{\lambda\mu} = j^e_\mu, \tag{37}$$

$$\partial_\lambda F^*_{\lambda\mu} = j^m_\mu, \tag{38}$$

with the local conservation laws for the electric and magnetic charges:

$$\partial_\mu j^{e,m}_\mu = 0, \tag{39}$$

we immediately see the invariance of these equations under the exchange of the electric and magnetic fields (Hodge star duality) provided that at the same time electric and magnetic charges and currents (and masses if they are different) are also interchanged:

$$e \leftrightarrow g, \qquad j^e_\mu \leftrightarrow j^m_\mu \tag{40}$$

together with $\mu_e \leftrightarrow \mu_m$ in Eqs.(16), (17), (20), (21), (24), (25), and $\lambda_e \leftrightarrow \lambda_m$ in Eqs.(24), (25).

The corresponding quantum field theory with electric e_i and magnetic g_i charges is selfconsistent if both charges are quantized according to the famous Dirac relation [6]:

$$e_i g_j = 2\pi n_{ij} \qquad (41)$$

when n_{ij} is an integer. For $n_{ij} = 1$ we have the Dirac quantization condition (8) in terms of the elementary electric and magnetic charges. But J.Schwinger [8] showed another possibility for the charge quantization condition when instead of Eq.(8) we have:

$$eg = 4\pi. \qquad (42)$$

The results (8) and (42) depend on the choice of the string singularity line. We prefer to consider the Dirac semi-infinite string and the Dirac relation (8) as a charge quantization condition.

If the fundamental electric charge e is so small that it corresponds to the perturbative electric theory, then magnetic charges are large and correspond to the strongly interacting magnetic theory, and vice versa. But below we consider some small region of e, g values (we hope that it exists) which allows us to employ the perturbation theory in both, electric and magnetic, sectors.

When non-trivial dyons – particles with both electric and magnetic charges simaltaneously – are present, then the analogue of the Dirac relation becomes a little more complicated and it then reads:

$$e_i g_j - e_j g_i = 2\pi n_{ij} \qquad (43)$$

which is duality invariant (see for example the reviews [5], [17] and the references there).

The relation (43) has the name of the Dirac–Schwinger–Zwanziger [6,8,9] quantization condition. But in this paper the theory of dyons is not exploited.

4 Propagators

At the same time as the partition function (11) let us consider the generating functional with external sources $J_\mu^{(A)}, J_\mu^{(B)}, \eta$ and ω:

$$Z[J^{(A)}, J^{(B)}, \eta, \omega] =$$

$$= \int [DA][DB][D\Phi][D\bar\Phi][D\Psi][D\bar\Psi] e^{-S+(J^{(A)},A)+(J^{(B)},B)+(\bar\eta,\Phi)+(\bar\Phi,\eta)+(\bar\omega,\Psi)+(\bar\Psi,\omega)} \qquad (44)$$

where

$$(J, A) = \int d^4 x J_\mu(x) A_\mu(x), \quad \text{and} \quad (\bar\eta, \Phi) = \int d^4 x \bar\eta(x) \Phi(x), \quad \text{etc.} \qquad (45)$$

Using this generating functional it is not difficult to calculate the propagators of the fields considered in our model.

Three "bare" propagators of the gauge fields A_μ and B_μ:

$$Q_{\mu\nu}^{0(A)} = < A_\mu A_\nu > = \frac{\delta^2 Z[J^{(A)}, J^{(B)}, \eta, \omega]}{\delta J_\mu^{(A)} \delta J_\nu^{(A)}},$$

$$Q_{\mu\nu}^{0(B)} = < B_\mu B_\nu > = \frac{\delta^2 Z[J^{(A)}, J^{(B)}, \eta, \omega]}{\delta J_\mu^{(B)} \delta J_\nu^{(B)}},$$

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$$Q_{\mu\nu}^{0(AB)} = <A_\mu B_\nu> = \frac{\delta^2 Z[J^{(A)}, J^{(B)}, \eta, \omega]}{\delta J_\mu^{(A)} \delta J_\nu^{(B)}} \qquad (46)$$

are presented in Fig.1 (see Fig.1(a)) together with the propagators $Q_{\mu\nu}^{(A)}$, $Q_{\mu\nu}^{(B)}$ and $Q_{\mu\nu}^{(AB)}$ determined by Fig.1(b).

Propagators (46) were calculated by authors of Ref.[16] in the momentum space:

$$Q_{\mu\nu}^{0(A,B)}(q) = \frac{1}{q^2}(\delta_{\mu\nu} + \frac{q^2 + M_{A,B}^2}{M_{A,B}^2}\frac{q_\mu q_\nu}{(n\cdot q)^2} - \frac{1}{(n\cdot q)}(q_\mu n_\nu + q_\nu n_\mu)), \qquad (47)$$

$$Q_{\mu\nu}^{0(AB)} = \frac{i}{q^2}\epsilon_{\mu\nu\rho\sigma}\frac{q_\rho n_\sigma}{(n\cdot q)}. \qquad (48)$$

The dot on the diagrams of Fig.1(a,b) corresponds to the following operator:

$$\Lambda_{\mu\nu} = iq^2\epsilon_{\mu\nu\rho\sigma}\frac{q_\rho n_\sigma}{(n\cdot q)}. \qquad (49)$$

Considering QED with Lagrangian (16) for electrons (monopoles are absent in this case) it is easy to see that the "bare" $D_{\mu\nu}^0(q^2)$ and the "dressed" (renormalized) $D_{ren,\mu\nu}(q^2)$ photon propagators obey the following relation presented in Fig.2:

$$D_{ren,\mu\nu}(q) = D_{\mu\nu}^0(q) + D_{\mu\kappa}^0(q)\Pi_{\kappa\lambda}(q)D_{ren,\lambda\nu}(q). \qquad (50)$$

Here the contribution of the electron loop is described by the operator $\Pi_{\kappa\lambda}(q)$ given by the following expression:

$$\Pi_{\kappa\lambda}(q) = e^2 \int \frac{d^4k}{(2\pi)^4}Tr[\gamma_\kappa G(k)\Gamma_\lambda(k, k-q)G(k-q)] \qquad (51)$$

where γ_κ is the Dirac matrix, Γ_λ is the renormalized vertex and $G(k)$ is the "dressed" propagator of the electron. Taking into account the transversality of the photon self-energy tensor we have:

$$\Pi_{\kappa\lambda}(q) = (q^2\delta_{\kappa\lambda} - q_\kappa q_\lambda)\Pi(q^2). \qquad (52)$$

The "dressed" propagators $Q_{ren,\mu\nu}^{(A)}$, $Q_{ren,\mu\nu}^{(B)}$ and $Q_{ren,\mu\nu}^{(AB)}$ containing the contributions of the electric (thin lines) and magnetic (thick lines) charged particle loops are presented in Fig.3.

The Lagrangians (16) and (17) contain the interaction terms $j_\mu^e A_\mu$ and $j_\mu^m B_\mu$ where j_μ^e and j_μ^m are the electric and magnetic currents:

$$j_\mu^e = e\bar{\Psi}\gamma_\mu\Psi \qquad (53)$$

and

$$j_\mu^m = g\bar{\Phi}\gamma_\mu\Phi. \qquad (54)$$

The interactions in the Lagrangians (20)-(23) are given by $j_\mu^e A_\mu$ and $j_\mu^m B_\mu$ as well as by "seagull" terms $e^2 A_\mu A_\mu \Psi^+\Psi$ and $g^2 B_\mu B_\mu \Phi^+\Phi$, respectively, but now we have:

$$j_\mu^e = e(\Psi^+\partial_\mu\Psi - \Psi\partial_\mu\Psi^+) \qquad (55)$$

and

$$j_\mu^m = g(\Phi^+\partial_\mu\Phi - \Phi\partial_\mu\Phi^+). \qquad (56)$$

The simplest electron-electron, monopole-monopole and electron-monopole interactions corresponding to the Lagrangians $L_{e,m}$ given by Eqs.(16) and (17) are shown in Fig.4.

Now we are ready to obtain the renormalization group equations for the effective fine structure constants when both, electric and magnetic charges are present in our field system.

5 Renormalization Group Equations for the Electric and Magnetic Fine Structure Constants

The Gell–Mann–Low RG equation (3) can be obtained by the calculation of the "dressed" propagators. In the case of QED the relation shown in Fig.2 gives us:

$$D(q) = Z_3^{-1} D^0(q) \tag{57}$$

where the renormalization constant Z_3 is related, in its turn, with $\Pi(q^2)$ determined by Eqs.(51),(52):

$$Z_3 = 1 - \Pi(\mu^2). \tag{58}$$

Here μ is the energy scale: $q^2 = \mu^2$.

The Gell–Mann–Low β-function is given by the following expression:

$$\beta(\alpha(p)) = -\frac{\partial \log Z_3}{\partial \log \mu^2}, \tag{59}$$

or

$$\beta(\alpha(p)) = -\frac{\partial \log (1 - \Pi(\mu^2))}{\partial \log \mu^2}. \tag{60}$$

In the one–loop approximation of perturbation theory (see for example [18]) we can write:

$$\beta(\alpha(p)) \approx \frac{\partial \Pi(\mu^2)}{\partial \log \mu^2} \approx -\frac{\partial \Pi(M^2)}{\partial \log M^2} \tag{61}$$

where M is the momentum cut–off.

Let us consider now the renormalization group equations for α and $\tilde{\alpha}$ when both (electric and magnetic) charges are present in our theory.

Fig.3 shows the contributions of the electric (thin lines) and magnetic (thick lines) charged-particle loops to the "dressed" propagators $Q_{ren,\mu\nu}^{(A)}$ and $Q_{ren,\mu\nu}^{(B)}$. We consider also the "dressed" propagators in loops assuming the theory of the case a) with Lagrangians $L_{e,m}$ given by Eqs.(16) and (17).

Introducing the renormalization constants Z_3 and $\tilde{Z}_3$ by the following relations:

$$Q_{ren,\mu\nu}^{(A)} = Z_3^{-1} Q_{\mu\nu}^{(A)}, \tag{62}$$

$$Q_{ren,\mu\nu}^{(B)} = \tilde{Z}_3^{-1} Q_{\mu\nu}^{(B)}, \tag{63}$$

$$Q_{ren,\mu\nu}^{(AB)} = (Z_3 \tilde{Z}_3)^{-1/2} Q_{\mu\nu}^{(AB)}, \tag{64}$$

it is not difficult to calculate Z_3 and $\tilde{Z}_3$ according to the diagrams shown in Fig.3. These diagrams demonstrate the following relations:

$$Q_{ren,\mu\nu}^{(A,B)} = Q_{\mu\nu}^{(A,B)} + Q_{\mu\kappa}^{(A,B)} \Pi_{\kappa\lambda}^{(e,m)} Q_{ren,\lambda\nu}^{(A,B)} + Q_{\mu\kappa}^{(AB)} \Pi_{\kappa\lambda}^{(m,e)} Q_{ren,\lambda\nu}^{(AB)} \tag{65}$$

which mean:

$$Z_3^{-1} Q_{\mu\nu}^{(A)} = Q_{\mu\nu}^{(A)} + Q_{\mu\kappa}^{(A)} \Pi_{\kappa\lambda}^{(e)} Z_3^{-1} Q_{\lambda\nu}^{(A)} + Q_{\mu\kappa}^{(AB)} \Pi_{\kappa\lambda}^{(m)} (Z_3 \tilde{Z}_3)^{-1/2} Q_{\lambda\nu}^{(AB)}, \tag{66}$$

$$\tilde{Z}_3^{-1} Q_{\mu\nu}^{(B)} = Q_{\mu\nu}^{(B)} + Q_{\mu\kappa}^{(B)} \Pi_{\kappa\lambda}^{(m)} \tilde{Z}_3^{-1} Q_{\lambda\nu}^{(B)} + Q_{\mu\kappa}^{(AB)} \Pi_{\kappa\lambda}^{(e)} (Z_3 \tilde{Z}_3)^{-1/2} Q_{\lambda\nu}^{(AB)}, \tag{67}$$

Using the expressions (47) and (48) obtained in Ref.[16] for propagators $Q_{\mu\nu}^{0(A,B,AB)}$ and the limit $M_{A,B} \to \infty$ in Eq.(47), it is possible to show that the following relations are valid:

$$Z_3^{-1}Q_{\mu\nu}^{(A)} = Q_{\mu\nu}^{(A)} + \Pi^{(e)}Z_3^{-1}Q_{\mu\nu}^{(A)} + \Pi^{(m)}(Z_3\tilde{Z}_3)^{-1/2}Q_{\mu\nu}^{(A)}, \qquad (68)$$

$$\tilde{Z}_3^{-1}Q_{\mu\nu}^{(B)} = Q_{\mu\nu}^{(B)} + \Pi^{(m)}\tilde{Z}_3^{-1}Q_{\mu\nu}^{(B)} + \Pi^{(e)}(Z_3\tilde{Z}_3)^{-1/2}Q_{\mu\nu}^{(B)}, \qquad (69)$$

or

$$Z_3 = \frac{1 - \Pi^{(e)}(\mu^2)}{1 - \Pi^{(m)}(\mu^2)(Z_3\tilde{Z}_3)^{-1/2}}, \qquad (70)$$

$$\tilde{Z}_3 = \frac{1 - \Pi^{(m)}(\mu^2)}{1 - \Pi^{(e)}(\mu^2)(Z_3\tilde{Z}_3)^{-1/2}}. \qquad (71)$$

Here $\Pi^{(e)}(\mu^2)$ and $\Pi^{(m)}(\mu^2)$ are given at $q^2 = \mu^2$ by the photon self–energy tensors $\Pi_{\mu\nu}^{(e)}$ and $\Pi_{\mu\nu}^{(m)}$ corresponding to the electron and monopole loops, respectively (see bubbles in Fig.3):

$$\Pi_{\mu\nu}^{(e,m)}(q) = (q^2\delta_{\mu\nu} - q_\mu q_\nu)\Pi^{(e,m)}(q^2) \qquad (72)$$

where $\Pi^{(e)}$ (or $\Pi^{(m)}$) is described by Eqs.(51),(52) with e, μ_e (or g, μ_m) as a charge and a mass of the electron (or monopole).

J.Schwinger was the first (see Refs.[8]) who investigated the renormalization problem of the electric and magnetic charges in QEMD.

Considering the "bare" charges e_0 and g_0 and the renormalized effective charges e and g, we must distinguish between two opposite cases. It was shown in Refs.[8,11,12]:

$$e/g = e_0/g_0, \qquad (73)$$

while other authors [13,14] obtained the following result:

$$eg = e_0g_0. \qquad (74)$$

S.Coleman [15] analysed the case when the extended t'Hooft–Polyakov monopole is shrunk to zero size. The effective theory describing the interaction between such objects in QEMD tells us something about the renormalization effects. The consistency condition gave the following result:

$$eg = (Z_3\tilde{Z}_3)^{1/2}e_0g_0 = 2\pi, \qquad (75)$$

or

$$Z_3\tilde{Z}_3 = 1. \qquad (76)$$

This means that the Dirac relation (8) is valid not only for the "bare" charges e_0 and g_0, but also for the renormalized effective charges e and g.

We have actually already rederived this result in the Zwanziger formalism, since we can obtain it by using (70) and (71). Let us in fact multiply (70) on both sides with $1/\sqrt{Z_3} - \Pi^{(m)}(\mu^2)/\sqrt{\tilde{Z}_3}$ and (71) by $1/\sqrt{\tilde{Z}_3} - \Pi^{(m)}(\mu^2)/\sqrt{Z_3}$ and then add the resulting equations. The last ones become by the cancellation of Π terms giving

$$\sqrt{Z_3} + \sqrt{\tilde{Z}_3} = 1/\sqrt{Z_3} + 1/\sqrt{\tilde{Z}_3}, \qquad (77)$$

from which it is easily seen that Eq.(76) follows.

Using this result we obtain the following important relations:

$$Z_3 = \tilde{Z}_3^{-1} = \frac{1 - \Pi^{(e)}(\mu^2)}{1 - \Pi^{(m)}(\mu^2)}. \tag{78}$$

RG equations for the fine structure constants α and $\tilde{\alpha}$ immediately follow from Eq.(78):

$$\frac{\mathrm{d}(\log \alpha(p))}{\mathrm{d}t} = -\frac{\partial \log Z_3}{\partial \log \mu^2}, \tag{79}$$

or

$$\frac{\mathrm{d}(\log \alpha(p))}{\mathrm{d}t} = -\frac{\partial \log(1 - \Pi^{(e)}(\mu^2))}{\partial \log \mu^2} + \frac{\partial \log(1 - \Pi^{(m)}(\mu^2))}{\partial \log \mu^2}$$

$$= \beta^{(e)}(\alpha) - \beta^{(m)}(\tilde{\alpha}) \tag{80}$$

and

$$\frac{\mathrm{d}(\log \tilde{\alpha}(p))}{\mathrm{d}t} = -\frac{\partial \log \tilde{Z}_3}{\partial \log \mu^2} = \frac{\partial \log Z_3}{\partial \log \mu^2} = \beta^{(m)}(\tilde{\alpha}) - \beta^{(e)}(\alpha). \tag{81}$$

Here the analytical expressions for β–functions are given by the same Eq.(60):

$$\beta^{(e,m)}(\alpha \text{ or } \tilde{\alpha}) = -\frac{\partial \log\left(1 - \Pi^{(e,m)}(\mu^2)\right)}{\partial \log \mu^2}, \tag{82}$$

but now these β–functions contain electron (e, μ_e) and monopole (g, μ_m) parameters, respectively.

The obtained RG equations (80) and (81) obey the following equality:

$$\frac{\mathrm{d}(\log \alpha(p))}{\mathrm{d}t} = -\frac{\mathrm{d}(\log \tilde{\alpha}(p))}{\mathrm{d}t} \tag{83}$$

which corresponds to the Dirac relation:

$$\alpha(t)\tilde{\alpha}(t) = \frac{1}{4} \qquad \text{(for all t)} \tag{84}$$

valid for the renormalized electric and magnetic fine structure constants at the arbitrary scales.

6 The beta–functions

The functions $\beta^{(e,m)}$ are given perturbatively by the expressions similar to Eq.(4):

$$\beta^{(e)}(\alpha) = \beta_2^{(e)}(\frac{\alpha}{4\pi}) + \beta_4^{(e)}(\frac{\alpha}{4\pi})^2 + ... \tag{85}$$

and

$$\beta^{(m)}(\tilde{\alpha}) = \beta_2^{(m)}(\frac{\tilde{\alpha}}{4\pi}) + \beta_4^{(m)}(\frac{\tilde{\alpha}}{4\pi})^2 + ... \tag{86}$$

The perturbative expansions (85) and (86) coincide with the series (4) calculated in QED, at least on the level of the two–loop approximation. The monopole(electric) loops inside the electric(monopole) loops appear only on the level of the three–loop approximation. Of course, these β–functions are different if we consider magnetic scalar particles instead of electric fermions, or vice versa. The corresponding coefficients $\beta_2^{(e)}$, $\beta_4^{(e)}$ of the series

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(85) or $\beta_2^{(m)}$, $\beta_4^{(m)}$ of the series (86) are given by Eqs.(5) or (6) depending on the type of the charged particles.

If both matter fields are electrically- and magnetically-charged fermions or both are scalars then we have the same expressions for the functions $\beta^{(e,m)}$ and we can write the following equations for the cases a) and b):

$$\frac{d(\log \alpha(p))}{dt} = -\frac{d(\log \tilde{\alpha}(p))}{dt} = \beta_2 \frac{\alpha - \tilde{\alpha}}{4\pi}\left(1 + 3\frac{\alpha + \tilde{\alpha}}{4\pi} + ...\right). \tag{87}$$

The last equations show that the one–loop approximation works with accuracy $\stackrel{<}{\sim} 30\%$ if both α and $\tilde{\alpha}$ obey the following requirement:

$$0.25 \stackrel{<}{\sim} \alpha, \tilde{\alpha} \stackrel{<}{\sim} 1. \tag{88}$$

But strictly speaking, we don't know the exact behaviour of the whole asymptotic series (87).

In Refs.[19] and [20] the behaviour of the effective fine structure constants was investigated in the vicinity of the phase transition point in compact (lattice) QED by the Monte–Carlo simulation method. The following result was obtained [19,20]:

$$\alpha_{crit}^{lat} \approx 0.20 \qquad \text{and} \qquad \tilde{\alpha}_{crit}^{lat} \approx 1.25. \tag{89}$$

These values almost coincide with the borders of the perturbation theory requirement (88). In consequence, assuming that the phase transition couplings (89) may be described by the one–loop approximation with accuracy not worse than $(30 - 50)\%$, we have tried to calculate them in the Higgsed monopole model (see Ref.[21]). The aim of the last paper was to confirm, in general, the idea of the approximate "universality" (regularization independence) of the phase transition couplings. The result obtained in [21]:

$$\alpha_{crit}^{lat} \approx 0.18_5 \qquad \text{and} \qquad \tilde{\alpha}_{crit}^{lat} \approx 1.35 \tag{90}$$

is in accordance with the lattice result (89). It seems that the idea of the approximate "universality" for the first-order phase transitions is really confirmed.

7 Conclusions

The aim of this paper was to obtain the renormalization group equations for the electric and magnetic renormalized fine structure constants using the Zwanziger formalism for QEMD. The result (see Eqs.(80) and (81)):

$$\frac{d(\log \alpha(p))}{dt} = -\frac{d(\log \tilde{\alpha}(p))}{dt} = \beta^{(e)}(\alpha) - \beta^{(m)}(\tilde{\alpha}) \tag{91}$$

confirms the Dirac relation $\alpha(t)\tilde{\alpha}(t) = 1/4$ existing at the arbitrary scale.

According to the philosophy given in the Introduction it is possible to consider the perturbation theory for $\beta^{(e)}(\alpha)$ and $\beta^{(m)}(\tilde{\alpha})$ simultaneously if both α and $\tilde{\alpha}$ are sufficiently small. Then the functions $\beta^{(e,m)}$ are given perturbatively by the usual series (4) or (7). The calculations in QED (see Section 1) have shown that the perturbation theory works up to $\alpha \stackrel{<}{\sim} 1$. Due to the Dirac relation (9), such a requirement leads to the following

condition: $\tilde{\alpha} \gtrsim 0.25$. In consequence, we have QEMD RG equations with beta-functions $\beta^{(e,m)}$ considered perturbatively if both α and $\tilde{\alpha}$ obey the following requirement:

$$0.25 \lesssim \alpha, \tilde{\alpha} \lesssim 1. \tag{92}$$

We have just this case in the vicinity of the phase transition point for the compact (lattice) QED: $\alpha_{crit}^{lat} \approx 0.2$ and $\tilde{\alpha}_{crit}^{lat} \approx 1.25$ (see Refs.[19] and [20]).

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Captions for figures:

Fig.1
(a) "Bare" propagators of gauge fields A_μ (thin wavy line) and B_μ (thin dashed line) describing a photon in two (non–dual and dual) states existing in the Abelian gauge theory with both electric and magnetic charges.

(b) Propagators of gauge fields A_μ (thick wavy line) and B_μ (thick dashed line) describing the contribution of dual transformations of the photon.

Fig.2
"Dressed" propagator (double line) of the photon in QED.

Fig.3
"Dressed" propagators of gauge fields A_μ (double wavy line) and B_μ (double dashed line) containing the contributions of the electrically- (thin lines) and magnetically-charged (thick lines) particle loops.

Fig.4
The interaction of the electric (thin line) and magnetic (thick line) currents.