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GENERAL DESCRIPTION OF A GEOMETRICAL RECONSTRUCTION PROGRAM FOR LARGE CHAMBERS ("1-6" VERSION)

by

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1. PURPOSE OF THE PROGRAM

Processing bubble chamber film measurements calls for a large volume of computing. It is convenient to divide this computing into three stages:

1) geometrical reconstruction of the event;
2) identification of the reaction channel;
3) storing and statistical processing of events identified.

This processing is carried out at the JINR by a program system written for the M-20 computer, specially connected for this purpose with the MINSK-2 machine which is used for receiving and storing on magnetic tape the data from the semi-automatic measuring devices. The processing itself is carried out on the M-20 and BESM-EM.

At present a third version of the processing program system is being completed*).

The working of the first of these programs -- a geometrical reconstruction program -- is described briefly in this report.

The new geometrical reconstruction program for bubble chamber tracks that has been written at the Laboratory for Computing Technique and Automation of the JINR takes into account the inhomogeneity of the magnetic field in the chamber, ionization losses, multiple scattering, and bremsstrahlung (the latter for electrons and positrons).

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*) A description of the first experimental data processing system can be found in a special collection[1]) issued in 1961.

The second version of our processing system for data from bubble chambers is described in a series of preprints [2-11]) issued in 1963-1966.

The programme is designed for processing data from:

a 2 m propane chamber,

a 1 m propane chamber,

a 1 m hydrogen chamber;

and can easily be adapted for use with any other chamber of similar construction.

Taking into account the special features of the bubble chambers at our Institute, the geometrical reconstruction program assumes that the optical axes of the stereo-cameras are parallel to each other and perpendicular to the surfaces separating the optical media. Deviations from the perpendicularity of the axes to the media are considered to be small and are taken into account when introducing the reference crosses. (The program is intended for between 3 and 30 reference crosses.)

In view of the fact that the JINR chambers are filled with liquid hydrogen or propane and it is possible to fill them with a freon-propane mixture, the algorithm of the program takes both ionization losses and multiple scattering into account.

Measurement of the magnetic field in large chambers has shown that its inhomogeneity may reach 20%; the program therefore takes into account the real field in the chamber, which is defined in the form of a table for the three field components, with the possibility of taking into account the field symmetry.

In order to determine the track parameters, a function χ^2 is set up, which takes into account the multiple scattering matrix. The function includes the trajectory computed -- the integral of the Lorentz equation in the real magnetic field allowing for losses. If the first approximation is well chosen, only one Newtonian iteration will be necessary for the final determination of the parameters involving the minimum χ^2.

The program is designed for processing events with not more than 15 tracks, photographed by a stereo-camera with not more than 6 objectives.

For the convenience of physicists, the topology of the event (number of ν^0 particles, γ rays, stopped particles, etc.) is defined in the form adopted at Dubna for all previous geometrical programs.

In order to save computer time and facilitate the work, the program is written in blocks in IS-2 machine language for the M-20, BESM-ZM, and BESM-4.

Programs can be used for processing data from a given chamber by introducing a special table of constants.

The program is so made that it can easily be used for on-line operation of PUOS semi-automatic measuring instruments with the BESM-ZM or BESM-4. The program was written by a group of colleagues from the Computing Technique and Automation and High-

Energy Laboratories with the participation of mathematicians from the Atomic Physics Institute at Alma Ata.

2. ORGANIZATION OF THE PROGRAM

Figure 1 shows the organization of the "1-6" program.

Let us consider the operation of each of the basic blocks of the geometrical reconstruction program.

Block No. 1

Transformation to a system of coordinates related to the optical axes (optical system). Compensation for deformation of the film.

The transformation to the optical system (x, y) from a system of coordinates (ξ, η) of the measuring instrument is made assuming rotation and translation, together with linear deformation along the x and y axes:

$$x = K_x(\xi\cos\varphi - \eta\sin\varphi) + x_0$$
$$y = K_y(\xi\sin\varphi + \eta\cos\varphi) + y_0 \ .$$

The transformation coefficients are:

$\left.\begin{matrix} K_x \\ K_y \end{matrix}\right\}$ deformation of the film along the x and y axes;

φ the angle of rotation when transforming from the system of coordinates of the measuring instrument to the optical system of coordinates;

x_0, y_0, the shift of the coordinates, are determined by the least squares method, using the measured coordinates of the reference crosses and their actual position in the optical system of coordinates.

Measurements carried out on film have shown that the deformations along and across the roll of film are different; this is taken into consideration by introducing two different coefficients.

The program allows the possibility of using as reference crosses both the crosses marked inside the volume of the chamber and those on the clamped glass of the camera. It should be noted that the use of reference crosses inside the volume of the chamber is preferable, since this increases the accuracy of reconstruction of the space coordinates.

The program features include testing of the quality of the measurements of the reference crosses in order to exclude erroneous determination of the above-mentioned transformation coefficients.

A detailed description of the algorithm of this block is given in another paper[12].

Block No. 2

Reconstruction of the space coordinates of points on the track

Reconstruction of the space coordinates of points measured on the track is always carried out using the two optimal (for the track concerned) stereo-projections which are selected when scanning.

The space coordinates of the corresponding points are reconstructed by means of the following formulae:

$$Z = \frac{B_r - B_l - (x_l \varphi_l - x_r \varphi_r)}{x_l \varphi_{1l} - x_r \varphi_{1r}}$$

$$X = Z x_l \varphi_{1l} + x_l \varphi_l + B_l$$

$$Y = \frac{y_l + y_r}{(Z\varphi_{1l} + \varphi_l)^{-1} + (Z\varphi_{1r} + \varphi_r)^{-1}}, \qquad (2.1)$$

which are obtained by the method of maximum likelihood.

The symbols used are clear from Fig. 2.

The functions $\varphi(r)$ and $\varphi_1(r)$, describing the optics, may be defined in an arbitrary form. They may be obtained either as a result of studying photographs of test objects or from any model of an optical system.

The reconstruction of the space coordinates of non-corresponding points is carried out in two steps:

i) The coordinates (X_r, Y_r) of near-corresponding points are determined in the right-hand photograph as shown in Fig. 3.

ii) The space coordinates are found using the formulae (2.1).

This algorithm has been used since 1963 in large-scale computing at Dubna for processing data from propane and hydrogen chambers.

A detailed description of the algorithm and the program is given in a preprint by Ivanchenko et al.[14].

Block No. 3

Computing the magnetic field at points in the chamber

In large chambers it is practically impossible to make the magnetic field constant over the whole volume, and, therefore, a special block has been included in the program for computing the magnetic field at given points on the track. The initial values of the magnetic field are measured at points $< X, Y, Z >$ and are defined for each point in the form of three components H_x, H_y, H_z.

The program is designed for not more than 320 points; the accuracy of recording of the field is 1 gauss.

If the magnetic field has one or two planes of symmetry, the program allows the possibility of defining the field in part of the chamber only and extrapolating it by symmetry to the remaining volume.

The magnetic field at each point of the track is computed by a linear interpolation from the nearest point in the chart of the magnetic field.

Block No. 4

Determination of track parameters in zero approximation

For the optimal determination of the particle parameters p, β, tg α, which is carried out in Block No. 5, it is necessary to obtain the most accurate values of these parameters, but without a great loss of computer time, and this is done in Block No. 4.

Here and subsequently:

- p is the particle momentum at the initial point;
- β is the angle between the projection of the momentum vector on to the plane XOY and the X axis;
- tg α is the tangent of the angle between the track and the plane XOY.

The momentum is determined from the mean curvature of the parabola which approximates to the projection of the track, taking into account ionization losses and the inhomogeneity of the magnetic field, as shown below.

By integrating the equation of motion in an inhomogeneous magnetic field and allowing for slowing down, we obtain the relation between the angle of turn and the projected arc length in the form of an expansion

$$\beta = \beta' s + \frac{1}{2}\beta'' s^2 + \frac{1}{6}\beta''' s^3 ,$$

where the derivatives of β with respect to s (β', β'', β''') are obtained from the Lorentz equation and are functions of p, p', $\vec{H}$, $\vec{H}'$.

The trajectory can be written as

$$y_{(s)} = \int_0^s \sin\beta\, ds$$
$$x_{(s)} = \int_0^s \cos\beta\, ds .$$

A transformation is made to a system of coordinates based on the computed track trajectory, and in this system a parabola is fitted to the trajectory, of the form

$$y = a(x - \tfrac{L}{2})^2 + c \quad ,$$

where L is the distance between the beginning and end of the trajectory. From the requirement that the function

$$Q^2 = \int_0^{s_K} [y_{(s)} - a(x_{(s)} - \tfrac{L}{2})^2 - c]^2 ds$$

be a minimum, the following expression for a is obtained:

$$a = \left\{ \frac{1}{\beta'} - \frac{\beta''}{(\beta')^2} s_K + \left[\frac{(\beta'')^2}{(\beta')^3} - \frac{6}{7} \frac{\beta'''}{(\beta')^2} - \frac{3}{56}\beta' \right] s_K^2 \right\} .$$

On the other hand, by using the least squares method to approximate the points measured to a similar parabola in the system of coordinates of the track we obtain:

$$a_e = \frac{(N+1)\sum_{i=0}^{N} y_i (x_i - \frac{L}{2})^2 - \sum y_i \sum (x_i - \frac{L}{2})^2}{(N+1)\sum_{i=0}^{N} (x_i - \frac{L}{2})^4 - [\sum (x_i - \frac{L}{2})^2]^2} .$$

By equating $a = a_e$, we obtain

$$a_e = a = F(\rho, \rho', \vec{H}, \vec{H}', s_K) .$$

By solving this expression as a function of p, we have

$$P = P_e + \left[\frac{0{,}15}{\cos^2\alpha}\left(H'R_e + \frac{1}{\cos\alpha}\,\frac{BD}{H} + 2B\sin\alpha\right) - \frac{1}{2\cos\alpha}\left(\frac{dP}{ds}\right)\right] s_K +$$

$$+ \frac{0.3}{7\cos^2\alpha}\left[\frac{R_e}{\cos\alpha}H'' - \frac{10}{3}\,\frac{d^2P}{ds^2} + 3\,tg\alpha\,B' - \frac{2\sin\alpha}{R_e}\,\mathcal{D} +\right.$$

$$\left. + \frac{1}{2}\,tg\alpha\,\frac{BH'}{H} + \frac{1}{\cos^2\alpha}\,\frac{B'D}{H} + \frac{2}{\cos^2\alpha}\,\frac{B\mathcal{D}'}{H} + \frac{3}{8}\,\frac{\cos\alpha}{R_e}H\right] s_K^2,$$

where

$$P_e = \frac{0{,}3R_e}{\cos\alpha}$$

$$R_e = \frac{1}{2a_e}$$

$$H = (H_x\cos\beta + H_y\sin\beta)\,tg\alpha + H_z$$

$$H' = \left[\left(\frac{d}{ds}H_x\cos\beta + \frac{d}{ds}H_y\sin\beta\right)\right]tg\alpha - \frac{dH_z}{ds}$$

$$B = H_y\cos\beta - H_x\sin\beta$$

$$B' = \left(\frac{d}{ds}H_y\right)\cos\beta - \left(\frac{d}{ds}H_x\right)\sin\beta$$

$$\mathcal{D} = H_x\cos\beta + H_y\sin\beta$$

$$\mathcal{D}' = \left(\frac{d}{ds}H_x\right)\cos\beta + \left(\frac{d}{ds}H_y\right)\sin\beta\,.$$

This formula makes it possible to determine p with a systematic error not exceeding 2% in the most unfavourable cases.

$tg\,\alpha$ is determined by making a least squares fit of the measured points in the plane (Z, s) to a straight line of the form $Z = Z_0 + s\,(tg\,\alpha)$.

β in the track system is determined by the formula

$$\beta = - arc\,tg \frac{L}{2(R_e + c)} .$$

A detailed description of the algorithm for this Block is given elsewhere[15].

Block No. 5

Accurate determination of track parameters

The Lorentz equation in a slowing-down medium can be written as

$$\frac{d\vec{P}}{dt} = \frac{e}{c}[\vec{v}\vec{H}] + \frac{\vec{P}}{P}\frac{dP}{dt} .$$

Changing to the variables p, d, β, s, we have:

$$\frac{d\alpha}{ds} = \frac{e}{pc}(H_y \cos\beta - H_x \sin\beta)$$

$$\frac{d\beta}{ds} = \frac{e}{pc}[-H_z + (H_x \cos\beta + H_y \sin\beta) tg\alpha] .$$

By integrating these equations in a real magnetic field with initial values of the parameters

$$
\begin{aligned}
p \text{ beg} &= \hat{p} + \Delta p \\
\beta \text{ beg} &= \hat{\beta} + \Delta\beta \\
(\mathrm{tg}\,\alpha) \text{ beg} &= \widehat{\mathrm{tg}}\,\alpha + \Delta\,\mathrm{tg}\,\alpha \\
Y \text{ beg} &= \hat{Y} + \Delta Y \\
Z \text{ beg} &= \hat{Z} + \Delta Z
\end{aligned}
$$

and retaining only the linear terms for the corrections sought Δp, $\Delta\beta$, $\Delta\,\mathrm{tg}\,\alpha$, ΔY, ΔZ, we shall have the following expressions for the projections of the trajectory on to the XOY plane

$$y_{(s)} = y_0 + y_{(s)}\Big|_{\hat{\beta}_0,\hat{p}_0,\hat{y}_0} + \frac{\partial y}{\partial \beta}\Big|_{\hat{\beta}_0\,\hat{p}_0,\hat{y}_0} \Delta\beta + \frac{\partial y}{\partial p}\Big|_{\beta_0,p_0,y_0} \Delta p + \Delta y_0$$

and on to the YZ plane

$$Z_{(s)} = Z_0 + \Delta Z_0 + Z_{(s)}\Big|_{\hat{\alpha}_0\hat{p}_0,\hat{z}_0} + \frac{\partial z}{\partial \alpha}\Big|_{\hat{\alpha}_0\hat{p}_0,\hat{z}_0} \Delta\alpha \,,$$

where

$\left.\begin{array}{l} Y_0,\ Y \\ Z_0,\ Z \end{array}\right\}$ are the optimal value and the coordinates of the initial track points measured experimentally.

The required corrections are determined from the condition that the following functions shall have a minimum value

$$\chi^2_{xy} = (\bar{y} - \bar{y}_e)(G + D_y E)^{-1}(\bar{y} - \bar{y}_e)$$

$$\chi^2_z = (\bar{z} - \bar{z}_e)(G + D_z E)^{-1}(\bar{z} - \bar{z}_e),$$

where

G is the multiple scattering matrix,

D_y is the variance of the errors in the measurement of the coordinates,

$\bar{X},\bar{Z}$ are the vectors determining the points of the theoretical curve,

$\bar{X},\bar{Z}_e$ are the vectors determining the points of the measured curve.

The maximum systematic error of the procedure mentioned due to substituting sums for integrals and using other approximations to simplify computation, is not greater than 0.1% of the nominal value of the momentum.

The track parameters are determined for the mass of the particle designated by the physicists when scanning. If the identification is not indicated, then the calculation is carried out on assumptions of three masses: p, π, K.

A simple algorithm is available for processing tracks that are very short or that are at a very large angle to the plane XOY. The direction of the track is determined by the end points, and the momentum for stopping particles by the track length. For rather long tracks stopping in the chamber, the momentum is determined from the track length, and the angles by the procedure already described, applied for a fixed momentum.

A specific block in the program is intended for electron processing. It has an algorithm previously developed at the Nuclear Problems Laboratory and the Laboratory for Computing Technique and Automation for the 1 m propane chamber.

This algorithm is described by Budarov et al.[16].

3. TESTING OF THE PROGRAM

In order to test the geometrical reconstruction program a special program generating fictitious tracks was set up. The initial data for this generating program are as follows:

i) momentum vector of the particle at the initial point;
ii) coordinates of the initial point;
iii) magnetic field in the chamber;
iv) characteristic of the medium, i.e. slowing-down capacity;
v) characteristics of the optical system;
vi) variance of the errors of measurement and multiple scattering.

Thus we can construct a track of any length and any direction with or without taking into account the measuring errors and multiple scattering which may be introduced by the Monte Carlo method.

The first series of tests was carried out on tracks generated without taking into account measuring errors and multiple acattering.

This testing showed that:

1. the first approximation finds the momenta with an error not exceeding 2% in the most unfavourable case;

2. the algorithm used in the block for the accurate determination of the track parameters is sufficiently linear for α and β within wide limits (0.1 radian) even under conditions of inhomogeneity of the magnetic field (up to 25%) and of momentum loss of the particle (up to 50%). For the momentum, the algorithm is linear in a region representing not less than 2% of the nominal value, with the inhomogeneity of the magnetic field and value of the losses mentioned above.

The value of the momentum is found by the algorithm with an accuracy not worse than 0.1%.

The second series of tests consists of statistical checking. The mean value of the reconstructed momentum was determined from 76 tracks, at least 1 m in length, generated in propane with $p = 800$ MeV/c, taking into account measuring errors and multiple scattering, and was found to be

$$799.8 \pm 1.8 ,$$

which shows the stability of the algorithm under difficult conditions.

Third series of tests. Blocks 4 and 5 considered together.

1. 800, 1500 and 3000 MeV/c proton tracks were generated in a propane chamber taking into account the errors, and parameters were computed for 16 tracks at each momentum (in all 48) by the first approximation program (Block No.4) and subsequently by the accurate algorithm (Block No.5).

In all cases, the difference between the parameters determined in the first approximation and the results from the accurate algorithm did not fall outside the region of linearity of the accurate algorithm.

2. The mean values of χ^2_{xy} and χ^2_z were checked and were found to be equal to their theoretical values.

We consider, however, that the testing of the program should be continued and the quantity of test tracks should be increased to 1000, as was done by Solmitz when setting up the geometrical reconstruction program for the Alvarez group.

Since the algorithms applied in the first two Blocks were previously used for large-scale computing, there is no need to check them so carefully. However, it should be taken into account that the accuracy of the reconstructed coordinates, which finally determines the accuracy of the measurements in the chamber, depends not only on the quality of the algorithm but also on the accuracy with which the constants of the stereo-camera/chamber-optics system were determined.

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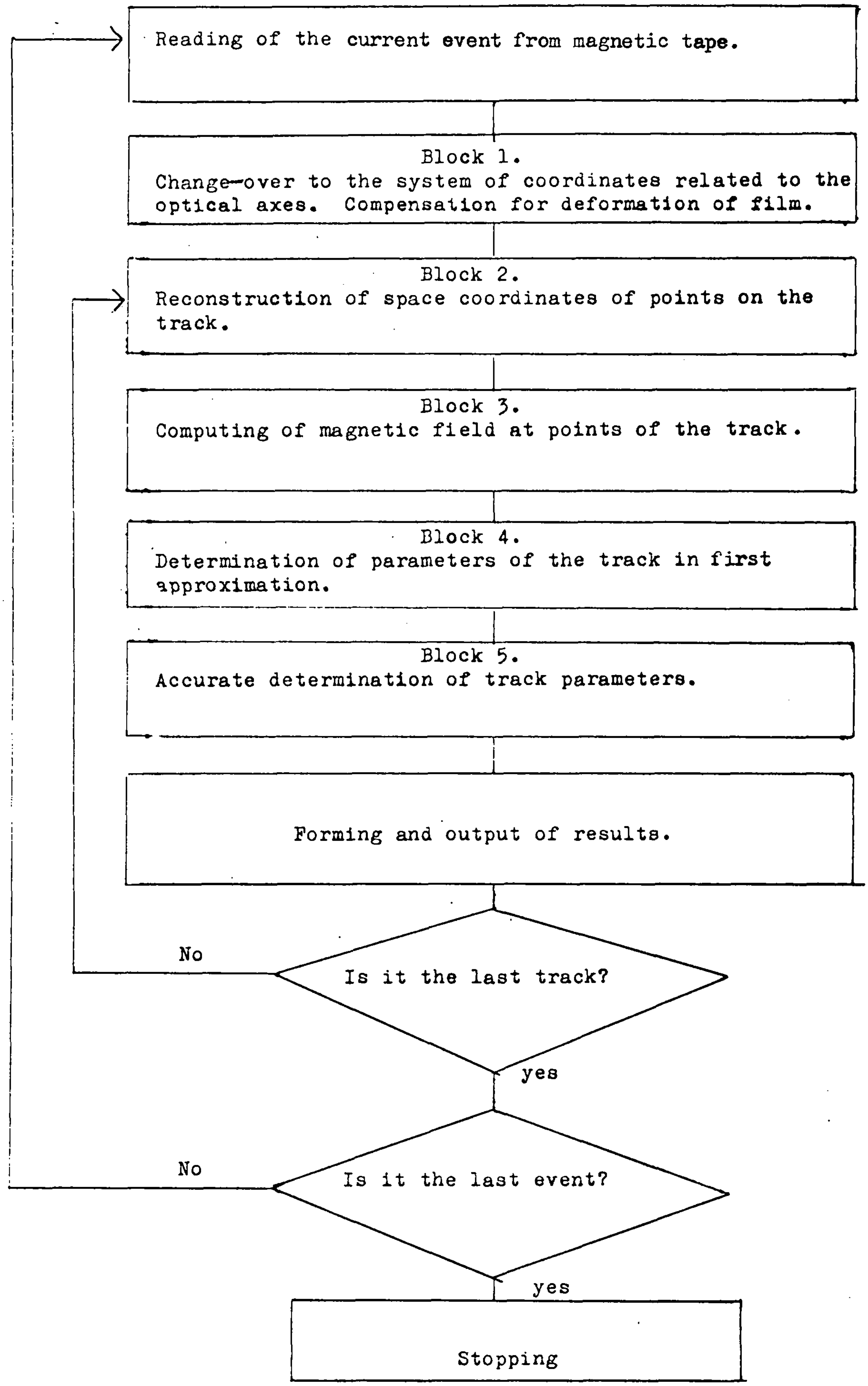
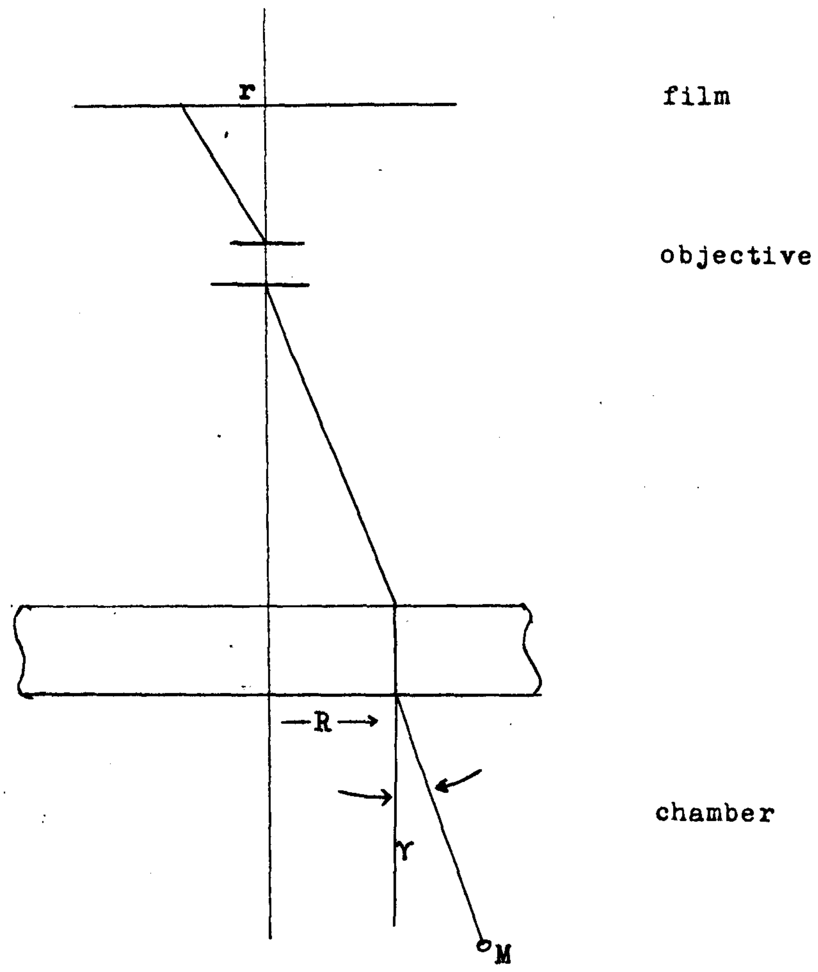


Block diagram of the geometrical reconstruction programme for large chambers (version 1-6)

Fig. 1.



Projection of space point M on to frame.

$$R(r) = r\,\varphi\,(r)$$

$$\mathrm{tg}\,\gamma = r\,\varphi_1(r)$$

Fig. 2.

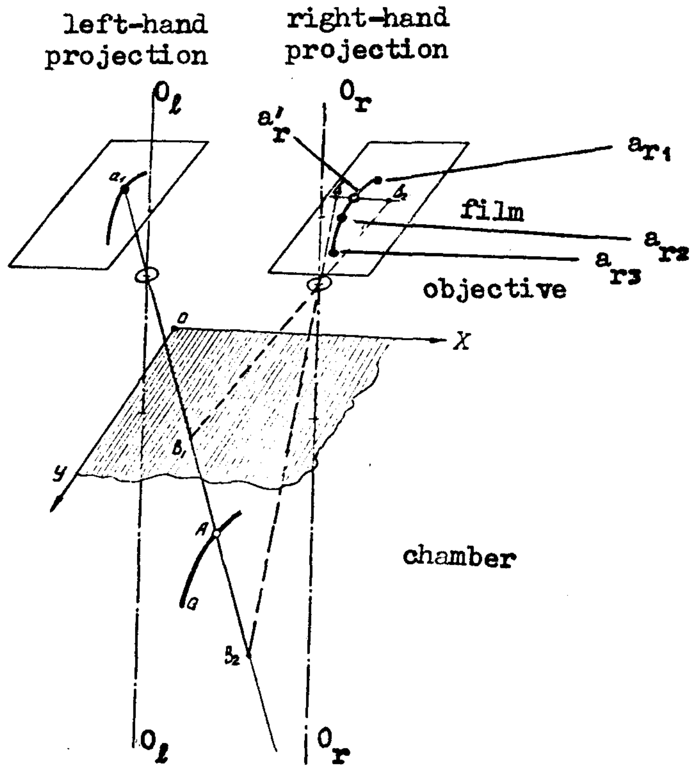


Fig. 3

Diagram of the determination of a near-corresponding point

O_l and O_r	- Axes of left-hand and right-hand stereo-cameras
XOY	- lower surface of the glass top of chamber
a_l	- point measured on the left-hand projection of the track Q
a_{r1}, a_{r2}, a_{r3}	- points measured on the right-hand projection of the track Q
B_1 B_2	- the straight line all points of which are projected on to the point a_l in the left-hand projection

$b_1\ b_2$	- the line which is the projection of $B_1\ B_2$ on to the right-hand frame; alternatively image of point a_ℓ in the right-hand frame.
$a'r$	- the point quasi-corresponding to a_ℓ ; a'_r lies on the intersection of $B_1\ B_2$ with the projection of the track, more accurately on the intersection of $b_1\ b_2$ with the parabola approximating to the points a_{r1}, a_{r2}, a_{r3}.
A	- space point on track Q, which is projected on to a_ℓ and a'_r (a_ℓ - is measured, a'_r - is found by computation).