JOINT INSTITUTE FOR NUCLEAR RESEARCH, DUBNA

Report P13 - 5751

INFLUENCE OF ULTRASOUND ON VAPOUR BUBBLE DYNAMICS IN LIQUID HYDROGEN, 2

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Dubna 1971

Translated at CERN by N.M. Harigel and G. Harigel
(Original: Russian)
Not revised by the Translation Service

(CERN Trans. Int. 71-9)

Geneva
October 1971

SUMMARY *

Theoretical considerations of the dynamics of the vapour bubble in liquid upon the ultrasonic field is given. The following assumptions are made : a bubble is spherical and homogeneous; vapour is in thermodynamical equilibrium with the surface layer of the liquid; liquid should be incompressible and isotropic. The system of equations takes into account heat and mass exchange between a bubble and the liquid. The state equation contains the second and third virial coefficients. The numerical solutions of the system of equations are given for the case of liquid hydrogen. Since the solutions are studied in the asymptotical region where the size of the bubble is 10^{-3} cm, the initial bubble is taken equal 5.10^{-4} cm, and the surface tension is neglected. The properties of solutions are discussed with the various values of thermodynamical and acoustical parameters corresponding to the working conditions of the ultrasonic bubble chamber. When the ultrasound is switched on, the initial bubble begins growing and after the finite number of pulsations dynamical equilibrium is established and the average radius becomes constant. The mechanism of the bubble growth is rectified heat diffusion. By taking into account the property of real gas one obtains a larger bubble compared to that of an ideal gas. It is shown that the qualitative behaviour of small and big bubbles has the peculiarity due to the presence of the surface and volume terms in the equations. These peculiarities and the sign of vapour specific heat conductivity along the phase equilibrium curve are of importance when the dynamical equilibrium pulsations of the bubble are established. The natural frequency pulsations greatly affect the bubble behaviour.

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* The above summary in English was provided by the authors, L.G. Tkachev and V.D. Shestakov.

INTRODUCTION

At the present time attempts are made to build ultrasonic bubble chambers (USBC)[1), 2)]. In connection with this, the influence of an ultrasonic field upon the dynamics of a vapour bubble in liquid hydrogen was theoretically considered [3), 4)]. In the present work, in distinction from [3), 4)], the vapour is not considered an ideal gas; as a result of this, the system of equations describing the behaviour of bubbles is different. As was to be expected, qualitative description of the behaviour of a single vapour bubble in a liquid under the influence of an ultrasonic field, did not change : as before, the mechanism for bubble growth is rectified heat diffusion. If one considers the characteristics of a real gas, then the bubble grows to a considerable large size. This explains the dominant role played by the volumetric factors during the establishment of pulsations of the bubble, which are in equilibrium.

FORMULATION OF THE PROBLEM

As in paper [3)], we shall limit ourselves to a consideration of a spherical vapour bubble in an incompressible liquid, making radial pulsations under the influence of the sinoidal ultrasonic field. The temperature field around the bubble is assumed to be isotropic, and the bubble itself-- homogeneous. The amplitudes and the frequencies of the pulsations of the bubble are limited by the condition of quasi-equilibrium of evaporation (condensation)*

$$\dot{M} < 4\pi R^2 \frac{aP' \sqrt{\mu}}{\sqrt{2\pi R_B T'}} , \qquad (1)$$

* The "μ" was omitted in formula below.

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where μ-- gram mole, $\dot{M}$ - speed of change of the mass of the bubble, R_B-- universal gas constant, R-- radius of the bubble, P' and T'-- pressure and temperature of gas in the bubble, a-- constant of accommodation.

The present work examines fairly large bubbles. For the description of their behaviour, terms conditioned by surface tension are inessential.

The condition (1) means that the vapour in the bubble is in thermodynamic equilibrium with the surface layer of the liquid

$$P'(t) = P(R,t),\ T'(t) = T(R,t), \tag{2}$$

where P(R,t) and T(R,t)--pressure and temperature in the liquid on the surface of the bubble at instant t.

The equation, defining the behaviour of a bubble, follows from the laws of conservation of momentum, mass and energy. The continuity equation in an incompressible liquid

$$r^2.\dot{r} = U_R.R^2 \tag{3}$$

allows us to integrate the Euler equation along the space variable. As a result we have :

$$R.\dot{U}_R + 2U_R.\dot{R} - \frac{1}{2}U_R^2 = \frac{P' - P_\infty}{\rho}, \tag{4}$$

where r--point coordinate in the liquid, $\dot{r}$ and U_R-- the speeds of the liquid in the points r and R correspondingly $(U_R = \lim_{r\to R} \dot{r})$; ρ-- density of the liquid, P_∞ -- pressure in the liquid at infinity from

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the bubble $P_\infty = P_o - P_1 \sin(2\pi ft)$. Here P_o-- static pressure, P_1 and f-- amplitude and frequency of the ultrasonic field. Equation (4) changes to the well known Rayleigh equation, if one assumes $U_R = \dot{R}$. In general, U_R and $\dot{R}$ are in correlation, following from the law of conservation of mass during evaporation and condensation on the surface of the bubble

$$\dot{R} = U_R + \frac{\dot{M}}{4\pi R^2 \rho} \tag{5}$$

Equations (4) and (5) determine R(t), if P'(t) is known. The vapour pressure is determined by the law of conservation of energy in the system under consideration

$$dE_v + dE_L = - 4\pi R^2 P' U_R dt \,, \tag{6}$$

where the increase of the internal energy of the vapour and of the liquid are correspondingly equal to :

$$dE_v = \varepsilon_v \, dM + \left(\frac{\partial E_v}{\partial \rho'}\right)_{M,T} d\rho' + c_v M.dT' \tag{7}$$

$$dE_L = - \varepsilon_L \, dM - 4\pi R^2 \, k \, \frac{\partial T}{\partial R} \, dt \tag{8}$$

Here ε_v and ε_L-- specific internal energy of the vapour and the liquid, ρ' and c_v-- density and specific heat of the vapour at constant volume, k-- coefficient of heat conductivity of the liquid, $\partial T/\partial R$-- temperature gradient in the liquid at the surface of the bubble. Taking into account that,

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$$\varepsilon_v - \varepsilon_L = L - P'\left(\frac{1}{\rho'} - \frac{1}{\rho}\right), \tag{9}$$

where L-- heat of vaporization, the equation of conservation of energy can be written in the following way :

$$L.dM + c_s\rho' \, dT = 4\pi R^2 \, k. \, \frac{\partial T}{\partial R} \, , \tag{10}$$

where c_s-- specific heat of vapour along the phase equilibrium, equal to

$$c_s = c_v + T'\left(\frac{dP'}{dT'}\right)_{\rho'} \cdot \frac{d\left(\frac{1}{\rho'}\right)}{dP'} \, . \tag{11}$$

Thermodynamic values of P', T' and ρ' are related to each other by the equation of state

$$P' = R_B \, T' \, \frac{\rho'}{\mu} \left[1 + B\,\frac{\rho}{\mu} + C\left(\frac{\rho'}{\mu}\right)^2\right], \tag{12}$$

with μ-- gram mole, B and C-- virial coefficients, which are established functions of temperature [5]. In the approximation of ideal gas, $B \equiv C \equiv 0$.

Formula (10) gives the equation determining the pressure of the vapour in a bubble

$$\frac{dP'}{dt} = \frac{3}{R} \cdot \frac{k.\frac{\partial T}{\partial R} - \rho' \, L \, \dot{R}}{L\,\frac{d\rho'}{dP'} + c_s\rho' \, \frac{dT'}{dP'}} \, , \tag{13}$$

where the derivatives $\frac{d\rho'}{dP'}$ and $\frac{dT'}{dP'}$ are measured along the curve of

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phase equilibrium.

In order to calculate the gradient of temperature of the liquid at the surface of the bubble, it is necessary to examine the equation of heat conductivity in the liquid

$$\frac{\partial T}{\partial t} + \nu \frac{\dot{R} - U_R \nu^3}{R} \cdot \frac{\partial T}{\partial \nu} = \frac{D}{R^2} \nu^4 \frac{\partial T}{\partial \nu^2} \qquad (14)$$

where D-- heat diffusivity of the liquid, $\nu = \frac{R}{r}$ -- dimensionless point coordinate in the liquid. In calculating (14), equation (3) was used.

The equations (4), (5), (13), (14) allow us to solve the problem of the behaviour of a vapour bubble in an ultrasonic field. The initial and boundary conditions are selected in the following manner : $R(o) = 5.10^{-4}$ cm, $P(o) = P_o$, $T(r=\infty,t) = T(\nu = 0, t) = T_\infty$; initial speed $\dot{R}(o)$ and diffusion of temperature in the liquid $T(\nu, t = 0)$ were determined, in accordance with paper [6], by supposing that with $t < 0$ the ultrasonic field is non-existent. Thermodynamic values L, k, c_γ are considered known functions along the curve of phase equilibrium [7), 8)].

As in paper [3], the problem is solved numerically. Results are shown as functions of R(t), M(t) or $\bar{R}(t)$, where the bar indicates the average value in time for the period of the ultrasonic field.

DISCUSSION OF THE RESULTS

Equations, describing the behaviour of the bubble, change

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into the corresponding equations in paper [3], if one supposes $U_R \equiv \dot{R}$ in (4) and $B \equiv C \equiv 0$. Exchange of formula (4) for the Rayleigh equation leads to a negligibly small change in the results in the examined range of temperatures : from 24 to 28 K. The influence of the characteristics of vapour on the behaviour of a bubble is more considerable, as can be seen from diagram 1, where the functions of R(t) are shown : the curve I corresponds to real gas, the curve II and III-- to ideal. The difference between curves II and III is explained by the method of assigning experimental values for the specific heat of vapour c_p and c_v, related to each other by the equation of state. If one uses the equation of state for the ideal gas and the numerical values for c_p (c_v), one gets then curve II (curve III). For the real gas, one gets, naturally, the single result -- curve I. In this case, the bubble becomes larger and its behaviour is characterized by peculiarities, which the small bubbles did not have.

Let us look in more detail at the characteristics of an ideal and real gas, utilizing equation (10). Present in this equation are surface as well as volumetric terms, proportional correspondingly to R^2 and R^3. In examining this qualitative behaviour, one can disregard the volumetric terms : sufficient number of small bubbles is present in the course of one cycle of the ultrasonic field, then

$$L\dot{M} = 4\pi R^2 k \frac{\partial T}{\partial R} . \qquad (15)$$

It is easy to show that this formula gives us the inequality $\dot{R}\dot{M} > 0$ (fig. 2a), which means that in the phase of growth (compression) of the bubble, its mass also grows (decreases). Here we can describe the qualitative behaviour of the bubble in terms of rectified and static heat diffusion, irrespective of the fact whether or not the vapour is an ideal gas.

When considering large bubbles, one can disregard the surface terms in equation (10), then

$$L\dot{M} + c_s M\dot{T} = 0 . \tag{16}$$

Taking into account that always $\dot{R}\,\dot{T} < 0$, we conclude that the sign of the product $\dot{R}\,\dot{M}$ depends on the sign of the specific heat of the vapour c_s.

The specific heat of an ideal gas $c_s = c_p - \frac{L}{T}$ can be positive as well as negative. The specific heat of real gas along the curve of phase equilibrium, calculated by using (11), according to experimental facts [5], is always negative.

Therefore, the specific heat of ideal and real gases, in the range of considered temperatures (in the environment of 26 K), have along the curve of phase equilibrium opposite signs. As result of this, in the first instance, $\dot{R}\,\dot{M} > 0$ and in the second-- $\dot{R}\,\dot{M} < 0$ (fig. 2b). Note, that in spite of the above, in both instances $\dot{\bar{R}}\,\dot{\bar{M}} > 0$; that is, generally, the mass of the vapour increases, if the average size of the bubble increases.

In the case of $\dot{R}\,\dot{M} < 0$ the conditions of thermodynamic equilibrium for a sufficiently large vapour bubble require condensation of the vapour in the course of its growth during the lowering of the temperature; and conversely, evaporation of the liquid in the course of the compression of the bubble during the raising of its temperature. Apparently, the transfer of energy, accompanying the transfer of a substance from one phase into another, and the

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transfer of energy, conditioned by rectified heat diffusion, are directed into opposite directions. As a result of this, dynamic equilibrium of sufficiently large bubbles in an ultrasonic field is established only when both currents compensate each other. The new way of establishing dynamic equilibrium of the pulsating bubble brings along with it, as we shall see, a qualitative new dependence between the significance of P_1, P_o and $\bar{R}_\infty = \lim_{t\to\infty} \bar{R}(t)$.

In figures 3 and 4 are shown the dependencies of $\bar{R}(t)$ during the various values of the amplitude of the ultrasonic field for the frequencies f = 400 kHz and f = 40 kHz. As the amplitude P_1 grows, the "life span" of the initial bubble grows and becomes infinitely large, when the amplitude reaches the value of the diffusion threshold $P_1 \geq P_{diff}$. Here, the value of the asymptotic radius $\bar{R}_\infty$ is rather small, so that the volumetric terms, in comparison with the surface ones, do not play a significant role in the creation of the dynamic equilibrium Therefore, $\bar{R}_\infty$ grows, as the amplitude of the ultrasonic field increases (fig. 3, P_1 = 1.1 bar, P_1 = 1.2 bar). In the case of large amplitudes of an ultrasonic field, the behaviour of the bubble is complicated not only by the fact that the volumetric effects play a significant role, but also on account of resonance effects, which increase according to the growth of the bubble and the approach of its frequency to the frequency of the ultrasonic field. When the frequencies are multiples of each other, the pulsations of the bubble on its own frequency become especially intensive. This leads either to the appearance of resonance levels on the curvature $\bar{R}$ (t), or to the disruption of the condition of quasi-equilibrium (1). The curves, corresponding to P_1 = 2.4 bar and P_1 = 2.03 bar in fig. 3, are examples of the fact that the resonance level has reached an equilibrium, because only due to strong pulsations on its own frequency one and the same value for $\bar{R}_\infty$, as well as for the

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amplitudes of the ultrasonic field, can be attained.

The curves, shown in fig. 4, demonstrate the role of volumetric factors during the creation of dynamic equilibrium of large bubbles : with the increase of the amplitude of an ultrasonic field, the value of $\bar{R}_{\infty}$ decreases, contrary to the instances when equilibrium is established as a result of equality of static and rectified diffusion. Such behaviour of the asymptotic radius of the bubble can be explained by the following rough calculations. In case of dynamic equilibrium, all physical values M(t), T(t), T'(t), etc... become periodic functions of the period of an ultrasonic field τ. Let us divide the equation (10) by M and integrate it from to to $t + \tau$.

$$\int_{t}^{t+\tau} L \frac{\dot{M}}{M} dt + \int_{+}^{t+\tau} c_s \dot{T}' dt = \int_{t}^{t+\tau} \frac{4\pi R^2}{M} k \frac{\partial T}{\partial R} .$$

When examining the qualitative behaviour of a bubble, one can consider that the thermodynamic values L, c_s, k are independent of temperature, and, consequently, of time. Then, the left side of equation (17) becomes 0, so that :

$$\int_{t}^{t+\tau} \frac{1}{R} \frac{\partial T}{\partial R} dt = \int_{t}^{t+\tau} \frac{1}{R^2} \frac{\partial T(\nu=1,t)}{\partial \nu} dt \approx 0.$$

Expanding into a series of sub-integral functions $\frac{1}{R^2}$, $\frac{\partial T(\nu=1,t)}{\partial \nu}$ and preserving in the expansion the more essential harmonics, one can show that the approximating formula

$$\bar{R}_{\infty} P_1 \approx \text{const.}$$

is valid, with which the results shown in fig. 4 agree very well.

In reference to relation (19), as well as the curves in figs. 4 and 5, one can mention the following. The decrease of P_1, f or the increase P_o leads to a decrease in the speed of the pulsation of the bubble. Since the volumetric terms in equation (10) are proportional to the derivative ones in time, they decrease faster, than the surface ones. Therefore, the relative role of rectified heat diffusion grows and a dynamic equilibrium is established during the larger values of $\bar{R}_\infty$. Naturally, with a decrease of amplitude and the speed of the pulsations of the bubble, the curve $\bar{R}(t)$ grows slower.

In fig. 6 are shown the dependencies $\bar{R}$ (t), derived during the various temperatures of the liquid T_∞ during the condition when the excess of static pressure in the liquid above the pressure of the saturated vapour is constant : $P_o - P_s = 0.5$ bar, and when the amplitude and the frequency of the ultrasonic field are also constant. As seen from the curves, the resonance effects during the amplitudes 2 - 3 bar are very significant. Due to this it is difficult to indicate the temperature, optimal for the technical realisation of USBC. From the point of view of the dynamics of bubble nuclei, because of smaller surface tension, higher temperatures are prefered.

It is essential to note that the obtained results pertain to an ideal, homogenous bubble. As seen from fig. 1, the general picture of the behaviour of a bubble in an ultrasonic field depends primarily upon the description of the characteristics of the vapour; therefore, in the future, it would be of interest to observe the dynamics of heterogenous bubbles.

In conclusion, the authors are grateful to V.A. Akulichev, V.N. Alekseev, B.A. Zhukov, G.I. Selivanov, and A.I. Filippov for their valuable discussions.

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Рукопись поступила в издательский отдел
13 апреля 1971 года.

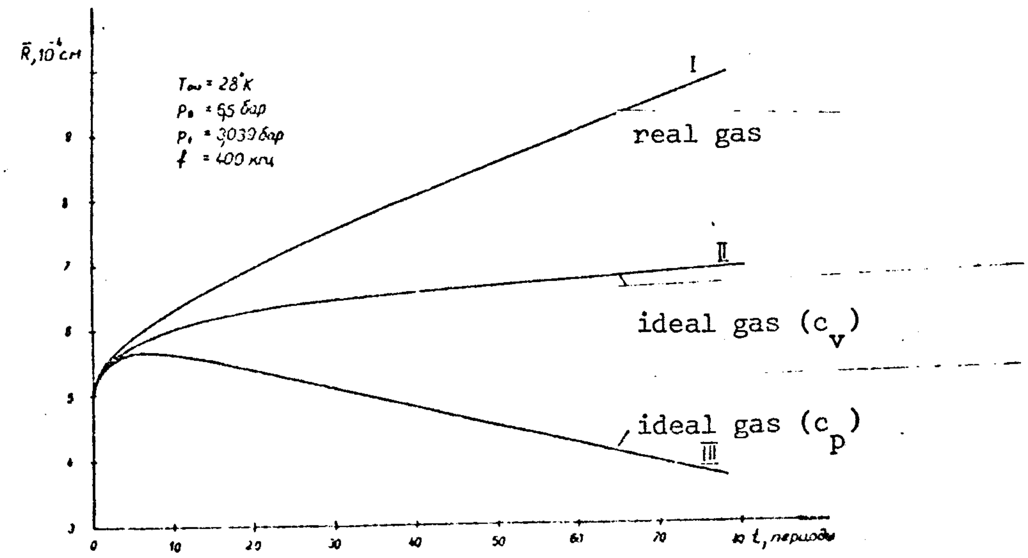
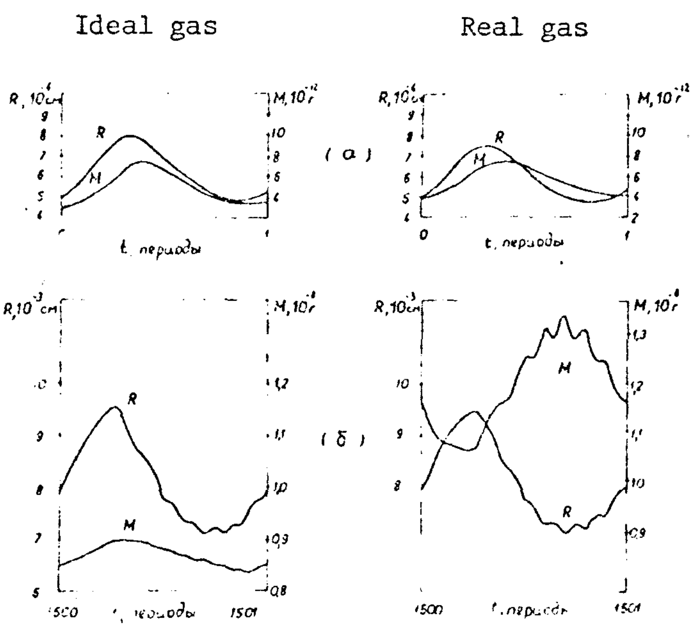

$\bar{R}, 10^{-4}$ см
T_{∞} = 28°K
p_0 = 55 бар
p_1 = 3039 бар
f = 400 кгц
I
real gas
II
ideal gas (c_v)
ideal gas (c_p)
III
0
10
20
30
40
50
70
t, периоды

Fig. 1



$f = 40$ kHz; $T_\infty = 26.15$ K; $P_o = 4.6$ bar ; $P_1 = 2.03$ bar.

Fig. 2 Functions of R(t) and M(t) for small (a) and for large (b) bubbles during the period of the ultrasonic field.

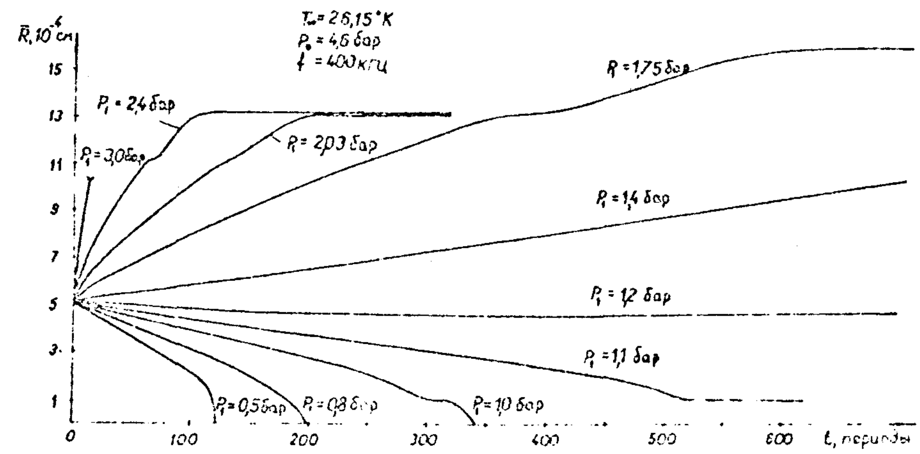


Fig. 3 Functions of $\bar{R}(t)$ for the various amplitudes of the ultrasonic field.

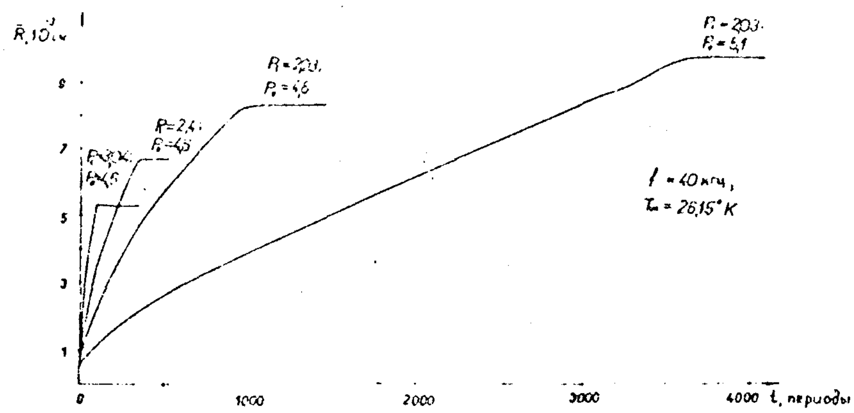


Fig. 4 Functions of $\bar{R}(t)$ during the various amplitudes of the ultrasonic field and during various static pressures.

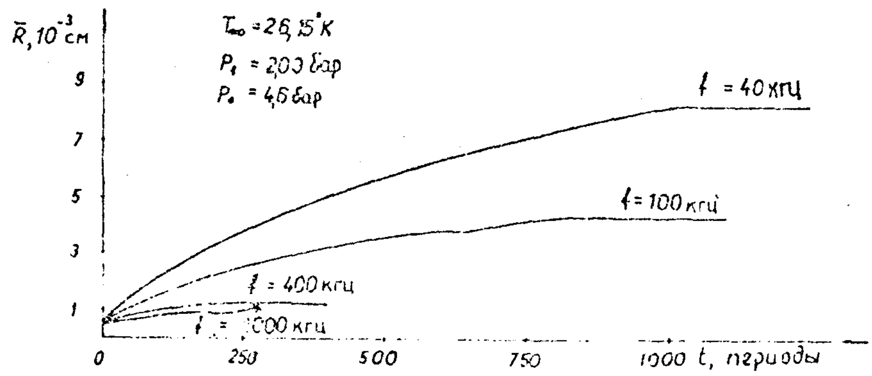


Fig. 5 Functions of $\bar{R}(t)$ during the various frequencies of the ultrasonic field. The star on the curvature indicates that further examination of the corresponding event is impossible with the condition of quasiequilibrium (1).

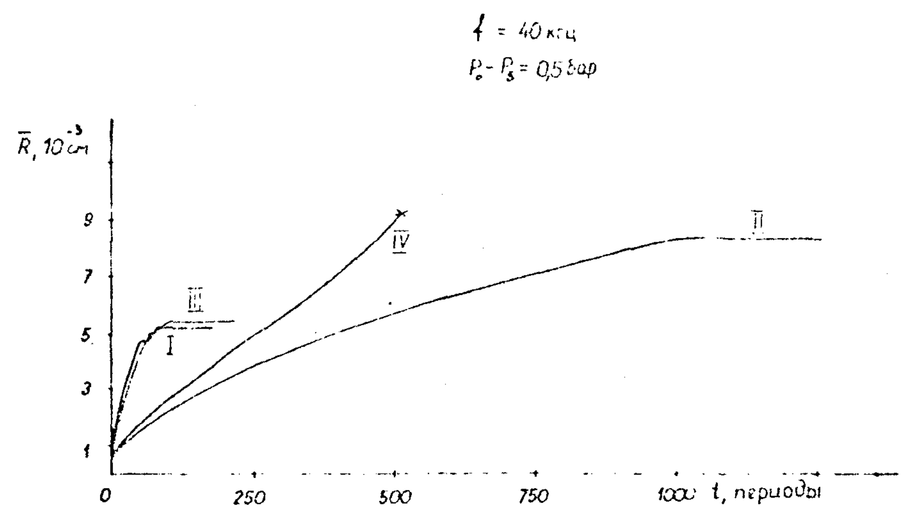


Fig. 6 Functions of $\bar{R}(t)$ during various temperatures

$$P_1 = 2.03 \text{ bar} \begin{cases} T_\infty = 24 \text{ K (I)} \\ T_\infty = 26.15 \text{ K (II)} \end{cases}$$

$$P_1 = 3.04 \text{ bar} \begin{cases} T_\infty = 26.15 \text{ K (III)} \\ T_\infty = 28 \text{ K (IV)} \end{cases}$$

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