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Anatomy of Mixing-Induced CP Asymmetries in Left-Right-Symmetric Models with Spontaneous CP Violation

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Abstract

We investigate the pattern of CP violation in $K^0-\bar{K}^0$, $B_d^0-\bar{B}_d^0$ and $B_s^0-\bar{B}_s^0$ mixing in a symmetrical $SU(2)_L \times SU(2)_R \times U(1)$ model with spontaneous CP violation. We calculate the phases of the left and right quark mixing matrices beyond the small phase approximation and perform a careful analysis of all relevant restrictions on the model's parameters from Δm_K , Δm_B , ϵ , ϵ'/ϵ and the CP asymmetry in $B_d^0 \rightarrow J/\psi K_S^0$. We find that, with current experimental data, the mass of the right-handed charged gauge boson, M_2 , is restricted to be in the range 2.75 to 13 TeV and the mass of the flavour-changing neutral Higgs boson, M_H , in 10.2 to 14.6 TeV. This means in particular that the decoupling limit $M_2, M_H \rightarrow \infty$ is already excluded by experiment. We also find that the model favours opposite signs of ϵ and $\sin 2\beta$ and is excluded if $\sin 2\beta > 0.1$.

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1 Introduction

In this paper we investigate in numerical detail an attractive extension of the standard model (SM), the Spontaneously Broken Left-Right model (SB-LR). The model turns out to be very constrained since, despite the larger number of bosons, symmetries strongly limit the new Yukawa couplings.

We show that a large fraction of parameter space is already excluded by conservative bounds arising from the B_d and B_s mass differences, the ϵ parameter for CP violation in the K system, and the sign of ϵ' . Even if current theoretical uncertainties persist in the K system, the expected experimental progress in B physics will soon bring conclusive tests of the model.

It is well known that CP is a natural symmetry of pure gauge theories with massless fermions.¹ As a result, CP violation actually probes the least known sector of unified theories, namely the scalar and Yukawa couplings. With the current development of dedicated accelerators to probe CP violation in the B system, it is important to study possible departures from the "Standard Model" based on the group $SU(2)_L \times U(1)$. Models based on the group $SU(2)_L \times SU(2)_R \times U(1)$, and more specifically those exhibiting spontaneous CP violation, offer the advantage of a well-defined, and actually quite constraining context, largely testable experimentally, while presenting a structure significantly different from the Standard Model. Before going into any details, let us stress already that the "left-handed" nature of the charged couplings in the SM, together with the absence of CP violation in the neutral channels, is extremely constraining: for instance electric dipole moments, intrinsically a LR transition, are strongly suppressed. In the same line, the scalar potential for the SM seems incompatible with a first order electroweak transition, thereby hampering low-temperature baryogenesis.

While very close in many aspects to the SM, and for this reason a natural extension, "LR" models significantly depart from it and provide a rich structure both for laboratory CP violation and baryogenesis [1], but also possibly in the leptonic sector, cf. [2]; the last two aspects will not be discussed in this paper.

By LR model we understand in general a description of electroweak interactions based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$. While such a group structure suggests low-energy parity restoration, a necessary condition for this is the equality of gauge couplings, $g_L = g_R$. This is not necessarily requested, and could actually prove a difficulty, notably in a cosmological approach: the persistence of an exact discrete symmetry to low-energy can lead to the formation of domains corresponding to different orientations of the breaking, and consequently to difficulties with the walls-energy. Some grand unified models, where P, but *not* $SU(2)_L \times SU(2)_R \times U(1)$ is broken at a very high scale, lead to a low-energy structure where $g_L \neq g_R$. While in this paper we will for simplicity set $g_L = g_R$, the results are easily adapted to the more general case, as, due to the high mass of W_R , the combination in use is generally g_R^2/M_R^2 , or, for the mixing terms, $g_L \cdot g_R \cdot \sin \zeta$, Ref. [3], where ζ is the mixing-angle between L and R bosons.

Some symmetries are however needed in order to constrain the scalar and Yukawa sectors of the theory. We thus request P as a symmetry of the Lagrangian (possibly, as stated above, in a weaker form where the interchange of fermions $f_L \rightleftharpoons f_R$ is accompanied by $g_L \rightleftharpoons g_R$).

¹This stems from the unitary nature of the groups, or equivalently the fact that gauge couplings are real. Anomalies can bring in T or CP violation, but such "strong CP violation" is only defined with respect to the determinant of fermion masses.

Even the definition of this symmetry is not without ambiguities: indeed, with in general non-diagonal mass matrices, the L and R partners are not uniquely defined, namely, a rotation U in flavour space can be allowed for, namely $f_L \rightleftharpoons U f_R$. In order to restrict the model further, we implement the attractive feature of spontaneous CP violation. This means that the Lagrangian must be symmetrical under CP (or CP generalized to include $g_L \rightleftharpoons g_R$), which is later broken by "misaligned" phases of the vacuum expectation values. Under these hypotheses, Ecker and Grimus have shown in Ref. [4] that, except for an exceptional case (which will not be considered here), the Yukawa couplings can be parametrized in terms of two real symmetrical matrices. As a result, all phases of the model can be related to a unique phase affecting vacuum expectation values (noted α below) and calculated exactly. This point is important, as it relates baryogenesis, and in particular the sign of the matter-antimatter asymmetry, to low-energy CP violation [1]. In practice, for the present analysis, four parameters are added to the Standard Model, but in counterpart, its single CP phase is now predicted.

The shorthand SB–LR will refer from now on to this "Spontaneously Broken Left-Right model". An important result of our analysis is that the SB–LR is in some sectors *more restrictive* than the SM itself. Indeed, while the SM is a subset of LR obtained by sending the R sector masses to infinity, a similar procedure applied to the SB–LR yields additional constraints since the CKM phase δ is no longer independent, but predicted within the model. For instance, we find that the CP violating phase in the resulting SM is too small, $|\delta| < 0.25$ or $|\delta - \pi| < 0.25$, whereas the global fit of [5] yields $\delta = 1.0 \pm 0.2$. Hence the SM limit of the SB–LR is inconsistent by 3.5σ with current experiments. This has the important consequence that the SB–LR is actually testable, and distinct from the SM: experimental bounds cannot be indefinitely evaded by simply sending the R sector to infinite masses: scalars and vectors in the range (2–20) TeV are definitely needed.

Experimental constraints on the SB–LR, mainly from the K system, have been thoroughly investigated in [3]. Since then, many SM parameters, in particular the CKM angles and the top quark mass, have been measured much more accurately, and the perspective of finding non-standard CP violation in the B system at the B factories, the Tevatron or the LHC has prompted a number of new analyses of the SB–LR, for instance [6, 7], which, however, all use on a certain approximation for calculating the phases of the left and right CKM matrices. We thus feel that a comprehensive analysis of the constraints from measured CP conserving and violating observables in both the K and the B system is timely, which in particular uses exact expressions for the CKM phases. The main new results of our analysis can be summarized as follows:

- the small phase approximation fails for CKM matrix elements involving the 3rd generation;
- the role of the Higgs bosons, neglected in most analyses, is crucial;
- the decoupling limit of the model, $M_2, M_H \rightarrow \infty$, is experimentally excluded, which implies *upper bounds* on M_2 and M_H ;
- the SB–LR favours opposite signs of the CP violating observables $\text{Re } \epsilon$ and $a_{\text{CP}}(B \rightarrow J/\psi K_S)$, which are both expected to be positive in the SM; hence, the model cannot accomodate both the experimentally measured ϵ and the SM expectation $a_{\text{CP}}^{\text{SM}}(B \rightarrow J/\psi K_S) \approx 0.75$ and is excluded if a_{CP} will be measured to be larger than 0.1.

The paper is organized as follows: in Sec. 2 we define the model underlying our calculations. In Sec. 3 we calculate the phases of the left and right quark mixing matrix. In Sec. 4 we formulate strategies for measuring and/or constraining the SB–LR parameters. In Sec. 5 we calculate B mixing in the SB–LR and incorporate constraints from the measurement of the CP asymmetry in $B^0 \rightarrow J/\psi K_S$. In Sec. 6 we discuss constraints on the model from K physics. In Sec. 7 we combine the constraints from both K and B observables. In Sec. 8, finally, we summarize and conclude.

2 Definition of the Model

We begin with a reminder, namely how the extended gauge group $SU(2)_L \times SU(2)_R \times U(1)$ cascades down to the unbroken electromagnetic subgroup $U(1)_{\text{em}}$ through the following simple symmetry-breaking pattern:

$$\begin{array}{ccc}
 SU(2)_L \times SU(2)_R \times U(1) & & \\
 \begin{array}{ccc} T_L^i & T_R^i & S \\ g_L & g_R & g_1 \end{array} & & \\
 \underbrace{\hspace{10em}} & & \\
 \begin{array}{ccc} SU(2)_L \times & U(1)_Y & \\ T_L^i & Y/2 = T_R^3 + S & \\ g_L & g' & \end{array} & & \\
 \underbrace{\hspace{10em}} & & \\
 U(1)_{\text{em}} & & \\
 Q = T_L^3 + Y/2 & & \\
 e & &
 \end{array}$$

Listed underneath each subgroup factor is our nomenclature convention for their associated generators and coupling constants.

We next specify the quark and scalar content of the model. The quarks transform under the unbroken gauge group as

$$q_{Li} = \begin{pmatrix} U_i \\ D_i \end{pmatrix}_L \sim (2, 1, 1/6), \quad q_{Ri} = \begin{pmatrix} U_i \\ D_i \end{pmatrix}_R \sim (1, 2, 1/6), \quad (2.1)$$

where i is a generation index. The generation of quark masses in the $SU(2)_L \times SU(2)_R \times U(1)$ model requires at least one scalar bidoublet Φ , i.e. a doublet under both $SU(2)$, corresponding to two standard doublets:

$$\Phi = \begin{pmatrix} \phi_0^1 & \phi_1^+ \\ \phi_2^- & \phi_0^2 \end{pmatrix} \sim (2, \bar{2}, 0).$$

As usual, the quarks are given masses by a spontaneous breakdown of the symmetry such that Φ acquires the VEV

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v & 0 \\ 0 & w \end{pmatrix}. \quad (2.2)$$

The quark mass matrices read

$$M^{(u)} = \frac{v}{\sqrt{2}}\Gamma + \frac{w^*}{\sqrt{2}}\Delta, \quad M^{(d)} = \frac{w}{\sqrt{2}}\Gamma + \frac{v^*}{\sqrt{2}}\Delta. \quad (2.3)$$

In general, both v and w are complex, which is the source of spontaneous CP violation. The phases can of course be redefined by a gauge rotation of the L or R fields, and, in particular, the phase *difference* of v and w can be rotated away.² Here we follow the notations of [4] and define *two* phase combinations:

$$\left| \frac{w}{v} \right| = r, \quad \arg(vw) = \alpha, \quad \arg(-vw^*) = \lambda. \quad (2.4)$$

Although the relevant diagrams are the same with an enlarged Higgs sector, only in the minimal case are the couplings of scalar fields to quarks determined by masses and mixing angles only. In this case, neglecting for the moment the contributions from the triplets, which do not couple directly to the quarks, there is a flavour-conserving neutral scalar field Φ_1 , the analogue of the SM Higgs, and a single charged scalar field Φ^\pm as well as two neutral scalar fields Φ_2, Φ_3 with flavour-changing couplings to quarks.

Further fields are needed to achieve the complete breakdown³ from $SU(2)_L \times SU(2)_R \times U(1)$ to $U(1)_{em}$; the simplest choices respecting LR symmetry are either two doublets (one L and one R), or two triplets. This latter choice is usually preferred when dealing with the leptonic sector, since the quantum numbers of these triplets allow for the generation of heavy neutrino Majorana masses directly related to LR symmetry breaking; b decays, however, do not sensitively depend on this precise structure of the scalar sector. The triplets are

$$\chi_L = \begin{pmatrix} \chi_L^{++} \\ \chi_L^+ \\ \chi_L^0 \end{pmatrix} \sim (3, 1, 2), \quad \chi_R = \begin{pmatrix} \chi_R^{++} \\ \chi_R^+ \\ \chi_R^0 \end{pmatrix} \sim (1, 3, 2)$$

and acquire the VEVs

$$\langle \chi_{L,R} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ v_{L,R} \end{pmatrix}.$$

In the remainder of this paper we shall assume $|v_L|^2 \ll |v|^2 + |w|^2 \ll |v_R|^2$.

The spontaneous breakdown of $SU(2)_L \times SU(2)_R \times U(1)$ to $U(1)_{em}$ generates the charged W boson mass matrix

$$M_{W^\pm}^2 = \begin{pmatrix} \frac{g_L^2}{4} (2v_L^2 + |v|^2 + |w|^2) & -g_L g_R v^* w / 2 \\ -g_L g_R v w^* / 2 & \frac{g_R^2}{4} (2v_R^2 + |v|^2 + |w|^2) \end{pmatrix} \equiv \begin{pmatrix} M_L^2 & M_{LR}^2 e^{-i\lambda} \\ M_{LR}^2 e^{i\lambda} & M_R^2 \end{pmatrix}.$$

The eigenvalues

$$M_1^2 = M_L^2 \cos^2 \zeta + M_R^2 \sin^2 \zeta + M_{LR}^2 \sin 2\zeta,$$

$$M_2^2 = M_L^2 \sin^2 \zeta + M_R^2 \cos^2 \zeta - M_{LR}^2 \sin 2\zeta,$$

²This rotation was performed in [8].

³Spontaneous breaking of CP without undue fine tuning may require the introduction of further fields, for instance singlets uncoupled to the fermions [9]. We will not discuss here the details of the scalar Lagrangian, which is not critical for work.

and eigenvectors

$$\begin{pmatrix} W_1^+ \\ W_2^+ \end{pmatrix} = \begin{bmatrix} \cos \zeta & -e^{i\lambda} \sin \zeta \\ e^{-i\lambda} \sin \zeta & \cos \zeta \end{bmatrix} \begin{pmatrix} W_L^+ \\ W_R^+ \end{pmatrix}$$

of this mass matrix correspond to the physical charged W bosons in the $SU(2)_L \times SU(2)_R \times U(1)$ model. The W_L - W_R mixing angle is defined as

$$\tan 2\zeta = -\frac{2M_{LR}^2}{M_R^2 - M_L^2},$$

and the charged current reads (with $g \equiv g_L \equiv g_R$ and without displaying unphysical scalars and charged Higgs contributions):

$$\begin{aligned} \mathcal{L}_{cc} = & -\frac{g}{\sqrt{2}} \bar{U}_i \left[\cos \zeta (V_L)_{ij} \gamma^\mu P_L - e^{-i\lambda} \sin \zeta (V_R)_{ij} \gamma^\mu P_R \right] D_j W_{1\mu}^+ \\ & -\frac{g}{\sqrt{2}} \bar{U}_i \left[e^{i\lambda} \sin \zeta (V_L)_{ij} \gamma^\mu P_L + \cos \zeta (V_R)_{ij} \gamma^\mu P_R \right] D_j W_{2\mu}^+. \end{aligned}$$

3 Quark Mixing

The Yukawa interaction part of the Lagrangian reads

$$-\mathcal{L}_Y = \Gamma_{ij} \bar{q}_{Li} \Phi q_{Rj} + \Delta_{ij} \bar{q}_{Li} \tilde{\Phi} q_{Rj} + \text{h.c.} \quad (3.1)$$

with $\tilde{\Phi} = \sigma_2 \Phi^* \sigma_2$. As discussed in the introduction, the crucial feature of the SB-LR with spontaneous CP violation is that P invariance coupled to spontaneous CP violation restricts the coupling matrices Γ and Δ : apart from one special case, see Ref. [4], which we shall not consider here, both matrices may be taken real and symmetric. We will work in this basis until further notice. After spontaneous symmetry breaking one obtains the quark mass matrices $M^{(u)}$ and $M^{(d)}$ of Eq. (2.3). The diagonalisation of $M^{(u)}$ normally requires a bi-unitary transformation, involving two unitary matrices acting separately on the L and R spinors; two more matrices are then needed for $M^{(d)}$. In this special case however, the matrices $M^{(u)}$ and $M^{(d)}$ are *symmetrical in the chiral basis* and can therefore be diagonalized by only two unitary matrices U, V , so that

$$M^{(u)} = U D^{(u)} U^T, \quad M^{(d)} = V D^{(d)} V^T, \quad (3.2)$$

where $D^{(u,d)}$ are diagonal mass matrices. Note that in the SB-LR the signs of the quark masses are observable, so that the entries in $D^{(u,d)}$ need in principle not be positive. This is a difference to the SM, where the sign of the quark masses in the Lagrangian,

$$-m_i (\bar{q}_{Li} q_{Ri} + \bar{q}_{Ri} q_{Li}),$$

can always be absorbed into the phase of q_R , which is not observable, since right-handed quarks have no charged weak interactions. This is of course no longer possible in the SB-LR. For the moment, we thus need to keep track of the possible signs of the masses; they will later be absorbed into the right quark mixing matrix so that the model has the standard mass

terms in the Lagrangian and the standard quark propagator, $1/(p_\mu \gamma^\mu - m)$, $m \geq 0$. As the CKM phases can only depend on mass ratios, we thus have a $2^5 = 32$ -fold multiplicity of solutions corresponding to the different possible choices of the quark mass signs. Fortunately, as we shall see later, phenomenological constraints remove most of these solutions.

Diagonalizing the mass matrices introduces two quark mixing matrices, one for the left, and one for the right sector; the crucial feature in the present case of spontaneous CP violation is that in the special basis where Δ and Γ are symmetrical, Eq. (3.2) implies that the left and right mixing matrices are complex conjugate to each other:

$$K \equiv K_L = U^\dagger V = K_R^*.$$

The remainder of this section will be devoted to the calculation of the phases of that matrix, but first we would like to make the connection to a more “standard” quark basis in which the left mixing matrix contains only one phase, δ (this choice is obviously not unique, and the numerous parametrisations of the Kobayashi-Maskawa matrix attest to this; the procedure below is, however, general). In order to do so, we first rewrite K as

$$K = \zeta^u \tilde{K} \zeta^{d*},$$

where $\zeta^{u,d}$ are diagonal matrices with entries ζ_i such that $(\zeta_i)^2 = \text{sign}(m_i)$. These phases are introduced to redefine the masses as positive. The advantage of introducing \tilde{K} is that for $\alpha = 0$, i.e. for the case of no CP violation, and for a suitable choice of gauge, all entries are real. Redefining now the phases of the quark fields by γ_i^u, γ_j^d , $1 \leq i, j \leq 3$, one can bring \tilde{K} into a standard form V_L (i.e. with only one phase left):

$$V_L = e^{-i\gamma^u} \tilde{K} e^{i\gamma^d}.$$

The right mixing matrix then reads

$$V_R = \eta^u e^{-i\gamma^u} \tilde{K}^* e^{i\gamma^d} \eta^d = \eta^u e^{-2i\gamma^u} V_L^* e^{2i\gamma^d} \eta^d.$$

Here $\eta = \zeta^2$ are diagonal matrices with entries ± 1 and specify the signs of the quark masses. Note that the phase matrices introduce only five independent phases in V_R , as only differences $\gamma_i^d - \gamma_j^u$ enter. V_R can then be written as

$$V_R = \begin{pmatrix} (V_L^*)_{11} e^{2i\alpha_1} & (V_L^*)_{12} e^{i(\alpha_1 + \alpha_2 + \epsilon_1)} & (V_L^*)_{13} e^{i(\alpha_1 + \alpha_3 + \epsilon_1 + \epsilon_2)} \\ (V_L^*)_{21} e^{i(\alpha_1 + \alpha_2 - \epsilon_1)} & (V_L^*)_{22} e^{2i\alpha_2} & (V_L^*)_{23} e^{i(\alpha_2 + \alpha_3 + \epsilon_2)} \\ (V_L^*)_{31} e^{i(\alpha_1 + \alpha_3 - \epsilon_1 - \epsilon_2)} & (V_L^*)_{32} e^{i(\alpha_2 + \alpha_3 - \epsilon_2)} & (V_L^*)_{33} e^{2i\alpha_3} \end{pmatrix} \quad (3.3)$$

with the five independent phases α_i, ϵ_i , in which we have also absorbed the signs of the quark masses; the 6th phase, hidden in $(V_L^*)_{ij}$, is the usual unique surviving phase of the SM model. Note that the phases of V_L and V_R depend on the parametrization chosen for V_L .

We also would like to mention that in addition to different parametrizations of V_L , also different conventions for the phase in W_L - W_R mixing are used in the literature. Most papers, notably [3] and [4] (and ours), keep the phase explicitly in the Lagrangian, whereas [8] prefers

to shuffle it into V_R . This corresponds to a choice of gauge $\lambda^{[8]} = 0$, while Ref. [4] uses $v = v^*$ instead, leading to

$$e^{i\lambda^{[EG]}} = -e^{-i\alpha}.$$

Denoting matrices in the different conventions by $V_L^{[8]}$ for Ref. [8] and $V_L^{[EG]}$ for Ref. [4], we then have to identify

$$V_R^{[8]} = e^{i\alpha} V_R^{[EG]}. \quad (3.4)$$

As later on we would like to use formulas given in Ref. [8], we also have to convert the phases δ_1 , δ_2 and γ of $V_R^{[8]}$ into our language. We find that one has to identify:

$$\begin{aligned} \gamma - \delta_2 &= 2\alpha_1 + \alpha, \\ \gamma + \delta_2 &= 2\alpha_2 + \alpha, \\ \gamma - \delta_1 &= \alpha_1 + \alpha_2 + \epsilon_1 + \alpha, \\ \gamma + \delta_1 &= \alpha_1 + \alpha_2 - \epsilon_1 + \alpha. \end{aligned}$$

Note that the system is degenerate and contains only three independent relations.

We can now parametrize the matrix K as

$$K = \begin{pmatrix} (V_L)_{11} e^{-i\alpha_1} & (V_L)_{12} e^{-i/2(\alpha_1+\alpha_2+\epsilon_1)} & (V_L)_{13} e^{-i/2(\alpha_1+\alpha_3+\epsilon_1+\epsilon_2)} \\ (V_L)_{21} e^{-i/2(\alpha_1+\alpha_2-\epsilon_1)} & (V_L)_{22} e^{-i\alpha_2} & (V_L)_{23} e^{-i/2(\alpha_2+\alpha_3+\epsilon_2)} \\ (V_L)_{31} e^{-i/2(\alpha_1+\alpha_3-\epsilon_1-\epsilon_2)} & (V_L)_{32} e^{-i/2(\alpha_2+\alpha_3-\epsilon_2)} & (V_L)_{33} e^{-i\alpha_3} \end{pmatrix}. \quad (3.5)$$

It is clear that the phases will be functions of r and α , and that CP violation is characterized by $y = r \sin \alpha$, with $r \sim O(m_b/m_t)$. This fact led the authors of [10, 4] to calculate \tilde{K} in a linear expansion in y . Later on, in [8], the full solutions for the phases for mixing between the first two generations were calculated and it was found that the linear approximation works perfectly well for these entries. It is, however, to be expected that the linear approximation breaks down for the third generation matrix elements, where the natural ‘‘smallness’’ of the expansion parameter y can be upset by enhancement factors m_t/m_b , cf. also Ref. [11]. In this paper we calculate the full phases beyond the small phase approximation, which, as shown in [8], amounts to solving the matrix equation

$$(1 - r^2)\tilde{W} \tilde{D}^{(u)} \tilde{W} + (r^2 e^{i\alpha} - e^{-i\alpha})\tilde{D}^{(u)} = 2ir \sin \alpha \tilde{K} \tilde{D}^{(d)} \tilde{K}^T, \quad (3.6)$$

which is equivalent to a system of 12 (real) coupled equations. The unknowns in these equations are six real parameters characterizing the unitary symmetrical matrix \tilde{W} and the six phases of the mixing matrix \tilde{K} . $\tilde{D} = \zeta D \zeta = \eta D$ are the diagonal mass matrices including the signs of the quark masses. In order to solve (3.6), it is convenient to replace the independent variables r and α by new variables β and β' , defined as [8]

$$\beta = \arctan \frac{2r \sin \alpha}{1 - r^2}, \quad e^{i\beta'} = \frac{1 - r^2 e^{-2i\alpha}}{|1 - r^2 e^{-2i\alpha}|}. \quad (3.7)$$

This transformation makes the dependence on one variable, β' , trivial:

$$\tilde{K} = e^{-i\beta'/2} K',$$

where K' is solution of

$$\cos \beta W' \tilde{D}^{(u)} W' - \tilde{D}^{(u)} = i \sin \beta K' \tilde{D}^{(d)} K'^T, \quad (3.8)$$

and W' is still unitary and symmetric. For the “natural” choice $r \sim O(m_b/m_t)$, to be motivated in the next section, β' is negligibly small, and we shall neglect it in our final results.⁴ The phases are thus to an excellent approximation functions of only one variable, β . As shown in Ref. [8], the above equation has solutions only for a restricted interval in β ,

$$\tan \frac{\beta}{2} \leq \frac{m_b}{m_t}.$$

We have solved (3.8) by a polynomial expansion as suggested in [8]. The input parameters are the quark masses renormalized at one common scale which we choose to be \bar{m}_t ; we use the following values, in the $\overline{\text{MS}}$ scheme:

$$\begin{aligned} \bar{m}_t(\bar{m}_t) &= 170 \text{ GeV}, \\ \bar{m}_b(\bar{m}_t) &= 2.78 \text{ GeV} \quad \Leftrightarrow \quad \bar{m}_b(\bar{m}_b) = 4.25 \text{ GeV}, \\ \bar{m}_c(\bar{m}_t) &= 0.63 \text{ GeV} \quad \Leftrightarrow \quad \bar{m}_c(\bar{m}_c) = 1.33 \text{ GeV}, \\ \bar{m}_s(\bar{m}_t) &= 0.060 \text{ GeV} \quad \Leftrightarrow \quad \bar{m}_s(2 \text{ GeV}) = 110 \text{ MeV}, \\ \bar{m}_d(\bar{m}_t) &= 0.0030 \text{ GeV} \quad \Leftrightarrow \quad m_s/m_d = 20.1, \\ \bar{m}_u(\bar{m}_t) &= 0.0017 \text{ GeV} \quad \Leftrightarrow \quad m_u/m_d = 0.56. \end{aligned} \quad (3.9)$$

The value of \bar{m}_s is a compromise between recent lattice calculations as summarized in [12] and QCD sum rule calculations [13].

One more input are the angles of the CKM matrix. The particle data group [14] quotes for the entries determined from tree-level processes, which receive only tiny corrections from LRS contributions:

$$\begin{aligned} |V_{ud}| &= 0.9740 \pm 0.0010, & |V_{us}| &= 0.2196 \pm 0.0023, \\ |V_{cb}| &= 0.0395 \pm 0.0017, & |V_{ub}/V_{cb}| &= 0.08 \pm 0.02. \end{aligned} \quad (3.10)$$

For an exactly unitary matrix where all entries lie within the above specified error range, we fix

$$|V_{us}| = 0.2219, \quad |V_{ub}| = 0.004, \quad |V_{cb}| = 0.04. \quad (3.11)$$

In addition, we have to specify the value of the phase δ for the case of no CP violation, i.e. $\delta(\beta = 0) = 0$ or π . We will label the corresponding set of solutions of (3.8) as class I and class II solutions, respectively. This induces another two-fold multiplicity in addition to the 32-fold one from the different quark mass signs, so that we finally have to deal with 64 different

no.	m_t	m_b	m_c	m_s	m_d	m_u
1	+	+	+	+	+	+
2	+	+	+	+	-	+
3	+	+	+	-	+	+
4	+	+	+	-	-	+
5	+	+	-	+	+	+
6	+	+	-	+	-	+
7	+	+	-	-	+	+
8	+	+	-	-	-	+
9	+	-	+	+	+	+
10	+	-	+	+	-	+
11	+	-	+	-	+	+
12	+	-	+	-	-	+
13	+	-	-	+	+	+
14	+	-	-	+	-	+
15	+	-	-	-	+	+
16	+	-	-	-	-	+
17	-	+	+	+	+	+
18	-	+	+	+	-	+
19	-	+	+	-	+	+
20	-	+	+	-	-	+
21	-	+	-	+	+	+
22	-	+	-	+	-	+
23	-	+	-	-	+	+
24	-	+	-	-	-	+
25	-	-	+	+	+	+
26	-	-	+	+	-	+
27	-	-	+	-	+	+
28	-	-	+	-	-	+
29	-	-	-	+	+	+
30	-	-	-	+	-	+
31	-	-	-	-	+	+
32	-	-	-	-	-	+

Table 1: Identification of solutions of (3.6) with quark mass signatures. We choose m_u to be always positive.

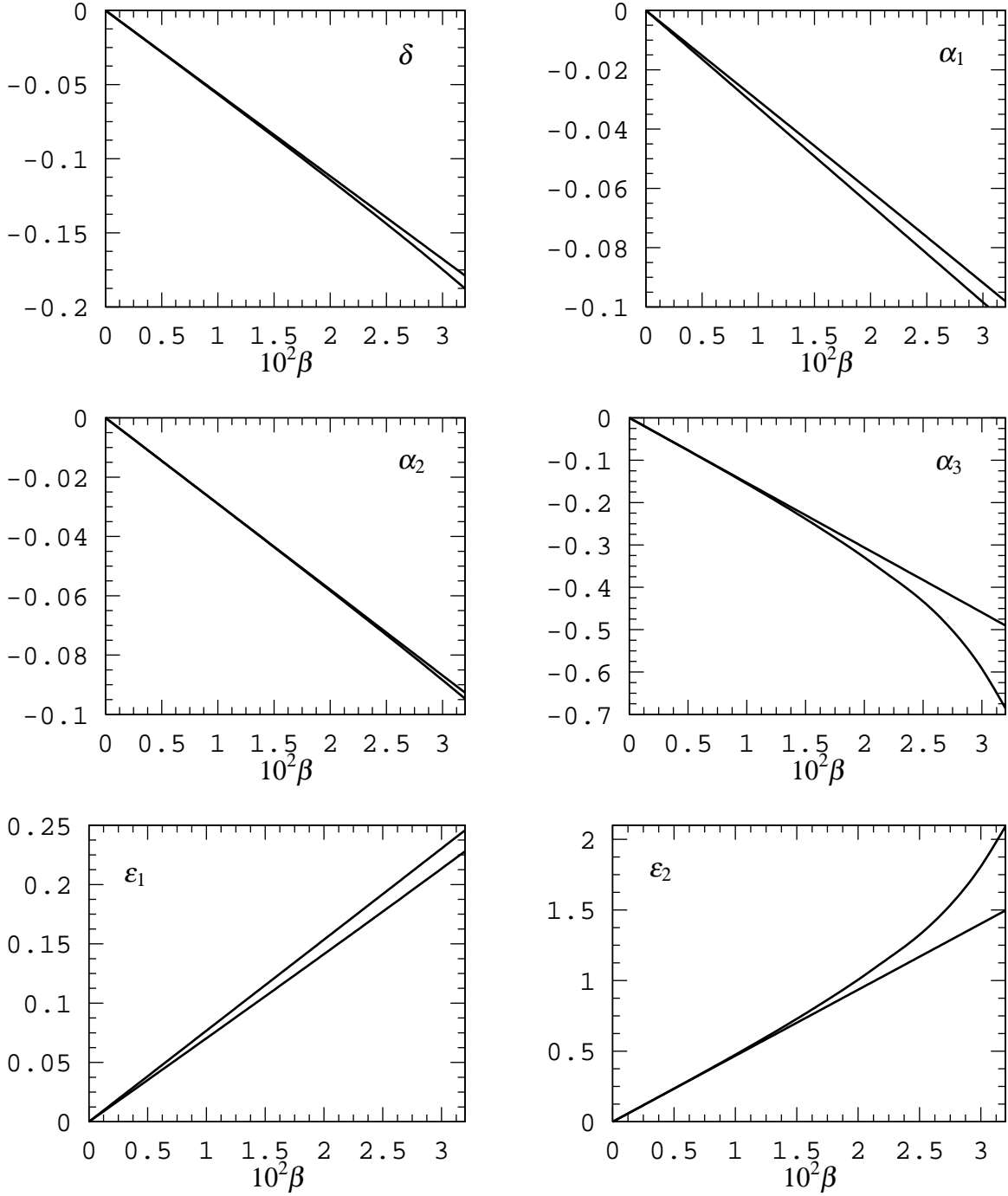


Figure 1: Independent phases of the CKM matrices acc. to (3.3) as functions of β in the Maiani convention, for $\delta(\beta = 0) = 0$, i.e. class I, and $\beta' = 0$, for positive quark masses. The straight lines are the phases calculated in the small phase approximation, the curves are the full results.

solutions of (3.8). In Tab. 1 we give the explicit identifications of the solutions with the mass signatures.

Before presenting results, we would also like to stress that K' and \tilde{K} are independent of the phase-convention for V_L – they only depend on the modulus $|V_L|$. The phase-convention for V_L enters just in the extraction of the six phases δ , α_i and ϵ_i from Eq. (3.5). In the following, we will always use the Maiani convention

$$V_L = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

with, from the experimental results, Eq. (3.10),

$$\theta_{12} = 0.2218 \pm 0.0031, \quad \theta_{23} = 0.0395 \pm 0.0017, \quad \theta_{13} = 0.0032 \pm 0.0008.$$

Our preferred values for CKM matrix elements (3.11) correspond to

$$\theta_{12} = 0.2238, \quad \theta_{23} = 0.0400, \quad \theta_{13} = 0.004, \quad (3.12)$$

so that

$$V_L(\delta = 0) = \begin{pmatrix} 0.9751 & 0.2219 & 0.0040 \\ -0.2219 & 0.9743 & 0.0400 \\ 0.0050 & -0.0399 & 0.9992 \end{pmatrix}$$

and

$$V_L(\delta = \pi) = \begin{pmatrix} 0.9751 & 0.2219 & -0.0040 \\ -0.2216 & 0.9743 & 0.0400 \\ 0.0128 & -0.0381 & 0.9992 \end{pmatrix}.$$

Note that $|V_{td}|$ is quite sensitive to the value of δ . A full analysis of the impact of CKM angle and quark mass uncertainties on our results is beyond the scope of this paper.

In Fig. 1 we plot the independent phases as functions of β in the allowed range $0 \leq \beta \leq 2m_b/m_t = 0.0327$, both the full results and the small phase approximation, for $\delta(\beta = 0) = 0$, i.e. class I, and positive quark masses. From the figure it is evident that the two phases characteristic for the 3rd generation, α_3 and ϵ_2 , deviate significantly from the small phase approximation for large β , whereas for the other phases the agreement between the full and the approximate result is very good. This feature is also observed for the other 63 solutions. It is also evident that the phases characterizing mixing between the first two generations are rather smallish, whereas α_3 and ϵ_2 can become large. This reflects the impact of enhancement factors $\sim m_t/m_b$ that overcome the smallness of β and invalidate the small phase approximation. The numerical results for all the 64 solutions are available from the authors as MATHEMATICA file.

Finally, we would like to comment on the approximate formulas for the sinus of certain parametrization-independent combinations of CKM entries relevant for B mixing, σ_d and σ_s , to be defined in Sec. 5, which were originally derived in [11] and used in [7] and follow-up papers. These approximate formulas rely on the small phase approximation and retain only leading terms in the ratios of quark masses. In Fig. 2 we plot $\sin \sigma_d$ as function of β calculated from the full solutions and from the approximate formula given in [11]. Apparently, the structure

⁴ β' is, however, in general not negligible in the large-mixing region.

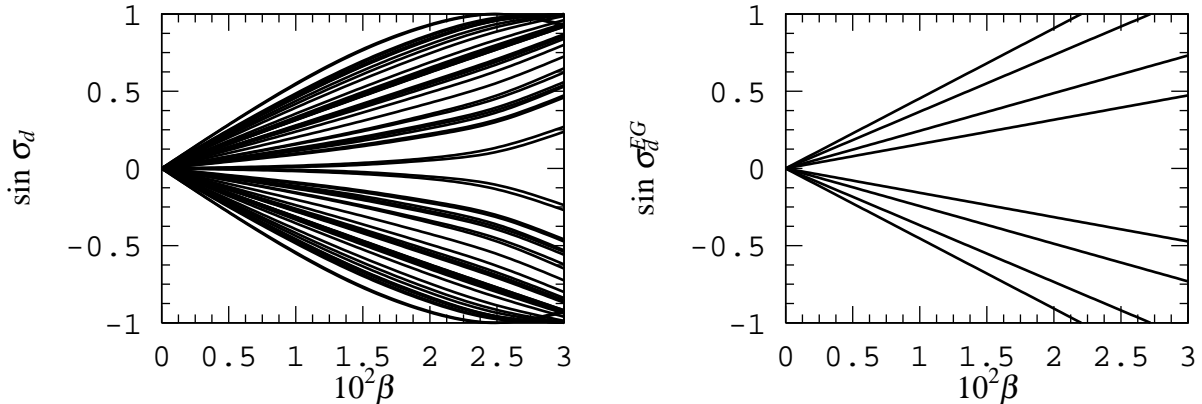


Figure 2: $\sin \sigma_d$, defined in (5.9), as function of β calculated from the full solution, left, and in the approximation derived in Ref. [11], right. Note that with the approximate formula, the value of the sinus can become larger than 1 for certain choices of the mass signs.

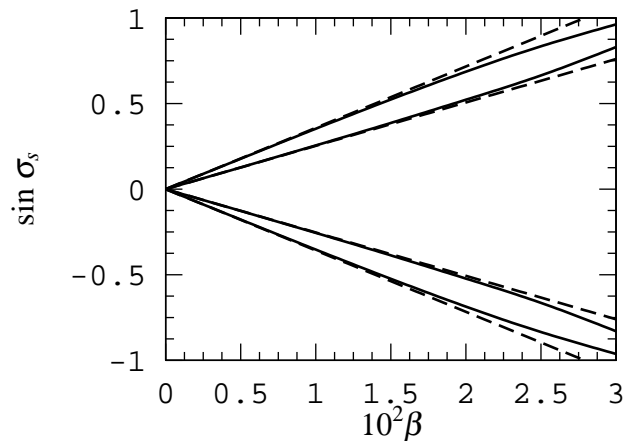


Figure 3: Same as previous figure, but for $\sin \sigma_s$. Dashed lines are the small phase approximation.

of the full solution is richer than can be reproduced by a simple approximation formula. Also, for a number of quark mass signatures and large β , the formula for $\sin \sigma_d$ predicts values larger than 1 and thus cannot be used in that region. On the other hand, for $\sin \sigma_s$, shown in Fig. 3, the approximation works quite well and only fails for large β .

4 The SB–LR Parameter Space and Strategies for Constraints

The new parameters of the SB–LR are the following (numerical values will be discussed in later sections):

- $M_2 \sim O(1 \text{ TeV})$, the mass of the predominantly right-handed weak gauge boson;

- ζ , the mixing angle between W_R and W_L , $\zeta \geq 0$;
- g_R , the coupling of W_R . As discussed in the introduction, although low-energy parity restoration would require $g_R = g_L$, this is not necessarily requested if parity, but not $SU(2)_L \times SU(2)_R \times U(1)$ is broken at a higher scale, and $g_R \neq g_L$ may be preferable to avoid domain-wall formation. For definiteness we will however set here $g_R = g_L$, as the results on boson masses can be easily adapted;
- $0 \leq r \leq 1$ and $0 \leq \alpha \leq \pi$, parametrizing the spontaneous breakdown of CP symmetry, i.e. the VEV of the bidoublet Φ ;
- extra Higgs masses which, in principle, are quite arbitrary. However, since they are associated with neutral flavour changing currents, the prejudice is usually to have heavy extra Higgses with masses $M_H \sim O(10 \text{ TeV})$. In this case, they cannot mix significantly with the lighter ones and must be nearly degenerate (their splitting being at most of the order of the weak scale), see also [4]. We will in the present study neglect the mass differences, and allow only one single mass parameter for the heavy Higgs bosons, M_H , and also assume $M_H > M_2$. A full study, allowing extra Higgses lighter than the right-handed bosons could some day be needed; it would however be sensitive to the fine details of the scalar potential.

Actually not all of these parameters are independent, but they observe the following relations and constraints:

$$\zeta = \left(\frac{m_W}{M_2}\right)^2 \frac{2r}{1+r^2}, \quad (4.1)$$

$$\frac{m_b}{m_t} > \left| \frac{r \sin \alpha}{1-r^2} \right|, \quad (4.2)$$

$$\frac{M_H}{M_2} < 13. \quad (4.3)$$

The second of these relations originates from quark mixing, see Sec. 3, and the third one comes from requiring convergence of the perturbation expansion [15].

Let us now discuss for what type of processes we expect measurable effects due to the SB–LR. First of all, SB–LR implies strong constraints on the CKM phase δ even in the decoupling limit of the model, i.e. $M_2, M_H \rightarrow \infty$. An inspection of all sets of quark mixing phases shows that $|\delta^{\text{SB-LR}}| < 0.25$ for class I solutions and $|\delta^{\text{SB-LR}} - \pi| < 0.25$ for class II solutions. One can make use of this fact to exclude the SM model limit of the SB–LR model experimentally by measuring δ from Δm_{B_d} or ϵ . The extraction of δ from these measurements, however, to date involves considerable theoretical uncertainties. Nevertheless, a recent global fit of SM CKM parameters [5], which includes conservative theory error estimates, finds $\delta^{\text{SM}} = 1.0 \pm 0.2$, so that we conclude that *the decoupling limit $M_2, M_H \rightarrow \infty$ of the SB–LR is excluded by 3.5σ* .

Let us next discuss the case of finite M_2 and M_H . As for M_2 , it enters either directly via a W_2 propagator or indirectly in ζ via W_L – W_R mixing. Obviously, with an expected modification of the amplitude of size $(m_W/M_2)^2 \sim O(10^{-3})$ it is in practice impossible to observe either of these effects in tree-level decays. In loop-induced processes, however, the situation changes, and we expect measurable or even sizable effects for the following cases:

- the suppression factor $(m_W/M_2)^2$ is partially compensated by large matching coefficient functions (tree-level Wilson-coefficients) of the effective Hamiltonian, radiative corrections or hadronic matrix elements (e.g. chiral enhancement in K mixing⁵);
- W_L - W_R mixing is enhanced by large quark mass terms from spin-flips, e.g. $\zeta \rightarrow \zeta m_t/m_b$ in $b \rightarrow s\gamma$ [17]; this affects all top-dominated penguin-diagrams and is thus expected to be important for processes with direct CP violation;
- the SM amplitude is forbidden or heavily suppressed (electric dipole moment of the neutron).

As for the Higgses, their contribution to SM tree-level decays is also heavily suppressed by factors $(m_W/M_H)^2 \sim 10^{-4}$ or smaller from the propagator. On the other hand, the neutral FC Higgs contributes to $\Delta F = 2$ processes at tree-level and the charged Higgs contributions get enhanced by m_t/m_b in b penguins; roughly speaking, their contribution is similar in size to that of W_R for K mixing and becomes dominant in B mixing.

Based on these relations, we distinguish two regions in parameter space with qualitatively different phenomenological consequences. These are

- the “natural” region with $r \sim O(m_b/m_t) \sim 0.02$, which implies $\zeta \sim O(10^{-4})$;
- the “large mixing” region with $\zeta \sim 10^{-3}$ so that $r \sim (0.1-1)$.

The small r region is called “natural”, because it has been argued to explain the observed smallness of the CKM mixing angles [18]. With the expected size of $M_2 \sim O(1 \text{ TeV})$, one then has a rather small mixing angle $\zeta \sim 10^{-4}$, which severely restricts the possible impact of SB-LR contributions on penguin-induced processes and on CP asymmetries from direct CP violation. On the other hand, condition (4.2) is fulfilled for any α , which can thus vary freely within 0 and π . In the natural region, ζ effectively decouples from mixing-induced CP violating processes and we can choose r and M_2 as independent variables.

In the large mixing region, on the other hand, we *require* ζ to be close to its maximum experimentally allowed value $\zeta \approx 0.003$ [3, 8]. Now r can become large, and consequently, via (4.2), α is restricted to values close to 0 or π . This is the region where one might expect sizable SB-LR effects to show up in penguin-induced processes. In this case it is more appropriate to choose ζ and M_2 as independent variables and determine r from Eq. (4.1).

In the present paper we restrict ourselves to mixing-induced CP violating effects and thus work consistently in the natural region, fixing⁶ $r = m_b/m_t$. The remaining independent parameters are then β , M_2 and M_H . In addition, we have a 64-fold multiplicity of CKM phases from the different possible choices for quark mass signs and the value of the phase δ in the limit of no CP violation, $\delta = 0$ or π . The observables we analyse in this paper are Δm_K , $\Delta m_{B_{d,s}}$, $a_{CP}(B \rightarrow J/\psi K_S)$, ϵ and ϵ'/ϵ . Another possible observable from the B_s system is $\Delta\Gamma_s$, which was analyzed in [19]. Another potentially powerful constraint can in principle be obtained from the neutron electric dipole moment (EDM). The LR model contributions to the

⁵The fact that there is no such chiral enhancement for B mixing led some authors to conclude that the SB-LR would not have much impact on these processes, cf. e.g. [16]; indeed, it is the Higgs contribution that is dominant in B mixing.

⁶ As long as $r \leq m_b/m_t$, its exact value does not matter.

EDM are discussed in Refs. [8] and [20], taking into account not only the sum of the quark EDM's as done in previous calculations, but also specific hadronic terms which involve W_L – W_R exchange between the quark lines of a neutron. It is indeed an interesting feature of the SB–LR model that it allows CP violation in this sector already within a 1-generation context. Crucial to such contributions is the presence of LR mixing, since the EDM is basically a LR transition and hence suppressed in the SM. In Ref. [20], the following bound was obtained:

$$\left| \zeta \sin(\gamma - \delta_2) \right| = \frac{2r}{1+r^2} \left(\frac{m_W}{M_2} \right)^2 \left| \sin \{2\alpha_1(\alpha) + \alpha\} \right| \leq 3 \cdot 10^{-6}.$$

Yet, this bound is not without criticism: it is obtained as a sum of large terms of opposite sign and thus comes with considerable uncertainty. In addition, it is to be supposed that there are also large contributions from gluonic matrix elements, i.e. strong CP violation, which might upset the bound. We thus refrain from taking it into account in our analysis.

Most of the observables we analyze in this paper are related to the matrix element

$$\langle M^0 | \mathcal{H}_{\text{eff}}^{\Delta F=2} | \bar{M}^0 \rangle = 2m_M \left(M_{12}^{\text{SM}} + M_{12}^{\text{LR}} + M_{12}^{\text{LD}} \right), \quad (4.4)$$

$F = S, B$ and $M^0 = K^0, B_d^0$ and B_s^0 . M_{12}^{SM} stands for the SM contribution, M_{12}^{LR} for the SB–LR contribution and M_{12}^{LD} for $(\Delta F = 1)^2$ contributions, which are negligible in B mixing, but expected to be sizable in K mixing [21]. We also introduce the mixing angles $\phi_M^{B_q}$ and ϕ_M^K ,

$$\phi_M^{B_q} = \arg M_{12}^{B_q}, \quad \phi_M^K = \arg M_{12}^K. \quad (4.5)$$

In both the B and the K system the mixing between flavour eigenstates can be described in terms of these mixing angles to an excellent accuracy; the only quantities to be considered in this paper, for which that approximation is not sufficient, are ϵ and ϵ'/ϵ . Note also that the mixing angles are convention-dependent quantities.

As for CP conserving quantities, constraints can be derived from Δm , the mass difference between mass eigenstates. For both K's and B's, one has

$$\Delta m = 2 |M_{12}|.$$

The experimental mass differences in the K and B system provide in principle a powerful constraint on $|M_{12}^{\text{LR}}|$; for the K system, however, the size of long-distance contributions to M_{12}^{LD} is not very well known, so in this case one usually makes the reasonable assumption⁷ that the LR contribution should at most saturate Δm_K . We thus constrain the LR parameters by requiring

$$2|M_{12}^{\text{K,LR}}| < \Delta m_K^{\text{exp}}. \quad (4.6)$$

For Δm_{B_d} , as LD contributions are negligible, theory is in a better shape and we can require

$$2|M_{12}^{\text{B}}| = \Delta m_{B_d}^{\text{exp}}. \quad (4.7)$$

We also investigate the CP violating observables $a_{\text{CP}}(B \rightarrow J/\psi K_S)$, to be defined in Sec. 5.2, which depends on both the K and B mixing angles, and ϵ and ϵ'/ϵ for the K system, which will be investigated in Sec. 6. These three observables vanish for $\beta \rightarrow 0$, in contrast to $\Delta m_{K,B}$. From the analysis of ϵ in particular, we shall conclude that the SB–LR is excluded in the decoupling limit $M_2, M_H \rightarrow \infty$.

⁷This assumption appears reasonable unless there are large cancellations between the different contributions to M_{12} .

5 $B^0-\bar{B}^0$ Mixing in the SB–LR

In this section we follow largely the notations and conventions of Ref. [22].

5.1 Constraints from Δm_{B_d} and Predictions for Δm_{B_s}

In the SM, M_{12} is dominated by box-diagrams with W_L and top exchange and given by

$$\begin{aligned} M_{12}^{\text{SM}} &= \frac{1}{32\pi^2 m_B} G_F^2 m_W^2 (\lambda_t^{LL})^2 S(x_t) \eta_2^B(\mu) \langle B^0 | [(\bar{d}b)_{V-A}(\bar{d}b)_{V-A}](\mu) | \bar{B}^0 \rangle \\ &\equiv \frac{1}{32\pi^2 m_B} G_F^2 m_W^2 (\lambda_t^{LL})^2 S(x_t) \hat{\eta}_2^B(\mu) [\alpha_s^{(5)}(\mu)]^{-6/23} \langle B^0 | [(\bar{d}b)_{V-A}(\bar{d}b)_{V-A}](\mu) | \bar{B}^0 \rangle. \end{aligned} \quad (5.1)$$

The hadronic matrix element is parametrized as usual as

$$\langle B^0 | [(\bar{d}b)_{V-A}(\bar{d}b)_{V-A}](\mu) | \bar{B}^0 \rangle = -2 \left(1 + \frac{1}{N_c}\right) B_B(\mu) f_B^2 m_B^2 e^{-i\phi_{CP}^B}, \quad (5.2)$$

where the bag-factor $B_B(\mu)$ describes the deviation from vacuum saturation ($B_B^{\text{vac}} = 1$) and ϕ_{CP}^B is an arbitrary phase describing the transformation behaviour of B flavour eigenstates under CP transformations:

$$(\text{CP})|B^0\rangle = e^{i\phi_{CP}^B}|\bar{B}^0\rangle, \quad (\text{CP})|\bar{B}^0\rangle = e^{-i\phi_{CP}^B}|B^0\rangle.$$

It goes without saying that physical observables must be independent of that phase. Often, the tacit choice $\phi_{CP}^B = \pi$ is made. f_B is the leptonic decay constant of the B meson, defined as

$$\langle 0 | (\bar{d}b)_A | \bar{B}^0 \rangle = i f_B p_\mu,$$

and the Inami–Lim function S is defined as

$$S(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(x_t - 1)^2} - \frac{3}{2} \frac{x_t^3}{(1 - x_t)^3} \ln x_t$$

with $x_t = \bar{m}_t(\bar{m}_t)^2/m_W^2$. For the CKM factors, we use the notation

$$\lambda_t^{AB} = V_{tq}^{A*} V_{tb}^B$$

with $A, B = L, R$. Numerical values of input parameters are given in Table 2. In the SB–LR, there are several additional contributions, notably the tree-level neutral Higgs exchange and box-diagrams with W_R and unphysical scalar exchanges, all of which are dominated by the top quark. Taking into account only the leading contributions in M_2 and M_H , one finds:⁸

$$\begin{aligned} M_{12} &= M_{12}^{\text{SM}} + M_{12}^{W_1 W_2} + M_{12}^{S_1 W_2} + M_{12}^H \\ &\equiv M_{12}^{\text{SM}} + M_{12}^{\text{LR}} \end{aligned} \quad (5.3)$$

⁸Box-diagrams with charged Higgses are suppressed relative to the L–R box-diagrams by roughly a factor $(M_2/M_H)^2$ and thus can be neglected as long as $M_2 \ll M_H$. Note also that M_{12}^{LR} as given in (5.3) is gauge-dependent; subsequent formulas are given in the t’Hooft–Feldman gauge. The diagrams restoring gauge-invariance have been calculated in [25, 26] and were found to be small in that gauge.

$\bar{m}_t(\bar{m}_t)$	$\eta_2^{B,LO}(m_b)$	$\hat{\eta}_2^{B,LO}$	$\hat{\eta}_2^{B,NLO}$	$B_B(m_b)$	\hat{B}_B^{NLO}	$f_B\sqrt{\hat{B}_B^{NLO}}$
$(170 \pm 5) \text{ GeV}$	0.86	0.580	0.551	0.9 ± 0.1	1.4 ± 0.1	$(207 \pm 42) \text{ MeV}$

Table 2: Theoretical input parameters for B mixing. Numbers are taken from [23, 24].

f_{B_s}/f_B	B_{B_s}/B_B
1.14 ± 0.08	1.00 ± 0.03

Table 3: Matrix elements for B_s , from [24].

with [11]

$$M_{12}^H = -\frac{\sqrt{2}G_F}{m_B M_H^2} \bar{m}_t(\bar{m}_t)^2 \eta^H(\mu) \lambda_t^{LR} \lambda_t^{RL} \langle B^0 | O_S(\mu) | \bar{B}^0 \rangle,$$

$$M_{12}^{W_1 W_2 + S_1 W_2} \simeq -\frac{G_F^2 m_W^4}{8\pi^2 M_2^2} B_B^S(\mu) f_B^2 m_B \left[\frac{m_B^2}{\bar{m}_b(\bar{m}_b)^2} + \frac{1}{6} \right] \lambda_t^{LR} \lambda_t^{RL}$$

$$\times \left\{ 4\eta_1^{LR}(\mu) F_1(x_t, \beta) - \eta_2^{LR}(\mu) F_2(x_t, \beta) \right\} e^{-i\phi_{CP}^B}$$

with $\beta = m_W^2/M_2^2$.

Let us first discuss the Higgs contribution. To leading logarithmic accuracy, the operator $O_S = (\bar{d}P_L b)(\bar{d}P_R b)$ renormalizes multiplicatively with an anomalous dimension that just compensates that of the factor m_t^2 such that $m_t^2 O_S$ is RG-invariant. The LO short-distance correction η^H can thus be written as

$$\eta^H(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_s(m_t)} \right]^{24/23}, \quad \eta^H(m_b) = 2.0.$$

On the other hand, the matrix element of O_S can be written as⁹

$$\langle B^0 | O_S(\mu) | \bar{B}^0 \rangle = -\frac{1}{2} f_B^2 m_B^2 B_B^S(\mu) \left[\frac{m_B^2}{\bar{m}_b(\bar{m}_b)^2} + \frac{1}{2N_c} \right] e^{-i\phi_{CP}^B}, \quad (5.4)$$

where the bag-factor $B_B^S(\mu) \sim O(1)$, $\mu \sim O(m_b)$, contains the full scale-dependence of the matrix-element. To our knowledge, $B_B^S(m_b)$ has never been estimated by any non-perturbative method. In the appendix we calculate the ratio B_B^S/B_B both to leading order in a $1/N_c$ expansion and from QCD sum rules. The results agree with

$$\frac{B_B^S(m_b)}{B_B(m_b)} = 1.2 \pm 0.2. \quad (5.5)$$

⁹ Note that our expressions for M_{12}^{LR} and the matrix element of O_S are by a factor of 2 smaller than those quoted in [11], which is due to the fact that the authors of that paper use a definition of f_B which makes it by a factor of $\sqrt{2}$ smaller than ours. This is also the source of a factor of 2 discrepancy in the expression for $|M_{12}^{LR}/M_{12}^{SM}|$ in Ref. [7].

Motivated by the results in Tab. 3, which indicate a small SU(3)-breaking for the bag-factor in the SM, we will use (5.5) also for the matrix elements over B_s^0 .

As for $M_{12}^{W_1 W_2 + S_1 W_2}$, its operator structure is more complicated than that of the Higgs contribution. The LO short-distance corrections $\eta_{1,2}^{LR}$ have been calculated in an approach suggested by Novikov, Shifman, Vainshtein and Zakharov [27], which in Ref. [28] was shown to be equivalent to the by now standard effective theory approach. From the results in [4, 11], one finds

$$\eta_1^{LR}(m_b) \approx 1.8, \quad \eta_2^{LR}(m_b) \approx 1.7. \quad (5.6)$$

F_1 and F_2 are in general complicated functions of x_t , x_b and $\beta = (m_W/M_2)^2$. However, in the limit $x_b \rightarrow 0$ and for $M_2 \geq 1.4$ TeV, they are to within 5% accuracy approximated by

$$F_1 \simeq \frac{x_t}{1-x_t} + \frac{x_t \ln x_t}{(1-x_t)^2} - x_t \beta \ln \beta,$$

$$F_2 \simeq \frac{x_t^2}{1-x_t} + \frac{2-x_t}{(1-x_t)^2} x_t^2 \ln x_t - x_t \ln \beta.$$

Numerically, $|4F_1| \ll |F_2|$. We thus approximate

$$\eta_2^{LR} F_2 - 4\eta_1^{LR} F_1 \approx \eta_2^{LR} (F_2 - 4F_1).$$

We are now in a position to calculate M_{12} in the SB-LR and to investigate its impact on phenomenology. Following [11], we write (5.3) as

$$M_{12} = M_{12}^{\text{SM}} (1 + \kappa e^{i\sigma_q}), \quad (5.7)$$

$$\text{with } \kappa \equiv \left| \frac{M_{12}^{\text{LR}}}{M_{12}^{\text{SM}}} \right|, \quad (5.8)$$

$$\sigma_q \equiv \arg \frac{M_{12}^{\text{LR}}}{M_{12}^{\text{SM}}} = \arg \left(-\frac{V_{tb}^R V_{tq}^{R*}}{V_{tb}^L V_{tq}^{L*}} \right). \quad (5.9)$$

Note that the phase σ_q is convention-independent and a physical observable. The minus sign in the definition of σ_q comes from the fact that, putting all CKM factors equal one, M_{12}^{SM} and M_{12}^{LR} have different relative sign. κ is nearly independent of the flavour of the spectator quark. Numerically, we find¹⁰

$$\kappa = \frac{B_B^S(m_b)}{B_B(m_b)} \left[\left(\frac{7 \text{ TeV}}{M_H} \right)^2 + \eta_2^{LR}(m_b) \left(\frac{1.6 \text{ TeV}}{M_2} \right)^2 \left\{ 0.051 - 0.013 \ln \left(\frac{1.6 \text{ TeV}}{M_2} \right)^2 \right\} \right], \quad (5.10)$$

which describes the full solution to within 5% accuracy for $M_H > 7$ TeV and $M_2 > 1.4$ TeV.

Let us now investigate the predictions of the SB-LR for and the constraints from, respectively, the experimental data for Δm_B . For the remainder of this section we consider $|M_{12}|$ as a function of only two variables, κ and β , instead of expressing κ in terms of M_2 and M_H . The mass difference in the B_d^0 system has been measured as [29]

$$\Delta m_{B_d} = (0.472 \pm 0.016) \text{ ps}^{-1}, \quad (5.11)$$

¹⁰Corrections in $(1 - \eta_2^{LR}/\eta_1^{LR})$ are smaller than neglected terms in $1/M_2^4$.

whereas for the mass difference in the B_s^0 system, there exists only a lower bound [30]:

$$\Delta m_{B_s} > 12.4 \text{ ps}^{-1}; \quad (5.12)$$

the SM expectation is [5]

$$\Delta m_{B_s}^{\text{SM}} = (14.8 \pm 2.6) \text{ ps}^{-1}. \quad (5.13)$$

In the SM and with our values for the CKM angles, Eq. (3.12), (5.11) restricts the phase δ as $\delta^{\text{SM}} = 1.17 \pm 0.44$ (taking into account that the measured value of ϵ implies $\delta^{\text{SM}} > 0$), where the error comes mainly from $f_B^2 \hat{B}_B$. Taking into account the theoretical uncertainties on $f_B^2 \hat{B}_B$ and m_t , (5.11) translates into

$$\left| (V_{tb}^L V_{td}^{L*})^2 (1 + \kappa e^{i\sigma_d}) \right| = (6.7 \pm 2.7) \cdot 10^{-5}. \quad (5.14)$$

From this result we may derive a constraint for κ : in the worst case of negative relative sign, κ evidently cannot be larger than

$$\kappa^{\text{max}} - 1 = \frac{(6.7 \pm 2.7) \cdot 10^{-5}}{|(V_{tb}^L V_{td}^{L*})^2|},$$

which roughly translates into

$$\kappa < 3,$$

which we will use in the following. For B_s mixing, on the other hand, we obtain from (5.12) the lower bound

$$\left| (V_{tb}^L V_{ts}^{L*})^2 (1 + \kappa e^{i\sigma_s}) \right| > 9.6 \cdot 10^{-4}. \quad (5.15)$$

For the ratio of mass differences, the theory error is much smaller:

$$\begin{aligned} \frac{\Delta m_{B_s}}{\Delta m_{B_d}} &= \frac{m_{B_s}}{m_{B_d}} \left(\frac{f_{B_s}}{f_{B_d}} \right)^2 \frac{\hat{B}_{B_s}}{\hat{B}_{B_d}} \left| \frac{(V_{tb}^L V_{ts}^{L*})^2 (1 + \kappa e^{i\sigma_s})}{(V_{tb}^L V_{td}^{L*})^2 (1 + \kappa e^{i\sigma_d})} \right| \\ &= (1.31 \pm 0.19) \left| \frac{(V_{tb}^L V_{ts}^{L*})^2 (1 + \kappa e^{i\sigma_s})}{(V_{tb}^L V_{td}^{L*})^2 (1 + \kappa e^{i\sigma_d})} \right|, \end{aligned} \quad (5.16)$$

which translates into the bound

$$\left| \frac{(V_{tb}^L V_{ts}^{L*})^2 (1 + \kappa e^{i\sigma_s})}{(V_{tb}^L V_{td}^{L*})^2 (1 + \kappa e^{i\sigma_d})} \right| > 17.2. \quad (5.17)$$

In Fig. 4 we plot the left-hand side of Eq. (5.14), normalized by the central value on the right-hand side, as function of κ for several values of β . In Fig. 5 we plot $|(V_{ts}^L V_{tb}^{L*})^2 (1 + \kappa e^{i\sigma_s})|/0.039^2$, which is expected to be 1 in the SM. From the plots we conclude the following:

- the decoupling limit $\kappa \rightarrow 0$ of the SB–LR is excluded; this is due to the fact that the SM phase as extracted from Δm_{B_d} is rather large, $\delta^{\text{SM}} \sim 1$, whereas the SB–LR predicts values close to 0 or π ; this result depends, however, on our specific choice of the CKM angles, Eq. (3.12); yet, we shall derive the experimental exclusion of the decoupling limit unambiguously from the analysis of ϵ in the next section;

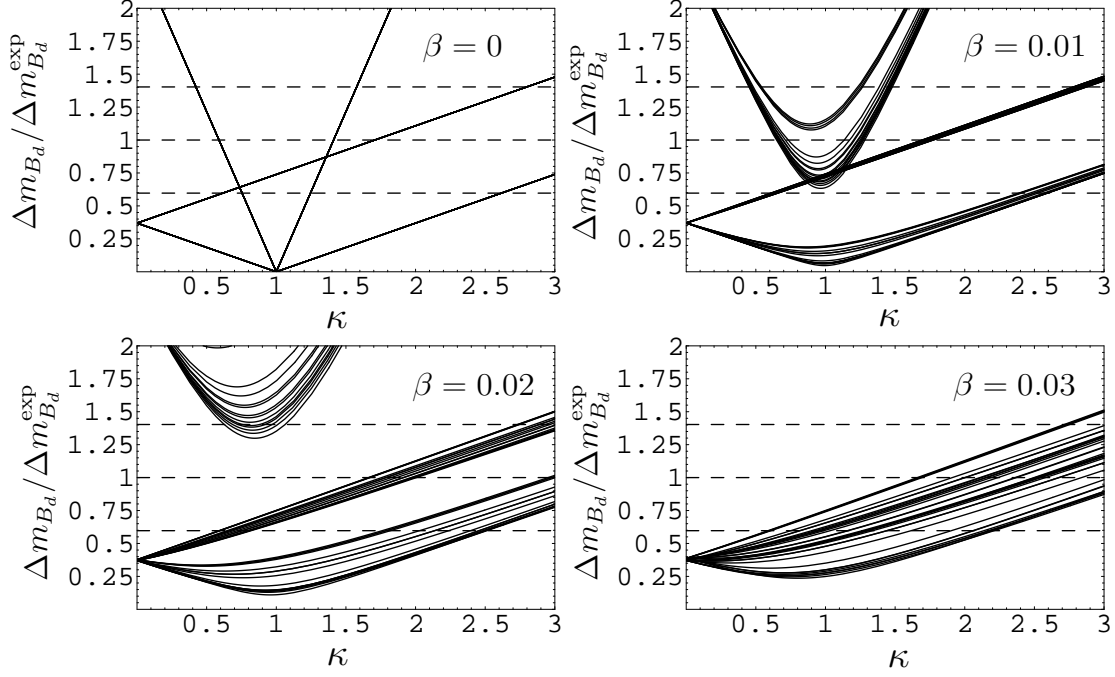


Figure 4: Constraints from Δm_{B_d} . Predictions of the SB-LR as functions of κ for different values of β , $\beta = 0, 0.01, 0.02, 0.03$. Dashed lines are the experimental result and theory errors. The lower curves are class I solutions, the upper curves are class II.

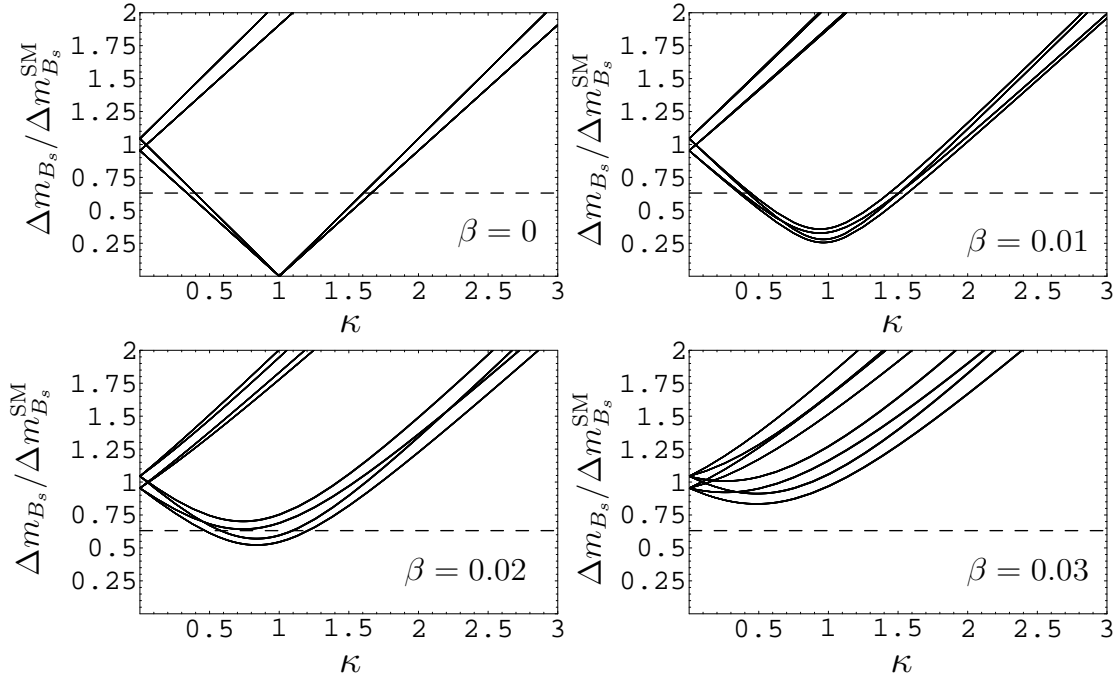


Figure 5: Like the previous figure, but for Δm_{B_s} . Normalization of vertical axis corresponds to the SM expectation $|V_{ts}| = 0.039$. Dashed line is the lower bound on Δm_{B_s} .

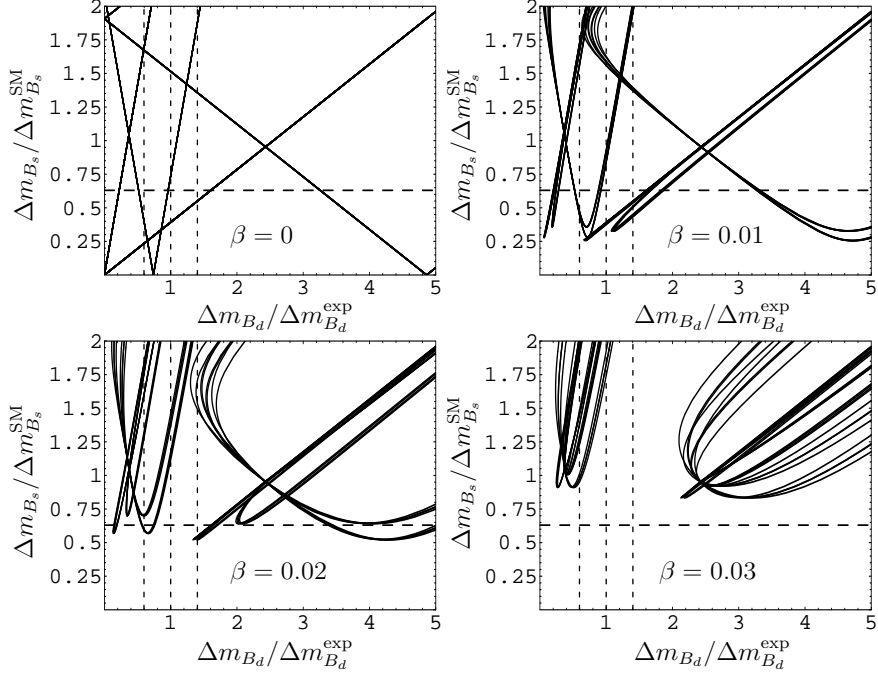


Figure 6: Correlation between predictions for Δm_{B_d} and Δm_{B_s} for different values of β as function of κ in $[0, 3]$. Short dashes denote experimental result and theory error for Δm_{B_d} , long dashes denote lower bound on Δm_{B_s} . The ensemble of lines in the left parts of the plots are class I solutions, the ones in the right halves are class II.

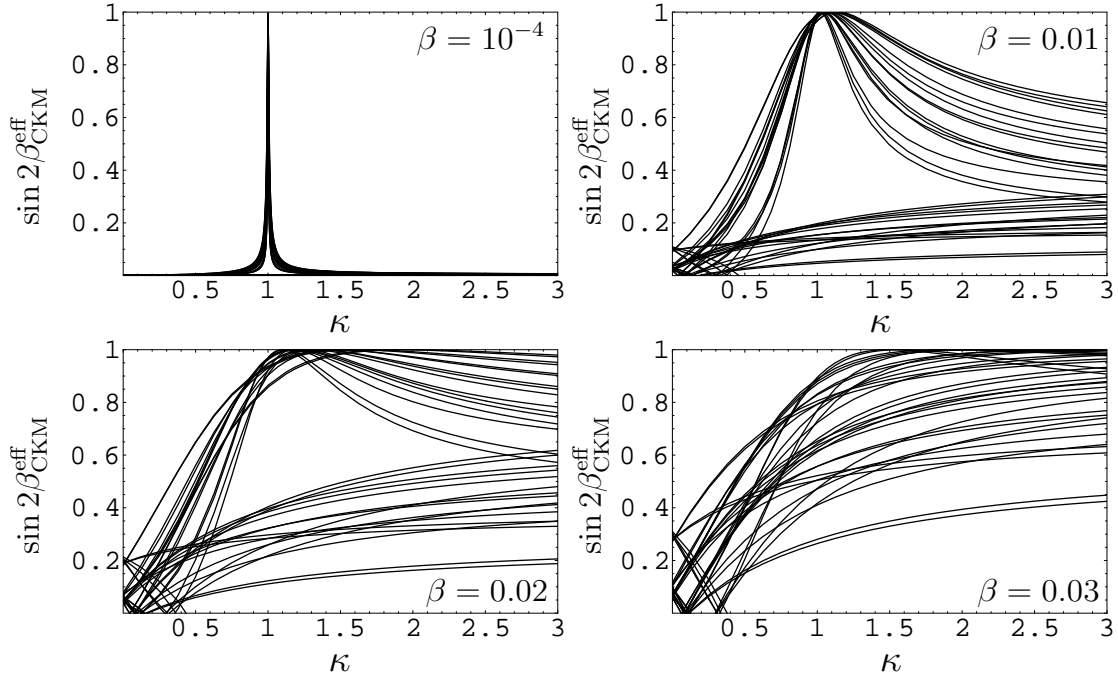


Figure 7: $\sin 2\beta_{\text{CKM}}^{\text{eff}}$ in the SB-LR as function of κ for several values of β , $\beta = 10^{-4}$, 0.01, 0.02, 0.03. Only solutions yielding positive values of $\sin 2\beta_{\text{CKM}}^{\text{eff}}$ are shown.

- class II solutions are excluded for $\beta \geq 0.021$;
- class I solutions require $\kappa > 0.52$, class II solutions $\kappa > 0.42$. This means that the Higgs contributions to κ are essential, see Eq. (5.10).

We next plot the correlations between predicted values for Δm_{B_s} and Δm_{B_d} , Fig. 6. This plot, too, illustrates the exclusion of the decoupling limit of the SB–LR, which corresponds to the two crossing-points of the different classes of solutions, best visible for $\beta = 0$. It is also evident that the SB–LR can comfortably accomodate any non-standard value of Δm_{B_s} as well as the expected one, Eq. (5.13). There is in particular a large fraction of class I and II solutions that predict very large values of Δm_{B_s} , so that a measurement of Δm_{B_s} close to its SM expectation will effectively constrain the parameter space of the SB–LR.

5.2 Constraints from $B_d^0 \rightarrow J/\psi K_S^0$

Other interesting constraints can be obtained from the measurement of the CP asymmetry in $B_d^0 \rightarrow J/\psi K_S^0$. Recently, the CDF collaboration has reported the following result [31]:

$$a_{\text{CP}} = \frac{\Gamma(\bar{B}_d^0(t) \rightarrow J/\psi K_S^0) - \Gamma(B_d^0(t) \rightarrow J/\psi K_S^0)}{\Gamma(B_d^0(t) \rightarrow J/\psi K_S^0) + \Gamma(\bar{B}_d^0(t) \rightarrow J/\psi K_S^0)} = (0.79_{-0.44}^{+0.41}) \sin(\Delta m_B t), \quad (5.18)$$

and with 90% probability $a_{\text{CP}}/\sin(\Delta m_B t) > 0$. In the SM, a_{CP} measures just $\sin 2\beta_{\text{CKM}}$ with

$$\beta_{\text{CKM}} = \arg\left(-\frac{V_{cd}^L V_{cb}^{L*}}{V_{td}^L V_{tb}^{L*}}\right); \quad (5.19)$$

the SM model expectation is $\sin 2\beta_{\text{CKM}}^{\text{SM}} = 0.73_{-0.06}^{+0.05}$ [5].

In the SB–LR, however, we have to interpret the measurement differently. For any B decay into a final CP eigenstate f_{CP} , one can define a convention and parametrization invariant quantity λ by

$$\lambda \equiv \left(\frac{q}{p}\right)_B \frac{\bar{A}_{f_{\text{CP}}}}{A_{f_{\text{CP}}}}$$

with the amplitudes $A_{f_{\text{CP}}} = A(B^0 \rightarrow f)$ and $\bar{A}_{f_{\text{CP}}} = A(\bar{B}^0 \rightarrow f)$ and the mixing amplitude $(q/p)_B \simeq -\exp(-i\phi_M^B)$, defined in (4.5). In the case of vanishing direct CP violation, the time-dependent CP asymmetry can be written as

$$a_{\text{CP}} = \text{Im } \lambda \sin(\Delta m_B t),$$

so that we have

$$\text{Im } \lambda(B^0 \rightarrow J/\psi K_S^0) = 0.79_{-0.44}^{+0.41}. \quad (5.20)$$

The expression for λ itself reads

$$\lambda(B^0 \rightarrow J/\psi K_S^0) = \exp(-i\phi_M^B) \eta_{J/\psi K_S^0} \left(\frac{V_{cb}^L V_{cs}^{L*}}{V_{cb}^{L*} V_{cs}^L}\right) \exp[i(-\phi_{\text{CP}}^B + \phi_{\text{CP}}^K)] \exp(i\phi_M^K).$$

$J/\psi K_S^0$ is CP odd, hence $\eta_{J/\psi K_S^0} = -1$, and the K mixing phase was defined in Eq. (4.5). The imaginary part reads

$$\sin 2\beta_{\text{CKM}}^{\text{eff}} \equiv \text{Im} \lambda(B^0 \rightarrow J/\psi K_S^0) = \sin \left[2\beta_{\text{CKM}} + \arg(1 + \kappa e^{i\sigma_d}) - \arg\left(1 + \frac{M_{12}^{\text{K,LR}}}{M_{12}^{\text{K,SM}}}\right) \right]. \quad (5.21)$$

The contribution from K mixing, the third term in square brackets, is numerically small and can be neglected for the moment.

In Fig. 7 we plot $\sin 2\beta_{\text{CKM}}^{\text{eff}}$ as function of κ for several values of β , where we only show solutions that yield $\sin 2\beta_{\text{CKM}}^{\text{eff}} > 0$. It is obvious that the SM expectation $\sin 2\beta_{\text{CKM}}^{\text{eff}} \approx 0.75$ can be accommodated by a number of solutions. It is also visible that for $\beta < 0.03$, there are roughly two branches of solutions, one with small $\sin 2\beta_{\text{CKM}}^{\text{eff}} < 0.4$, the other one spanning all possible values between 0 and 1. A measurement of $\sin 2\beta_{\text{CKM}}^{\text{eff}}$ around its SM expectation would favour either $\kappa \approx 0.6$ or $\kappa > 1.2$. Any more detailed analysis requires to take into account the constraints from K mixing.

6 Constraints from the K System

While the K system was the first one to be analysed in SB–LR, it remains plagued by theoretical uncertainties.¹¹ The main observables to be considered are obviously Δm_K , ϵ and ϵ' . The formulas for Δm_K are analogous to those for Δm_B discussed in the last section. To be specific, we use the LO QCD corrected formulas for M_{12}^{LR} of Ref. [4] with the bag-factor $B_K^S = 1$; radiative corrections to M_{12}^{SM} are taken from Ref. [23]; we also use $\hat{B}_K = 0.89$ [24]. As for ϵ , in the SM, it is usually written as

$$\epsilon = \frac{1}{\sqrt{2}} e^{i\pi/4} \left(\frac{\text{Im} M_{12}}{\Delta m_K} + \xi_0 \right) \quad (6.1)$$

with

$$\xi_0 = \frac{\text{Im} a_0}{\text{Re} a_0}, \quad a_0^* = \langle \pi\pi(I=0) | -i\mathcal{H}_{\text{eff}}^{|\Delta S|=1} | \bar{K}^0 \rangle_{\text{weak}}, \quad (6.2)$$

where the matrix element in (6.2) does contain only the weak phase, but no strong final-state rescattering phases. As the derivation of this formula includes some relations and approximations that need not be valid in the SB–LR, we rederive it, following the transparent discussion given in Ref. [32].

The parameter ϵ measures essentially the phase-difference between M_{12} and Γ_{12} , where Γ_{12} is the matrix element of the decay matrix Γ over the K^0 and \bar{K}^0 states. Introducing

$$\delta\theta_{M/\Gamma} = \arg M_{12} - \arg \Gamma_{12},$$

one finds for the hypothetical case of no CP violation

$$\delta\theta_{M/\Gamma} = \pi,$$

which essentially follows from the fact that $\Delta m_K \equiv m_L - m_s > 0$, but $-\Delta\Gamma_K \equiv \Gamma_L - \Gamma_S < 0$. Note that only the phase-difference $\delta\theta_{M/\Gamma}$ is an observable and convention-independent

¹¹It should not be forgotten that the same is actually true for the SM.

quantity, but not $\arg M_{12}$ and $\arg \Gamma_{12}$ separately, which depend on the K analogue of the arbitrary phase ϕ_{CP} , introduced in (5.2) for B's, and on the s and d quark phases, i.e. on the parametrization of the quark mixing matrices. For the analysis of ϵ , it proves convenient to choose $\phi_{CP} = \pi$, which we shall use in the remainder of this section. One then has — still for the case of no CP violation and using the Maiani parametrization of the CKM matrix — $\arg M_{12} = 0$ and $\arg \Gamma_{12} = \pi$. In the real CP violating world, $\delta\theta_{M/\Gamma}$ is only slightly different from π : making use of the fact that K decays are dominated by the 2π channel with isospin 0 (the famous $\Delta I = 1/2$ rule), a measure of CP violation in the interference of mixing and decay, i.e. of $\delta\theta_{M/\Gamma}$, is given by ϵ , which is defined in such a way as to contain no effects from direct CP violation and which can be written as [32]:

$$\epsilon = -\frac{x}{4x^2 + 1} \{1 + (2x)i\} \sin \delta\theta_{M/\Gamma}, \quad (6.3)$$

$$\text{with } x = \frac{\Delta m_K}{\Delta \Gamma_K} = 0.478 \pm 0.002 \approx 0.5,$$

so that

$$\epsilon \approx \frac{1}{2\sqrt{2}} e^{i\pi/4} (-\sin \delta\theta_{M/\Gamma}). \quad (6.4)$$

Γ_{12} can, in contrast to M_{12} , not be calculated accurately from theory, and one exploits the dominance of decays into the $2\pi(I=0)$ final state to derive¹²

$$\arg \Gamma_{12} \approx -2 \arg (a_0 e^{i\pi/2}). \quad (6.5)$$

Combining Eqs. (6.4) and (6.5), we finally obtain

$$\epsilon = \frac{1}{2\sqrt{2}} e^{i\pi/4} \sin (\arg M_{12} + 2 \arg a_0). \quad (6.6)$$

In contrast to Eq. (6.1), this formula also allows to demonstrate explicitly that ϵ does not depend on the parametrization of the quark mixing matrices: M_{12} contains the generic CKM factor $(\lambda_i^{AB})^2$, $A, B \in \{L, R\}$, $i \in \{u, c, t\}$, whereas a_0 contains the factor λ_i^{AB*} , so that phase-redefinitions of V_L and V_R cancel in the sum (6.6).

The experimental result for ϵ , $|\epsilon| = (2.280 \pm 0.013) \cdot 10^{-3}$ [14], implies

$$\arg M_{12} + 2 \arg a_0 = (6.449 \pm 0.037) \cdot 10^{-3}. \quad (6.7)$$

In the SM, both terms in the above sum are small so that \arg can be replaced by the ratio of imaginary to real part. In addition, as $\arg M_{12} \approx 0$ in our phase-convention, one also has $\text{Re } M_{12} \approx |M_{12}|$, which is just $\Delta m_K/2$, so that one can approximate (6.6) by (6.1) to excellent accuracy. As $2 \arg a_0 \ll \arg M_{12}$, this term can safely be neglected in the SM; in the SB-LR this is no longer true: the contributions of W_R to a_0^{LR} have been calculated in Refs. [4, 8], but involve considerable theoretical uncertainties. From [4, 8], we find

$$2|\arg a_0^{\text{LR}}| < 0.005 \cdot \left(\frac{1 \text{ TeV}}{M_2}\right)^2.$$

¹²The factor $e^{i\pi/2}$ comes from the fact that $\arg \Gamma_{12}$ is related to the matrix element of \mathcal{H}_{eff} , whereas we have defined a_0^* as matrix element of $-i\mathcal{H}_{\text{eff}}$.

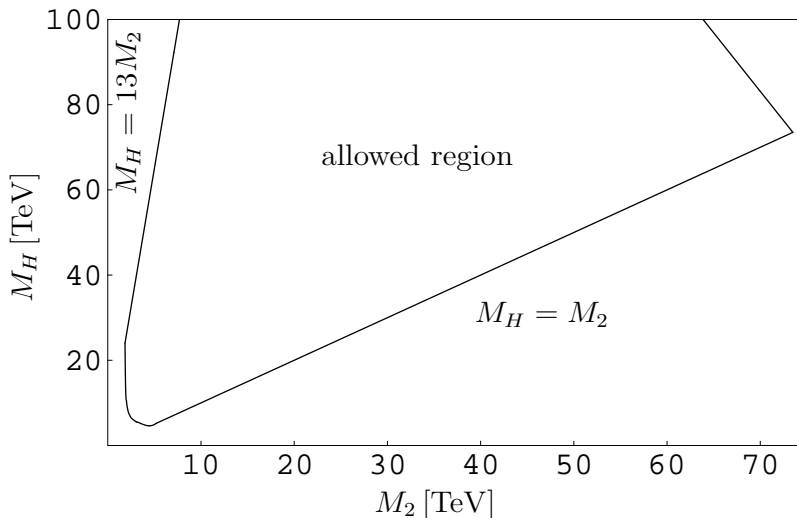


Figure 8: Allowed values for M_2 and M_H from the K physics constraints.

In view of the theoretical uncertainty of a_0^{LR} , which involves a number of only poorly known hadronic matrix elements, we prefer to include it into the uncertainty of $\arg M_{12}$ in (6.7). Assuming that the unknown Higgs contributions are not larger than those from W_R , we include twice the value of $2|\arg a_0^{\text{LR}}|$ in the uncertainty of $\arg M_{12}$ and thus find the constraint

$$6.375 \cdot 10^{-3} - 0.01 \cdot \left(\frac{1 \text{ TeV}}{M_2}\right)^2 < \tilde{\theta}_M < 6.523 \cdot 10^{-3} + 0.01 \cdot \left(\frac{1 \text{ TeV}}{M_2}\right)^2 \quad (6.8)$$

$$\text{with } \tilde{\theta}_M = \left| \frac{2 \text{ Re } M_{12}}{\Delta m_K} \right| \arg M_{12},$$

which also takes into account 2 experimental standard deviations and where we have rescaled $\text{Re } M_{12}$ to its experimental value.

Finally, the expression for ϵ' reads

$$\epsilon' \approx \frac{1}{\sqrt{2}} \exp^{i\pi/4} \frac{\text{Re } a_2}{\text{Re } a_0} (\xi_2 - \xi_0),$$

where ξ_2 is defined in analogy to ξ_0 for the $2\pi(I=2)$ final state. In view of the large theoretical uncertainties associated with the precise value of $\text{Re}(\epsilon'/\epsilon)$, we only require the SB–LR to predict a positive value.

Let us now discuss the constraints to be obtained from the three observables Δm_K , ϵ and ϵ' . First, we consider the decoupling limit $M_2, M_H \rightarrow \infty$. In this limit we find

$$\tilde{\theta}_M < 2.9 \cdot 10^{-3},$$

which is less than half of the experimental value and is related to the smallness of the standard CKM phase δ in the SB–LR. From this result, which is insensitive to the exact value of the uncertain CKM angle θ_{13} , we firmly conclude that the *decoupling limit* $M_2, M_H \rightarrow \infty$ is *experimentally excluded*.

We next investigate the allowed region in the space of mass-parameters imposed by the constraint (6.8) and the one on Δm_K , Eq. (4.6). The result is plotted in Fig. 8. We find in particular the following lower bounds on the extra boson masses:

$$M_2 > 1.85 \text{ TeV}, \quad M_H > 5.2 \text{ TeV}. \quad (6.9)$$

The bound for M_2 is in the ballpark of the usually obtained values, cf. Ref. [4], the one on M_H is smaller, the reason being that, in contrast to all previous analyses, we did not assume charm quark dominance for M_{12}^H , but also included the top quark contributions which can destructively interfere with the charm quark ones, thus lowering the limit on M_H . The experimental limit on $\tilde{\theta}_M$, i.e. ϵ , implies also (not very constraining) upper bounds on the extra boson masses:

$$M_2 < 73.5 \text{ TeV}, \quad M_H < 230 \text{ TeV}. \quad (6.10)$$

We recall that all these limits and bounds are to be modified by the inclusion of B physics constraints. This concludes our discussion of constraints from K physics.

7 Combining Constraints from K and B System

Combining all the constraints from Δm_K , Δm_{B_d} , Δm_{B_s} , ϵ , ϵ' and $\sin 2\beta_{\text{CKM}}^{\text{eff}}$, our main finding is that, although the values of the CP conserving observables can be reproduced by a large range of input parameters, this is not the case for the CP violating ones: the crucial point is a strong anti-correlation between the signs of $\text{Re } \epsilon$ and $\sin 2\beta_{\text{CKM}}^{\text{eff}}$, which are both known to be positive from experiment. We illustrate this point in Fig. 9, where we plot the values of ϵ (to be precise: $\epsilon \cdot e^{-i\pi/4}$) vs. $\sin 2\beta_{\text{CKM}}^{\text{eff}}$ for all sets of input parameters (n , β , M_2 , M_H) with $2 \text{ TeV} \leq M_2 \leq 50 \text{ TeV}$ and $M_2 \leq M_H \leq 13M_2$ that pass the cuts on the mass differences (with a 50% uncertainty on Δm_{B_d} to account for the uncertainty of the CKM angles) and on the sign of $\text{Re } \epsilon'$. It is obvious that only a few sets of input parameters can reproduce the observed sign of both ϵ and $\sin 2\beta_{\text{CKM}}^{\text{eff}}$. We find that the class I quark mass signature no. 31 is the only one to accomplish that, and thus the only one of the initial 64 signatures to survive all cuts. A closer inspection shows that the maximum possible $\sin 2\beta_{\text{CKM}}^{\text{eff}}$ correlated with $\tilde{\theta}_M$ in the range given in (6.8) is

$$\sin 2\beta_{\text{CKM}}^{\text{eff,max}} = 0.1,$$

which is incompatible with the SM expectation $\sin 2\beta_{\text{CKM}}^{\text{SM}} \approx 0.75$. The exclusion of all quark mass signatures except for one also cuts deeply into the allowed range for M_2 and M_H . For fixed M_2 (and β), we have the following constraints on M_H :

- a lower bound from $\Delta m_K^{\text{LR}} < \Delta m_K^{\text{exp}}$;
- an upper bound from $\sin 2\beta_{\text{SM}}^{\text{eff}} > 0$ (because $\sin 2\beta_{\text{SM}}^{\text{eff}} < 0$ for $M_H \rightarrow \infty$);
- a lower bound from the upper limit on $\tilde{\theta}_M$, Eq. (6.8);
- an upper bound from the lower limit on $\tilde{\theta}_M$.

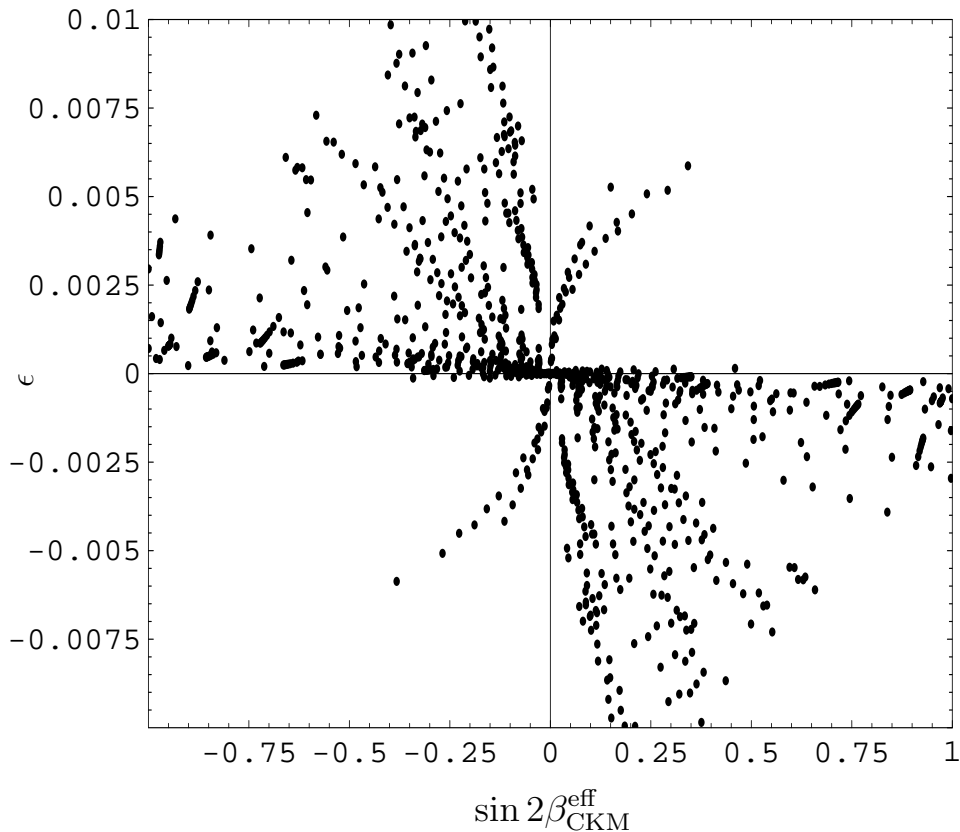


Figure 9: Allowed values for the CP violating parameters ϵ and $\sin 2\beta_{\text{CKM}}^{\text{eff}}$ whose input parameters pass the Δm cuts and yield the correct sign of $\text{Re}\epsilon'$.

The allowed region in (M_2, M_H) (also taking into account the constraints from Δm_B) thus gets very much restricted, as shown in Fig. 10. We find the bounds

$$2.75 \text{ TeV} < M_2 < 13 \text{ TeV}, \quad 10.2 \text{ TeV} < M_H < 14.6 \text{ TeV},$$

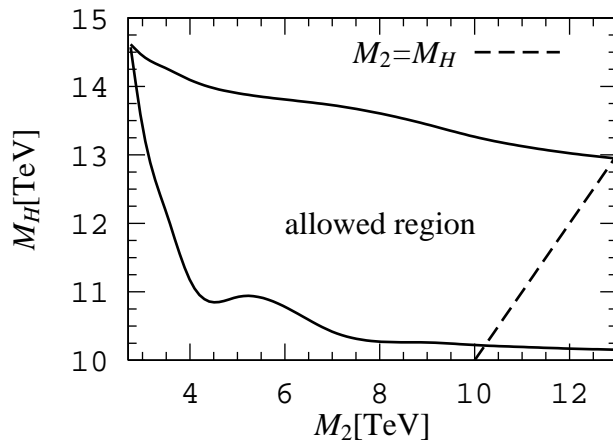


Figure 10: Allowed region in (M_2, M_H) , taking into account all constraints.

which improve considerably on those from K physics alone, Eqs. (6.9) and (6.10). The predictions for Δm_{B_s} are in the range $(0.6 - 1.1) \Delta m_{B_s}^{\text{SM,exp}}$, i.e. a measurement of Δm_{B_s} close to its SM expectation would not pose any additional constraint.

As for β , we find that

$$0.009 < \beta < 0.0327,$$

i.e. the combination of K and B constraints only results into a lower bound on β , but does not improve the maximum one, which is still given by (4.2).

8 Summary and Discussion

In this paper we have investigated in detail the present status of the left-right symmetrical model with spontaneous CP violation, based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$. The parameter space of this model includes the masses of the predominantly right-handed charged gauge boson, M_2 , of FC neutral and charged Higgs bosons, which we have assumed to be degenerate with a common mass M_H , as well as the parameter β , measuring the size of the VEV of the Higgs bidoublet Φ , which characterizes the spontaneous breakdown of CP symmetry, and n , the 64 different quark mass signatures, which are observable in the SB-LR. In contrast to previous publications, e.g. [4, 3, 6, 7], in which the constraints on the model from K and B physics were treated separately, ours is the first one to consider them in a coherent way and using the exact results for the CKM phases instead of the small phase approximation. We have concentrated on experimental constraints imposed by the mass differences $\Delta m_{K,B}$ and observables describing CP violation, i.e. ϵ , ϵ' and $a_{\text{CP}}(B \rightarrow J/\psi K_S)$. In view of large theoretical uncertainties, we only use the sign, but not the absolute value of $\text{Re}(\epsilon'/\epsilon)$ as a constraint, and we do not use the electric dipole moment of the neutron. Our main finding is that, although the K and B constraints can be met *separately* by a large range of input parameters, it is their *combination* that restricts the model severely. We find in particular that the CP violating observables ϵ and $\sin 2\beta_{\text{CKM}}^{\text{eff}}$ are crucial: the sets of input parameters that pass the constraints imposed by the meson mass differences $\Delta m_{K,B}$ yield to a large majority *opposite signs* of ϵ and $\sin 2\beta_{\text{CKM}}^{\text{eff}}$. The combination of all constraints yields the following results:

- all but one quark mass signatures are excluded, only class I solution no. 31 survives;
- $a_{\text{CP}}(B \rightarrow J/\psi K_S) \equiv \sin 2\beta_{\text{CKM}}^{\text{eff}} < 0.1$, which is compatible with the present experimental result (5.18), but incompatible with the SM expectation 0.75;
- predictions for Δm_{B_s} are in the range $(0.6 - 1.1) \Delta m_{B_s}^{\text{SM,exp}}$;
- the masses of the extra bosons are restricted to

$$2.75 \text{ TeV} < M_2 < 13 \text{ TeV}, \quad 10.2 \text{ TeV} < M_H < 14.6 \text{ TeV};$$

- the value of β is restricted to $0.009 < \beta < 0.0327$.

We would like to stress that our findings are largely independent of the details of the scalar potential: the relevant neutral Higgs vertices can be obtained essentially from the requirement

of gauge-invariance of S matrix elements, as discussed in Ref. [25]. This does not apply to the charged Higgs vertices, which in principle *do* depend on the specifics of the scalar potential: we thus have imposed the condition $M_H > M_2$ in order to suppress all contributions from charged Higgses (in particular box-diagrams).

We also would like to stress that our study does not claim to be exhaustive as we did not allow the most crucial SM input parameters, the CKM angles and quark masses, to float within their presently allowed ranges. Taking into account these uncertainties would certainly affect the phases of the CKM matrices and thus mainly show up in the CP violating observables, which, as we have shown, are crucial. It is thus not to be excluded that an analysis of the input parameter uncertainties would result in increasing the viable LR parameter ranges, but we doubt that it will change the anticorrelation between the signs of ϵ and $\sin 2\beta_{\text{CKM}}^{\text{eff}}$, which implies a small maximum value of $\sin 2\beta_{\text{CKM}}^{\text{eff}}$ attainable in the model.

Another limitation of the present analysis is that we have kept the ratio of Higgs VEVs, r , constant and equal to m_b/m_t . As stressed before, this quantity governs the amount of mixing between L and R bosons. For most observables, the relevant parameter is $\beta \sim 2r \sin \alpha$, on which our analysis is based. Reducing the value of r while keeping β fixed (remember that $\tan[\beta/2] \leq m_b/m_t$) would not affect our conclusions. Increasing r , however, has an impact on the imaginary parts of a_0 and a_2 , and from there on the values of ϵ and ϵ' . Such an increase is strongly disfavoured if we take into account the constraint from the neutron's EDM. Our most important result, namely the bound on $\sin 2\beta_{\text{CKM}}^{\text{eff}}$ is, however, not affected by these considerations.

Acknowledgements

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Appendix

A Non-factorizable Contributions to B_B^S/B_B

We estimate the ratio of bag-factors B_B^S/B_B , which enters the matrix element M_{12} describing $B^0-\bar{B}^0$ mixing, by two methods, the $1/N_c$ expansion and QCD sum rules. Some aspects of the $1/N_c$ expansion for B_B have also been discussed in [33].

To leading order in the $1/N_c$ expansion, it follows from (5.2) that $B_B = 3/4$ and from (5.4) that $B_B^S = 1/(1 + m_b^2/(6m_B^2))$, so that

$$\frac{B_B^S}{B_B} \stackrel{N_c \rightarrow \infty}{=} \frac{4}{3} \frac{1}{1 + \frac{1}{6} \frac{m_b^2}{m_B^2}} \approx 1.2. \quad (\text{A.1})$$

As factorization is exact in the large N_c limit, B_B becomes scale-independent and it is not clear at what scale (A.1) is valid. Another approach more suited to include the scale-dependence is provided by QCD sum rules [34]. QCD sum rules for B_B have already been discussed in [35]; our results for B_B^S are new. We consider the correlation functions

$$\Pi(p^2, p'^2) = i^2 \int d^4x d^4y e^{i(px-p'y)} \langle 0 | T j_B^\dagger(x) O^{SM}(0) j_B^\dagger(y) | 0 \rangle,$$

$$\Pi^S(p^2, p'^2) = i^2 \int d^4x d^4y e^{i(px-p'y)} \langle 0 | T j_B^\dagger(x) O^S(0) j_B^\dagger(y) | 0 \rangle,$$

with the currents and operators

$$j_B = (m_b + m_d) \bar{d} i \gamma_5 b,$$

$$O^{SM} = (\bar{d}b)_{V-A} (\bar{d}b)_{V-A},$$

$$O^S = (\bar{d}b)_{S-P} (\bar{d}b)_{S+P}.$$

In the standard QCD sum rules philosophy, Π and Π^S are on the one hand calculated in a local operator product expansion in the deep Euclidean region $p^2, p'^2 \ll 0$ and on the other hand analytically continued to the Minkowskian region by dispersion relations, saturated by the hadronic ground state. Equating these two representations yields — after some technicalities which are well known to the experts and which we refrain from describing in this appendix — QCD sum rules for B_B and B_B^S , respectively. It turns out that the leading non-factorizable contributions come from the dimension 5 mixed condensate $\langle \bar{d}\sigma g G d \rangle$, which is enhanced by a factor m_b ; the sum rules read:

$$\frac{m_B^4 f_B^2}{(p^2 - m_B^2)(p'^2 - m_B^2)} \frac{8}{3} f_B^2 m_B^2 B_B = \Pi_{\text{fact}} + \Pi^{(5)} + \dots,$$

$$\frac{m_B^4 f_B^2}{(p^2 - m_B^2)(p'^2 - m_B^2)} 2 f_B^2 m_B^2 B_B^S = \Pi_{\text{fact}}^S + \Pi^{S,(5)} + \dots$$

Here the dots denote subleading non-factorizable contributions from $O(\alpha_s)$ perturbation theory and the gluon, four-quark and higher condensates, which we neglect in our estimate.

We can now subtract the factorizable parts, perform Borel-transformation and continuum subtraction (some of the technicalities mentioned above), and with $B \equiv 1 + \Delta B$ we obtain

$$\frac{\Delta B_B^S}{\Delta B_B} = \frac{4}{3} \frac{1}{\frac{m_B^2}{m_b^2} + \frac{1}{6}} \frac{\hat{B} \Pi_{\text{non-fact}}^{S,(5)}}{\hat{B} \Pi_{\text{non-fact}}^{(5)}}. \quad (\text{A.2})$$

Unfortunately, both ΔB in the numerator and denominator are scale-dependent so that (A.2) suffers from a large scale-uncertainty. Following Ref. [36], we thus introduce the LO RG-invariant quantities (M^2 is the Borel-parameter)

$$\hat{\Pi}(M^2) = \left[\alpha_s(M^2/m_b) \right]^{-\gamma_B/(2\beta_0)} \hat{B} \Pi(M^2),$$

$$\hat{\Pi}^S(M^2) = \left[\alpha_s(M^2/m_b) \right]^{-\gamma_{BS}/(2\beta_0)} \hat{B} \Pi^S(M^2),$$

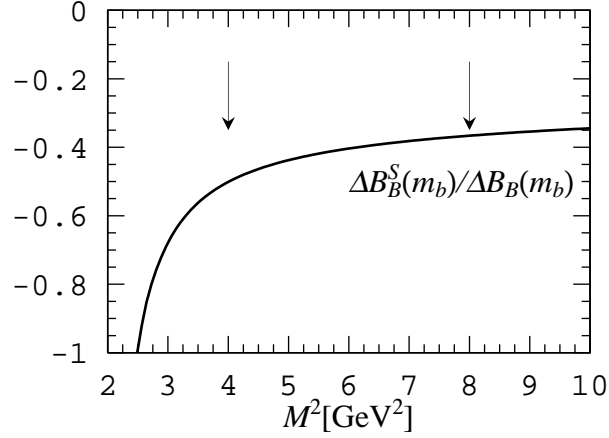


Figure 11: QCD sum rule for $\Delta B_B^S(m_b)/\Delta B_B(m_b)$ as function of the Borel-parameter M^2 ; input parameters: one-loop pole mass $m_b = 4.8 \text{ GeV}$, $s_0 = 34 \text{ GeV}^2$. The arrows denote the fiducial region for the Borel-parameter.

where we have in particular taken into account that the natural scale of QCD sum rules for heavy hadrons is $\mu = M^2/m_b$, see the discussion in [36]. This allows us to calculate $\Delta B_B^S/\Delta B_B$ directly at the scale $\mu = m_b$ with

$$\begin{aligned} \frac{\Delta B_B^S(m_b)}{\Delta B_B(m_b)} &= \frac{4}{3} \frac{1}{\frac{m_B^2}{\bar{m}_b(m_b)^2} + \frac{1}{6}} \left[\frac{\alpha_s(m_b)}{\alpha_s(M^2/m_b)} \right]^{(\gamma_{BS} - \gamma_B)/(2\beta_0)} \\ &\times \left(-\frac{1}{4} + \frac{1}{2} \frac{\int_{m_b^2}^{s_0} ds \frac{(s+m_b^2)(m_b^2-s)}{s^2} e^{-s/M^2}}{\int_{m_b^2}^{s_0} ds \frac{(s+m_b^2)m_b^2}{s^2} e^{-s/M^2}} \right) \end{aligned} \quad (\text{A.3})$$

with $\beta_0 = 11 - 2/3 \cdot 4$, $\gamma_{BS} = -8$ and $\gamma_B = 2$; $s_0 \approx 34 \text{ GeV}^2$ is the continuum threshold.

In Fig. 11 we plot the sum rule as function of M^2 ; the dependence on the M^2 as well as on s_0 and m_b is very mild; we find

$$\frac{\Delta B_B^S(m_b)}{\Delta B_B(m_b)} = -(0.43 \pm 0.07), \quad (\text{A.4})$$

which, with $\Delta B_B(m_b) = -(0.1 \pm 0.1)$ from lattice calculations [24], yields

$$\frac{B_B^S(m_b)}{B_B(m_b)} = 1.16 \pm 0.14. \quad (\text{A.5})$$

This result agrees perfectly well with that from $1/N_c$ expansion, Eq. (A.1). We thus quote as our final result

$$\frac{B_B^S(m_b)}{B_B(m_b)} = 1.2 \pm 0.2. \quad (\text{A.6})$$

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