Geneva, June 14, 1966

Institute for Theoretical and Experimental Physics

State Committee for Atomic Energy of the USSR

Report No. 408

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ON THE FORM OF BEAM LOCATOR ELECTRODES

Moscow 1966

(Translated by E. Fischer, CERN)

[CERN-trans 66-4]

Abstract

Rules have been derived showing how to choose the shape of electrodes measuring the displacements of centre of gravity of the beam of charged particles in a chamber of arbitrary cross-section. The paper is a continuation of [1]

Recently several articles and reports have been published (ref. 2-5) on the determination of the form of electrodes for the measurement of the beam position (so-called "beam-locator electrodes"). In the present article theoremes are discussed which allow to solve this problem exactly for vacuum chambers of arbitrary cross section.

In ref. 1 has been shown that the difference of the potentials induced by a beam of charged particles in two conducting surfaces A and B which are isolated from ground and which are of arbitrary form can be found from the formulae :

$$\psi_A - \psi_B = \int \rho \; (x, y, z) \cdot \varphi \; (x, y, z) \; dx \; dy \; dz \qquad (1)$$

where ψ_A and ψ_B are the potentials which appear on the surfaces A and B when a beam passes by, $\rho(x, y, z)$ is the charge density in the beam and $\varphi(x, y, z)$ is the potential at the point (x, y, z) which is originated by having one single positive charge on the surface A and one single negative charge on the surface B (in absence of a beam).

Let us now look at the most interesting case for our purpose where the beam density over the length of electrodes and somewhat beyond them (see below) does not depend on the coordinate z (the longitudinal direction of the beam). Equ. 1 can then be written in the form :

$$\psi_A - \psi_B = \iint dx\, dy\, \rho(x, y) \int \varphi(x, y, z)\, dz = \iint \rho(x, y)\, \phi(x, y)\, dx\, dy \qquad (2)$$

In equ. (2) enters the potential integrated over z

$$\phi\ (x, y) = \int \varphi\ (x, y, z)\ dz \qquad (3)$$

Equ. (2) follows from equ. (1) only in those cases where the beam density ρ does not depend on z in the region where φ is essentially different from 0 - that is roughly speaking in a region which exceeds the length of the electrodes by one chamber width. Usually this condition is largely fulfilled.

The function $\varphi(x, y, z)$ fulfils the Laplace equation

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0 \qquad (4)$$

with the boundary conditions

$$\varphi_A = \text{const.},\ \varphi_B = \text{const.},\ \varphi_{C,\ D\ldots} = \text{const.} \qquad (5)$$

The potentials φ_A and φ_B are determined by the charges of the electrodes A and B (unit charges of opposite sign). The potentials of all other electrodes are either 0 (if they are at ground) or are determined by the geometry of the problem (their mutual capacities).

The vacuum chamber is assumed to be on ground so that for sufficiantly large absolute values of z (far away from the electrodes) φ and $\partial\varphi/\partial Z$ are equal 0.

In order to clarify the properties of the function $\phi(x, y)$ we integrate equ. 4 over z from $-\infty$ to $+\infty$. Remembering that

we find
$$\int_{-\infty}^{\infty} \frac{\partial^2\varphi}{\partial Z^2}\, dz = \left.\frac{\partial\varphi}{\partial z}\right|_{-\infty}^{\infty} = 0$$

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \qquad (6)$$

the function ϕ (x, y) fulfils, consequently, the two-dimensional Laplace equation with, generally speaking, very complicated boundary conditions.

For our further discussion we limit ourselves to the case where the measuring system contains inside the vacuum chamber a closed grounded electrode (a screen) the section of which does not depend on z over a sufficiently long distance. We assume now that the walls of the screen are partly replaced by electrodes (among which the electrodes A and B) as is, for instance, shown in fig. 1. We assume furthermore that the screen goes very close to the electrodes partically without gap. The system composed of the screen and the electrodes is placed inside the vacuum chamber. The latter one is not drawn on figure 1.

Under the above formulated conditions the form of the border of the region in which the potential $\varphi(x, y, z)$ is different from zero does not depend on z and the problem is very much simplified. It is natural to characterize a point of the border line not by three but by two coordinates, for instance by the coordinate z and the angle α as it is shown in figure 1. We now integrate the potential $\varphi(x, y, z)$ over z along the border lines. The integration over the screen is zero. In the region beyond the screen the potential φ and together with it the integral $\int \varphi \, dz$ are also zero, because under our conditions the field produced by the electrodes A and B beyond the screen is negligably small. We obtain therefore

$$\phi_A(\alpha) = \varphi_A \, \ell(\alpha) \; ; \; \phi_B(\alpha) = \varphi_B \, \ell(\alpha); \quad \phi_{C,\, D\ldots} = \varphi_{C,\, D\ldots} \, \ell(\alpha) \quad (7)$$

The potential ϕ and together with it the difference of the potentials on the beam locator electrodes is determined by the solution equ. (6) under the boundary conditions equ. (7). We note that if one varies (in dependence on α) the lengths of the electrodes the boundary conditions can depend in an arbitrary way on the angle α while the potentials φ_A and φ_B do not, of course, depend on the angle α.

It follows from equ. (1) (for details see ref. 1) that one must create with the help of the beam locator electrodes A and B in the space a potential which depends linearly from x if one wants to measure the abscisse of the centre of gravity of the beam. This can be done either by giving the electrodes A and B the form of endless plates parallel to the ordinate axis or by varying linearly with respect to x the potential on the border surrounding the beam. For electrodes the form of which does not depend on z only the first way is possible. This is practically without interest. The exact solution of the problem of measuring the beam position by two beam locator electrodes is in this case impossible (an approximation of the solution with the help of electrodes of finite dimensions is possible and is being used in large scale). The second way leads immediately to simple condition : <u>the form of the electrodes which measure the beam displacement along the x-axis must be chosen in such a way that they appear for an observer looking along the y-axis to be triangles</u> as it is shown in figure 2a, b and c. It should be emphasized that the fulfilment of this condition allows an accurate solution of the problem as it has been posed for vacuum chamber of arbitrary cross-section. No additional conditions on the electrode length has to be added. It had, however, been assumed that the charge intensity in the beam does not depend on z over a sufficient length (see above) and that the screen is sufficiently long.

We add several remarks :

1) In the geometry as shown in figure 2c the electrodes A and B serve for measuring the beam displacement along the x-axis. The projection is formed by straight lines for an observer looking along the y-axis. The electrodes C (on the top) and D (on the bottom) serve for the measurement of the ordinate of the centre of gravity of the beam. The projections of these electrodes must appear to be formed by straight lines for an observer looking down the x-axis, but for an observer looking down the y-axis (as shown in the figure) they have, generally speaking, a complicated form. Between the electrodes appear free spades which, strictly speaking, should be filled in by the screen. If the free spaces are not big, and if the vacuum chamber is close to the electrodes, it is not absolutely necessary to fill the free spaces.

2) If the electrodes are symmetrically arranged, the existance of the plates C and D have no influence on the measurement of the beam displacement along the X-axis by means of the electrodes A and B. In fact, if one brings unit charges of opposite signs onto the electrodes A and B, the potentials of the plates C and D do not change and remain zero. One can therefore recon them to be connected to the screen, and the plates A and B have for an observer looking down the y-axis the form of triangles as it has been asked for.

3) The electrodes 2a, b and c solve the problem of measuring the abcissa of the centre of gravity of the beam independently of its ordinate. In order to convince oneself on this, it is sufficient to know that the equipotentials of the field integrated over z are straight lines parallel to the y-axis. Under this condition the difference of the potentials $\psi_A - \psi_B$ as determined by equ. 1 does not depend on the ordinate of the beam (and not, in particularly, on the distribution function over the y-axis).

4) It has been essential for our discussion that on a section perpendicular to the z-axis the screen and the electrodes form a contour the form of which does not depend on z over the whole range in which potentials essentially different from ~~form~~ zero appear after unit charges have been brought onto the beam locator electrodes. If there is a big gap between the vacuum chamber and the electrodes and if the electrodes are short (so that stray ~~straight~~ field effects play an important rôle) then one must place between the electrodes the grounded screen as it has been shown in figure 2. In the case of long electrodes, one can drop the screen without influencing the results.

When discussing the rôle of the screen, one must keep the following in mind : the screen is necessary in order to make zero the potential which appears on the extention of the contour of the electrodes (see figure 3) after bringing unit charges onto the electrodes A and B. Only in this case is integral $\int \varphi \, dz$ taken along the border lines equal to the potential of the electrode multiplied by its length $\ell(\alpha)$. As can be seen from the figure already a very short grounded screen is sufficient in order to make with sufficient accuracy the potential on the extension of the contour of the electrodes to zero.

5) The signal, measured from the locator electrodes, is proportional to the product of the beam displacement and its charge. In order to obtain the beam displacement itself, it is necessary to divide the potential difference by a signal proportional to the intensity of the beam. Such a signal can be obtained by any electrode fully surrounding the beam (see ref. no. 1).

If some conditions are fulfilled, a signal proportional to the beam intensity can be furnished from the potential as originated on the locator electrodes themselves. In order to clarify ~~clairfy~~ this question, let us have a look at figure 4. We consider a system of electrodes surrounding the beam. If the electrodes are galvanically connected, they form one single ring electrode, the potential of which does not depend on the position of the beam (on the displacement of the beam from point 0 to point 1). If we use ~~signal~~ for the intensity signal the sum of the potentials of the electrodes, then we obtain in this case four times the potential of such an electrode : 4 U (for the case of four electrodes as shown on the figure).

If the beam is displaced, the general charge on the electrodes isolated from ground remains zero. The charges of its parts A, B, C and D, however, are not separately equal 0. If, for instance, the beam goes onto point 1, the charge of the electrode B will increase and the charge of the electrode A decrease.

If now the electrodes are separated and the beam displaced again, then not only the sum of the charges remains zero, but also the charge of every electrode separately, The potential of the electrodes are not equal to each other any more (which gives the possibility to measure the beam displacement) and the sum, generally speaking, is not equal to 4 U.

We shall show that the sum of the potentials of the electrodes does not depend of the position of the beam if the capacities of all electrodes are equal. In fact, if the capacities are equal, the transfer of charge from one electrode to another does not perturb the sum of potentials on the electrodes and, by transfer of charge one can arrive at the same distribution which appears if the electrodes are galvanically connected.

In this way, one can measure the intensity of the beam by means of the sum of the potentials of the electrodes surrounding the beam if the capacity of all electrodes are equal to each other (independently on the size and the position of these electrodes). On the size and position of the electrodes depends their contribution to the sum of potentials and also the error which is made when the capacities are not equal. As it follows from this deduction, the capacity of the locator electrodes is their full capacity to ground in the real scheme with all electrodes in place.

The formulated rule is correct only in the cases where the sum electrode obtained from galvanical connection of all individual electrodes surrounds fully the beam. The "tongs" of the screen (see figure 1 or fig. 2c) and the gaps between the electrode perturb the accuracy of the proof. This is, however, in most of the cases unimportant.

References :

1) L.L. Gol'din, PTE, in press, 1966

2) A.A. Kuz'min, S.S. Kurotshkin, PTE, no. 4, 1962, p. 126

3) I.P. Karabekov, M.A. Martirossian, PTE, no. 5, 1964, p. 36.

4) A.J. Sherwood, IEEE Transaction, volNs-12, no. 3, 925, (1965)

5) G. Schneider, Private Communication, 1965, CERN.

11th January, 1965.

Figure captions :

Fig. 1 Signal electrodes and screen. (The vacuum chamber surrounding the system is not shown).

Fig. 2 Different forms of signal electrodes (seen in projection).

Fig. 3 Stray field in presence (left) and in absence (right) of a screen.

Fig. 4

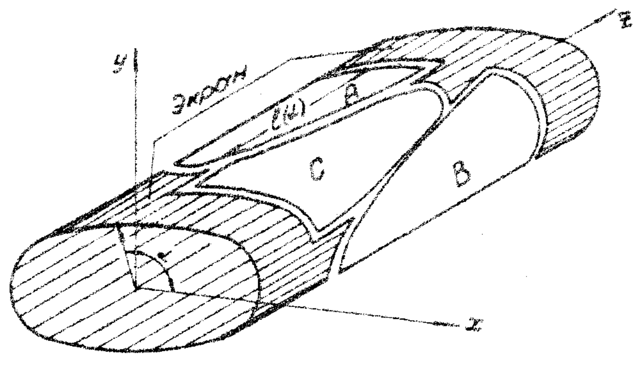

Рис.1. Сигнальные электроды и экран.
(Вакуумная камера, охватывающая измерительную систему, на рисунке не указана).

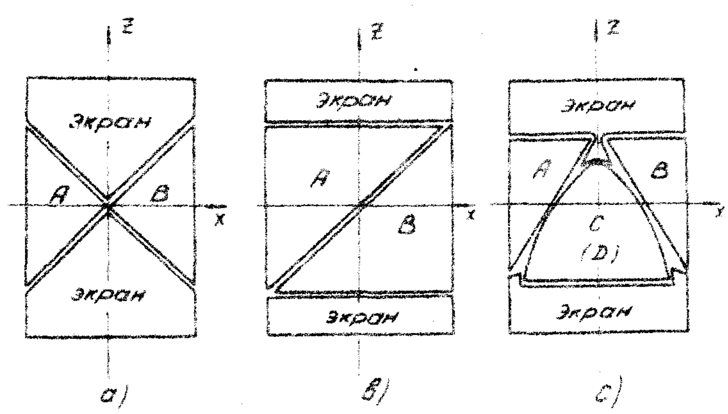

Рис.2. Различные формы сигнальных электродов (вид в плане).

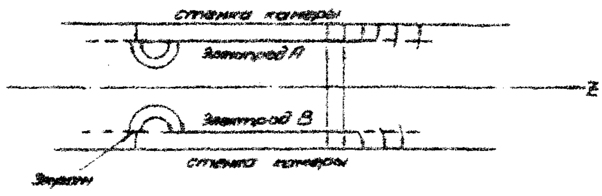

Рис.3. Краевое поле при наличии (слева) и при отсутствии (справа) экрана.

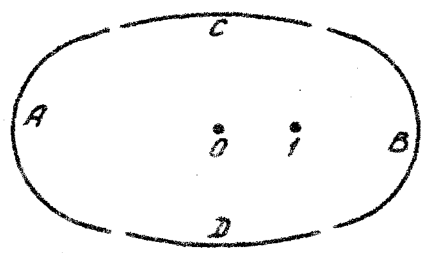

Рис.4.