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## Implications of Results from $Z$ - and $WW$ -Threshold Running\*

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### Abstract

One year of  $Z$ - and  $WW$ -Threshold running of TESLA can provide the possibility to measure electroweak precision observables to an extremely high accuracy. At the  $Z$  peak  $\mathcal{O}(10^9)$   $Z$  bosons and about  $6 \times 10^8$   $b$  quarks can be collected. We employ the expected uncertainties  $\Delta M_W = 6$  MeV and  $\Delta \sin^2 \theta_{\text{eff}} = 0.00001$  and demonstrate in this way that very stringent consistency tests of the Standard Model (SM) and the Minimal Supersymmetric Standard Model (MSSM) will be possible. The indirect determination of the Higgs-boson mass within the SM can reach an accuracy of about 5%. The  $6 \times 10^8$   $b$  quarks can be used to investigate various  $b$  physics topics.

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# Implications of Results from $Z$ - and $WW$ -Threshold Running

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One year of  $Z$ - and  $WW$ -Threshold running of TESLA can provide the possibility to measure electroweak precision observables to an extremely high accuracy. At the  $Z$  peak  $\mathcal{O}(10^9)$   $Z$  bosons and about  $6 \times 10^8$   $b$  quarks can be collected. We employ the expected uncertainties  $\Delta M_W = 6$  MeV and  $\Delta \sin^2 \theta_{\text{eff}} = 0.00001$  and demonstrate in this way that very stringent consistency tests of the Standard Model (SM) and the Minimal Supersymmetric Standard Model (MSSM) will be possible. The indirect determination of the Higgs-boson mass within the SM can reach an accuracy of about 5%. The  $6 \times 10^8$   $b$  quarks can be used to investigate various  $b$  physics topics.

## 1 Theoretical basis

Electroweak precision observables (POs) provide an excellent tool to either distinguish different models from each other or to perform internal consistency tests of a specific model. LEP2, SLD and the Tevatron have reached an accuracy of  $\Delta M_W = 42$  MeV for the  $W$ -boson mass and  $\Delta \sin^2 \theta_{\text{eff}} = 0.00018$  for the effective leptonic mixing angle [2].  $\Delta M_W$  will be improved by new data from these experiments and by data from the future LHC. A future  $e^+e^-$  linear collider (LC) might be able to reach an even better accuracy for the POs. At TESLA it is planned to realize a high luminosity of  $\mathcal{L} \approx 7 \cdot 10^{33} \text{cm}^{-2} \text{s}^{-1}$  at low energies. In a one-year run of TESLA at the  $2M_W$  threshold  $M_W$  can be pinned down to  $\Delta M_W = 6$  MeV. A one-year run at the  $Z$  pole would provide  $\mathcal{O}(10^9)$   $Z$  bosons and could thus determine the effective leptonic mixing angle up to  $\Delta \sin^2 \theta_{\text{eff}} = 0.00001$  [1]. This low-energy run scenario with high luminosity we will refer to as ‘GigaZ’ in the following. In Tab. 1 we show the expected experimental accuracies for  $M_W$ ,  $\sin^2 \theta_{\text{eff}}$ , the top-quark mass  $m_t$ , and the (lightest) Higgs-boson mass  $m_h$  in this scenario in comparison to LEP2/Tevatron, the LHC, and a LC with lower luminosity.<sup>1</sup>

In this paper we compare the theoretical predictions for  $M_W$  and  $\sin^2 \theta_{\text{eff}}$  in different scenarios with the expected experimental uncertainties. We also investigate the indirect determination of the Higgs-boson mass in the SM. In addition, during the one-year run at the  $Z$  peak, about  $6 \times 10^8$   $b$  quarks can be collected, allowing us to explore the potential for constraining the unitarity triangle and rare  $b$  decays.

In order to calculate the  $W$ -boson mass in the SM and the MSSM we use

$$M_W = \frac{M_Z}{\sqrt{2}} \sqrt{1 + \sqrt{\frac{4\pi\alpha}{\sqrt{2}G_F M_Z^2} \frac{1}{1 - \Delta r}}}}, \quad (1)$$

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<sup>1</sup> In the case of the SM  $m_h$  denotes the Higgs-boson mass, in the case of the MSSM  $m_h$  denotes the mass of the lightest  $\mathcal{CP}$ -even Higgs boson.

|                              | LEP2/Tevatron | LHC     | LC       | GigaZ    |
|------------------------------|---------------|---------|----------|----------|
| $M_W$                        | 30 MeV        | 15 MeV  | 15 MeV   | 6 MeV    |
| $\sin^2 \theta_{\text{eff}}$ | 0.00018       | 0.00018 | 0.00018  | 0.00001  |
| $m_t$                        | 4 GeV         | 2 GeV   | 0.2 GeV  | 0.2 GeV  |
| $m_h$                        | ?             | 0.2 GeV | 0.05 GeV | 0.05 GeV |

Table 1: Expected precisions of today's and (possible) future accelerators for  $M_W$ ,  $\sin^2 \theta_{\text{eff}}$ ,  $m_t$ , and the (lightest) Higgs-boson mass  $m_h$ .

where the loop corrections are summarized in  $\Delta r$  [3]. The quantity  $\sin^2 \theta_{\text{eff}}$  is defined through the effective couplings  $g_V^f$  and  $g_A^f$  of the  $Z$  boson to fermions:

$$\sin^2 \theta_{\text{eff}} = \frac{1}{4Q_f} \left( 1 - \frac{\text{Re } g_V^f}{\text{Re } g_A^f} \right), \quad (2)$$

where the loop corrections are contained in  $g_{V,A}^f$ . In our analysis we include the complete one-loop results for  $M_W$  and  $\sin^2 \theta_{\text{eff}}$  in the SM and the MSSM [3,4] as well as the leading higher-order QCD and electroweak corrections [5–8]. In order to allow a direct comparison between the virtual effects within the SM and the MSSM, we do not include the recent electroweak two-loop results in the SM [9], for which so far no counterpart exists in the MSSM. In the following we will neglect the theoretical uncertainties due to unknown higher-order corrections, but will concentrate on the impact of an improved accuracy of both the observables  $M_W$  and  $\sin^2 \theta_{\text{eff}}$  as well as of the input parameters  $m_t$ ,  $m_h$ , etc. entering the theoretical predictions.

In the SM the Higgs-boson mass is a free parameter. Contrary to this, in the MSSM the masses of the neutral  $\mathcal{CP}$ -even Higgs bosons are calculable in terms of the other MSSM parameters. The largest corrections arise from the  $t$ - $\tilde{t}$ -sector, where the dominant contribution reads:

$$\Delta m_h^2 \sim \frac{m_t^4}{M_W^2} \log \left( \frac{m_{t_1}^2 m_{t_2}^2}{m_t^4} \right) \approx \frac{m_t^4}{M_W^2} \log \left( \frac{(M_{\text{SUSY}}^2 + m_t^2)^2 - m_t^2 X_t^2}{m_t^4} \right). \quad (3)$$

Here  $m_t X_t = m_t (A_t - \mu \cot \beta)$  denotes the non-diagonal entry in the  $\tilde{t}$ -mass matrix. Since the one-loop corrections are known to be very large, we use the currently most precise two-loop result based on explicit Feynman-diagrammatic calculations [10, 11], where the numerical evaluation is based on Ref. [12].

## 2 Numerical analysis of different scenarios

In order to visualize the potential of the improved accuracy obtainable at GigaZ, we show in Fig. 1 the regions in the  $\sin^2 \theta_{\text{eff}} - M_W$ -plane which are allowed in the SM and the MSSM. They are compared with the experimental values, taking into account

today's and possible future accuracies. The allowed region of the SM prediction corresponds to varying  $m_h$  in the interval  $90 \text{ GeV} \leq m_h \leq 400 \text{ GeV}$  and  $m_t$  within its experimental uncertainty, while in the region of the MSSM prediction besides the uncertainty of  $m_t$  also the SUSY parameters are varied. As can be seen in the figure, GigaZ will provide a very sensitive test of the theory via the precision observables. If the experimental values of the latter should stay within the present  $1\text{-}\sigma$  bounds it will be difficult to distinguish between the SM and the MSSM from the precision data, but there will be a high sensitivity to deviations from both models.

A possible obstacle for future PO analyses is the uncertainty in  $\Delta\alpha$ . While today's most precise theory-driven determination ( $\delta\Delta\alpha = 0.00017$ ) [13] could still be a limiting factor for the precision tests, it can be shown [14] that this would no longer be the case with an optimistic future expectation of  $\delta\Delta\alpha = 0.000075$  [15].

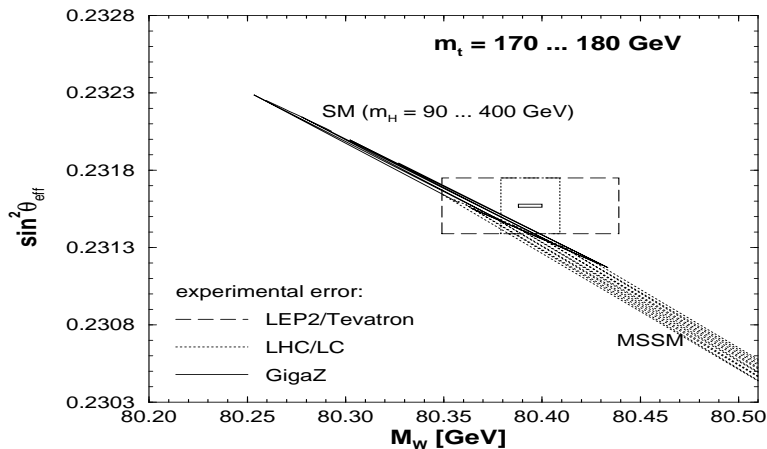


Figure 1: Theoretical prediction of the SM and the MSSM in the  $\sin^2 \theta_{\text{eff}} - M_W$ -plane compared with expected experimental accuracies at LEP2/Tevatron, the LHC and GigaZ.

In the following scenario we assume that SUSY will have been discovered and can be measured at a LC, resulting in a precision of  $\Delta M_{\text{SUSY}} \approx 0.1\%$ . We also assume that the lightest Higgs boson will have been discovered, where we take (as an example)  $115 \text{ GeV} \leq m_h \leq 116 \text{ GeV}$ . In Fig. 2 we compare the expected accuracies of the LHC and GigaZ with the prediction of the MSSM for  $M_W$  and  $\sin^2 \theta_{\text{eff}}$ . The high resolution of GigaZ is shown to have a drastic effect on the sensitivity with which the model can be probed.

### 3 Indirect determination of $m_h$

Assuming the SM to be valid, we have performed an indirect determination of the Higgs-boson mass separately from the predictions for  $M_W$  and for  $\sin^2 \theta_{\text{eff}}$ . Here we have made use of the numerical formulas given in Ref. [16]. As shown in Tab. 2, the accuracy at GigaZ would lead to a large improvement of the indirect limits on  $m_h$ . In the most optimistic case, where we have assumed  $\delta\Delta\alpha = 0.000075$  [15] and

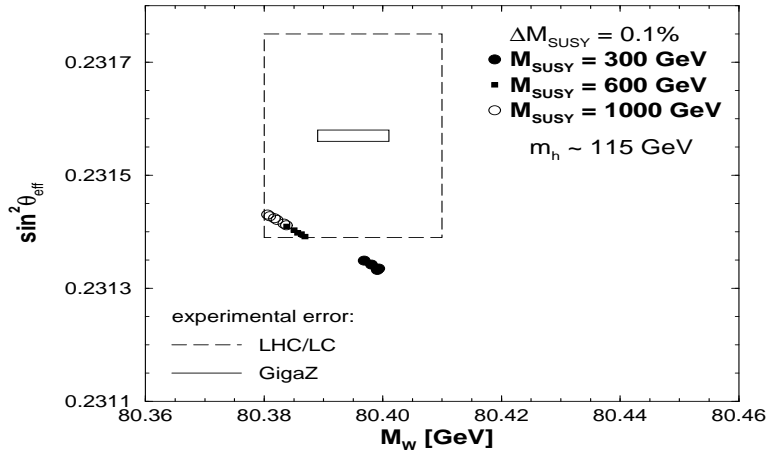


Figure 2: The expected accuracy of the LHC and GigaZ is compared with the prediction of the MSSM for  $m_h \approx 115$  GeV and  $M_{\text{SUSY}} = 300, 600, 1000$  GeV.

$\Delta m_t = 0.2$  GeV,  $m_h$  can be predicted up to an accuracy of about 5%, which would provide an extremely sensitive test of the SM.

|                              | LEP2/Tevatron | LHC | LC  | GigaZ | GigaZ (most opt. case) |
|------------------------------|---------------|-----|-----|-------|------------------------|
| $M_W$                        | 63%           | 33% | 30% | 16%   | 12%                    |
| $\sin^2 \theta_{\text{eff}}$ | 39%           | 38% | 37% | 13%   | 6%                     |

Table 2: Expected precisions of indirect determinations of the Higgs-boson mass  $\Delta m_h/m_h$  via  $M_W$  and  $\sin^2 \theta_{\text{eff}}$  for the expected uncertainties for these observables at different accelerators.

#### 4 $b$ Quark Physics with $2 \times 10^9$ $Z$ Bosons

GigaZ offers interesting possibilities for  $b$  quark physics. Assuming a sample of  $2 \times 10^9$   $Z$  bosons produced per year at GigaZ, one ends up with  $6 \times 10^8$   $b$  or  $\bar{b}$  quarks. To evaluate the  $b$ -physics potential of GigaZ, one has to compare it to the second generation  $b$  physics experiments such as LHC-b or BTeV.

As a first step towards a comparison we collect a few parameters. The number of  $b$  quarks produced per year at a second generation hadronic fixed target experiment such as LHC-b or BTeV is typically  $10^{12}$  to  $10^{13}$  [17]. However, this large number is contained in an enormous background, the signal-to-noise ratio being typically  $S/N \approx 5 \times 10^{-3}$ , to be compared to  $S/N \approx 0.15$  at GigaZ.

Another advantage of GigaZ over LHC-b / BTeV is the efficiency of the flavor tag, i.e. to discriminate between  $b$  and  $\bar{b}$ . This tag is essential for performing  $\mathcal{CP}$  violation studies. Using polarized beams at GigaZ will result in a large forward-backward asymmetry between  $b$  and  $\bar{b}$  quarks which – together with the cleaner

environment – will allow for an efficiency of the flavor tag of about 60% at GigaZ. The corresponding number at the hadronic fixed target experiments is about 6%.

In both GigaZ as well as LHC-b / BTeV all bottom flavored hadrons are accessible. In order to estimate the production rate for each species one may use the  $b$  quark branching ratios obtained from LEP or from theoretical estimates [18]. These are listed in Tab. 3.

| mode                          | $b$ branching ratio | mode                      | $b$ branching ratio         |
|-------------------------------|---------------------|---------------------------|-----------------------------|
| $b \rightarrow B_u$           | 40%                 | $b \rightarrow B_d$       | 40%                         |
| $b \rightarrow B_s$           | 12%                 | $b \rightarrow \Lambda_b$ | 8%                          |
| $b \rightarrow B^{**}$        | $\approx 25\%$      | $b \rightarrow B_c$       | $\approx 10^{-3} - 10^{-4}$ |
| $b \rightarrow (bcq)$ -baryon | $\sim 10^{-5}$      |                           |                             |

Table 3: Rule-of-thumb numbers for  $b$  quark branching fractions.

One of the most important physics topics is the precision measurement of  $\mathcal{CP}$  violation in  $B$  decays, assuming that the CKM sector of the SM survives the test at the first generation  $B$  physics experiments. The second generation will provide precise measurements of the CKM angles  $\alpha$ ,  $\beta$ ,  $\gamma$  and of the  $B_s$  mixing phase  $\delta\gamma$ .

The “gold-plated way” to access the angle  $\beta$  is to measure the time dependent  $\mathcal{CP}$  asymmetry in  $B \rightarrow J/\Psi K_s$ , which is related to  $\beta$  with practically no hadronic uncertainty. While at the first generation experiments the uncertainty  $\sigma(2\beta)$  will be about 8%, the precision achieved at LHC-b is expected to be  $\sigma(\sin 2\beta) \approx 1.5\%$  [17]. This has to be compared to  $\sigma(\sin 2\beta) \approx 4\%$  at GigaZ [19], thus the large statistics of LHC-b wins by a small margin.

The extraction of the CKM angle  $\alpha$  is more severely affected by hadronic uncertainties. One way, which has been studied in some detail, is to measure the  $\mathcal{CP}$  asymmetries in the decays  $B \rightarrow \pi\pi$ , from which  $\alpha$  can be obtained through an isospin analysis. This, however, requires a measurement of the decay  $B_d \rightarrow \pi^0\pi^0$ , which is estimated to have a small branching ratio and is very hard to identify at LHC-b / BTeV. The prospects of extracting  $\alpha$  at LHC-b in this way have been studied and yield an uncertainty  $\sigma(\alpha) \sim 3^\circ - 10^\circ$ , depending on the value of  $\alpha$ . The crucial point is whether a measurement of  $B_d \rightarrow \pi^0\pi^0$  will be possible at GigaZ, but detailed studies have not yet been performed.

Determining  $\gamma$  at the first generation experiments will be extremely difficult, since the  $B_s$  states are not accessible. At LHC-b one can determine  $\gamma$  as well as  $\delta\gamma$  from  $B_s$  decays, the typical uncertainties being  $\sigma(\gamma) \sim 6^\circ - 14^\circ$  depending on the  $B_s$  mixing parameter and strong phases. The relative angle  $\delta\gamma$  can be determined from a polarization analysis of the decay  $B_s \rightarrow J/\Psi\phi$  from which one expects typically  $\sigma(\delta\gamma) \sim 10^{-2}$ . However, such a measurement will be difficult at GigaZ. Other modes relevant for  $\gamma$  such as  $B \rightarrow DK$  modes have not yet been studied in detail.

Other  $b$  physics topics which can be covered by GigaZ are rare decays, heavy hadron spectroscopy (such as doubly heavy hadrons), and studies taking advantage of the large polarization of the  $b$  quarks from  $Z$  decays. None of these has been studied in detail yet, but clearly the large statistics of LHC-b / BTeV will be hard

to beat, at least in the modes which are easy to detect. Hence GigaZ can only have a chance using decay modes where a particularly clean environment is mandatory.

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