THE SEARCH FOR GRAVITATIONAL WAVES

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Abstract

The basic physics of g.w.'s emission, propagation and detection is outlined. The general features of candidate sources and detectors are discussed, showing how astrophysical emission events could be detected by devices not far from their Standard Quantum Limit. Two kinds of ground based detectors currently under active development, Km size interferometers and mK cold massive mechanical resonators, promise to approach such a SQL. Their concepts are described together with the perspectives for optimal sensitivity. Current and perspective strategies for a confident identification of g.w. signals with networks of few detectors are commented in conclusion.

1. INTRODUCTION

In Newtonian gravitation masses, as sources of gravitational field, act at a distance and their motion does not generate any velocity or acceleration dependent action. A field equation is written, the Poisson equation, to determine the static scalar Newtonian potential when the matter density distribution is given, but of course it is not Lorentz invariant.

Various attempts had been made, after the Maxwell equations for electromagnetism, to let masses in uniform motion generate sort of "gravitomagnetic" field and masses in accelerated motion generate "waves" propagating the gravitational field at the velocity of light. This is uniquely accomplished by Einstein's theory of General Relativity, GR, which presently is satisfactorily confirmed by observations and experimental tests. According to GR, the gravitational interaction is propagated by a tensor field: the metric tensor

[GR1-6]. The source of the gravitational field is not only the matter energy momentum, but also the gravitational field itself, as it carries as well energy and momentum. For this reason Einstein equations are non linear; by contrast Maxwell equations are linear, because the electromagnetic field does not contribute to its own sources.

Einstein's GR shows all the "gravitomagnetic" and wave propagation effects expected from an analogy with electromagnetism. A mass in rotation produces a static dipolar gravitational field which acts on spinning masses in motion much alike it would do the dipolar magnetic field of a rotating charged body on a moving particle, which carries charge and spin. Gravitational waves are generated when masses are accelerated, propagate freely at the speed of light and are absorbed when other masses are put in motion by their action.

The full GR equations are needed, when one attempts to calculate the emission of the strongest sources, as the coalescence of neutron star binaries and supernova explosions. Because of the difficulties in such calculations, for which the necessary numerical methods are still under development, the predictions of source luminosities need further study. By contrast both the free propagation and the action on "laboratory" masses to be used as g.w. detectors is adequately treated in a linearized weak field approximation.

It was Einstein himself who in 1916, using such an approximation immediately after proposing the 1915 field equations, gave the first wave solutions "... by procedures similar to those used for retarded potentials in electrodynamics. Thus it follows that the gravitational field propagates with the velocity of light". In two subsequent papers, see ref. [GR4], the whole problem of g.w. emission, propagation and absorption is solved, as Einstein gives: i) the "quadrupole" formula for the power irradiated by moving masses as g.w.'s sources ii) the energy lost in g.w.'s by such sources iii) the propagation in free space of "plane" waves, which come as transverse, travelling at c-velocity and with two polarization states iv) the action of g.w.'s on mechanical system and the power they absorb from the wave. All this is basically what is still used for most of the calculations about sources and detectors.

So the basic physics of g.w.'s is understood by some three quarters of a century, while we are still waiting for a detection. I will try here to give an idea of what are the general features of emission processes and detection procedures and the consequent experimental problems.

A throughfull discussion [GR6] shows that, in the accessible non quantum regime, GR is the theory of gravitation which agrees to parts in a thousand of the post-Newtonian effects with the available observations and tests, which are now many more than the three classical ones proposed by Einstein himself - the gravitational redshift of light, the deflection of light by the Sun and the precession of the perihelion of Mercury. Before the discovery of the binary pulsar PSR1913+16, all these tests had to do, in e.m. language, with the "electric" component of the field, as they consisted of accurate measurements of the post-Newtonian effects generated by the Sun as a static field source. Of course Lorentz transformations make confident that also the "gravitomagnetic" effects and the c-velocity wave propagation of the field must be there, even if not yet directly tested. The study of PSR1913+16, as a system of two neutron stars in close orbit, has given recently a splendid confirmation of the process of emission of g.w.'s as predicted by GR, through the energy lost by this system in the emission process itself. Of course it remains of great importance to perform direct experimental tests of the "gravitomagnetic" field and of the process of absorption of g.w.'s, but it would be surprising if of one would find any disagreement with GR.

The observation of astrophysical sources has been an increasing motivation for the efforts, which are dedicated to the detection of g.w's by a quarter of a century and which should soon meet success. Observations of g.w's may become a very direct probe of the structure and evolution of matter in the universe as gravitation is the driving force on the cosmic scale. A few possibilities are by a long time intensively studied and concern processes as supernovae and coalescence of binary system, in which, as neutron stars and black holes are involved, the gravitational force gets uniquely comparable in intensity to nuclear and subnuclear forces. Also one may obtain informations on the deep interior of high density objects, which are opaque to e.m. waves or to neutrinos. Another exciting possibility is that the detection of a g.w.'s cosmic background should be telling about the very first instants after the Big Bang, possibly very close to the Planck time.

The physics of the detection is also exciting. I will try to show that we need detectors that, while of large dimensions or mass, must be operated in regimes where quantum effects are important. This poses great challenges to experimentalists, as for instance to look for the change by one quantum of vibration at Khz frequencies in a mass of few tons. And when this will be achieved, we may even think of venturing beyond. The detection of g.w.'s is in the somewhat unusual condition in which one is looking at a "classical" force, the g.w.'s, with a "quantum " system, the detector. So there is no limit in principle to the resolution one can achieve, if the quantum detector is suitably prepared and operated. Such methods pose interesting questions about quantum measurement theory [GR7] and a variety of proposals for actual experimentation with simple model systems are already around.

These notes are intended for newcomers and I cannot go any deep in so many aspects of the subject. Also the literature is enormous and I am not attempting to review it, not even for historical accounts. So I suggest a few General Readings to whoever is interested in a deeper understanding of the physics of gravitation and of detectors. For whom who may want to see the actual shaping up of current research efforts, I make specific reference to papers in a recent Conference, whose proceedings are in press. Sect.2 gives basic ideas; the framework of the detection problem is outlined in Sect.3, with the aid of order of magnitude calculations. Sect.4 and Sect.5 are dedicated respectively to introduce to the detectors currently in operation or under construction and to the perspectives for most efficient detection technologies and strategies.

2. GRAVITATIONAL WAVES

The tensor $g_{\mu\nu}$ represents in geometric language the metric of space-time, so that the invariant infinitesimal interval between space-time events* is $ds^2=g_{\mu\nu}dx^\mu dx^\nu$. In a weak field

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{1}$$

$h_{\mu\nu}$ represents the small, $|h_{\mu\nu}|<<1$, "ripples" of space time imposed by the small effect of gravitation on a metric tensor $\eta_{\mu\nu}$, which is flat in the absence of gravitation. From Einstein's field equations, in the linearized weak field approximations, when masses are in slow motion $v/c<<1$ and the sources are of small dimensions with respect to the g.w. wavelength $\lambda_{gw} >> |\vec{x}'|$ one obtains [GR all] the wave equation to first order in $h_{\mu\nu}$, with the ordinary d'Alembert operator

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\tilde{h}_{\mu\nu} = -\frac{16\pi G}{c^4}T_{\mu\nu} \tag{2}$$

and the gauge condition

$$\frac{\partial \tilde{h}_{\mu}^{\nu}}{\partial x^{\nu}} = 0 \tag{3}$$

where

$$\tilde{h}_{\mu\nu} = h_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}h_{\lambda}^{\lambda} \tag{4}$$

* repeated indexes are summed; greek indexes=0,1,2,3; latin indexes =1,2,3; $\eta_{\mu\nu}\equiv$diag. (1,–1,–1,–1)

and $T_{\mu\nu}$ is the energy momentum tensor of matter, $G = 6.673\times10^{-11} m^3kg^{-1}s^{-2}$ the gravitational constant, $c = 2.998\times10^8 ms^{-1}$ the velocity of light. In the language of forces and potentials, $h_{\mu\nu}$ represents the weak gravitational potential, as it can be immediately seen by making contact with the Newtonian gravitation of a static mass density distribution ρ. In the approximation $T_{\mu\nu}$ has only one non zero component, $T_{00}= \rho c^2$ and Eq (2) reduces to

$$\nabla^2\tilde{h}_{00} = \frac{16\pi G}{c^2}\rho \tag{5}$$

and its general solution is

$$\tilde{h}_{00}(\vec{x}) = -\frac{4G}{c^2}\int\frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|}d^3x' \tag{6}$$

We recognize in Eqs (5,6) the Poisson equation and its solution for the Newtonian potential

$$\Phi \equiv \frac{c^2}{4}\tilde{h}_{00} \tag{7}$$

In the wave zone, at distances $R \sim |\vec{x}| \gg \lambda_{gw}$ from the source, we have the retarded potentials solution

$$\tilde{h}_{\mu\nu}(\vec{x},t) = -\frac{4G}{c^4}\int\frac{T_{\mu\nu}\left[\vec{x}',t-(\vec{x}-\vec{x}')/c\right]}{\left|\vec{x}-\vec{x}'\right|}d^3x' \tag{8}$$

where the integral is extended on the volume of the source. In the approximation indexes are raised and lowered with the unperturbed metric $\eta_{\mu\nu}$, $T_{\mu\nu}$ obeys the approximate local non covariant law

$$\frac{\partial T_{\mu}^{\nu}}{\partial x^{\nu}} = 0 \tag{9}$$

which gives the conservation of energy and momentum of matter alone, disregarding the contribution from radiation. It is obvious from Eq (8) that g.w.s propagate at the velocity of light.

2.1 Plane waves

A monochromatic, linearly polarized plane wave propagates in vacuum along the $x_1 \equiv x$ direction as the simplest solution of Eq. (1) for $T_{\mu\nu} = 0$. Wave packets of any kind can be obtained by superposition as we are in the linear approximation. The 10 components of the symmetric tensor $\tilde{h}_{\mu\nu}$ can be reduced [GR4] ultimately to only two independent ones by the application of the gauge condition and by the freedom to perform further infinitesimal coordinate transformations, which leave the metric unaffected and the field weak. One can make a particular choice so that $\tilde{h}_{\mu\nu} = 0$, $\tilde{h}_{kk} = 0$. Thus $\tilde{h}_{\lambda}^{\lambda} = 0$, $\tilde{h}_{\mu\nu} = h_{\mu\nu}$ and only two spatial components survive to give the so called transverse - traceless, TT, gauge in which the wave is represented as

$$h_{\mu\nu}^{TT}(x,t) = \left(h_+ e_{\mu\nu}^+ + h_\times e_{\mu\nu}^\times\right) \text{sen } \omega\left(t - \frac{x}{c}\right) \tag{10}$$

with

$$e_{\mu\nu}^+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad e_{\mu\nu}^\times = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{11}$$

The plane wave appears as the linear superposition of two independent polarization states. In fact $e_{\mu\nu}^+ e^{\times\mu\nu} = o$ and we see that the rotation matrix

$$u_\alpha^\beta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & \text{sen}\theta \\ 0 & 0 & -\text{sen}\theta & \cos\theta \end{pmatrix} \tag{12}$$

transforms $e^+_{\mu\nu}$ into

$$e_{\mu\nu} = u_\mu^\alpha u_\nu^\beta e_{\alpha\beta}^+ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos 2\theta & \text{sen} 2\theta \\ 0 & 0 & \text{sen} 2\theta & -\cos 2\theta \end{pmatrix} \tag{13}$$

so that for $\theta = \pi / 4$ the polarization state e_{μ}^{+} is rotated into the other one $e_{\mu}^{\times}$. One can then construct circularly polarized states, left and right, as

$$c_{\mu\nu}^{\pm} = e_{\mu\nu}^{+} \pm i\, e_{\mu\nu}^{\times} \tag{14}$$

and see that, under rotation of an angle θ in the plane orthogonal to the propagation direction, one has

$$c'^{\pm}_{\mu\nu} = u_{\mu}^{\alpha} u_{\nu}^{\beta} c_{\alpha\beta} = e^{\pm 2i\theta} c_{\mu\nu}^{\pm} \tag{15}$$

so that a phase shift 2θ is obtained for a rotation θ: "gravitons" are massless, spin two particles and have appropriately only two helicity states.

2.2 Emission from small distant sources.

Let the source be at the distance d and assume $d >> \lambda_{gw} >> |\vec{x}'|$, so that $|\vec{x} - \vec{x}'| \cong d$. The integral of T_{00} over the matter distribution gives the total mass, which is conserved. When the conservation law Eq (9) is imposed, after some manipulation [GR2, GR3], only the component $T_{00} = \rho c^2$ survives to enter Eq (8) and give the non-zero components $\tilde{h}_{ik}$ where i, k run only on space indexes

$$\tilde{h}_{ik}(d,t) = -\frac{2G}{c^4 d} \frac{\partial^2}{\partial t^2} \int \rho\left(\vec{x}, t - d/c\right) x'_i \, x'_k \, d^3x' \tag{16}$$

Far from the source we see a plane wave, so we can use the simple form $h_{\mu\nu}^{TT}(\vec{x},t)$ above. If we recall the definition of the quadrupole moment of a mass distribution

$$Q_{ik} = \int \rho(\vec{x}') \left(x'_i \, x'_k - \frac{|\vec{x}'|^2}{3} \delta_{ik} \right) d^3x' \tag{17}$$

we obtain

$$2h_{+}^{TT} = h_{22}^{TT} - h_{33}^{TT} = -\frac{2G}{c^4 d} \left[\frac{d^2 Q_{22}}{dt^2} - \frac{d^2 Q_{33}}{dt} \right]_{t-d/c} \tag{18a}$$

$$h_{\times}^{TT} = h_{23}^{TT} = h_{32}^{TT} = -\frac{2G}{c^4 d} \left[\frac{d^2 Q_{23}}{dt^2} \right]_{t-d/c} \tag{18b}$$

This is a relevant point: the lowest multipole at which g.w. are emitted is the quadrupole, the "electric" one in e.m. language. The monopole and the "electric" and "magnetic" dipole are forbidden; higher multipoles are of higher orders in $1/c$. In the approximation, the absence of monopole is consequent to the conservation of total mass in the source, the absence of "electric" and "magnetic" dipoles is linked to the conservation for the source respectively of total momentum and angular momentum and to the equivalence between inertial and gravitational mass, Box 1.

BOX 1 In electromagnetism a multipole expansion gives the e.m. luminosity L_{em} of a system of charges q_i mass m_i coordinates $\vec{x}_i$ and velocity $\vec{v}_i$ as

$$L_{em} = \frac{2}{3c^3}\left(\frac{d^2\vec{D}}{dt^2}\right)^2 + \frac{2}{3c^3}\left(\frac{d^2\vec{M}}{dt^2}\right)^2 + \frac{1}{20c^5}\left(\frac{d^3\tilde{Q}}{dt^3}\right)^2$$

where

$$\vec{D} = \Sigma_i q_i \vec{x}_i, \vec{M} = \Sigma_i q_i \left(\vec{x}_i \wedge \frac{\vec{v}_i}{c}\right), \tilde{Q} = \Sigma_i q_i \left(x_\alpha^i x_\beta^i - \frac{x^2}{3}\delta_{\alpha\beta}\right)$$

are respectively the electric and magnetic dipole and the electric quadrupole moments. If the charges have the same charge to mass ratio $w = q_i/m_i$, then $\frac{d\vec{D}}{dt}$ and $\vec{M}$ become proportional respectively to the total momentum and to the total angular momentum. As they are conserved $\frac{d^2\vec{D}}{dt^2}$ and $\frac{d^2\vec{M}}{dt^2}$ vanish: neither electric nor magnetic dipole radiation are allowed.

In gravitation a similar multipole expansion gives immediately vanishing "electric" and "magnetic" dipole contributions: this is due to the equivalence between gravitational and inertial mass, as for all bodies the "charge" to mass ratio is identical.

We note in passing that theories of gravitation, other than GR, allow monopole and/or dipole radiation [GR 6].

The intensity I(t) of g.w.'s is given [GR2] by

$$I(t) = \frac{c^3}{16\pi G}\left[\left(\frac{dh_+^{TT}}{dt}\right)^2 + \left(\frac{dh_\times^{TT}}{dt}\right)^2\right] \quad (19)$$

In terms of the quadrupole moments we get

$$I(t) = \frac{G}{16\pi c^5 d^2}\left[\left(\frac{d^3 Q_{23}}{dt^3}\right)^2 + \frac{1}{4}\left(\frac{d^3 Q_{22}}{dt^3} - \frac{d^3 Q_{33}}{dt^3}\right)^2\right]_{t-d/c} \quad (20)$$

and the total power emitted by the source , the luminosity

$$L_{gw} = \frac{G}{5c^5}\left|\frac{d^3 Q_{ik}}{dt^3}\right|^2 \quad (21)$$

where the units for I (t) and L_{gw} are respectively W/m^2 and W.

2.3 Interaction with matter

The transverse-traceless gauge makes $h_{\mu\nu}$ look very simple, but such a reference frame has nothing to do with the laboratory, in which we want to detect the effects of g.w.s.

We can always null a gravitational field at a single space-time location, in consequence of the equivalence between inertia and gravitation, but not on two separate locations. So we need at least two test masses, to look for their relative motion, or an extended body, to look how it gets strained: gravity manifests itself as a "tidal" force. Specifically let us consider two mirrors A and B in free fall with light bouncing between them. We measure their space separation l as $2l = c(t_A + t_B)$, where $t_A + t_B$ is the time taken for the light to make the round-trip.

In the gravitational field the two mirrors describe space time trajectories $x_A^\mu(\tau)$ and $x_B^\mu(\tau)$, where τ is the proper time, and their separation is $l^\mu(\tau) = x_B^\mu(\tau) - x_A^\mu(\tau)$. As they are in free fall, they follow "geodesics" of space-time and their geodesic separation $l^\mu(\tau)$ follows the equation [GR1, 3, 8]

$$\frac{D^2 l^\mu}{D\tau^2} + R^\mu_{\alpha\beta\gamma} l^\alpha \frac{dx_A^\beta}{d\tau}\frac{dx_A^\gamma}{d\tau} = 0 \quad (22)$$

where D indicates the covariant derivative and $R^\mu_{\alpha\beta\gamma}$ is the Riemann curvature tensor. The Riemann tensor depends on second derivatives of the metric and is non zero only in the presence of gravitation, while is zero everywhere for purely inertial fields. It can be seen [GR3, 8] that in the TT gauge one gets the non zero components of the Riemann tensor as

$$R_{\mu 0 \nu 0} = -\frac{1}{2c^2}\frac{d^2}{dt^2}h^{TT}_{\mu\nu} \tag{23}$$

The Riemann tensor is invariant under gauge transformations. Let us choose a "laboratory" frame, so called Fermi coordinates, by which we attach to the mirror A a Cartesian system, using three orthogonal gyroscopes, and a clock. It can be seen that a plane g.w. propagating orthogonal to and linearly polarized parallel to the direction of the line joining the two mirrors, gives a change in their spatial separation Δl

$$\frac{d^2}{dt^2}\left(\frac{\Delta l}{l}\right) \cong \frac{1}{2}\frac{d^2 h_+}{dt^2} \tag{24}$$

We have dropped the index TT in $h_+(t)$ which now indicates wave packets of any kind: Eq. (24) is obtained form Eq. (22) and Eq. (23) considering that the

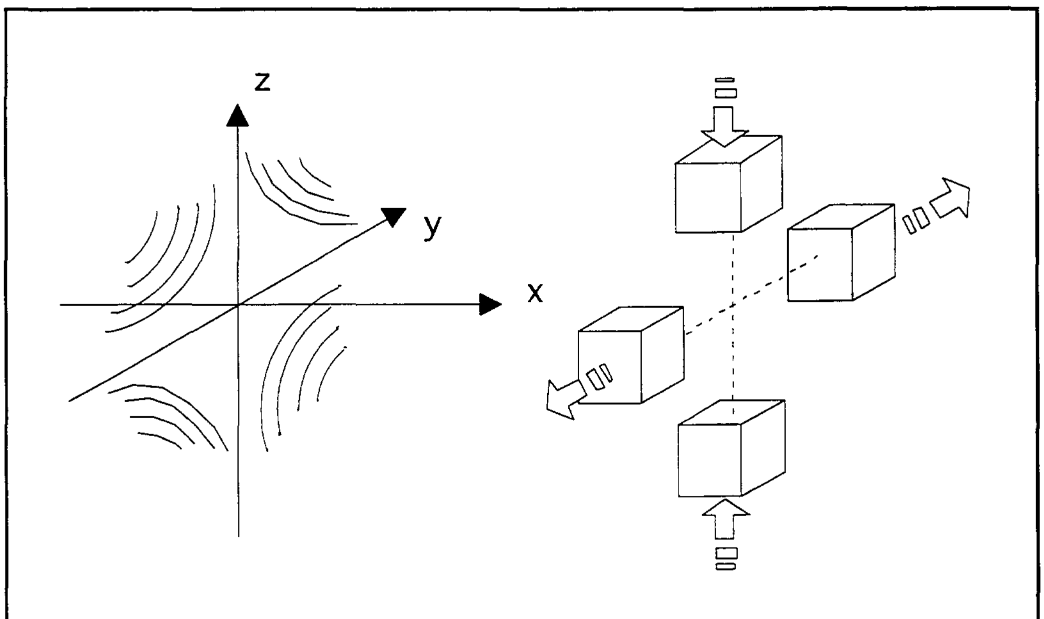

Fig.1 Lines of force of a plane linearly polarized g.w. propagating along the x axis and its effect on a "cross" of free masses.

laboratory frame gives flat space-time at A, that in the approximation proper time coincide with the clock coordinate time and that the covariant derivative reduces to the ordinary derivative.

As $\Delta l / l << 1$, because $|h_{\mu\nu}| << 1$, Eq. (24) integrates immediately as

$$\frac{\Delta l}{l} = \frac{h_+}{2} \tag{25}$$

The mirrors wiggle under the wave with amplitude Δl. The light, unaffected by g.w.s in the Fermi gauge, undergoes with the round trip in respect to the unperturbed situation the additional phase shift $\Delta\phi(t) = \frac{4\pi l}{\lambda} h_+(t)$.

It is interesting to notice [GR 8] that in the TT gauge we would have found $\Delta l \equiv 0$, because of a cancellation between the Riemann tensor term and a term coming from the covariant derivative which in TT cannot be approximated as done for the laboratory frame. In this so called "riding the wave" TT gauge, the mirrors are seen to stay still, while now it is the motion of photons which is affected by the g.w.'s. Of course one calculates the same phase shift $\Delta\phi$. The discussion should dissipate the suspicion that g.w.'s alter sticks, clocks and photons to effectively null their own effects.

If the two mirrors are bound together with a spring giving a simple harmonic oscillator with frequency $\omega_0 / 2\pi$ and resonance width ω_0 / Q, we simply add to the right of Eq (24) the elastic and dissipative forces, as

$$\frac{d^2}{dt^2}\left(\frac{\Delta l}{l}\right) = \frac{1}{2}\frac{d^2 h_+}{dt^2} - \frac{\omega_0}{Q}\frac{d}{dt}\left(\frac{\Delta l}{l}\right) - \omega_0^2\left(\frac{\Delta l}{l}\right)^2 \tag{26}$$

Eq(26) shows that the g.w. acts by forcing the harmonic oscillator.

We have just seen the principle of operation of two relevant kinds of detectors, currently under active development: i) laser interferometry with quasi free mirrors at large separations and ii) large mass, high Q cryogenically cooled mechanical resonators. In the e.m. language they both work as detectors of the "electrical" components of the g.w. field.

For completeness it must be noticed that in principle one can make detectors which sense the "magnetic" components. These would be a pair of gyros in free fall. From a geodesic deviation equation for spinning bodies, analogous to Eq (22), it has been calculated that their axis of rotation could be checked, using polarized light, to wiggle by $\Delta\vec{S}$ one in respect to the other. One has $\Delta\vec{S} / |\vec{S}| \cong h_+$, where $\vec{S}$ is the angular momentum of each gyro. The "magnetic" effects, when sensed by "magnetic" antennae, are of the same order of magnitude of the "electric" effects, as sensed by "electric" antennae.

Multiplying both sides of Eq(24) by the mass M of the mirror, we see that the g.w. is exerting the force

$$F_{gw} = \frac{Ml}{2}\frac{d^2h_+}{dt^2} \qquad (27)$$

Eq(27) shows that F_{gw} is tidal, in that it is proportional to the separation l, and shows how F_{gw} is connected to the "ripples" in space-time represented by h_+. At one time the force F_{gw} squeezes in the y direction and pulls in the z direction; a semiperiod afterwards F_{gw} does the opposite, Fig. 1. The F_{gw} induced by the other polarization component $h_\times$ would be the same, only tilted by 45° in the y-z plane.

2.4 Observation of g.w.'s in emission

The binary system PSR1913+16 has been studied [GR 6] for more than 20 years (PSR means pulsar, the numbers give the direction in the sky: $19^h13'$ right ascension, +16° declination). One of the two is a pulsar, a neutron star in rotation, which gives pulses at $\nu = 17Hz$ with an intrinsic $\frac{d\nu}{dt} = 2.5 \times 10^{-15} Hz/s$ and is an extremely stable mechanical clock. The pulses show large Doppler modulations, ascribed in a standard way to the orbiting around an unseen companion with period $P \cong 7^h45'$, at a distance $d \cong 1.4 \times 10^6 km$ with a typical velocity $v \cong 10^{-3}c$. The system is an ideal "laboratory" for testing GR, as comparatively large GR effects can be revealed on the orbital motion and on the propagation of light. For instance the precession of the perihelion, which for Mercury is the famous 43" per century, here amounts to 4.2° per year! A stringent agreement is found with GR, used to order up to $(v/c)^8$ if one assumes both stars to be neutron stars of mass 1.4 M_O, where M_O is the mass of the Sun (this is extremely plausible as such a mass has been invariably found for all the other known neutron stars). A crucial ingredient in the successful fit of the observations is the decrement in orbit period ΔP due the spiralization of one star unto the other as the system looses energy through g.w.'s. The decrement ΔP cumulated to about 12 s in some 15 years. Quadrupole g.w. emission is tested, through the time dependence of ΔP, to a few parts in a thousand, a very stringent test. This is the only available experimental observation concerning g.w.'s and has been among the motivation for awarding the Nobel prize to R.A. Hulse and J.H. Taylor.

3. SOURCES LUMINOSITY AND DETECTORS SENSITIVITY

In Eq. (21) the luminosity scales with $\frac{5c^5}{G} \cong 2 \times 10^{53} W \cong 10^6 M_O c^2 / s$, sometimes called the "luminosity of the universe" as it is of the order of the total power emitted as light by all the galaxies: the effects on detectors are expected very small.

3.1 Source luminosities

If the source has mass M, linear dimensions R and its internal motion develops on characteristic times t, so that we have a characteristic internal velocity $v \cong R/t$, we use Eq (21) and dimensional arguments to get

$$L_{gw} \cong \frac{c^5}{G}\left(\frac{R_S}{R}\right)^2\left(\frac{v}{c}\right)^6 \tag{28}$$

where $R_S = 2GM/c^2$ is the so called Schwarzschild radius of the mass distributions M. The large factor c^5/G is now upstairs, but let's see what price we paid. The factor in v/c says that we need internal velocities close to that of light. The factor in R_S/R is even more demanding. Roughly speaking R_S represents the linear dimension at which matter must collapse to become a black-hole, a system so dense that not even light can leave it, because the escape velocity is larger than c. For instance for the Sun $R_{SO} = 3$ km, for the Earth $R_{S\oplus} \cong 1cm$. A 1.4 M_O neutron star has $R \cong 10$ km, so that even for such a compact system $R_S/R \cong 0.4$.

For g.w.'s of frequency ν_{gw} and thus wavelength $\lambda_{gw} = c/\nu_{gw}$, using that $L_{gw} = 4\pi d^2 I$ and using Eqs. (19), (25) and (28), a dimensional evaluation gives for the strain $\frac{\Delta l}{l}$ on a detector at distance d

$$\frac{\Delta l}{l} \cong \frac{R_S}{R}\left(\frac{v}{c}\right)^3 \frac{\lambda_{gw}}{d} \tag{29}$$

Let us make two distinct cases: "laboratory" and "astrophysical" sources.

A "lab" source would be an elastically bound system in violent vibration; it would have $v \sim v_s$, with v_s the velocity of sound, emit at a frequency $\nu \cong v_s/R$, so that $\lambda_{gw} \cong cR/v_s$ and we get from Eq (29)

$$\left(\frac{\Delta l}{l}\right)_{lab} \cong \frac{R_S}{d}\left(\frac{v_s}{c}\right)^2 \tag{30}$$

for $v_s = 10^{-5}c$, $M \cong 10^3 kg$ at $d \cong 10$m we have $R_s \cong 10^{-24} m$ and we get

$$\left(\frac{\Delta l}{l}\right)_{lab} \cong 10^{-35} \tag{31}$$

Table 1a

Non continuous sources [GR5, A1]: metric perturbations h_+ on an optimally oriented detector from non continuos sources at ≅10 M*pc* (VIRGO cluster, 2500 galaxies); g.w.'s from rotating and orbiting systems are variously polarized in relation to the source angular momentum axis and the direction of propagation

Source	Time evolution of the signal	h_+	event rate $(year)^{-1}$
TYPE II SUPERNOVA			
axisimmetric hydrodynamic rotating collapse	~10ms pulse and damped oscillations	10^{-23}	10÷100
collapse of rotating neutron star to black hole self oscillations of remnant black hole	~1ms pulse as an overdamped oscillation at central freq. few kHz	10^{-21}	10
thermomechanical instabilities in remnant neutron star	(?)	10^{-21}	(?)
COALESCING BINARIES			
orbit spiralization of neutron stars	"chirping" train of oscillations, slowly increasing in freq. and ampl. from 20 Hz to 500 Hz in 100 s	10^{-21}	few
final merging	3ms train of ~10 damped oscillations	$5x10^{-21}$	few
STAR INFALL IN CENTRAL GALACTIC BLACK HOLE	impulsive freq. range < 1 Hz	$2x10^{-21}$	(?)

Table 1b

Other sources [GR5,A1]

Binary stars in the Galaxy at ~10 K_{pc} give periodic g.w.'s at $10^{-5}÷10^{-3}$ Hz with amplitudes $h_+≅10^{-23}÷10^{-21}$; A stochastic background of cosmological origin would give $h_{rms}≅10^{-21}÷10^{-23}$ in the frequency range $10^{-4}÷10^{-2}$ Hz, if of energy density $E_{gw}≅2x\ 10^{-8}$ in units of the closure energy density of the universe.

As an "astro" source, we consider a binary system of neutron stars; they will orbit at distance R with Keplerian velocity $v = \sqrt{\frac{GM}{R}}$ and we get

$$\left(\frac{\Delta l}{l}\right)_{astro} \cong \frac{R_S^2}{Rd} \tag{32}$$

The final coalescence of one such a system in the Andromeda galaxy at $d \cong 2 \times 10^{19} km$ with $R_S \cong 4km$, $R \cong 10km$ would give

$$\left(\frac{\Delta l}{l}\right)_{astro} \cong 10^{-19} \tag{33}$$

The two estimates are widely different, with the lab one so much smaller. But is even the "large" astro prediction detectable at all? After all, for a lab size detector, the Δl is smaller than a proton diameter.

3.2 The standard quantum limit in detectors sensitivity

Let us consider one of the detectors currently in operation, the sensitive core of which is a $M = 2300kg$, $Q \cong 2 \times 10^7$, $l = 3m$ Aluminium bar resonating in the fundamental mode at $\omega_0 / 2\pi \cong 1KHz$. At such high Q, the bar when hit keeps ringing for $Q/\omega_0 \cong 1.6 \times 10^3 s$. The modes are so well separated that the energy absorbed in one mode stays there for long times. The bar behaves like a "coherent" matter system: the energy states are those of a simple harmonic oscillator and are accordingly quantized in units ω_0. The best we may expect to do is to detect the g.w. energy absorbed, as it stimulates a transition between two such quantum states, the so called Standard Quantum Limit for the detection of pulses. The change in oscillation amplitude Δl will be of course $M\omega_0^2 \Delta l^2 \cong \hbar\omega_0$ and thus

$$\left(\frac{\Delta l}{l}\right)_{SQL}^{bar} \cong \frac{1}{l}\sqrt{\frac{\hbar}{M\omega_0}} \cong 10^{-21} \tag{34}$$

with the parameters given. Notice that, at the SQL, one is seeing the end faces of the bar wiggle by $\Delta l \cong 3 \times 10^{-21} m$; this is much less then the proton diameter, but it is not a contradiction, as this is the collective motion of a large number of particles.

For a laser interferometer under construction the SQL, as given by the vibrational modes of the masses M of the mirrors using Eq. (34), is about $\left(\frac{\Delta l}{l}\right)_{SQL} \cong 10^{-23}$, but we like to consider it by looking at the quantum limitations on the laser light. First we have a phase noise according to the uncertainty relation $\Delta\phi\Delta N \cong 1$, where ΔN is the uncertainty in the number of photons. As in a coherent state one has $\Delta N \cong \sqrt{N}$, the "shot" noise of photon, we see that the phase noise translates in the strain noise

$$\left(\frac{\Delta l}{l}\right)_{shot}^{int.} = \frac{\lambda_l}{2\pi l}\sqrt{\frac{\hbar\omega_l}{\eta P_l \Delta t}} \tag{35}$$

where λ_l, ω_l P_l are respectively the wavelength, frequency and power of laser light, η the quantum efficiency of the final light detector and Δt the measuring time.

Second we have, because of ΔN a fluctuating radiation force on the mirrors, as the photons of course carry momentum. If one takes both into account, optimizing for the laser power, and considers that the mirrors are actually suspended as pendula with proper frequency ω_0 much smaller than the range of frequencies of interest, one finds [GR 8]

$$\left(\frac{\Delta l}{l}\right)_{SQL}^{int.} \cong \frac{1}{l\eta^{\frac{1}{4}}}\sqrt{\frac{\hbar}{M\omega_0}} \tag{36}$$

which, apart for the factor $\eta^{\frac{1}{4}}$ is of course similar to Eq (24). However it appears at present that the optimal laser power needed would be out of reach and so the limit taken in consideration is that given by Eq (35).

We see from Eq. (25), and Eqs (31-36) and table 1a that the lab sources are excluded, but astrophysical sources are in reach, if intrinsic "quantum limits" in large systems of matter or light can be approached.

3.3 Conditions to approach the SQL

Let us see, following the specific example of bar detectors impulsively excited, what are the general conditions to approach the SQL. In a bar at temperature T, the end faces will fluctuate by Δl_{rms} given by $M\omega_0^2 \Delta l_{rms}^2 \cong k_B T$, which, even at liquid helium T's is much larger than Δl_{SQL}. However the high Q resonator will respond to a pulse of duration $\Delta t_p << Q/\omega_0$ with an immediate change in oscillation amplitude while, over the time Δt_p, the amplitude change Δl_{th}, expected form thermal noise, is given by $\Delta l_{th}^2 \cong \frac{\omega_0 \Delta t_p}{Q}\Delta l_{rms}^2$; so, at this short times, the energy signal to noise ratio in enhanced by the factor $Q/\omega_0\Delta t_p$ [GR 5]. If we ask $\Delta l_{th} < \Delta l_{SQL}$, to be in the SQL regime, we then see from the above that it must be

$$\frac{k_B T}{Q}\Delta t_p \leq \hbar \qquad (37)$$

the "thermal" action during Δt_p must be smaller than the quantum action. Notice that in Eq (37) the mass of the detector is absent. The meaning of Eq (37) is that, for sufficiently high Q/T and short Δt_p, one is able to detect an impulsive increase $\hbar\omega_0$ in the energy in the bar, on top of the much larger average thermal energy content $k_B T$.

The bar technology to fulfil Eq. (37) has already been demonstrated because predictions give $\Delta t_p \cong ms$, and massive bars have been successfully cooled [A2] to less than $T \cong 100mK$ showing Q values $Q \cong 2\times10^7$.

We need now an electromechanical transducer to transform the mechanical signal, a "phonon", in a electromagnetic signal, a "photon", which will be read by an amplifier and enter finally the data acquisition and analysis system. To complete the detection in SQL conditions, we would need to have the amplifier also be at its quantum limit in the sense that it should introduce the least amount of allowed energy noise [GR 4,5], which is in fact $\sim \hbar\omega$, where ω is the frequency at which the amplifier operates. Current technology does not give yet quantum limited amplifiers for $\omega \approx \omega_0 \cong kHz$ (cryogenically cooled HEMFET amplifiers show a noise of some $10^6\hbar\omega_0$). So one tries to shift the frequency of the photon in which the phonon had been transduced from ω_0 to a higher frequency ω where single photon counting technologies are available. This procedure does not introduce in principle additional noise, since shifting the frequency does not change the photon number. In terms of energy measured in numbers of "gravitons" of energy $\hbar\omega_0$ with $\omega_0/2\pi \cong 1Hz$,the process of emission in the Andromeda galaxy of a g.w. pulse of amplitude $h_{SQL} \cong 10^{-21}$ at the detector would be described as follows:

10^{68} gravitons→	10^{22} gravitons /m^2 →	1 phonon→	1 photon→	1 photon
from Andromeda	on the detector	at 1 kHz in the detector	at 1kHz in the transducer	at > GHz in the amplifier

The cryogenic resonant bars, which are currently the only detectors in continuous operation, make use either of amplifiers based on superconducting electronics, so called Superconducting Quantum Interference Devices, SQUID's, or of parametric amplifiers. Both operate at microwave frequency; so this is the frequency range of the final photon for detection. The best sensitivities currently achieved correspond to 10^4 quanta. The development of optomechanical transducer would easy the final amplifier problem, as single photon counting devices at optical frequencies are available.

3.4 Concept of interferometric detectors

The laser light of wavelength λ_l is split to enter the two arms of length l each of a "delay line" interferometer, bounces N times between the mirrors and is recombined in "dark fringe" conditions at the photodiode. Under the action of the g.w. the x, y arms change length by $\Delta x, \Delta y$ to give a total phase shift $\Delta\phi = \frac{4\pi N}{\lambda_l}(\Delta x - \Delta y)$. The tidal g.w. force moves the orthogonal arms in opposition; for instance for a h_+ g.w. propagating along the z axis, $\Delta x = -\Delta y = h_+ l/2$. In general one has to take account of: i) generic orientation of the g.w. direction of propagation and polarization; ii) the elastic forces, with resonance ω_0 and damping time constant Q/ω_0, which hold the mirrors in position and attenuate environmental disturbances; iii) the fact that $h_+(t)$ may change over the time the photons bounce back and forth. A simple analysis gives the response to a monochromatic g.w. $h_+ e^{i\omega_{gw} t}$

$$\Delta\phi(t) = \frac{4\pi Nl}{\lambda_l} \frac{sen\left(\frac{\omega_{gw} Nl}{c}\right)}{\left(\frac{\omega_{gw} Nl}{c}\right)} F(\theta,\xi,\varphi) \frac{\omega^2{}_{gw}}{\omega^2{}_{gw} - \omega_0^2 - i\omega_{gw}\frac{\omega_0}{Q}} h_+ e^{i\omega_{gw} t} \qquad (38)$$

with

$$F(\theta,\xi,\varphi) = \frac{1}{2}\left[\left(1+\cos^2\theta\right)\cos 2\xi \cos 2\varphi - \cos\theta sen 2\xi sen 2\varphi\right]. \qquad (39)$$

For highest sensitivity one maintains the "light storage time" smaller than the g.w. period, $\omega_{gw} << \frac{c}{Nl}$; on the other hand the vibration isolation from the environment increases as $\omega_{gw} >> \omega_0$; so the interferometer operates in the frequency range $\omega_0 < \omega_{gw} < \frac{c}{Nl}$ with phase shift sensitivity given by Eq. (38) which reduces to

$$\Delta\phi(t) = \frac{4\pi Nl}{\lambda_l} F(\theta,\xi,\varphi) h_+(t) \qquad (40)$$

In alternative the mirrors can be arranged in each arm to make up for a Fabry-Perot cavity of finesse f; one gets the same results with the substitution $N \rightarrow 2f/\pi$.

For the interferometers under construction the limiting noise sources are: i) the local seismic noise on the suspensions, which prevents operation below ~ 10÷100 Hz; ii) the thermal vibrational noise in the mirror masses and in the pendula wires in the range 50÷500 Hz; iii) the photon shot noise in the laser light, above 500 Hz. Other noise sources, as for instance the fluctuations in refractive index due to the residual gas along the light path and the laser instabilities in power, frequency and phase, are made unimportant with technologies under advanced development.

The French CNRS - Italian INFN collaboration VIRGO is setting up a l=3 km, two arms at right angles, Fabry-Perot interferometer at Cascina, near Pisa, Italy [GR 8]. The 20 kg mirrors are 35 cm wide and will have f=50. The monomode, single frequency Nd:YAG laser power P_l=20 W will be increased in the cavities to ~ 10^3 W by making it reenter the cavities, so called "recycling". Sensitivities are expressed as noise equivalent spectral metric perturbation $\tilde{h}_n(\nu)$, such that the r.m.s. minimum detectable metric perturbation h_{min} in the bandwidth $\Delta\nu$ is $h_{min} = \tilde{h}_n(\nu)\Delta\nu^{1/2}$. The sensitivity expected for VIRGO in the range 50÷500 Hz is $\tilde{h}_n = 3x10^{-23} Hz^{-1/2}$. For an impulsive signal, spread on a signal bandwidth $\Delta\nu$=100 Hz one would have a sensitivity $h_{min}=3x10^{-22}$.The sensitivity degrades proportionally to the frequency above about 500 Hz.

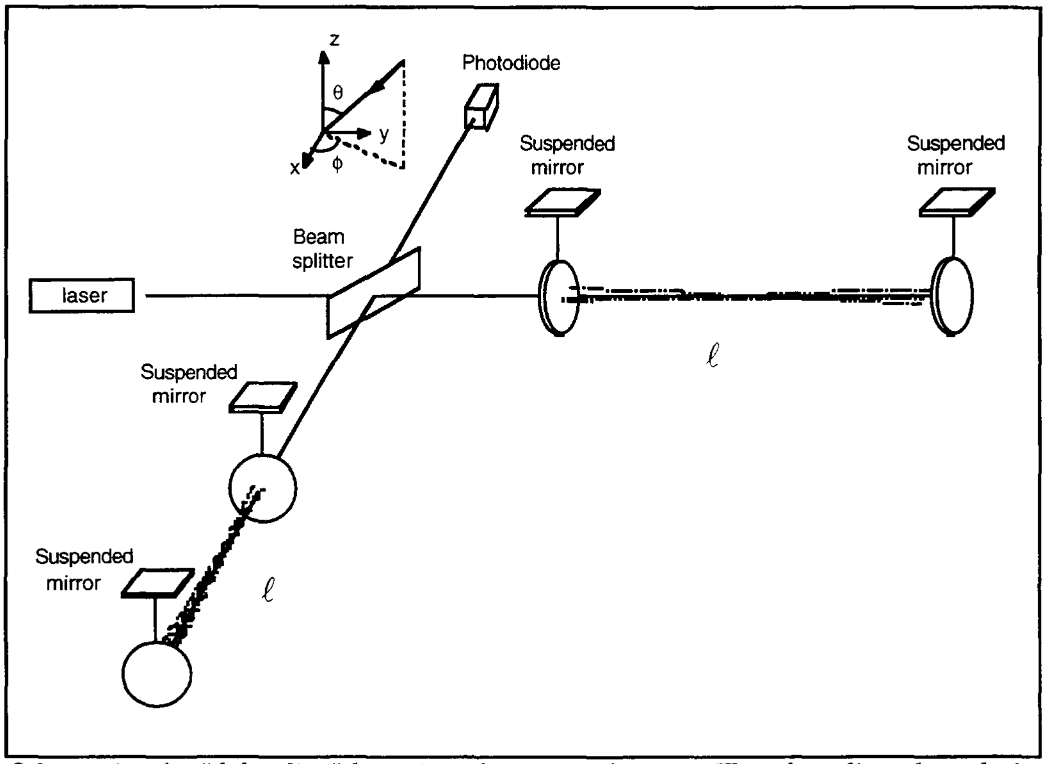

Fig. 2 Schematic of a "delay line" laser interferometer detector. The plane linearly polarized g.w. propagates in the direction at angles θ and ϕ; its polarization is at angle ξ in the orthogonal plane

It is relevant to notice that the environmental, thermal and photon shot noises, all act as displacement noises on Δl, so it is always an advantage to increase the arms length l to get a smaller noise on $\Delta l/l$. The mirror and suspensions thermal noise is currently foreseen as the actual limiting factor in an approach to the SQL, before one has to worry about the optimal laser power problem.

Table 2

Interferometric detectors

All are expected to be limited by seismic noise below ~100Hz, except VIRGO, for which the use of superattenuators will push the limit down to 10 Hz; h_{min} gives the expected sensitivity to short pulses.

Name [ref.]	Collaboration	Site	Arms length (Km)	h_{min}	TYPE
TENKO 100	Japan	ISAS	0.1	10-19	delay line
LIGO [A6]	U.S.A.	Washington St. Louisiana St.	4	$3x10^{-22}$	Fabry Perot
VIRGO [A7]	CNRS (FR) INFN (I)	Cascina (I- Pisa)	3	$3x10^{-22}$	Fabry Perot
GEO [A8]	Germany England	Hannover	0.6	10^{-21}	Fabry Perot

3.5 Concept of a cryogenic resonant bar detector

In a cylindrical bar detector of diameter d smaller than the length l, the g.w's enforce oscillations in the longitudinal odd harmonics at frequencies $v_n = nv_s / 2l$, with n = an odd integer and v_s the velocity of sound in the material. The cross section of the detector decreases as n^{-2}, with the order n of the harmonics. The fundamental mode is governed by the same Eq(26) for point like masses provided one makes the substitution $l \rightarrow 4l / \pi^2$ and uses $\omega_0 \cong \pi v_s / l$. A short g.w. pulse of duration $\Delta t_p \sim 1/\omega_0$ and amplitude h_+ on a bar initially at rest, gives

$$\frac{\Delta l}{l} = \frac{h_+}{4} F(\theta, \varphi) e^{-\frac{t\omega_0}{Q}} sen\omega_0 t \qquad (41)$$

where $F(\theta, \varphi)=sen_2\theta cos2\varphi$ is an antenna pattern factor Fig. 3. Thus the response in amplitude is similar to that of free masses, but the bar continues to vibrate for a number of periods of the order of the dissipation factor Q. The bar of mass M is coupled to an electromechanical transducer, made of one (or more) mechanical (and electrical) oscillators tuned to ω_0, which transforms in an electrical signal the mechanical oscillations. The INFN detectors of Table 3 all have the transducer mass m make for one arm of a capacitor biased at the electric field E_b.. For high Q coupled oscillators, energy is conserved in transferring from one to the other. Thus the transducer voltage change ΔV when the bar end face moves by Δl is $\Delta V = E_b \sqrt{\frac{M}{m}} \Delta l$.

This voltage is fed to the final amplifier and then to data acquisition and processing.

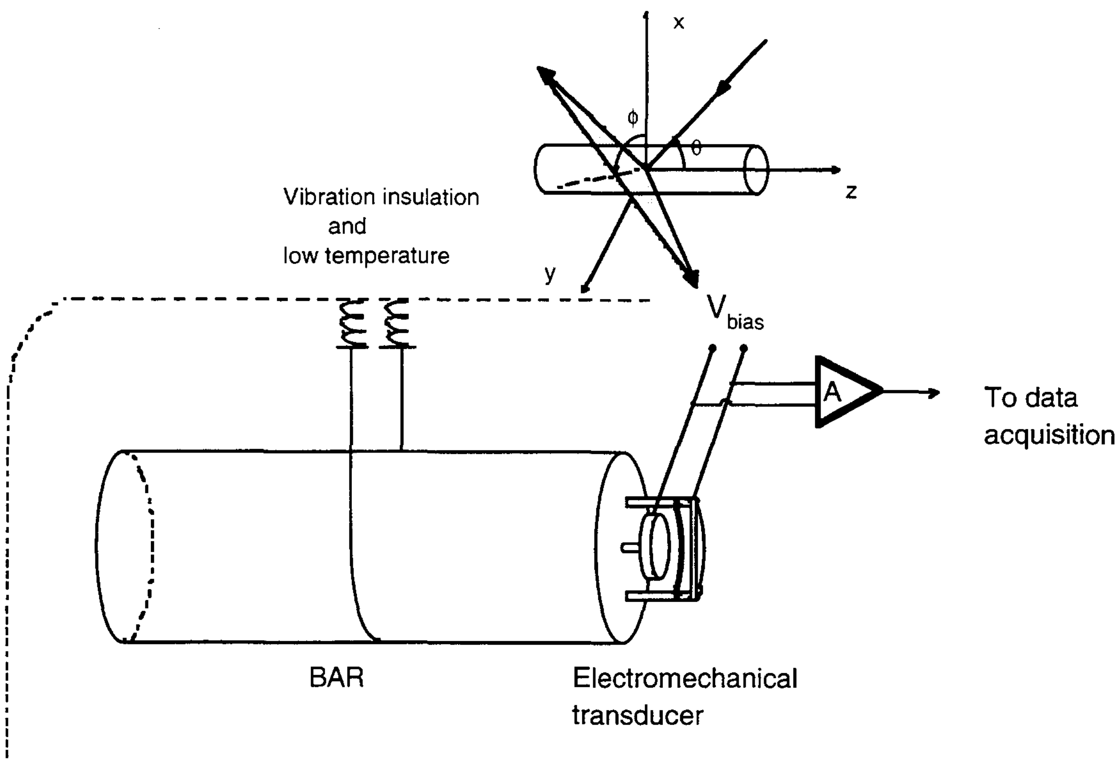

Fig. 3 Schematic of a resonant bar detector. The g.w. propagates at the angle ϑ in respect to the bar axis and is linearly polarized at the angle ϕ.

The noise sources which limit the sensitivity are: i) the vibrational thermal noise in the bar and in the transducer, which in energy contribute one k_BT each, where T is the temperature at which the detector is cooled; ii) the final amplifier energy noise k_BT_n. One takes care of other sources, as environmental vibrational and electromagnetic noise, cosmic rays etc., by the use of technologies still under active development.

With optimal filtering, the pulse energy sensitivity is usually expressed [GR5, A9] as an effective noise temperature of the system as:

$$T_{eff} = 2T\frac{\omega_0}{Q\Delta\nu} + T_n\left[\frac{2\left(\xi+\xi^{-1}\right)}{\beta\omega_0} + \frac{\beta\omega_0}{2\xi}\right]\Delta\nu \tag{42}$$

where $\Delta\nu$ is the postdetection bandwidth, ξ the ratio of the noise resistance of the amplifier to the output impedance of the transducer and β the ratio of electrical to mechanical energy in the transducer.

The first term, "narrow band" noise, is the thermal noise in the mechanical oscillator, which is smaller for larger $\Delta\nu$ as discussed for the SQL. The amplifier contributes the "wide band" noise both directly and also by a "back-action" effect on the mechanical resonators giving the two other terms. In the interplay between "narrow band" and "wide band" contributions, there is an optimal post detection bandwidth, $\Delta\nu_{opt} = \omega_0\left(2\pi Q\sqrt{\Gamma}\right)^{-1}$, where Γ is the ratio between the spectral power of wideband versus narrowband noise; one wants $\Gamma<<1$ and then it is seen that the actual detector bandwidth can be made much larger then the mechanical resonance width. The meaning of T_{eff} is that the changes ΔE in oscillation energy over the averaging time $(\Delta\nu)^{-1}$ due to noise follow a Boltzmann distribution $n(\Delta E) = n_0 \exp(\Delta E / k_B T_{eff})$; notice that it can be $T_{eff}<<T$.

It is useful to relate the energy ΔE_S absorbed by the detector to the g.w.'s spectral energy flux $f(\omega)$ via a cross-section $\sigma(\omega)$ as $\Delta E_S = \int f(\omega)\sigma(\omega)d\omega$. The spectral flux $f(\omega)$ is related to the g.w. intensity of Eq(19) by $\int_{-\infty}^{+\infty} I(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\omega)d\omega$. As the resonance is narrow we have

$$\Delta E_S = f(\omega_0)\Sigma \tag{43}$$

with $\Sigma = \int \sigma(\omega)d\omega$. In considering long bars of various d/l one finds [A10] that

$$\Sigma = 2.6\frac{G}{c^3}al^3Y \tag{44}$$

where Y is the elastic Young modulus of the material and $a = (\pi/4)\ (d/l)^2$. When $\Delta E_s = k_B T_{eff}$ we have a detection of g.w.'s energy at unity signal to noise ratio.

It is transparent the advantage on one side of using very low temperatures high Q materials, low noise amplifiers and tight energy coupling of the transducer chain (ultimately at the SQL one would have $k_B T_{eff} < \hbar\omega_0$) and on the other side of using bars of larger l, d/l and Young modulus.

3.6 Low frequency detectors

For g.w.'s detection at frequencies below some 10Hz, terrestrial seismic disturbances impose to look for detectors in space. The principle of operation is that of the measurement of the relative motion of two masses by means of electromagnetic tracking signals between the two. The masses can be celestial bodies or man made satellites orbiting in the solar system.

A notable exception is one of the first g.w. experiments ever performed, in which it was attempted to see g.w.'s excite the Moon normal modes of vibration, by means of a gravimeter landed by the Apollo 17 lunar mission, but the Moon is also too noisy and thus these attempts have been abandoned.

Table 3

Cryogenic bar detectors in operation or in advanced construction

Name [ref.]	Collaboration	Location	T (k)	T_{eff} (mk)	h_{min}	Status
EXPLORER [A4]	INFN Rome I, Rome II, Frascati, L'Aquila	CERN	2.0	5	$7x10^{-19}$	in operation
NAUTILUS [A2]	idem	Frascati	0.1			tested at T=1.5 K
AURIGA [A10]	INFN Padova, Legnaro, Trento, Ferrara	Legnaro	0.1			tested at T=300 K
ALLEGRO [A3]	U.S.A.	Baton Rouge	4.2	5	$7x10^{-19}$	in operation
UWA [A5]	Western Australia	Perth	~6	~10	$\sim 10^{-18}$	under test and calibration
- [A11]	U.S.A.	Stanford	0.1			under construction
UWA is made of Niobium, M=1.5 tons and has ν_0=700 Hz, $Q \cong 2x10^8$; all others are made of Al 5056 and have M=2.3 tons (except Stanford M=5 tons) l=3 m, $Q=2x10^7$, $\nu_0 \cong 900$ Hz.						

The current experimental efforts are summarized as follows. When the two test masses are at distances much larger than the g.w. wavelength, one looks at the Doppler shift $\Delta\nu/\nu$ in the frequency of the tracking light which is directly proportional to the g.w. amplitude of metric perturbation h(t) [GR5]. This method is used to: i) give limits to the stochastic background, by looking at the motion of pulsars relative to earth; here the $\Delta\nu/\nu$ is measured as a deviation from the accurately predictable frequencies of the pulsar pulse emission; for $\nu_{gw} \cong 10^{-8}$ Hz, it is currently obtained $E_{gw} < 10^{-6}$ of the closure energy density of the universe; the sensitivity deteriorates for higher frequencies; ii) attempt g.w.'s detection and limit the stochastic background in the frequency region $10^{-4} \div 10^{-2}$ Hz, by tracking the motion relative to earth of spacecraft's as Viking, Pioneer 10 and others.

The mission Ulysses has been giving sensitivities of the order $h=10^{-14}$ [A12], which correspond to a sensitivity in the displacement of the spacecraft of $\Delta l \sim 1$ cm at its distance $l \sim 8x10^{13}$ cm; future missions as Galileo and Cassini should improve the sensitivity by at least one order of magnitude. Still they cannot give interesting limits on the stochastic background as at best one may have $E_{gw} \leq 0.1$, in units of the closure energy density.

The Laser Interferometer Space Antenna project LISA [A13] is intended to develop a Michelson interferometer of $5x10^6$ Km arms length, to be launched possibly by the year 2015 by an ARIANE 5 rocket in heliocentric orbit. LISA works on the same principle as the earth based interferometers and should give a spectral sensitivity $\tilde{h}_n \cong 10^{-21} Hz^{-1/2}$ in the frequency range $10^{-4} \div 10^{-1}$ Hz. In particular LISA should be able to look for cosmic background g.w. radiation with an energy density sensitivity of $\sim 10^{-8}$ of the closure density in the range $10^{-3} \div 10^{-2}$ Hz.

4. PERSPECTIVES

As we see from Table 3, only recently we have been having a few detectors to operate, for extended periods of time, years, with sensitivities good enough at least for galactic supernova events. The field is in vigorous development, with other detectors under construction, and one may ask what are the expectations. There are two aspects: i) the directions to improve the sensitivity; ii) the strategies to maximize the chances of a detection and positive identification of g.w. signals.

4.1 Sensitivities

Taking as reference the end of the century, the sensitivities expected for the interferometers are those given by their approved final designs Fig. 4.

For cryogenic resonator detectors, a first generation has been already demonstrated with EXPLORER, ALLEGRO and UWA.

The directions for improvement can be seen by considering the energy ΔE_s absorbed by the detector in an impulsive event as in Eq(43).

Typical bar values are $\Sigma = 10^{-21}$ m^2 Hz. In considering mechanical resonators in form not only of slim bars but also of stumpy cylinders or of spheres, it is easily seen [A10] that Σ can be given, within a 10% approximation, by the same expressions Eq(44) when $a = \pi/6$ is used for a sphere.

So, recalling the discussion on the SQL, there is an additional route for improvement: increase the cross section. A linear dimension l=3 m appears to be a practical limit already. For present bars $a = \pi/100$, so a sphere would give a factor ~20 improvement. The use of Molybdenum for bars or spheres could be explored as the Q of small samples gave $Q \cong 10^7$ at low T's; in this case another factor of about 5 would be gained on the Young modulus in respect to the Al 5056 material. So one may get a factor 10^2, by fabricating a Mo sphere of 3 m diameter, weighting a few hundred tons.

Whichever the resonator , bar or sphere or else, the SQL is achieved for $\Delta E_s = \hbar\omega_0$. So in parallel one has a factor 10^4 improvement on the transducer + amplifier chain to approach the SQL, in respect to the performances of current detectors.

All in all a factor 10^6 better sensitivity in energy could be pursued for cryogenic resonators, which translates in a factor 10^3 in the minimum detectable metric perturbation h_{min}. Fig. 4 gives the sensitivities h_{min} for short, $\Delta t_p \cong (\omega_{gw})^{-1}$, impulsive signals for interferometers and cryogenic resonators. A last relevant point is that the situation presented above for mechanical resonator is somewhat idealized, because only white "brownian" thermal noise has been considered. Bar detectors invariably show an additional "excess" noise in form of rare, few per day, high energy events. These events distribute also Boltzmann like, but the characteristic energy $k_B T_{ex}$ is such that $T_{ex} \cong 1 \div 10$ K, to be contrasted with $T_{eff} \cong 10$ mK for the white thermal noise.

4.2 Detection Strategies

Quasi-continuous signal as the "chirps" from coalescing binaries and continuos signals have such marking signatures, that in principle they allow detection by a single detector, an interferometer in this case. Actually the LIGO pair of interferometers are oriented to maximize sky coverage. As optimal filtering techniques will be used to recover signals buried in the noise, it is crucial to predict with the greatest accuracy the shape of the signal, leaving as unknown to be searched for only it's amplitude and phase. To this effect much efforts are dedicated to calculate, using GR at higher orders of approximation, the g.w. emission of compact binary systems.

This is not so for impulsive signals, as they are expected to be rare and not much bigger than background. For a bar detector, even an optimal reconstruction with the new fast data acquisition [A14] would have difficulties in identifying revealing time structures in candidate signals, which would allow to sort signals confidently out of noise. In this respect a single detector is useless.

Operating cryogenic detectors show a postdetection bandwith ~1 Hz, so that even on intercontinental basis time delays due c-velocity travel time are not presently resolved. The current strategy thus is to have the bar detectors in coincidence within the available post detection bandwith. To maximize coincidence probabilities, the bar detector axes are oriented as close as possible parallel to a single direction in space, otherwise, due to antenna pattern effects, the same signal would excite differently the bars. Such coincidence protocols can be extended to interferometers, so that the ultracryogenic bars and the interferometers under construction, as they may have similar sensitivities for pulses in the KHz region, may constitute in the near future a global observatory to search for supernova events. An improvement could be, for such a network, to look for time delays between signals, due to c-velocity travel time. It has been recently proposed how to resolve rare pulses with submillisecond resolution with resonant detectors and the method has been in part demonstrated in room temperature tests [A10, 14].

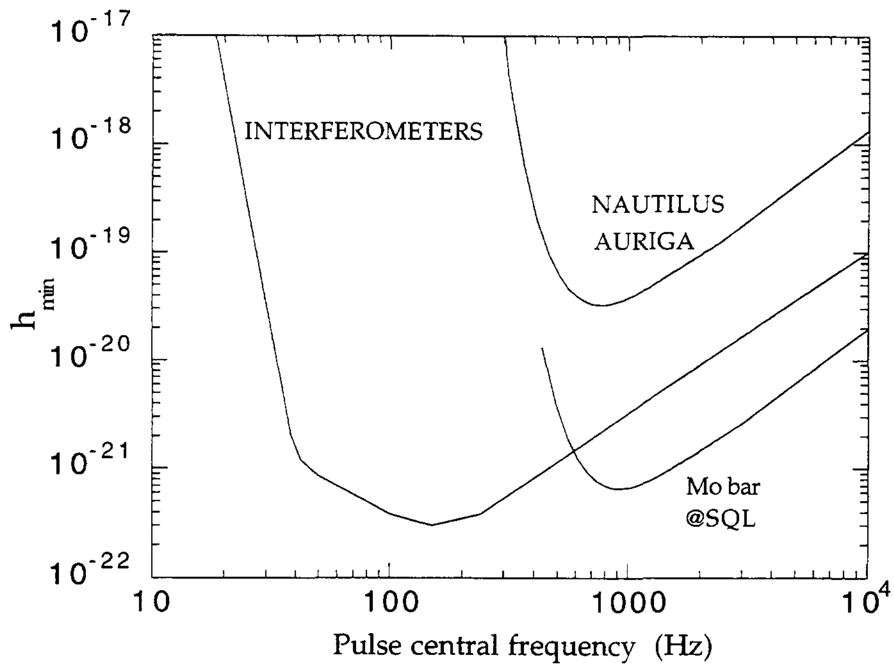

Fig. 4 Minimum detectable pulse amplitude h_{min} at unity signal to noise ratio versus the pulse central frequency, as expected for the interferometers under construction and for the ultracriogenic detectors. The sensitivity of a Molybdenum bar with d/l = 0.3 working at the SQL is also shown.

The supernova watch would be helped by parallel observations of optical and other e.m. signals. Strictly speaking this will not be a coincidence as the e.m. signals develop over time scales of a few hours after the event, while g.w.'s come out within fractions of a second. Neutrino signals would be on a much closer time scale, but, if the supernova event does not take place in the Galaxy, it looks that even the next generation of the neutrino detectors would be unable to make the detection. Another point is that the antenna pattern effects make the detectors blind for part of the day to some g.w.'s directions and polarization, while the detector rotates with the earth.

By contrast the 15 heaviest galaxies, which are within 5 Mpc and would give a few per year rate of detectable signals, are distributed in sort of double cone of 60° aperture just along the direction of the VIRGO cluster [A10], giving a strong anisotropy to the distribution of possible supernova sources in the sky.

So some attention has been dedicated recently: i) to sort of omnidirectional detector or array of detectors; ii) to extract from the combined responses of a (minimal)number of detectors in a network the (maximal) information on the incoming signal so to confidently tag it as a g.w. signal.

The combined response of interferometers has been analyzed to see the minimal number of them able to reconstruct amplitude, direction of propagation and polarization of the g.w. signal; the analysis uses the distinctive properties of g.w. signals to easy the solution of such an "inverse" problem.

For resonant detectors, the idea of a spherical detector has been recently revived [A15, 16]. The sphere has 5 degenerate low lying quadrupolar modes, which for a sphere of 3 m diameter would be still $\cong$ 1 KHz. Of course suitable combinations of the responses of the modes give omnidirectional sensitivity. An additional interesting feature is that the lowest monopole mode is at a close frequency, so that it could be used as a veto for non-GR excitations. The arrival time procedure of ref. [A10, 14] could be applied to a network of two spheres in two laboratories even as close as few hundred kilometers. The network of "2 spheres" would constitute a rather complete observatory giving amplitude, polarization, direction and velocity of propagation of the signal and one veto against spuria [A10].

A network of "6 bars" has been recently proposed [A10]. This is comparable in overall performance, as for onmidirectionality and total cross section and is somewhat more complete, as in principle it would allow vetoes both against non-traceless and against non-transverse signals. When the responses are recombined, the linear sum and a cubic combination are always zero and reflect respectively the tracelessness and the transversality of the Rienmann tensor carried by the wave; the quadratic sum gives the square of the pulse amplitude. These combinations correspond to invariants order rotations and thus are independent of the source direction and polarization. At least as an exercise, such a network continues to be studied, looking for suitable local or intercontinental arrangements, and, in conjunction with a complete numerical simulation of a bar detector it will be used to set up Monte Carlo's to evaluate detector confidence levels of g.w. signals of given characteristics in amplitude, direction and polarization.

ACKNOWLEDGEMENTS

I am grateful to Pierluigi Fortini and Stefano Vitale for continous discussions and to Roberto Onofrio for a critical reading.

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