BEYOND THE STANDARD MODEL

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1. INTRODUCTION

The attempts to go beyond the Standard Model have largely concentrated on the solutions to the 'hierarchy problem', the difficulty in field theory in keeping the observed states light in the presence of new physics at a very large mass scale (such as the Grand Unification scale, M_X, or the Planck scale, M_{Planck}, where gravity becomes strongly interacting). There are just two possible solutions to this problem, compositeness where the protection from the large mass scale occurs because some or all of the observed states are composite with soft form factors when coupling to the states beyond the Standard Model, or supersymmetry where the light states are prevented from getting a mass by a new symmetry, supersymmetry (SUSY). In these lectures, I will start with a review of the structure of effective field theories, in which there is a new scale of physics at a high scale, as the most direct introduction to the hierarchy problem and its solutions. I shall then present an overview of the two competing solutions, SUSY and compositeness. The remainder of the lectures is devoted to some of the detailed predictions of supersymmetry and a stage of (SUSY) unification beyond the Standard Model. The latter is certainly worth some study as these ideas offer the only *quantitative* prediction for the parameters of the Standard Model which come from physics beyond the Standard Model.

1.1 Motivation: effective field theories

It is very natural, given the success of the Standard Model, to ask why Nature should choose such a structure and what, if anything, lies beyond? Attempts at unification broadly divide into schemes in which one or more of the states of the Standard Model are composite and schemes in which the symmetry of the Standard Model is extended to enlarge the gauge group, and/or add supersymmetry and, most ambitiously, to include gravity in the unification. In this lecture I shall try to motivate extensions of the Standard Model via a 'bottom-up' approach in which the detailed structure of the low-energy theory is used to limit the possible forms of unification, rather than the 'top-down' approach which starts with a specific model for unification.

I find it quite remarkable that much of the Standard Model can be constructed starting from the assumption that it is an effective field theory descending from a fundamental theory at some high scale, M [1]. This follows because the effective Lagrangian describing the light fields, ϕ_{light}, obtained on integrating out the heavy fields [with mass of $O(M)$] may be written in a Taylor series ordered in inverse powers of M

$$\mathcal{L}(\phi_{light}, \phi_{heavy}) \to \mathcal{L}^{eff}(\phi_{light}) = \Sigma_n a_n O_n^{d_n} (\frac{1}{M})^{d_n - 4} , \tag{1}$$

where $O_n^{d_n}$ is a local combination of light fields labelled by its engineering dimension. The best-known example of this is the Fermi theory of weak interactions generated by a term such as $G_F(\overline{\mu}\gamma_\mu \nu)(\overline{\nu}\gamma^\mu e)$ which applies at energy scales much less than the W-boson mass and comes from integrating out the heavy W boson.

It may be seen from Eq. (1) that low-dimension terms dominate for energies $<< M$. This means the dominant terms of the effective theory describing scalar and fermion fields define a renormalizable theory.

$$\mathcal{L}^{eff} \sim \phi^4,\ \overline{\psi}\psi\phi,\ M^2\phi^2,\ M\overline{\psi}\psi . \tag{2}$$

If this effective Lagrangian has any light fields the large mass terms of Eq. (2) must be absent. The implications of this condition for the possible types of field are

- **Light fermions** If L has a chiral symmetry, light fermions are possible:

  $$\psi_L \to e^{i\alpha}\psi_L,\ \psi_R \to \psi_R \text{ forbids } M\overline{\psi}\psi \equiv M(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L) , \tag{3}$$

  where the subscripts L and R refer to the helicity components.

- **Light vector bosons** If L has a local gauge symmetry, light bosons are possible:

  $$A_\mu \to A_\mu + \partial_\mu\Lambda \text{ forbids } M^2 A_\mu A_\mu . \tag{4}$$

  In this case the leading dimension terms have $d_n = 4$ and define a renormalizable, local, gauge field theory of the gauge bosons interacting with scalars or fermions. One can even say more about the nature of the gauge boson coupling to fermions. The general coupling has the form $A^\mu(a\overline{\psi}_L\gamma_\mu\psi_L + b\overline{\psi}_R\gamma_\mu\psi_R)$. We have just argued that a light fermion should have no mass term $M\overline{\psi}\psi$ in this effective theory. Thus, if the fermion is massive, it must get its mass from spontaneous symmetry-breaking from a term $\overline{\psi}_L\psi_R\Phi$ where Φ is a scalar field which acquires a vacuum expectation value (vev), v. If the gauge symmetry is unbroken by this vev and the corresponding gauge boson is massless, then, necessarily, the transformation properties of ψ_L and ψ_R under this gauge symmetry must be the same to allow the mass term. This implies $a = b$ in the coupling of the massless gauge boson to the massive fermion, i.e. massless gauge bosons couple in a vectorlike manner. On the other hand, if the gauge symmetry is spontaneously broken by the vev of Φ, then there is no requirement that $a = b$ and hence massive gauge bosons may have parity-violating couplings[1)].

- **Light Scalars** If L has a spontaneously-broken global symmetry there are light Goldstone bosons:

  $$\phi \to \phi + a \text{ forbids } M^2\phi^2 , \tag{5}$$

  but ϕ cannot carry gauge quantum numbers, otherwise a $M'^2\phi^2$ term is allowed with $M' \sim gM$ where g is the gauge coupling.

1) This is likely but not inevitable — there are left–right symmetric models in which the massive boson couples in a parity-conserving way.

1.2 Supersymmetry

The only possible symmetry capable of keeping a Higgs boson light (as is needed in the Standard Model) is supersymmetry (plus a chiral symmetry). As discussed below, a supersymmetry transformation transforms a scalar state into its fermion partner and thus relates their masses:

$$\phi\,(J=0)\ \to \psi\,(J=\frac{1}{2})\ \text{ requires }\ m_\psi = m_\phi\ . \tag{6}$$

Thus, if the fermion mass m_ψ is forbidden by a chiral symmetry as discussed above, the scalar mass will also vanish.

The conclusion of all this is that an effective field theory describing scalar, fermion, and vector fields, light on a scale M associated with an underlying fundamental theory, must be a renormalizable gauge field theory — just the structure of the Standard Model — in which the massless gauge bosons have vectorlike couplings (as do the gluons and the photon) while the massive gauge bosons may have parity-violating couplings. However, to accommodate the light scalars needed for spontaneous symmetry-breaking, a stage of supersymmetric unification beyond the Standard Model is necessary and the breaking of this supersymmetry must not be much larger than the electroweak breaking scale (otherwise the scalar mass will be too large).

1.3 Do we really need supersymmetry?

Our arguments seem to imply that all scalars should be heavy... but what about the observed scalar mesons the π, K, η, etc? These are not present in the original QCD Lagrangian but appear as bound states of quarks. The arguments presented above for the absence of light scalar states apply to *elementary* scalars present in the original Lagrangian which have pointlike interactions. The distinction becomes clear only when calculating radiative corrections, for example to the scalar mass. The graph of Fig. 1 gives

$$\begin{aligned} m_\phi^2 &= \frac{h^2}{(2\pi)^4}\int \frac{d^4k}{(k^2)(k+p^2)} Tr(\frac{(1-\gamma_5)}{2}(k+p)k) \\ &\approx \frac{h^2}{16\pi^2}\int^{\Lambda^2} dk^2 \\ &\approx \frac{h^2}{16\pi^2}\Lambda^2\ , \end{aligned} \tag{7}$$

where Λ is a cut-off imposed to render the integral finite. In effective field theories it has a physical meaning, namely the scale at which new physics occurs. Clearly if $\Lambda^2 \approx M_X^2, M_{Planck}^2$ then the scalar ϕ will be very heavy and not appear in the low-energy Lagrangian. This is the 'hierarchy problem'. For composite scalars such as the η the calculation changes, since now at each vertex there should be included form factors describing the overlap of the composite scalar with the elementary fermions from which they are made. Taking this form factor to have the physically reasonable form $\exp(-k^2/\Lambda_{QCD}^2)$ we have

$$\begin{aligned} m_\phi^2 &\approx \int_o^\infty e^{-\frac{2k^2}{\Lambda_{QCD}^2}} \\ &\approx \Lambda_{QCD}^2\ . \end{aligned} \tag{8}$$

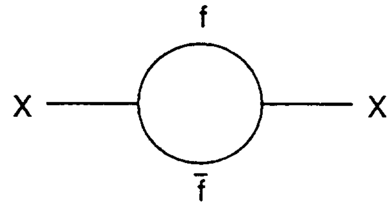

Figure 1: Graph generating scalar mass through a fermion loop.

1.4 Composite Higgs

But can the *Higgs* scalars of the Standard Model be composite? The answer is 'yes' and to demonstrate this forcibly note that with *no* Higgs scalar the W and Z bosons would still be massive! We know that the chiral symmetry of QCD is broken spontaneously with the pions and the pseudo Goldstone bosons associated with this breaking

$$SU(2)_L \otimes SU(2)_R \quad \underset{\pi^s\text{Goldstones}}{\overset{<\bar{u}u>=<\bar{d}d>=O(\Lambda^3_{QCD})}{\longrightarrow}} \quad SU(2)_{L+R} \, . \tag{9}$$

The pions, being made of quarks, will couple to the W and Z boson. Indeed using PCAC we know there is a coupling $(g_2 f_\pi/2)W^\mu \partial_\mu \xi$. Using this to evaluate the graphs of Fig. 2 gives

$$\frac{1}{q^2} + \frac{1}{q^2}.\frac{g^2 f_\pi^2 q^2}{4q^2}.\frac{1}{q^2} + ... = \frac{1}{q^2 - (\frac{g_2 f_\pi}{2})^2} \, , \tag{10}$$

which is the propagator of a *massive* state with mass

$$M_W = \frac{g_2 f_\pi}{2} \, . \tag{11}$$

Here $f_\pi \approx 100$ MeV is related to Λ_{QCD}. Inserting the numerical values for the couplings in this equation gives $M_W \approx 100$ MeV, far too small, and in any case the π's exist and have not been 'eaten' by the W and Z. However, this result does suggest that the mechanism could work if there is a new composite scale.

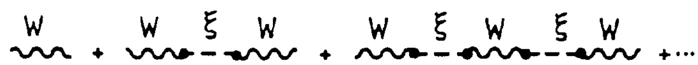

Figure 2: Graphs generating W mass when chiral symmetry is spontaneously broken. The line labeled ξ is the pion Goldstone mode.

1.5 Technicolour

We suppose there is a new strong 'technicolour' interaction with gauge group $SU(N_{TC})$. We also suppose there is a pair of techniquarks U, D which transform in the same way under $SU(2) \otimes U(1)$ as the u, d quarks. If technicolour is spontaneously broken in a manner analogous to QCD

$$SU(2)_L \otimes SU(2)_R \quad \underset{\pi^8_{TC}\ \text{Goldstones}}{\overset{<\overline{U}U> = <\overline{D}D> = O(\Lambda^3_{TC})}{\longrightarrow}} \quad SU(2)_{L+R} \tag{12}$$

the W and Z will acquire mass

$$M_W = \frac{g_2 f_{\pi,TC}}{2} \ . \tag{13}$$

Now, provided Λ_{TC} and $f_{\pi,TC}$ are scaled up appropriately ($f_{\pi,TC}$, $\Lambda_{TC} \approx 250$ GeV), M_W will be correct and the techni-interaction will be strong at a scale of $O(1$ TeV). This is the idea behind technicolour. An immediate implication is that there will be technihadrons at $O(1$ TeV) and pseudo Goldstone techni-π's etc. from 2 GeV upwards. I shall return to a discussion of techni-phenomenology shortly.

1.6 Top-quark condensate

One can even realize the composite Higgs idea *without* introducing any new fermions at all. The basic idea is that a strong binding force between top quarks (due to new physics at a high scale, Λ_c) may cause a dynamical breakdown of the electroweak symmetry through the formation of a top-quark condensate, without the need for an elementary Higgs scalar. In these theories there is still a massive scalar field left after symmetry breaking but it is a composite state, a top-quark–top-antiquark bound state [2].

1.7 Introduction to supersymmetry

There are several reasons why supersymmetry has been widely suggested as a likely extension of the Standard Model. In the first place supersymmetry provides the only possible (finite) symmetry beyond those based on Lie groups. It is also a necessary ingredient in string theories which many people think the best candidate for a 'Theory of Everything' unifying all the fundamental forces including gravity. While these points may suggest we consider supersymmetry as an underlying symmetry of Nature, the only reason we have for expecting a *low-energy* realization of the symmetry is the 'hierarchy problem', the difficulty discussed above of keeping the electroweak scale small compared to the unification scale without having the interactions become strong.

In these lectures I shall try to explain how supersymmetry explains the 'hierarchy problem' and determine the constraints the solution places on the supersymmetric spectrum. I shall also discuss what the phenomenology of the new supersymmetric states is likely to be and the prospects for finding experimental evidence for supersymmetry. First, however, I turn to a very brief introduction to supersymmetry.

The simplest possible supersymmetry algebra is given by the anticommutation relations involving spinorial generators Q_α, two-dimensional Weyl spinors [3].

$$\begin{aligned} \{Q_\alpha, Q_\beta\} &= \{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} = 0 \\ \{Q_\alpha, \overline{Q}_{\dot\beta}\} &= 2\sigma^\mu_{\alpha\dot\beta} P_\mu \end{aligned} \tag{14}$$

where σ^i are the 2×2 Pauli matrices and σ^0 is the unit matrix. The appearance of the momentum operator shows that the full supersymmetry algebra should include the generators of the Poincaré group giving a super-Poincaré algebra.

The (massless) representations of supersymmetry are easily obtained from Eq. (14) and consist of just two states of helicity λ and $\lambda + 1/2$. This immediately follows from the fact that, from Eq. (14), only one product of generators is non-vanishing. When building supersymmetric (SUSY) models the only multiplets that lead to consistent theories are left-handed 'chiral' supermultiplets with helicity (0,1/2) pairs, vector supermultiplets with helicity (1/2,1) pairs, and the gravitino supermultiplet with helicity (3/2,2) together with the conjugate fields.

It may now be seen how the hierarchy problem is solved in supersymmetric theories. Supersymmetric mass terms relate scalar and fermions within a supermultiplet as in Eq. (6). If the fermion mass is forbidden by a chiral symmetry (i.e. if the left- and right-handed states have different gauge quantum numbers), supersymmetry ensures that the scalar mass will also vanish. Radiative corrections may induce a scalar mass, but only at $O(M_{SUSY})$ where M_{SUSY} is the scale of supersymmetry breaking. Thus if $M_{SUSY} = O(1 \text{ TeV}) << M_{GUT}, M_{Planck}$ the hierarchy problem is solved. However, this implies that the zoo of SUSY states cannot be heavier than this scale, i.e. they are certainly in the range accessible to the next generation of accelerators.

The supersymmetry algebra of Eq. (14) may be extended by introducing more than one SUSY generator, $Q_\alpha^{a=1,..,N}$. These N-extended algebras are not used to construct low-energy supersymmetric theories because they have only non-chiral representations and thus lead to 'mirror' families of quarks and leptons, with the identical gauge couplings, but opposite helicity to their Standard Model partners. For example, because there are now *two* non-zero products of generators, the $N = 2$ algebra has the representation with helicities (–1/2,0,1/2). Since there is no evidence for such mirror states, essentially all work on low-energy SUSY has concentrated on the $N = 1$ supersymmetry case. In the next section I shall discuss how a $N = 1$ SUSY version of the Standard Model is built.

1.7.1 The supersymmetric spectrum

If the Standard Model is to be incorporated in a field theory with $N = 1$ supersymmetry, it is first necessary to assign the states of the Standard Model to $N = 1$ supersymmetric representations. Let us start with the $SU(3) \times SU(2) \times U(1)$ vector bosons. The only super-multiplet available that contains vectors is the vector supermultiplet containing states of helicity 1 and 1/2. Thus we must assign the vector bosons to sector multiplets transforming in the same way as the vectors under $SU(3) \times SU(2) \times U(1)$; this is shown in Table 1. It is clear that this means we have to introduce gauginos, spin-1/2 fermionic partners of the gauge bosons, which transform under the gauge group in exactly the same way as their vector partners.

Turning now to the assignment of the quarks and leptons to supermultiplets, it appears that there is the option of assigning them to vector supermultiplets, with vector partners, or to chiral supermultiplets, with scalar partners. However, the former possibility is not available, unless we increase the size of the gauge group, for the transformation properties of the quarks and leptons is different from the gauge bosons of the Standard Model (cf. Table 1). Thus we cannot identify any of the gauginos of Table 1 with the quarks or leptons and we have to assign the latter to *chiral* supermultiplets with the transformation properties of the quarks and leptons. These introduce scalar partners to the quarks and leptons transforming in the same way as their fermionic partners as shown in Table 1.

Table 1

SUSY states in the MSSM

	Spin	Name	SM partner
$\tilde{g}$	1/2	gluino	gluon
$\tilde{W}^{+-}$	1/2	Wino	W boson
$\tilde{Z}$	1/2	Zino	Z boson
$\tilde{q}$	0	squark	quark
$\tilde{l}^{+-}$	0	slepton	lepton
$\tilde{\nu}$	0	sneutrino	neutrino
$\tilde{H}$	1/2	Higgsino	Higgs boson

Finally, we must assign the Higgs bosons of the Standard Model to a supermultiplet. Having spin 0, they can only be assigned to a chiral supermultiplet and thus they must have fermionic partners called Higgsinos. However, the introduction of an $SU(2)$ doublet of fermions of definite helicity introduces an anomaly to the $SU(2) \times U(1)$ gauge theory. This is unacceptable, and to avoid it we are forced to introduce two Higgs chiral supermultiplets with *opposite* hypercharge (so that the Higgsinos have cancelling contributions to the anomaly) as shown in Table 1. It turns out that both Higgs multiplets are needed in supersymmetry to give quarks and leptons a mass, so this is indeed the minimal choice (i.e. attempts to identify the H_2 supermultiplet with a lepton supermultiplet without introducing an H_1 would have failed).

To summarize, the supersymmetric version of the Standard Model requires more than twice the number of particles: gauginos, partners for each of the gauge bosons, and squarks and sleptons, scalar partners for each of the quarks and leptons, and two Higgs doublets with fermionic 'Higgsino' partners.

1.7.2 Couplings in the supersymmetric model

Since the gauge multiplet structure of the new supersymmetric states is the same as that of their Standard Model partners, the interaction of gauge bosons with these states is fixed. Supersymmetry relates these gauge couplings to new couplings involving two superpartners in place of the original states (for example a quark–quark–gluon vertex is related to a squark–quark–gluino vertex). Thus the gauge-related interactions are fixed, given the multiplet structure of Table 1, and may be found, for example, in Ref. [3].

The remaining coupling that must be specified to complete the definition of the supersymmetric model corresponds to the Yukawa couplings and those couplings related to them by supersymmetry. These are most conveniently given by the 'F' term of the superpotential which is a polynomial in the chiral field ϕ (and does not involve their complex conjugates).

In terms of P the associated Yukawa interactions are given by $\Sigma_{i,j} \frac{\partial^2 P(\Phi)}{\partial \Phi_i \partial \Phi_j} |_B \ \psi_i \psi_j$, where the subscript B means the chiral fields Φ_i are to be replaced by their scalar components ϕ_i, and ψ_i are the corresponding fermionic components. There are scalar couplings related to these Yukawa couplings which are given by $\Sigma_i \mid \frac{\partial P}{\partial \Phi_i} \mid_B^2$. The couplings needed to give quarks and leptons a mass are contained in the superpotential given by

$$P = h_{ijk} l_i h_{2j} \overline{e}_k, h'_{ijk} q_i h_{2j} \overline{d}_k, h''_{ijk} q_i h_{1j} \overline{u}_k \ , \tag{15}$$

where l and e (q and $\overline{u}\overline{d}$) are the left-handed components of lepton-doublet and antilepton-singlet (quark-doublet and antiquark-singlet) chiral superfields, respectively, and $h_{1,2}$ are Higgs superfields.

The model with the minimal-multiplet content of Table 1, with only these terms in the superpotential, is known as the minimal supersymmetric standard model (MSSM). (As we shall see there are additional trilinear terms allowed by the gauge symmetry which give rise to non-standard supersymmetric versions of the Standard Model.) Note that each term in Eq. (15) gives three separate Yukawa couplings. For example the first term gives

$$L_{Yukawa} = l_i h_{2j} \overline{e}_k + \tilde{l}_i \tilde{h}_{2j} \overline{e}_k + l_i \tilde{h}_{2j} \tilde{\overline{e}}_k) \ , \tag{16}$$

where we denote by a supertwiddle the scalar partners to the quarks and leptons, namely the squarks and sleptons. The first term is the usual term in the Standard Model needed to give charged leptons a mass. The new couplings associated with the supersymmetric states, related to the first term by the operation of the supersymmetry generator, are given by the second and third terms.

Note that the last two terms involving the new supersymmetric states involve them in pairs. This is a general feature of the MSSM which preserves an R-parity [4], $R = (-1)^{B+L+S}$, under which the new supersymmetric states are all odd, while the Standard Model states are even. This has a profound effect on the phenomenology of the MSSM, for the new SUSY states may only be produced in pairs and the lightest supersymmetric state (the LSP) is stable.

There is one further coupling needed to complete the couplings of the minimal supersymmetric version of the Standard Model. In order to generate a mass for the Higgsinos associated with the Higgs doublets, $H_{1,2}$, it is necessary to add a term to the superpotential given by

$$P' = \mu H_1 H_2 \ . \tag{17}$$

In addition to giving a mass μ to the Higgsinos, this term plays an important role in determining the Higgs scalar potential and the pattern of electroweak symmetry-breaking. As we shall discuss in more detail in Section 3.3.5, the scalar term following from Eq. (17) aligns the vacuum expectation values (vevs) of the two Higgs fields so that the photon is left massless, obviously a crucial ingredient for a viable theory.

Before concluding this section let me just comment on the possible alternatives to the MSSM. In particular the couplings of Eq. (16) are not the only ones allowed by $SU(3) \otimes SU(2) \otimes U(1)$. The following terms are also allowed in the superpotential [5, 6]

$$[\lambda_{ijk} l_i l_j \overline{e}_k + \lambda'_{ijk} l_i q_j \overline{d}_k + \lambda''_{ijk} \overline{u}_i \overline{d}_j \overline{d}_k]_F \ . \tag{18}$$

As before, each of these terms gives three Yukawa couplings; for example the first term gives

$$\lambda_{ijk}(l_i l_j \bar{e}_k + l_i \tilde{l}_j \bar{e}_k + \tilde{l}_i l_j \bar{e}_k) \ . \tag{19}$$

It may be seen that these involve a single SUSY state and thus break the R-parity of the Standard Model. If any of the couplings are present, they have a significant effect on the phenomenology of SUSY for they allow for the production of a *single* SUSY state, and also the LSP is no longer stable. As a result, the normal SUSY signal, namely, missing energy and momentum from the escape of the neutral LSP, is no longer the case. Why then are these terms excluded in the construction of the MSSM?

The first point to note is that if the terms proportional to λ and λ' are simultaneously present, the nucleon is unstable because these terms violate lepton and baryon number, respectively. Since the Born amplitude is proportional to $(\lambda\lambda')/m_{\tilde{d}}^2$, and the squark mass is at most in the TeV range, the decay rate is quite unacceptably fast. The cure is to forbid these new terms by a discrete symmetry, known as matter parity, under which the quark and lepton superfields appearing in the superpotential change sign while the Higgs superfields are left invariant. Thus the last three terms of Eq. (18) change sign under this symmetry and are forbidden while the terms of Eq. (15) are invariant and allowed [7, 8]. Clearly this forbids all the terms of Eq. (18) and leaves just the couplings of the MSSM. Thus the matter parity leads to the R-parity of the MSSM.

However, there are more possibilities to stabilize the proton than to forbid all the terms of Eq. (18) [5]. Provided the terms proportional to λ and λ' are not simultaneously present, the Born term generating nucleon decay will be absent. It is possible to eliminate one or other of these operators by symmetries other than matter parity, provided one allows for the possibility that quarks and leptons transform differently [9]. Although this is not possible if the theory is embedded in SU(5), in which the quarks and leptons transform under discrete symmetries in the same way, it is possible in other GUTs and also in string unification, which need not be embedded in a GUT. Indeed a study of all Z_N symmetries ($N < 5$) shows that it is easy to obtain any of the following, all of which inhibit nucleon decay [10]

- Matter 'parity' $\tilde{\lambda} = \tilde{\lambda}' = \tilde{\lambda}'' = 0$; $\Delta B = \Delta L = 0$
- Lepton 'parity' $\tilde{\lambda}' = \tilde{\lambda}'' = 0$; $\Delta B \neq 0, \Delta L = 0$
- Baryon 'parity' $\tilde{\lambda} = 0$; $\Delta B = 0, \Delta L \neq 0$.

I shall not pursue these models further here, but wish to stress that such models should be considered when determining general tests for supersymmetry.

2. SUSY VERSUS COMPOSITE THEORIES: AN OVERVIEW

In this section I shall briefly review the implications of the extensions of the Standard Model for the precision tests of the theory that have been extensively performed using the recent high-precision data from LEP and SLAC. I shall also discuss the constraints on the Standard Model parameters, the masses and mixing angles, that may come from these extensions.

2.1 Top-quark condensate

As mentioned above, if the top quark is strongly bound, a top-quark condensate may form breaking the electroweak symmetry just as happens for technicolour. The top-quark condensate represents the extreme example of dynamical symmetry-breaking with minimal new physics. In the following discussion we parametrize the effect, at low energies, of the new strong interaction binding the top quark by a four-fermion interaction with coupling proportional to Λ_c^{-2}.

- **Precision tests** In the limit where the scale, Λ_c, of the new physics becomes much larger than the electroweak breaking scale it is found [2] that the low-energy theory contains just the states of the Standard Model including a (composite) Higgs scalar field which appears elementary at energy scales much less than Λ_c. The couplings of this effective Higgs field are identical to those of the Standard Model Higgs and the effective low-energy theory is indistinguishable from the Standard Model [11]. Thus the expectation is that, for a large unification scale, there should be negligible deviation from the Standard Model predictions in agreement with current measurements.
- m_t The top-quark condensate generates W and Z masses via Fig. (3b). Evaluating this graph gives

$$M_W^2 = \frac{g_2^2}{2} \frac{N_c}{(4\pi)^2} m_t^2 \ln(\frac{2}{m_t^2}) \tag{20}$$

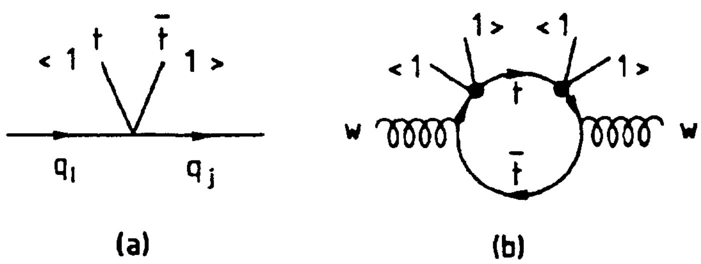

Figure 3: Quark and W, Z masses from a top-quark condensate.

This form will give an acceptable W mass for $m_t \leq 200$ GeV, (consistent with present LEP bounds) only for a very large cut-off scale $\Lambda > O(10^{10}$ GeV)! The top mass is generated by the graph of Fig. (3a). Equivalently, it is determined by the effective top-quark coupling, $g_t(\mu)\bar{t}_L t_R \phi^0$, describing the interaction, below the scale Λ_c, of the bound-state scalar doublet Φ which contains the Goldstone mode and massive scalar state discussed above. This is an effective coupling which, at low scales, replaces the four-fermion coupling and generates the top-quark mass when ϕ develops a vev. Since this coupling is generated by the new strong interaction, it is large at the scale Λ_c. However, this is corrected at low scales due to large radiative corrections proportional to $\log(\Lambda^2/m_t^2)$. These corrections may be calculated via the renormalization group (RG) for the effective top-quark coupling which is identical to the renormalization group for the top-quark coupling in the Standard Model [12]. The RG equations for the running coupling $g_t(\mu)$ and the QCD running coupling $g_3(\mu)$ are

$$16\pi^2 \frac{d\ln(g_t)}{dt} = \frac{9}{2} g_t^2 - 8 g_3^2$$

$$16\pi^2 \frac{d\ln(g_3)}{dt} = -7g_3^2$$
$$16\pi^2 \frac{d\ln(g_t/g_3)}{dt} = \frac{9}{2}g_t^2 - g_3^2 \ . \qquad (21)$$

The last term of Eq. (21) vanishes for $g_t^2 = \frac{2}{9}g_3^2$ corresponding to an infra-red, stable fixed point at which $m_t \approx 100$ GeV. How close to this fixed point the couplings get depends on the initial conditions and the 'distance' (Λ/μ) over which the couplings run. For initial conditions $g_t^2 >> \frac{2}{9}g_3^2$ there is an effective, low-energy fixed point with a larger value of g_t corresponding to the value $m_t = 230$ GeV. For $g_t^2 < \frac{2}{9}g_3^2$ the effective fixed-point structure is ineffective as one cannot cross the real fixed point corresponding to $m_t = 100$ GeV. However, as we have discussed, in the top-quark condensate model a large initial Yukawa coupling is necessary, so the effective fixed point governs the top-quark mass. The focusing effect of the RG equation is shown in Fig. 2.1, where it may be seen that, for widely varying initial values, the top mass at low energy scales is very close to the effective fixed point.

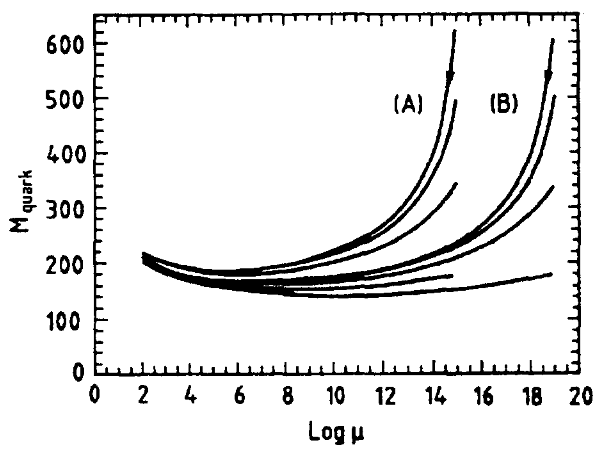

Figure 4: The running top-quark mass as a function of the scale μ. (A) $\Lambda = 10^{15}$ GeV. (B) $\Lambda = 10^{19}$ GeV.

Similarly one may analyse, using the RG, the running of the coefficient λ of the effective four-scalar interaction $\lambda \mid \phi \mid^4$, and hence determine the radiatively corrected scalar mass. The RG flow is shown in Fig. 5 and using it yields the predictions

$$m_t = 230 \pm 30 \text{ GeV}$$
$$m_\rho = 270 \pm 30 \text{ GeV} \ . \qquad (22)$$

- **Light fermion masses and mixing angles** The light-quark and lepton masses and mixing angles can be accommodated in this scheme through the introduction of four-fermion interactions coupling the top quark and top antiquark to the light quarks as in Fig. (3a). These interactions are postulated to come from new physics at the scale Λ_c and, with suitable choice for their coefficients, they can generate the observed structure. However, this is only a reparametrization of the theory and offers no understanding of this sector of the theory.

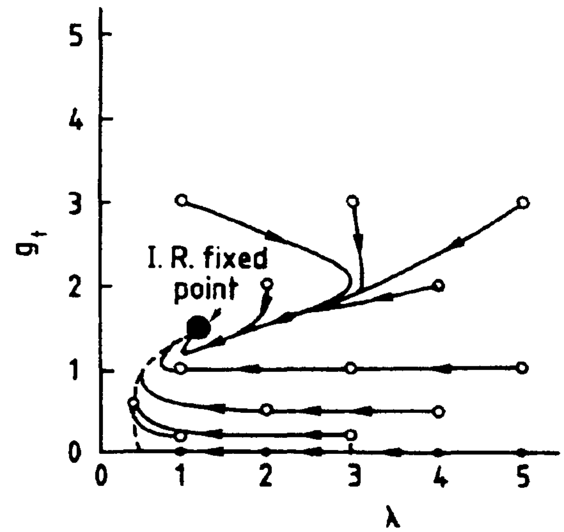

Figure 5: Full RG trajectories showing the joint evolution of the top-quark coupling g_t and the quartic scalar coupling λ.

Is the top-quark condensate scheme a reasonable alternative to the Standard Model? Obviously the top mass of Eq. (22) is too large in comparison with the recent LEP bounds, but it is possible to reduce it in versions which implement the mechanism in slightly different models, for example with a fourth generation, or with two Higgs doublets, or with supersymmetry... . However, in my opinion, there is a fatal flow with the scheme as implemented above, for it has a fine-tuning problem. The problem is that the expectation for the scale of electroweak breaking (and the associated top-quark mass) in a composite model of the Higgs is naturally of order of the composite scale Λ; in order to make it much less than Λ_c it is necessary to fine-tune the parameters of the theory. Recent work on top-quark condensation has been directed towards reducing the fine-tuning problem by reducing Λ_c. However, now the predictive power is lost, for the RG corrections discussed above become irrelevant. In this case the top mass and Higgs mass are determined by the (uncalculable) strong interaction dynamics.

2.2 Technicolour

In technicolour theories electroweak breaking is generated by a condensate of new quarks, 'techniquarks T_i', which are bound by a new 'technicolour interaction' and form a condensate [13]. As the techniquarks transform as electroweak doublets, this condensate breaks the electroweak symmetry in a manner analogous to the top-quark condensate. However, the technicolour models have the advantage that they avoid the need for fine-tuning.

Any attempt to study a composite structure is hindered by the inherently non-perturbative nature of the problem. Initial work on technicolour models attacked this problem by using QCD as a model theory and just scaling the results of QCD up in energy. However, much of the recent work on technicolour models has concentrated on elucidating possible differences between them and QCD in the hope that they will cure the problems encountered when trying to build realistic technicolour theories [14]. I have no time to review the details of technicolour models here, but shall concentrate on the expectation of these models for low-energy physics.

- **Precision tests** Unlike the top-quark condensate case technicolour models do not reduce to the Standard Model owing to the new 'technifermion' states. These states contribute to radiative corrections and so change the predictions of the Standard Model for precision measurements. In the case that technicolour behaves like QCD, present measurements rule out even one technifamily. However, this conclusion is sensitive to non-QCD-like effects and to new (light) states which may be associated with the new physics such as Majorana neutrinos, Z bosons, etc. so it may be possible to accommodate a technicolour scheme, but not in its simplest guise. In particular it has been argued [15] that all precision measurements are satisfied by a $SU(2)$ technicolour model with a light technineutrino (with mass 50–100 GeV) and a light technilepton (with mass 150 GeV).
- m_t Quark masses in technicolour theories result from a coupling of the Standard Model quark to the techniquark via a new 'Extended technicolour' (ETC) interaction. Thus chiral symmetry-breaking, initiated in the techniquark sector via the techniquark condensate, is communicated to the quark and lepton sector via radiative corrections involving ETC boson exchange as in Fig. 2.2.

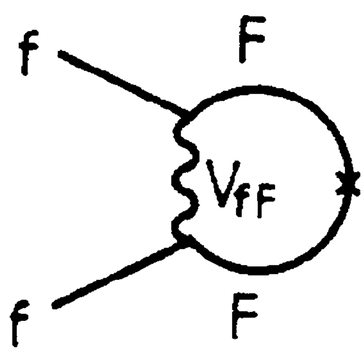

Figure 6: Light-quark mass generation in ETC theories via the coupling of the light quark, f, to the technicolour state, F, via an ETC boson, V_{fF}.

If (in analogy with QCD) one calculates this diagram assuming the absence of large corrections due to the strong technicolour interaction, one finds that the quark masses are given by

$$\frac{m_{q_i}}{M_W} \approx \frac{\Lambda_{TC}^2}{\Lambda_{ETC,i}^2}\,, \tag{23}$$

where Λ_{TC} and Λ_{ETC} are the scales at which the technicolour and extended technicolour interactions become strong. Since $\Lambda_{ETC} > \Lambda_{TC}$ the prediction is that the quarks (and leptons) should be much lighter than the W boson. Thus a heavy top quark does not fit easily into this picture. To accommodate it requires some new ingredient such as a combination of $t\bar{t}$ and $T\overline{T}$ condensates or non-QCD-like strong technicolour interactions which change the estimate of the radiative corrections generating the quark masses following from Fig. 2.2.

- **Light fermion masses and mixing angles** Apart from the top quark the remaining quark and lepton masses can be well described by Eq. (23). Mixing angles can be generated by ETC exchange graphs too, but the flavour-changing neutral currents generated by Born graphs involving ETC boson exchange turn out to be unacceptably large if the mass of the ETC bosons are chosen to give the correct light fermion masses in Eq. (23) [16]. To avoid this conflict, non-standard technicolour behaviour is needed changing the structure of Eq. (23) [15].

In summary, while technicolour theories are not dead, their saviour requires postulating non-QCD-like behaviour for the new technicolour interaction. As a result it is difficult, if not impossible, to extract reliable predictions from a theory due to the uncertainty in the strong-interaction effects. When compared to the predictive succes of supersymmetric models discussed below, it is difficult to avoid the conclusion that technicolour model-builders are swimming against the tide!

2.3 Supersymmetry

The alternative to a non-perturbative extension of the Standard Model involves a new symmetry, supersymmetry, to protect the Higgs scalar from large radiative corrections [3, 17]. As discussed above, the Minimal version of the Supersymmetric Standard Model (the MSSM) assigns the states of the Standard Model to supermultiplets which carry their $SU(3) \otimes SU(2) \otimes U(1)$ gauge quantum numbers. As a result, the spectrum of states is extended to include supersymmetric partners of the gauge bosons, the quarks and the leptons, namely the (fermion) gauginos, the (scalar) squarks and sleptons, respectively, together with two Higgsino (fermion) superpartners of the two Higgs scalar electroweak doublets needed in SUSY.

The solution to the hierarchy problem requires that these new states be light [$< O(1$ TeV$)$]. However, this implies that the zoo of SUSY states cannot be heavier than this scale, i.e. they are certainly in the range accessible to the next generation of accelerators.

- **Precision tests** A feature of the MSSM is that in the limit $M_{SUSY} \to \infty$, the MSSM $\to$ Standard Model with a *light* Higgs boson, $m_\rho^0 < 146$ GeV [18]. In this limit the predictions for the precision tests become those of the Standard Model, in agreement with current experiment within current errors. As the supersymmetric states become lighter, the predictions change in a way that depends on the details of the supersymmetric spectrum. For example, it is possible to maintain the excellent agreement with the S and T parameters while changing the prediction for $Z \to b\bar{b}$ to bring it into exact agreement with the current mean value by a careful choice of the ino masses [19].

- m_t By itself the MSSM does not determine the top mass. However, low-energy supersymmetry really only makes sense in the context of a unified theory (a GUT or a compactified string theory) and in this case there are large radiative corrections involving powers of $\log(M_X/M_W)$ where M_X is the GUT or compactification scale.

These favour a value for m_t close to the quasi-infra-red fixed-point (IRFP) given by $m_t \approx 200 \sin\beta$ where $\tan\beta = v_1/v_2$ and $v_{1,2}$ are the vevs of the two Higgs fields. At present we do not know β so the fixed-point result is still viable.

- **Light fermion masses and mixing angles** These are not determined in the MSSM (radiative corrections are small as the initial values are far from the IRFP) but, just as in the Standard Model, the Yukawa couplings of the theory may be chosen to give the observed masses and mixing angles. If one is to make predictions for these quantities, it is necessary to extend the symmetries of the model.

2.4 Phenomenology of the MSSM

As we have seen, the spectrum of supersymmetric states is constrained by the hierarchy problem to be quite low and accessible to experimental detection. The effects of supersymmetry may be detected either by their virtual effects on processes involving just the Standard Model states, or by direct production of the new states. We start with a discussion of the most important virtual effects of SUSY that have been identified.

2.4.1 New flavour-changing gauge interactions

The discussion of gauge-related interactions given in Section 1.7.2 applied to the current eigenstates. Owing to the non-degeneracy of quarks, we know that the charged, weak, gauge interactions can change flavour when expressed in terms of quark-mass eigenstates. There is an equivalent source of flavour-changing interactions in the new squark and slepton sector which we shall now discuss [3].

The connection between the current quark (or lepton) basis and the mass eigenstate quark (or lepton) basis is given by unitary transformations, $\mathbf{V}^{u,d}_{L,R}$. In terms of these the Cabibbo–Kobayashi–Maskawa mixing matrix is $\mathbf{U}_{CKM} = \mathbf{V}^{u\dagger}_{L}\mathbf{V}^{d}_{L}$. In general we have to define *different* unitary rotations $\tilde{\mathbf{V}}^{u,d}_{L,R}$ diagonalizing the squarks. Thus the interaction of the W bosons with squarks will be described by a new matrix $\tilde{\mathbf{U}}_{CKM} = \tilde{\mathbf{V}}^{u\dagger}_{L}\tilde{\mathbf{V}}^{d}_{L}$. Perhaps more significant is the immediate implication that SUSY theories have flavour-changing *neutral* currents (FCNC). For example, the gluino–squark–quark vertex has interactions given by $\mathbf{U}^{d\tilde{d}}_{CKM} = \mathbf{V}^{d\dagger}_{L,R}\mathbf{V}^{\tilde{d}}_{L,R}$ and $\mathbf{U}^{u\tilde{u}}_{CKM} = \mathbf{V}^{u\dagger}_{L,R}\mathbf{V}^{\tilde{u}}_{L,R}$ for the down and the up sectors, respectively. If SUSY were exact, $\mathbf{U}^{d\tilde{d}}_{CKM} = \mathbf{U}^{u\tilde{u}}_{CKM}$ and the FCNC would vanish. However, SUSY is not exact and this equality is broken at some level. The expectation for the resulting mixing matrices depends sensitively on the structure of the model at high scales, up to the Grand Unified or Planck mass. In the minimal-unification scheme the expectation is quite simple, namely $\mathbf{V}^{\tilde{u}}_{L,R} = \mathbf{V}^{u}_{L,R}$, $\mathbf{V}^{\tilde{d}}_{R} = \mathbf{V}^{d}_{R}$ and $\mathbf{V}^{\tilde{d}}_{L} = \mathbf{V}^{u}_{L}$ [20]. The reason for this asymmetrical result is that, owing to the large top Yukawa coupling, there are significant radiative corrections to the down-squark mass matrix proportional to the up-quark mass matrix. These dominate over the original (non-radiative) supersymmetric contribution which is just given by the down-quark mass matrix. The radiative contribution is diagonalized by the same rotation that diagonalizes the up-quark mass matrix and hence $\mathbf{V}^{\tilde{d}}_{L} = \mathbf{V}^{u}_{L}$. Using this gives $\mathbf{U}^{u\tilde{u}}_{CKM} = 1$ and $\mathbf{U}^{d\tilde{d}}_{CKM} = \mathbf{U}_{CKM}$. Non-minimal models may give both matrices non-vanishing, but the expectation is still [21] that they have the same order of magnitude as $\mathbf{U}_{CKM}$.

Although these effects introduce new sources of FCNC, it turns out that for SUSY masses in the range expected these do not lead to unacceptably large effects. For example, the box diagram of Fig. 7 gives a contribution to the $K_L - K_S$ mass difference due to wino FCNC. There is an equivalent graph involving gluinos, and since the gluino has QCD strength couplings it may be expected that this is the dominant contribution. However, in the minimal unification scheme discussed in Section 3.3.5, the gluinos and squarks are the heaviest states due to their QCD interactions, and their large mass suppresses this contribution. Using $\mathbf{U}^{d\tilde{d}}_{CKM} = \mathbf{U}_{CKM}$, the limit from the observed $K_L - K_S$ mass difference is [22], for the dominant contribution coming from the first two generations,

$$\frac{m^2_{\tilde{d}} - m^2_{\tilde{s}}}{(\mathrm{Max}(m^2_{\tilde{u}}, m^2_{\tilde{g}}))^2} \leq \frac{10^{-4}}{M^2_W} \,. \tag{24}$$

Since the $\tilde{d}, \tilde{s}$ mass difference comes from the quark-mass difference in the first two generations, this bound is satisfied for squark, gluino masses of more than 30 GeV and agrees with the expectation for squark masses following from the unification analysis discussed below. Similar conclusions follow for the Wino contribution of Fig. 7.

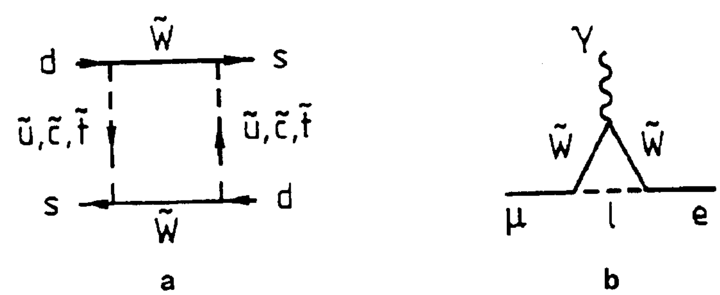

Figure 7: Graphs generating flavour-changing processes in supersymmetry.

Finally, in the slepton sector there are new lepton-number-violating processes coming from the fact that the sneutrinos, unlike the neutrinos, are massive and need to be diagonalized when determining the structure of the charged currents. The graph of Fig. 7 generates the process $\mu \to e\gamma$ and, imposing the experimental limits found for this process, gives the bounds [22] (assuming mixing angles of the same order as the CKM angles)

$$\frac{m^2_{\tilde{e}} - m^2_{\tilde{\mu}}}{(\mathrm{Max}(m^2_{\tilde{\mu}}, m^2_{\tilde{W}}))^2} \leq \frac{10^{-3}}{M^2_W} \,. \tag{25}$$

This gives a similar bound to that found for the gluino but, given the minimal model expectation for the mass spectrum, is a more severe limit. However, the choice of mixing angle used in this bound is quite arbitrary, so the only reasonable message to be drawn from the calculation is that lepton-number violation in the MSSM may easily be within the experimental bounds.

2.4.2 New sources of CP violation in SUSY

In the Standard Model the source of CP violation is the complex nature of the CKM matrix giving the CKM phase $\delta = \mathrm{Argdet}(U_{CKM})$. (The effect of T reversal on a constant, C, is $TCT_{-1} = C_\star$ so complex couplings act as a source of T or CP violation.)

In the MSSM there are additional complex couplings which may give rise to CP-violation [23]. With the minimal unification scheme of Section 3.3.5 there are two further complex couplings giving rise to CP-violating phases, namely $\phi_A = \text{Arg}(Am^*_{1/2})$ and $\phi_B = \text{Arg}(Bm^*_{1/2})$ [23]. Finally there is the effective theta parameter $\overline{\theta}$ specifying the strong CP violation. In this case it is given by $\overline{\theta} = \theta - \text{Arg}(\text{det}\lambda_u\lambda_d) - 3\text{Arg}(m_{1/2})$ where the λ's are the matrices of Yukawa couplings. These new sources of CP violation contribute to CP violation in various processes, the neutron and electron dipole electric moments, rare Kaon decays, rare B decays etc. The most sensitive of these to the SUSY CP-violation effects turns out to be the dipole electric moments.

The analysis of these moments starts with the $\Delta S = 0$ CP-violating Hamiltonian, written in terms of the elementary quark and gluon degrees of freedom as

$$H_{CPV} = \Sigma_i C_I(\overline{g}, \mu) O_i(\mu) \ , \tag{26}$$

where the O_i are the quark/gluon operators and $C_i(\overline{g}, \mu)$ the coefficients expresses in terms of the running strong coupling, which takes account of large radiative corrections, and the renormalization scale, μ. It was pointed out by Weinberg [24] that a significant contribution comes from the gluonic operator

$$O_g = f^{abc} G_a G_b \tilde{G}_c \ , \tag{27}$$

where $G_a^{\mu\nu}$ is the gluon field strength, $\tilde{G}_a^{\alpha\beta} = \epsilon^{\mu\nu\alpha\beta} G_{a,\mu\nu}$ and the Lorentz indices have been suppressed in Eq. (27). Subsequently, two further operators were indentified as significant [25], the colour dipole operator

$$O_q = \overline{q}\sigma_{\mu\nu} q T^a G_a^{\mu\nu} \tag{28}$$

and the electric dipole operator

$$O_\gamma = \overline{q}\sigma_{\mu\nu} q F^{\mu\nu} \ . \tag{29}$$

In SUSY these operators are all generated. For example, the coefficient C_q of the colour dipole operator is

$$C_q = \frac{2Am_u g_3^2}{4\pi m_{\tilde{g}}^2 \sin(\phi)} \ , \tag{30}$$

where ϕ gets contributions from both ϕ_A and ϕ_B. The net contribution is estimated to be [25, 26] $d_n^{Tot} \approx 10^{-22}\sin(\phi) ecm$ which, when compared to the experimental bound $| d_n | < 10^{-25} ecm$ requires $\sin(\phi) < 10^{-3}$. This is a strong constraint on the magnitude of $\text{Arg}(Am^*_{1/2})$ and $\text{Arg}(Bm^*_{1/2})$ and the question arises how natural is such a value in the context of a SUSY model? The expectation for many supergravity/string models is that these phases are very small, for A and B vanish at the Planck scale (if both A and B were related to trilinear terms in the superpotential they would vanish in the so-called no-scale models). Although radiative corrections will still generate a non-zero value for $\text{Arg}(Am^*_{1/2})$ at low scales, an estimate of these effects shows it is very small of $O(10^{-11})$ [26] and gives a neglible contribution to d_n. A similar result has been found in the case of the electron dipole electric moment [27]. The conclusion is that these dipole moments are quite sensitive to the new sources of CP violation in SUSY models.

2.4.3 Superparticle searches in the MSSM

The spin-1/2 sector has gluinos with mass $M_{\tilde{g}} = m_{1/2}$ at the unification scale radiatively corrected as in Fig. 11. The charginos $\tilde{W}^{\pm}$, $\tilde{H}^{\pm}$ mix via the 2×2 mass matrix

$$\begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \sin\beta & \mu \end{pmatrix} \tag{31}$$

and the neutralinos $(\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0)$ mix via the 4×4 matrix

$$\begin{pmatrix} M_1 & 0 & -M_Z \cos\beta \sin\theta_W & M_Z \sin\beta \sin\theta_W \\ 0 & M_2 & M_Z \cos\beta \cos\theta_W & -M_Z \sin\beta \cos\theta_W \\ -M_Z \cos\beta \sin\theta_W & M_Z \cos\beta \cos\theta_W & -\mu & \\ M_Z \sin\beta \sin\theta_W & -M_Z \sin\beta \cos\theta_W & -\mu & 0 \end{pmatrix} \tag{32}$$

where θ_W is the weak angle, $M_{1,2}$ ($= m_{1/2}$ at the unification scale) are the triplet and singlet neutralino supersymmetry-breaking masses and $\tan\beta = v_2/v_1$, the ratio of the Higgs vevs.

In the MSSM the LSP, $\tilde{\Lambda}$, is stable and its character determines much of the phenomenology of supersymmetry. A stable LSP should be neutral, otherwise, given the expected initial abundance from cosmological origins, it should have been found in nuclear matter. Thus we expect the LSP to be either a neutralino or a sneutrino, and, given the favoured spectrum discussed in Section 3.3.5, it is more likely to be a neutralino. It is often assumed that the LSP following from diagonalizing the matrix of Eq. (32) is the photino, $\tilde{\gamma}$, but this need not be the case. In the limit $M_1, M_2 \to 0$ (i.e. $m_{1/2}/m_0 << 1$) the photino is indeed the LSP but in the limit $\mu \to 0$ the LSP is the Higgsino. More generally one should allow for general mixing, denoting the mass eigenstates as χ_i^0.

2.4.4 Bounds on SUSY states

The vast bulk of supersymmetric (SUSY) phenomenology assumes the 'Minimal Supersymmetric Standard Model' (MSSM) which conserves what is known as R-parity [4]. In this model all the new states, the superpartners of the Standard Model states, are R-parity-odd while all the Standard Model states are R-parity-even. As a result, the new supersymmetric states can only be produced in pairs and a supersymmetric state cannot decay only to conventional states. This has a profound effect on the phenomenology of such states; in particular all experimental searches for the new supersymmetric states rely on pair production and most searches involve missing transverse momentum ($\not{p}_T$) as a signal for the production of the 'LSP' the lightest supersymmetric state which must be stable and neutral (for cosmological reasons).

- For weakly interacting sparticles the bounds are dominated by LEP following from the decay of the Z into a pair of sparticles. Thus $M_y \geq M_Z/2$ for $y = \tilde{l}, \tilde{H}, \tilde{\nu}, \tilde{q}, \tilde{q}, \tilde{W}$. For neutralinos one may still have lighter sparticles provided they couple less strongly to the Z.
- For strongly interacting sparticles, $\tilde{g}$ and $\tilde{q}$, the bounds are dominated by the hadronic colliders and, imply $m_{\tilde{q}} > 150$ GeV, for $m_{\tilde{g}} > m_{\tilde{q}}$ or $m_{\tilde{g}} > 100$ GeV, both at the 90% confidence level. To illustrate how these bounds are established and the prospects for finding strongly interacting sparticles at the SSC and the LHC, I shall discuss a case

study done for the LHC of gluino pair production and decay [28]. The basic process is

$$\begin{aligned} pp &\rightarrow \tilde{g}\tilde{g} \\ &\rightarrow (q\bar{q}\tilde{\Lambda})(q\bar{q}\tilde{\Lambda}) \equiv (E_T)_{Missing} + njet \ . \end{aligned} \quad (33)$$

However, if, as is expected from the radiatively corrected mass spectrum, the gluinos are heavier than several other superparticles, cascade decays become relatively important compared to the direct decay to the LSP, shown in Eq. (33). Such decays soften the $(E_T)_{Missing}$ which is the characteristic signal of this process.

Particularly at higher gluino mass, the cascade decays are very important. Using these various decay patterns, the cross-section has been determined via Monte Carlo simulations. It was found that the Standard Model background, coming principally from $t\bar{t}$ decays, is substantial. However, cuts, which rely on the different dependence on azimuthal angle of jets or on transverse energy distribution, are able to distinguish the signal from the background for gluinos with mass up to 1 TeV. The cascade decays also open up the possibility of signals which do not rely on $(E_T)_{Missing}$. For example the process

$$\tilde{g}\tilde{g} \rightarrow Z + Z + \tilde{X} \rightarrow 2(l^+l^-) + \tilde{X} \quad (34)$$

was studied [28]. Although the rates are quite small [some 30 events for a 750 GeV gluino after imposing an $(E_T)_{Missing}$ cut which eliminates the background] they provide a very clear signal, which would be a good second-generation experiment if the $(E_T)_{Missing}$ search provides evidence for gluinos. The results of the Monte Carlo study of this process may be found in the LHC study [28].

2.4.5 SUSY Higgs

Perhaps the most immediate test of SUSY will be in the Higgs sector, for *any* SUSY model requires a light Higgs ($m < 146$ GeV). In fact SUSY models require a rich structure of Higgs scalars, although many are expected to be much heavier, of the order of the SUSY-breaking scale, owing to the fact that SUSY requires two Higgs doublets, $H_{1,2}$. After the Higgs mechanism eliminates three components we are left with five Higgs states, four CP-even states H^0, h^0, H^+, H^- and one CP-odd state A^0. In terms of the original Higgs fields these are given by

$$\begin{aligned} H_1 &= \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(H^0 \cos\alpha - h^0 \sin\alpha + iA^0 \sin\beta - iG^0 \sin\beta) \\ H^- \sin\beta - G^- \cos\beta \end{pmatrix} \\ H_2 &= \begin{pmatrix} H^+ \cos\beta + G^+ \sin\beta \\ v_2 + \frac{1}{\sqrt{2}}(H^0 \sin\alpha + h^0 \cos\alpha + iA^0 \cos\beta + iG^0 \sin\beta) \end{pmatrix} \end{aligned} \quad (35)$$

where $G^{+,-,0}$ are the Goldstone fields which are 'eaten' by the Higgs mechanism. The scalar potential involving the neutral Higgs components driving the vacuum expectation values $v_{1,2}$ has the form

$$\begin{aligned} V(H_1^0, H_2^0) &= \frac{g^2 + g'^2}{8}(\mid H_1^0 \mid^2 - \mid H_2^0 \mid^2)^2 + \ m_1^2 \mid H_1^0 \mid^2 + \\ & m_2^2 \mid H_2^0 \mid^2 - m_3^2(H_1^0 H_2^0 \ + \ h.c. \end{aligned} \quad (36)$$

Note that the quartic couplings are determined in terms of the $SU(2)\otimes U(1)$ couplings in contrast with the Standard Model where the quartic coupling is unknown. It is this fact that results in an upper bound for the Higgs mass. Following from this potential (and the one describing the charged fields) we find

$$\begin{aligned}
m_A^2 &= m_1^2 + m_2^2 \\
m_H^{\pm} &= m_A^2 + M_W^2 \\
m_{H,h}^2 &= \frac{1}{2}(m_A^2 + M_Z^2 \pm \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta}) \\
\tan 2\alpha &= -\frac{m_A^2 + M_Z^2}{m_A^2 - M_Z^2}\tan 2\beta \ . \qquad (37)
\end{aligned}$$

We see from this that

$$\begin{gathered}
m_H^{\pm} \geq M_W \\
m_h \leq m_A \leq m_H \\
m_h \leq M_Z \cos 2\beta \leq M_H \ . \qquad (38)
\end{gathered}$$

However, the latter inequality applies at tree level using Eq. (36). The inclusion of radiative correction involving a top-quark loop gives a correction proportional to $h_t^2 m_t^2$ where h_t is the top Yukawa coupling of the Higgs to the top. The latter is also proportional to m_t so this radiative correction is proportional to m_t^4 and is large if the top quark is large. Including this term, the Higgs, h, can evade the bound of Eq. (38) but only by a finite amount, the final result being $m_h \leq 146$ GeV. To summarize, the Higgs sector in the MSSM is very rich and discovery of the Higgs states would be indirect evidence for the MSSM. More significantly there is a strong *upper* bound on the lightest Higgs which must apply in any SUSY model irrespective of the Higgs content[2)].

3. UNIFICATION HINTS

We have discussed the prospects for physics beyond the Standard Model which can be inferred from the structure of the theory at low energies without assuming any particular structure for the theory at high scales. Here we wish to discuss the implications for physics at low scales which follow from the assumption of a unifed theory at high scales. As we shall see, this generates a remarkably successful *quantitative* prediction for some of the parameters of the Standard Model. Indeed this is the only such prediction that has been reliably obtained and it is for this reason that I consider it worth while to devote some time to a discussion of the ideas involved.

3.1 Grand Unification

The original suggestion that there might be an underlying Grand Unified gauge field theory beyond the Standard Model was triggered by the observation of Georgi, Quinn and Weinberg [29] that, although the strong electromagnetic and weak interactions are quite different at low energies, the couplings, if continued to high energies, approach each other. This could be explained if there was an underlying simple gauge group with a single coupling constant from which the Standard Model emerged after symmetry-breaking Grand Unification [30].

2) Provided, of course, the model is not extended to include significant new interactions involving the Higgs which could alter the radiative corrections.

3.1.1 $SU(5)$

The prototype Grand Unified Theory is based on the group $SU(5)$ a rank-5 group with just enough neutral generators to accommodate those of the rank-5 Standard Model [31]. In $SU(5)$ the states of a single family are accommodated in just two representations $\overline{5}$ and 10. Thus the representation content of the Standard Model is simplified. The assignments to $\overline{5}$ and 10 are shown in Eq. (39)

$$
\begin{aligned}
SU(5) &\supset SU(3)\otimes SU(2)\otimes U(1) \\
\psi_{\overline{5}} &= \begin{bmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ \nu_e \end{bmatrix}_L \\
\chi^{10} &= \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & u_3^c & -u_2^c & -u^1 & -d^1 \\ -u_3^c & 0 & u_1^c & -u^2 & -d^2 \\ u_2^c & -u_1^c & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & e^+ \\ d^1 & d^2 & d^3 & e^+ & 0 \end{bmatrix}_L
\end{aligned}
\tag{39}
$$

where c denotes the charge conjugate and the numerical indices are $SU(3)$ indices. It may be seen that the quarks and leptons belong to the same multiplet. Of course $SU(5)$ must be broken and it is important to consider the pattern of the symmetry-breaking leading to the Standard Model. In the case of $SU(5)$, this is shown in Eq. (40).

$$
SU(5) \underset{<\Sigma_{24}^A>}{\longrightarrow} SU(3)\otimes SU(2)\otimes U(1) \underset{<H_5>}{\longrightarrow} SU(3)\otimes U(1) \tag{40}
$$

where one can see that the single 24-dimensional adjoint representation, Σ, leads to the breaking of $SU(5)$ to the Standard Model, $SU(3)\otimes SU(2)\otimes U(1)$, and subsequently the electroweak breaking is triggered by a doublet contained in a 5-dimensional Higgs representation, H_5. The most interesting prediction following from $SU(5)$ is for the weak mixing angle relating the two coupling constants associated with $SU(2)\otimes U(1)$. The prediction is[3)]

$$
\sin\theta_w{}^2 = \frac{T_r(T_{3L}^2)}{T_r(Q^2)} = 3/8 \ , \tag{41}
$$

but this applies at the unification scale, M_X. As we shall discuss, including radiative corrections to the couplings to run them down to low scales, one finds that the 3/8 initial value leads to quite acceptable values at laboratory energies for the weak mixing angle, but only in a supersymmetric GUT.

The Yukawa couplings of the theory are also restricted by $SU(5)$ in a way that gives predictions for fermion masses. This may be seen from the equation

$$
L_Y = h\overline{\psi}_{5R}^{p}\chi_{p,qL}^{10}H_5^q \Rightarrow m_e = m_d = h < h_5^5 > \ . \tag{42}
$$

The mass of the electron is equal to the mass of the down quark at the unification scale. The same is true for the heavier generations. As we shall discuss in the last lecture these

3) I leave it to the reader to derive this form!

predictions are not realistic apart from that of the third generation, so some modification is needed. It is possible to generate such a modification for example through the coupling

$$L'_Y = h'\overline{\psi}^p_R H^{sq}_{45p}\chi_{sqL} \Rightarrow m_q = 3m_e \ , \tag{43}$$

where H_{45} is a 45-dimensional representation of Higgs fields. The Yukawa coupling generates a relation between quark masses, which says that the electron would be 1/3 of the down-quark mass, a much more acceptable relation.

3.2 Unification and low-energy models

In this section I shall discuss features of low-energy models that descend from a stage of unification be it Grand Unification or string unification, paying particular regard to the question whether we can determine the parameters of the Standard Model. I start with a quick comparison of string versus Grand Unification expectations.

3.2.1 String versus Grand Unification

Perhaps the most compelling argument for physics beyond the Standard Model is just that, while hinting at an underlying unity of the fundamental forces, the Standard Model stops short of achieving this unity. The three forces derive from local gauge theories with gauge groups $SU(3)$, $SU(2)$, and $U(1)$ but no explanation is given for the origin of these different gauge factors nor for the choice of fermion and scalar representations. In addition, even without neutrino masses, there are twenty parameters needed to specify the model, (not a convincing ingredient for a 'Theory of Everything').

Grand Unified Theories (GUTs) seek to improve the situation by embedding the Standard Model gauge group in a larger gauge group with a single gauge-coupling constant. While the enlarged symmetry may relate several of the couplings of the Standard Model, it usually does so at the cost of introducing even more parameters, most of which are associated with the extended scalar sector needed to give the spontaneous symmetry-breaking necessary to break the GUT to the Standard Model. By contrast, unification based on (compactified) string theories offers the possibility of relating *all* couplings to a single parameter which is usually taken to be the Planck mass. However, this approach is hindered by the large number of candidate string vacua and at present it must be admitted that there is no understanding of how to select between such vacua. Given this ambiguity, it is probably premature to discuss any particular compactification scheme. Nonetheless, the effort that has gone into building specific models has not been wasted for it has shown how some of the prejudices for unification based on GUTs may not be true. I have attempted to summarize the situation in Table 2.

One of the most important differences in superstring unification is that, even if the gauge group is not Grand Unified below the compactification scale, the gauge couplings are related and there is a definite multiplet structure. Thus the good features of Grand Unification may be obtained without enlarging the gauge group and hence without encountering the problems associated with the need for spontaneous breaking of that group. A particular example of this is the need in $SU(5)$ for a 5 of Higgs which contains colour-triplet Higgs, partners of the usual Higgs doublet needed for electroweak symmetry-breaking. The colour triplets mediate proton decay and hence must be made heavy, of the order of the GUT scale, while the doublets must remain light. The difficulty in explaining this multiplet splitting is completely eliminated in 4D string theories without Grand

Unification, for in them there is no GUT requiring colour-triplet partners of the Standard Model Higgs doublets.

Table 2
Comparison of GUT and 4D superstring structures

GUTs	4D STRINGS
• GAUGE GROUP e.g. $SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$	 $G_{\text{'Hidden'}} \otimes G'_{\text{'Visible'}}$ G = E_8 or subgroup $G' = E_6$ ... $SU(3) \otimes SU(2) \otimes U(1)$ Unification of gauge couplings even without a GUT
• MATTER $SU(5) : \bar{5} + 10 \rightarrow 1$ family $SO(10) : 16 = (1 + \bar{5} + 10)$	 Definite multiplet structure even without a GUT e.g. 'level-1': only non-exotic $SU(3) \otimes SU(2)$ reps allowed
• FAMILIES Larger gauge structure? ...not very convincing	 3 generation examples known
• MASS HIERARCHY ⇒ SUSY GUTS	 ⇒ $(N = 1)$ SUSY can persist from superstring (No need for coloured Higgs)
• LOW-ENERGY STRUCTURE $(N = 1)$ SUSY B# ? L# ? ⇒ Discrete symmetry needed	 $(N = 1)$ SUSY B# ? L# ? ⇒ Discrete symmetry predicted
• PARAMETERS $g_i \rightarrow g_{GU}$ $m_b = m_\tau, \ldots$ Higgs sector? ...many undetermined parameters.	 In principle all predicted in terms of single parameter $M_P \equiv M_{Planck}$

It is stated in Table 2 that the solution to the mass hierarchy problem in both GUTs and 4D string theories requires a stage of low-energy supersymmetry. The reason for this is that radiative corrections to scalar masses drive the electroweak breaking scale associated with the mass of the Higgs doublet of scalars to the largest scale in the theory.

Thus in GUTs the expectation is that the W and Z masses should be at least of the order of the GUT scale, quite unacceptably large. Even in the Standard Model, without the new interactions introduced by Grand Unification, one may argue that gravitational radiative corrections, estimated with a cut-off of the order of the Planck scale, will drive the electroweak breaking scale towards the Planck scale. Although the calculation of these radiative corrections requires renormalization and the finite part could in principle be cancelled by the finite piece of the counter term, this requires an arbitrary and unnatural fine-tuning. In a theory such as the string in which the calculation of these mass corrections is finite, this problem is even more severe for there is no arbitrary parameter available for such fine-tuning.

3.3 Evidence of supersymmetry: unification hints

Much of the motivation for our low-energy effective field theory descending from a Theory of Everything (the superstring?) relied on the existence of low-energy supersymmetry to solve the hierarchy problem. Thus tests of the idea of an underlying unification must first test for the presence of supersymmetry without which the whole programme breaks down. To date no SUSY state has been found. However, this is not unexpected, for all the new states are expected to have a mass of the order of the supersymmetry breaking scale, and if this is large the new states will be beyond current experimental thresholds for production. However, the supersymmetry breaking scale cannot be taken to be too large without re-introducing the hierarchy problem so the new SUSY states should be accessible by the next generation of accelerators. Making the SUSY states too heavy to produce directly is not enough to avoid experimental detection, for even before the new SUSY states are produced in the laboratory they may manifest themselves as virtual corrections through radiative processes. In particular, one may worry that the precision tests of the Standard Model, which are in agreement with the the Standard Model predictions, already rule out a stage of low-energy supersymmetry. Remarkably, this is not the case because the effects of the new SUSY states tend to be very small and easily in agreement with the current experimental measurements. This happens even for a SUSY breaking scale, considerably less than the 1 TeV which is a rule-of-thumb limit coming from the requirement that the hierarchy problem be solved. This result may be seen as a consequence of a decoupling theorem that states that the MSSM becomes the Standard Model in the limit of large SUSY breaking.

I do not have time in these lectures to review these precision tests and their constraints on the SUSY spectrum. However, there is another very important effect of virtual SUSY states which I cannot pass over for it *does* have experimental support and is perhaps the main reason that supersymmetric models have been taken so seriously. This is the effect of SUSY states on Unification predictions. If one adds some assumptions about Grand Unification to the MSSM, the scale of supersymmetry breaking and the associated superpartner masses is constrained [32]. Since supersymmetry was introduced to allow for such unification, it seems reasonable to treat seriously such predictions for the mass scale of supersymmetry.

The prediction follows from the Grand Unified prediction for the gauge couplings such as occurs in $SU(5)$ together with the form of the running strong, weak, and electromagnetic couplings. In the Standard Model this was analysed by Georgi, Quinn, and Weinberg [29] using the RG equations for $\alpha_i = g_i^2/(4\pi), i = 2, 3$ and $\alpha_1 = \frac{5}{3}g_i^2/(4\pi)$, where

g_i are the $SU(3), SU(2), U(1)$ couplings

$$\alpha_i^{-1}(q^2) = \alpha_i^{-1}(\mu^2) + \beta_{0i} ln(q^2/\mu^2) + .. , \tag{44}$$

where the one-loop β function is simply evaluated knowing the light matter content

$$2\pi\beta_{0i} = \begin{pmatrix} 0 \\ \frac{-22}{3} \\ -11C^2 \end{pmatrix} + N_f \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3}C^2 \end{pmatrix} + N_H \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix} , \tag{45}$$

and N_f, N_H denote the number of families and Higgs doublets, respectively. In $SU(5)$ $C^2 = 1$ and $\alpha_i(M_X^2) = \alpha_G$; the same applies to many of the phenomenologically viable GUTs and superstring unification schemes as I shall shortly discuss. Given these equations, the low-energy couplings should evolve in energy to meet at the unification value α_G. However, the recent precision measurements of $\sin\theta_W$ are inconsistent with these predictions; the couplings fail to meet by more than six standard deviations [32].

The situation is quite different in the MSSM due to the effects of the new supersymmetric states on the evolution of the couplings. Including this gives [33]

$$2\pi\beta_{0i} = \begin{pmatrix} 0 \\ -6 \\ -9C^2 \end{pmatrix} + N_f \begin{pmatrix} 2 \\ 2 \\ 2C^2 \end{pmatrix} + N_H \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix} . \tag{46}$$

In Fig. 8 the evolution of the couplings (including two-loop effects) is shown and it may be seen that the couplings do meet in a point. This happens if the mass of the new supersymmetric states (assumed degenerate here) are low, $M_{SUSY} \approx 10^{2.5\pm1}$. On the basis of these results it is tempting to argue that there is evidence both for new forms of (relatively) light supersymmetric matter *and* an underlying unified theory[4)]. This conclusion is so dramatic that it needs to be evaluated with considerable caution. The most obvious reservation is that it relies on the extrapolation of the Standard Model, albeit extended to its supersymmetric version, some twelve orders of magnitude beyond the energy scale at which it has been tested. No theory has proved to be so robust in the past so it is understandable if one views this extrapolation with some caution. Even given the framework of Grand Unification there is considerable uncertainty, for the relative values of the couplings may differ in different unification schemes. Within the (large) class of theories which give the $SU(5)$ predictions there are corrections coming

4) It is perhaps appropriate to comment about the meaning of this fit, for the three gauge couplings are described in terms of three effective parameters M_X, α_X and the effective supersymmetry mass scale, meaning there will always be a fit and apparently no test of unification! However, this is not quite fair for the resulting values of the parameters must be reasonable if the scheme is to make sense. Thus M_X should be less than the Planck scale but large enough to inhibit proton decay in Grand Unified theories. Also we would like α_X to be within the perturbative domain although it may be sensible to contemplate non-perturbative unification. Finally, the supersymmetry mass scale must be large enough to explain why no supersymmetric states have been found and small enough to avoid the hierarchy problem (as we shall see the latter gives a very strong constraint). As we shall discuss, these conditions are indeed satisfied.

from virtual states with mass $\geq O(M_X)$ which have not been included in the analysis and which can affect the predictions substantially. These predictions are even more sensitive to additional light states beyond those needed for the minimal supersymmetric Standard Model. (Indeed it is possible to bring the predictions of non-supersymmetric $SU(5)$ into agreement with experiment if additional light Higgs doublets are added.) Furthermore, if the GUT should not break immediately to the Standard Model, there will be several intermediate scales of breaking and again the results will change substantially.

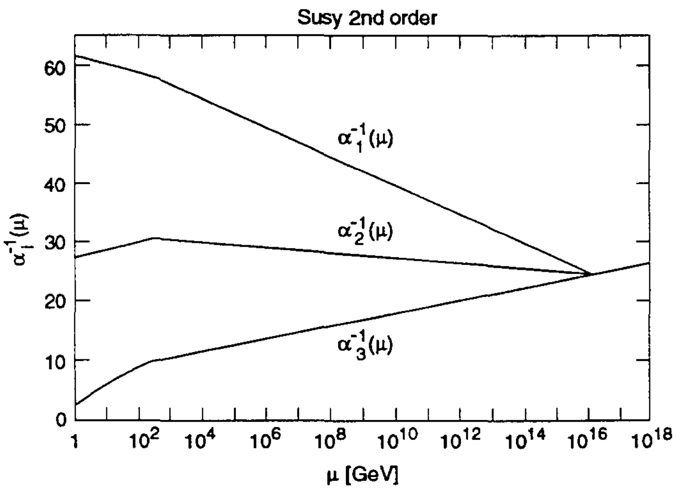

Figure 8: Evolution of running couplings. The break at low scales corresponds to the transition from non-supersymmetric to supersymmetric β functions.

Despite these caveats it is remarkable that the simplest possible extension of the Standard Model to include supersymmetry, coupled with the simplest assumption about Grand Unification, yields predictions in detailed agreement with experiment, provided the new supersymmetric states are very light. The main objection to the analysis of the likely mass of the new supersymmetric states is that, owing to large radiative corrections, the most reasonable supersymmetric spectrum is not degenerate. To discuss the likely effects of non-degenerate SUSY states I shall use the spectrum motivated by most realistic supergravity/superstring models of supersymmetry breaking discussed in Section 3.3.4.

3.3.1 Unification of gauge couplings

What are the implications of unification for the supergravity-inspired SUSY spectrum discussed in Section 3.3.4? Using this spectrum to determine when to change from the non-supersymmetric to the supersymmetric β-functions, we may repeat the analysis of the unification predictions [34]–[36]. In Figs. 9 and 10 we show the contours of constant α_3 which lead to unification. It may be seen that, allowing for the range $\alpha_3(M_Z) = 0.102$–0.12, values for the supersymmetry mass scale M_{SUSY} in the range (10^2–10^5) GeV are allowed [I define M_{SUSY} to be $Min(m_0, m_{\frac{1}{2}}, \mu)$]. Thus the effect of the non-degenerate spectrum is to increase the anticipated mass of the SUSY states (the bounds given above were for $\alpha_3 = 0.108 \pm 0.005$). The value obtained for the unification scale is $M_X \approx 3.10^{16}$ GeV.

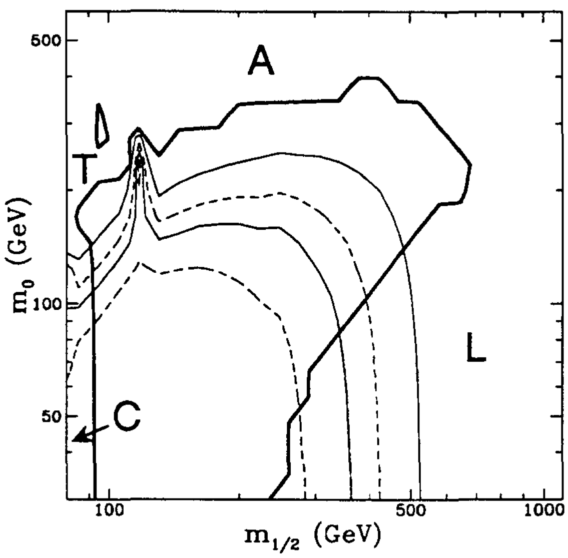

Figure 9: The contours of $\Omega_{LSP}h^2 = 0.5, 0.33, 0.25, 0.16$ from bottom to top, superimposed on the outline of the region allowed by all constraints. The regions ruled out by the various constraints are labelled by letters indicating: A = age of universe constraint ($\Omega_{LSP}h^2 > 1$), C = chargino mass bound from LEP, T = tachyonic top squarks, L = charged LSP. This plot is essentially Figure 6 from the paper Kane et al., Phys. Rev. D50 (1994) 3498.

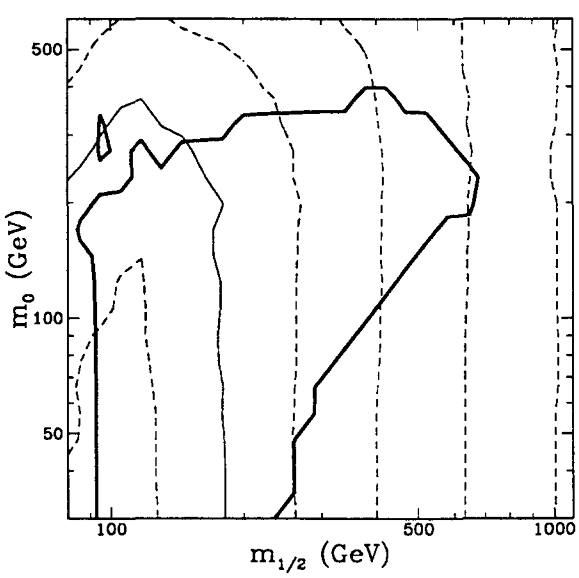

Figure 10: The same data set as Fig. 3 by the same authors, with the same outlined allowed region, but with contours of α_3. The contours are for the values (left to right) 0.132, 0.130, 0.128, 0.126, 0.124, 0.122. (The solid line is 0.130; all the rest are dashed.)

- **String Unification scale** How does this unification value fit in with the string expectation? The naive string unification scale M_{SU} is the Planck mass and hence

is too large. However, the threshold effects coming from including heavy states with mass of order of the compactification scale could reduce this scale; but a survey [37, 38] of all orbifold models shows that typically M_X is *increased* by the string threshold corrections Δ_i from the string unification scale M_{SU}. Only for very special assignments of the matter fields to (twisted) multiplets can the unification scale be reduced. What then are the implications of coupling unification for string theories? The first possibility is that the threshold corrections *do* conspire to lower M_X. At least the existence of an orbifold example shows this is possible [37] and to date the threshold effects in more general compactifications have not been completely worked out. A second possibility is that the string gives a gauge group larger than the Standard Model which preserves the ratios of gauge couplings and which subsequently breaks at an intermediate scale $M_X \approx 10^{16}$ GeV to the Standard Model. This group need not be Grand Unified. My favourite example is the group $SU(3)^3$ which is known to occur in some promising compactification schemes. Provided there are equal numbers of quark and lepton multiplets, the couplings renormalize in the same way between M_{SU} and M_X as required. Another possibility is to give up the minimal unification assumption and to add states beyond the minimal set. For example, in flipped $SU(5)$ it is necessary to evolve the $SU(5) \otimes U(1)$ couplings from M_{SU} to M_X before using the $SU(3) \otimes SU(2) \otimes U(1)$ evolution. Unfortunately this spoils the agreement with the low-energy couplings in the original version of the theory, but in non-minimal versions the effect of states with masses of order 3.10^{13} GeV can lead to acceptable results [39]. This example nicely illustrates both the additional predictive power of the string theory and the uncertainties introduced once non-minimal theories are introduced.
To summarize, the success of minimal unification predictions with $M_X \approx 3.10^{16}$ GeV can be understood within the framework of specific string models. However, since these models do not correspond to minimal string unification, some of the aesthetic appeal of simplicity is lost.

3.3.2 Supersymmetry breaking

So far the discussion of supersymmetric unification has relied only on the global supersymmetry aspects of the theory which involve the supermultiplet structure and the renormalizable couplings. However, most workers in the field assume that it is the locally supersymmetric version that is realized [40]. There are several reasons for this. In the first place the local version of the supersymmetric algebra necessarily includes the local version of the Poincaré group and hence includes a theory of gravity. The hope is that the ultimate Theory of Everything will involve a unification of all the fundamental forces including gravity, and so the local version seems desirable. Moreover, the superstring theories which offer the best hope of such a Theory of Everything naturally lead at low energies to local supersymmetric theories. In addition the local version offers a very plausible explanation for the origin of the scale of supersymmetry breaking consistent with the requirements of a solution to the hierarchy problem.

3.3.3 The hidden sector

In SUSY models it is normally assumed that supersymmetry is broken in a sector of the theory, the so-called 'hidden' sector, which has only gravitational interactions with the states of the Standard Model. Such a structure emerges quite naturally in superstring theories. For example, the original heterotic string generates an $E_8 \otimes E_6$ structure in which the E_8 plays the role of the 'hidden sector'. In general 4D string models the E_8 group is broken at the compactification scale to some subgroup which plays the role of the hidden sector. Apart from triggering supersymmetry breaking, the hidden sector plays no role in low-energy phenomenology, for the states of this sector are confined with mass of the order of the gaugino condensate scale ($\approx 10^{13}$ GeV, see below).

The most plausible origin for supersymmetry breaking in the hidden sector follows because the gauge interactions of the hidden sector are likely to be asymptotically free and become strong at the condensation scale Λ_c, below the compactification scale. Then, in analogy with what happens in QCD, it is likely that a fermion condensate, $< \lambda\lambda >$, forms, the fermions, λ, being the gauginos of the hidden sector [41, 42]. In local supersymmetry (but not in global supersymmetry) such a condensate breaks supersymmetry. The characteristic measure of this breaking is the mass, $m_{3/2}$, that it generates for the (spin- 3/2) gravitino, supersymmetric partner of the graviton and is given by $m_{\frac{3}{2}} \propto \frac{<\lambda\lambda>}{M_P^2}$. Since the scale determining the gaugino condensate is given by the scale, Λ_c, at which the gaugino binding becomes non-perturbative, we have $m_{\frac{3}{2}} \propto \frac{\Lambda_c^3}{M_P^2} = M_P \exp(-\frac{3}{2b_0 g^2})$, where b_0 is the coefficient of the one-loop β-function and the last equality follows from using the running of the gauge coupling from its value, g, at the Planck scale [43]–[45]. Thus the condensation scale can be much less than the Planck scale and hence the gravitino mass may easily be hierarchically smaller than the Planck scale.

This is the best explanation in supersymmetric models for the magnitude of the hierarchy. As we shall see, the presence of supersymmetry then guarantees that the supersymmetry-breaking effects in the visible sector are also small, of the order of the gravitino mass. It may be seen that the supersymmetric solution to the hierarchy problem has copied the essential feature of technicolour theories, namely symmetry breaking via a condensate. However, as we shall also discuss, in contrast with the extended technicolour theories, the existence of elementary scalar states in supersymmetry subsequently allows for a straightforward pattern of symmetry breaking in the visible sector and generation of quark and lepton masses.

3.3.4 The visible sector

Since supersymmetry breaking occurs in the hidden sector it can only be communicated to the visible sector via gravitational, flavour-blind interactions suggesting that there will be generated a common mass, $m_{1/2}$, for the gauginos of the Standard Model and another common mass, m_0, for the scalars (the expectation is that these are both of order $m_{3/2}$)[5]. In addition there must be a new term $\mu H_1 H_2$ in the superpotential giving a common Higgsino, Higgs mass at the unification scale. It is also found in specific supergravity models [48] that there are additional supersymmetry-breaking terms given by $(A_0 P_3 + B_0 P_2)$ where A_0 and B_0 are masses of order $m_{3/2}$, and P_3 and P_2 are the

5) Recently it has been observed that in string theories this universality of masses may be broken if the fields have different modular weights. Although possible, the necessity to avoid large flavour-changing neutral currents strongly constrains the amount of such non-universality and suggests that, in a viable model, flavour-blind masses at the unification scale is a good approximation [38].

trilinear and quadratic terms of the superpotential with the supermultiplets replaced by their scalar components. The expectation for A_0 and B_0 depends on the metric; in the case of the no-scale theories which descend from the string, A_0 vanishes at the unification scale. The gauge bosons and fermions of the Standard Model do not acquire mass at this stage owing to residual unbroken gauge and chiral symmetries.

The flavour independence of the supersymmetry-breaking terms is broken by radiative corrections involving the gauge and Yukawa couplings of the Standard Model. These corrections may be calculated explicitly and are most conveniently included via the renormalization group equations for the masses [47] and the A and B parameters using for initial values at M_X the common gaugino and scalar masses $m_{1/2}$ and m_0 and the common A_0 and B_0 parameters.

The degeneracy of the gauginos and scalars is broken by radiative corrections involving the gauge and Yukawa couplings of the Standard Model, and these may be calculated via the renormalization group equations for the masses [48] using for initial values at M_X the common gaugino and scalar masses $m_{1/2}$ and m_0. The only Yukawa coupling large enough to give a significant contribution to this evolution in the Standard Model is likely to be the one responsible for the top-quark mass and this is the only one kept in the subsequent analysis. In Fig. 11 the resultant spectrum for the superparticle masses is shown for a representative choice of the supersymmetry-breaking parameters [34]. It may be seen that those states with the larger gauge coupling are systematically heavier; the gluinos are heavier than the Winos and Bino, and the squarks are heavier than the sleptons.

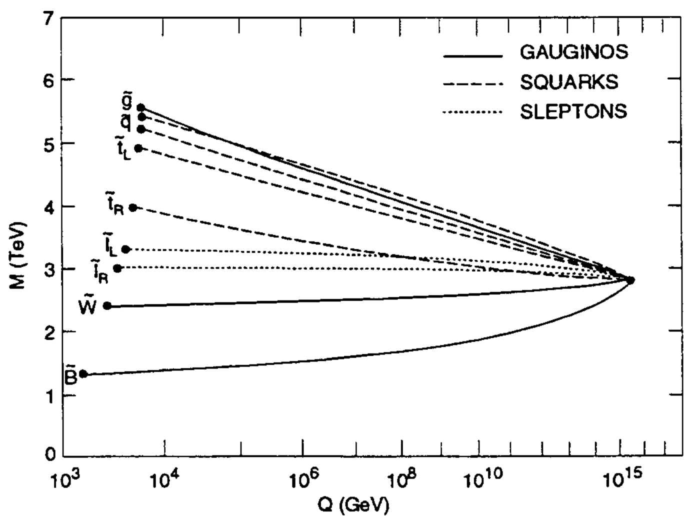

Figure 11: Running masses in the MSSM.

3.3.5 Unification of masses

We have shown that supersymmetry together with unification relations between gauge couplings gives a simple and entirely consistent picture. However, this is not all that can be said about supersymmetric unification. One of the main arguments for a low-energy supersymmetry was the need to explain the hierarchy of masses and in particular the difference between the Planck scale and the electroweak breaking scale. It is therefore reasonable to ask how the supersymmetric unification we have constructed explains this difference. Remarkably the theory contains a mechanism for symmetry breaking which automatically selects the correct breaking pattern, namely $SU(3) \otimes SU(2) \otimes U(1) \rightarrow SU(3) \otimes U(1)$. To see how this comes about we note that the potential describing the Higgs fields is largely determined by supersymmetry having the form [48]

$$V(H_1, H_2) = m_1^2 \mid H_1^0 \mid^2 + m_1^2 \mid H_2^0 \mid^2 + \mu B H_1^0 H_2^0 + \frac{\frac{3}{5}g_1^2 + g_2^2}{8}(\mid H_1^0 \mid^2 - \mid H_2^0 \mid^2)^2 \ . \quad (47)$$

Note that the coefficient of the quartic term is given in terms of gauge couplings by supersymmetry, whereas it is an unknown parameter in the Standard Model. The masses of the two Higgs fields are given by

$$\begin{aligned} m_1^2 &= m_{H_1}^2 + \mu^2 \\ m_2^2 &= m_{H_2}^2 + \mu^2 \end{aligned} \quad (48)$$

with

$$m_{H_1}^2(M_X) = m_{H_2}^2(M_X) = m_0^2 \ . \quad (49)$$

Using this we see that the Higgs mass terms are given via the RG equations in terms of the same mass parameters discussed above. The masses μ, m_1, m_2 evolve differently; for m_i we have [49]

$$\frac{dm_i^2}{d\log(Q)} = \frac{1}{8\pi^2}(-3M_2^2 g_2^2 - \frac{3}{5}M_1^2 g_1^2 + 3h^2\delta_{i2}(m_{\tilde{t}}^2 + m_{\tilde{t}^c}^2 + m_2^2 + A^2)) \quad (50)$$

where the M_i are the gaugino masses and the tilde denotes the supersymmetric state. Thus the difference between m_1, m_2 is due to the term involving the top Yukawa coupling, h. This term will drive m_2^2 negative if it is large enough, triggering electroweak breaking.

Now we can see why it is $SU(2)$ and not $SU(3)$ that is broken by this radiative breaking mechanism for, although the RG equations for both the Higgs scalars and the top (and bottom) squarks have destabilizing terms due to the top Yukawa coupling, only the squarks have large stabilizing terms due to QCD interactions [47]. Thus it is the Higgs $(mass)^2$ that is driven negative, and it is $SU(2)$ that is broken by the resultant Higgs vev.

From Eqs. (50) and (47) we see that the initial value of h may be chosen to give the correct value of M_Z. Thus to each point of the solution plane of Fig. 10 it is possible to assign a definite h (or equivalently m_t) needed to give the correct electroweak breaking scale. Using this the results of the analysis of electroweak breaking may be conveniently summarized by drawing contour plots of constant m_t in the $m_0, m_{1/2}$ [34]. These are shown by the dashed lines in Fig. 10 from which it may be seen that part of the previously allowed region is excluded by the requirement of acceptable radiative electroweak breaking coupled with the LEP bounds on the top mass. In Fig. 10 the origin of the variation with respect to m_0 is obvious, for larger m_0 requires larger h to drive m_2^2 negative at the correct scale. The same applies to increasing the value of μ. It may be seen from Fig. 10 that a heavy top quark leads to a very satisfactory explanation of both the existence of electroweak breaking and its magnitude for a wide range of supersymmetry breaking parameters.

This very pleasing self-consistency check of the MSSM shows how the electroweak breaking scale is generated from the unification scale, realizing the original aim in introducing supersymmetry. It also allows us to investigate the hierarchy problem for, from Eq. (50), one may see the Higgs mass squared, and hence the electroweak breaking scale is generated largely by the squark mass and has its natural value proportional to this mass. In this analysis this effect is manifested by the need to fine-tune, for large squark masses, the top quark Yukawa coupling to make the constant of proportionality small enough to get the correct electroweak breaking scale.

3.3.6 The hierarchy problem and the SUSY spectrum

The naturalness constraints may be made more precise by writing the relation between the W mass and the squark mass in a direct form. The renormalization group equations are just a convenient method for summing the large logarithms $\propto \log(M_x/M_W)$ that occur in calculating the radiative corrections to the Higgs mass. However, these radiative corrections relate on-shell masses and so it is clear that the solution to the renormalization group equations leads to relations relating on-shell quantities to on-shell quantities and free of uncertainties related to the definition of 'running' masses. The result of calculating these radiative corrections to the Higgs mass and finally expressing the result in terms of the associated W mass is given in the equation [34]

$$M_W^2 \approx \frac{3}{8\pi^2} h_t^2 (m_{\tilde{t}}^2 + m_{\tilde{t}c}^2) \log\left(\frac{M_X^2}{m_{\tilde{t}}^2}\right) \left[1 - \frac{8\pi^2}{6h_t^2 \log(\frac{M_X^2}{m_{\tilde{t}}^2})}\right] . \tag{51}$$

No symmetry makes the term in square brackets small for, as may be seen, it involves supersymmetry-*breaking* masses. Thus the expectation is that it is of order 1 and taking it to be much smaller corresponds to a fine-tuning. To quantify this we may rewrite Eq. (51) in the form

$$M_W^2 = c\left[1 - \frac{8\pi^2}{6h_t^2 \log(\frac{M_X^2}{m_{\tilde{t}}^2})}\right] M_W^2 \tag{52}$$

where

$$c = \frac{3h_t^2(m_{\tilde{t}}^2 + m_{\tilde{t}c}^2)}{8\pi^2 M_W^2} \log\left(\frac{M_X^2}{m_{\tilde{t}}^2}\right) . \tag{53}$$

In Ref. [50] 'reasonable' scales for the supersymmetry thresholds were estimated by demanding that the sensitivity of the electroweak breaking scale to any of the parameters of the Standard Model should be less than some value. In Eq. (52) the sensitivity of M_W to the dominant supersymmetry breaking mass, the top squark mass, is determined by the quantity c and the equivalent statement is that $1/c$ should be less than some value.

No fine-tuning would correspond to $c \approx 1$ but, more conservatively, one might use the value of $c = 10$ that was chosen by Barbieri and Giudice [50] as a measure of a reasonable theory. The requirement that the electroweak breaking scale be generated from the unification scale introduces more sensitivity to the top Yukawa coupling than is found in the MSSM without unification, as may be seen from the appearance of the large logarithm in Eq. (51). As a result, constraining $c \leq 10$ dramatically reduces the allowed region of parameter space. Note that the fine-tuning constraint is largely independent of the details of the unification at M_X and may be expected to give similar limits on the SUSY spectrum in any unification scheme having a very large M_X.

3.3.7 m_b/m_τ

We have seen how the simplest scheme of supersymmetric unification gives excellent agreement with the measured gauge couplings and, for a top mass within the experimentally allowed range, with the W mass. It is natural to ask what are the supersymmetric unification expectations for fermion masses?

In specific GUTs there are relations between fermion masses, the best-known being the $SU(5)$ relation $m_b = m_\tau$ [51]. As with the relation amongst gauge couplings, this relation applies at the unification scale and must be radiatively corrected. These corrections offer a further tantalizing piece of circumstantial evidence in favour of supersymmetric unification for, starting with the relation $m_b(M_X) = m_\tau(M_X)$, they can bring the prediction for m_b/m_τ into agreement with experiment using the same value for M_X found in the analysis of gauge couplings. This cannot be done in the non-supersymmetric case without adding new interactions [52]. Agreement in the supersymmetric case is possible only for a restricted range of parameters, further tying down the range of supersymmetric masses. In particular a running b quark mass, $m_b(m_b) = 4.2$, can only result if the top Yukawa coupling is large, $h_t \approx 1.25$. For $h_t > h_b$ the top quark is heavy, $m_t = 170 \pm 10$ GeV, and the μ parameter is also large, $\mu \approx 2m_0$. The situation is shown in Fig. 10.

3.3.8 Dark matter

In the MSSM the interactions respect an R-parity under which the new SUSY states are odd while the states of the Standard Model are even. As a result the SUSY states may only be pair-produced and the lightest supersymmetric state (LSP) is stable. The existence of the LSP is a prediction of the MSSM and, since its couplings are determined, the relic abundance of the LSP is also a prediction. The nature of the LSP and its abundance is determined in terms of the SUSY-breaking parameters introduced above and as we have seen there is only a small region allowed in this parameter space. Thus the range of possible relic abundance is strongly constrained and it is clearly of interest to see how this compares to the cosmological bounds on dark matter. This is shown in Fig. 10, where the contours of fixed relic abundance are drawn on the m_0, $m_{1/2}$ plane [53, 36]. We see that almost all of the allowed range has relic abundance less than the closure density and is therefore not ruled out. Perhaps more interestingly, over a substantial part of the allowed region, the LSP abundance is large enough to be the candidate for dark matter! The sceptic may view this as a coincidence but, to me, the remarkable overlap between these abundances provides further circumstantial evidence in favour of the need for supersymmetry.

3.4 String unification

In string theories the constraints of Unification are much more severe than in GUTs. A given four-dimensional string theory has a definite multiplet structure and, moreover, the unification scale, M_{SU}, is *known* in terms of the Planck scale. The one-loop running gauge coupling is given by [54],[39],[37]

$$\alpha_i^{-1} = k_i\alpha_{SU}^{-1} + \beta_{0i}\ln(\frac{M_{SU}^2}{\mu^2}) + \Delta_i \ . \tag{54}$$

Here k_i are the Kac-Moody levels of the $U(1)$, $SU(2)$ and $SU(3)$ factors and Δ_i are threshold corrections from loops involving massive (Kaluza–Klein) modes with mass of the order of the compactification scale. The tree-level gauge couplings are related by

$$g_1^2 k_1 = g_2^2 k_2 = g_3^2 k_3 = \frac{4\pi}{\alpha'} G_{Newton} \equiv g_{SU}^2 \tag{55}$$

and α' is the inverse of the string tension squared. The values $k_3 = k_2 = (3/5)k_1$ give the standard SU(5)-like predictions for the couplings [$C = 1$ in Eqs. (45) and (46)]. They arise in many 4D string models, both with Grand Unified gauge groups and in models which do not have a stage of Grand Unification. The string unification scale in the $\overline{MS}$ scheme is given by $M_{SU} = 0.7\, g_{SU}\, 10^{18}$ GeV. The simplest realization of minimal string unification is for the gauge group after compactification to be just $SU(3) \otimes SU(2) \otimes U(1)$ with the minimal particle content, i.e. there is no need for any additional heavy states. While no example of such a string theory has yet been constructed, it seems likely that something quite close to it can be found. However, the result found in the Section 3.3.1, $M_X \approx 3.10^{16}$ GeV, is difficult to satisfy, for a survey [37, 38] of all orbifold models shows that typically M_X is *increased* by the string threshold corrections Δ_i from the string unification scale M_{SU}. Only for very special assignments of the matter fields to (twisted) multiplets can the unification scale be reduced.

What then are the implications of coupling unification for string theories? The first possibility is that the threshold corrections *do* conspire to lower M_X. At least the existence of an orbifold example shows this is possible [37]. A second possibility is that the string gives a gauge group larger than the Standard Model which preserves the ratios of gauge couplings and which subsequently breaks at an intermediate scale $M_X \approx 10^{16}$ GeV to the Standard Model. This group need not be Grand Unified. My favourite example is the group $SU(3)^3$ which is known to occur in some promising compactification schemes. Provided there are equal numbers of quark and lepton multiplets the couplings renormalize in the same way between M_{SU} and M_X as required. Another possibility is to give up the minimal unification assumption and to add states beyond the minimal set. For example in flipped $SU(5)$ it is necessary to evolve the $SU(5) \otimes U(1)$ couplings from M_{SU} to M_X before using the $SU(3) \otimes SU(2) \otimes U(1)$ evolution. Unfortunately this spoils the agreement with the low-energy couplings in the original version of the theory, but in non-minimal versions the effect of states with masses of order 3.10^{13} GeV can lead to acceptable results [39]. This example nicely illustrates both the additional predictive power of the string theory and the uncertainties introduced once non-minimal theories are introduced.

To summarize, the success of minimal unification predictions with $M_X \approx 3.10^{16}$ GeV can be understood within the framework of specific string models. However, since these models do not correspond to minimal string unification some of the aesthetic appeal of simplicity is lost.

4. SUMMARY

Recent attempts to go beyond the Standard Model have concentrated on solving the hierarchy problem. Composite models offer the prospect of solution but specific examples fail to provide a convincing picture. Extending the symmetry of the Standard Model to include supersymmetry has been shown capable of protecting the hierarchy of masses. Substantial circumstantial support for this picture comes from the successful predictions of the gauge couplings at low scales, given that they are unified at high scales. Furthermore, the pattern and magnitude of electroweak breaking is predicted if there is some unification of the SUSY-breaking masses. Also the quark and lepton masses, and the relic abundance of the lightest supersymmetric state are in good agreement with the idea of SUSY unification. Although this is only circumstantial evidence, it does require a very low mass scale for the new SUSY states, less than a TeV, which means that these states should be accessible at LHC or SSC energies.

The phenomenology of the new SUSY states is largely dictated by their gauge couplings, which are uniquely specified. However, there is an ambiguity in defining the Yukawa and related scalar couplings and this gives rise to two classes of SUSY phenomenology. The first class has an R-symmetry which means the lightest SUSY state (the LSP) must be stable. The characteristic signal for SUSY relies on the missing energy signals for SUSY decay to the LSP. The second R-parity violating class has no stable LSP and so the signals of SUSY may change. Luckily, even in this case, the new SUSY states should be visible at the SSC or the LHC for masses in the range needed to solve the mass hierarchy problem. Thus these accelerators provide a means to directly check the tantalizing but circumstantial evidence for SUSY which has emerged from a study of supersymmetric unification.

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