SUPERFLUIDITY

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Abstract
The paper contains a brief and elementary introduction to the phenomenon of superfluidity in liquid helium-4. The experimentally observed properties of superfluid ^{4}He are described, and outline explanations are given in terms of the existence of a Bose condensate and a special form of excitation spectrum. The quantization of superfluid circulation is described, and the role played by free quantized vortex lines is explained.

1. INTRODUCTION

A fluid composed of electrically neutral particles that can, in certain circumstances, flow without friction is called a superfluid. There is a close analogy to superconductivity, which relates to frictionless flow of the electrons in certain metals at a low temperature. Two superfluids are known in terrestrial nature: liquid ^{4}He at a temperature below about 2 K; and liquid ^{3}He at a temperature below about 2 mK. The neutron fluid in a neutron star may also be a superfluid.

All superfluids owe their properties to some type of Bose condensation. In liquid ^{4}He, in which the atoms are themselves bosons, the Bose condensation is more or less straightforward; in liquid ^{3}He, in which the atoms are fermions, there is, loosely speaking, a Bose condensation of Cooper pairs, as in the electron fluid in a superconductor. However, superfluid ^{3}He is quite complicated, even in comparison with a conventional superconductor, because the Cooper pairs are formed in states with non-zero angular momentum and non-zero total nuclear spin. In this paper we shall concentrate on the simplest superfluid, formed from liquid ^{4}He. Liquid ^{4}He, in either its normal phase or its superfluid phase, is used as a coolant in practical applications: most commonly in superconducting magnets and SQUID systems.

It is not practicable to include in this paper a complete set of references. Much of the material is covered in reference [1], although sometimes from a slightly different point of view. A few other references are included, especially where the material is not covered in reference [1].

2. THE THERMODYNAMIC PROPERTIES OF LIQUID ^{4}He

The phase diagram is shown in Fig. 1. There is no conventional triple point, and no solid phase exists at pressures below about 25 atm. There are two liquid phases, helium I and helium II, separated by the "λ-line". Helium I is a conventional (normal) liquid; helium II is a superfluid.

The specific heat along the vapour pressure line is shown in Fig. 2. There is a sharp peak at the λ-line, of a type found in other systems at a temperature that marks the onset of some type of ordering process: for example, at the Curie point of a ferromagnetic.

3. THE ELEMENTARY SUPERFLUID PROPERTIES OF HELIUM II

Superfluid helium will flow without friction even through a very narrow channel, provided that the flow velocity is less than some critical value; it will flow easily even in anadsorbed film. The critical velocity depends on channel width (increasing with decreasing

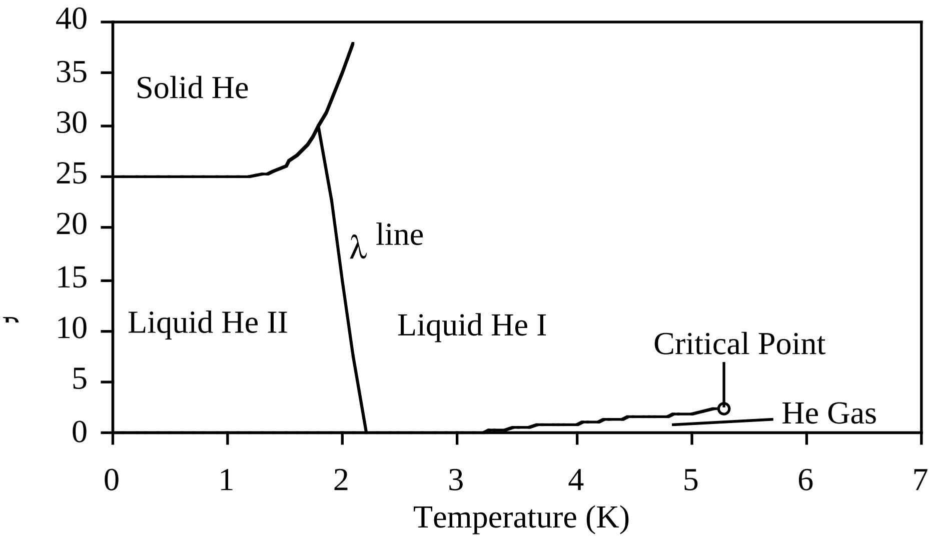

Fig. 1 The phase diagram for ^{4}He

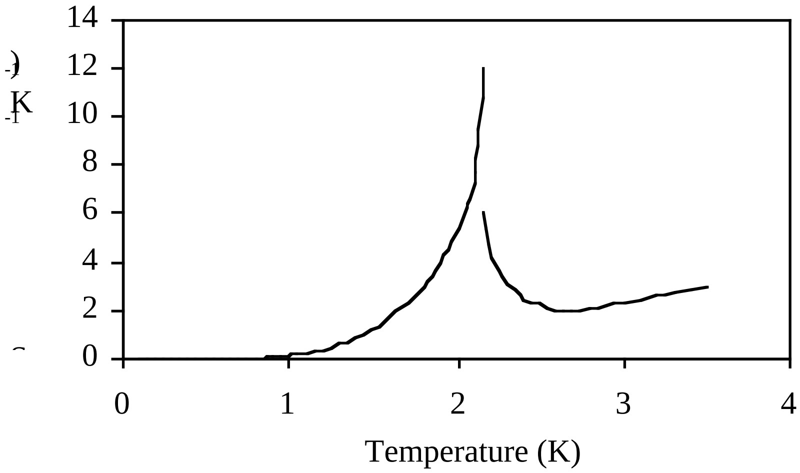

Fig. 2 The heat capacity of ^{4}He

width), and is typically a few cm per second. This frictionless flow suggests that the helium can behave as though it has no viscosity. However, an observation of the damping in the liquid of a disc oscillating at small amplitude in its own plane leads to a conventional viscosity equal in order of magnitude to that of Helium I. The behaviour appears to be somewhat irrational.

The thermal conductivity of helium I is very low. However, that of superfluid helium is high and unconventional: the heat flow is generally proportional to neither the cross-sectional area nor the temperature gradient. The heat conductivity is very high indeed at low heat currents, but the non-linearity means that it can be much reduced at high heat currents. Detailed investigation shows that heat conduction in superfluid helium obeys a wave equation ($\nabla^2 T = \left(1/c_2^2\right)\partial^2 T/\partial t^2$) rather than the usual diffusion equation ($K\nabla^2 T = \rho C \partial T/\partial t$); thus temperature fluctuations do not diffuse, but instead they propagate as a wave (called *second sound*).

When superfluid helium flows through a narrow channel, it emerges at a lower temperature: the *mechanocaloric effect*. Closely related to this effect is a large thermomechanical effect; a temperature gradient along a narrow channel containing superfluid helium leads to a large pressure gradient.

4. HELIUM AS A QUANTUM LIQUID: BOSE CONDENSATION

The fact that helium does not form a solid even at T=0, except at high pressure, is clearly indicative of quantum effects. Each atom in the liquid is confined by its neighbours; this confinement leads to a large zero-point energy, which increases with increasing liquid density; the zero-point energy therefore tends to keep the atoms apart to an extent that prevents solidification, the attractive forces between the helium atoms being quite weak.

The third law of thermodynamics requires that the entropy of the helium, even in its liquid state, must tend to zero as the temperature tends to zero. Superfluid helium at T=0 must therefore be an *ordered liquid*. We have noted that the peak in the heat capacity at the λ-line is characteristic of an order-disorder transition. It must surely follow that superfluidity is associated with this ordering process.

What type of ordering is possible in a liquid? To obtain a useful hint let us look at the predicted properties of a hypothetical ideal gas at very low temperatures (hypothetical because all real gases liquefy at a low temperature). The properties depend on the statistics of the particles: ^{4}He atoms are bosons, so we look at the properties of an ideal Bose gas. What we find predicted is *Bose condensation*. Below a certain critical temperature a finite fraction of the particles occupy the lowest quantum state in the vessel containing the gas, this fraction increasing to unity at $T \to 0$. The ordering at T=0 takes the form of putting all the particles in the same quantum state.

Can a similar effect occur in a liquid? And if so, is it responsible for superfluidity?

5. THE TWO-FLUID MODEL

Before we answer these questions we shall introduce a phenomenological model that describes the observed properties of superfluid helium, even when they appear to be somewhat irrational.

According to the two-fluid model superfluid helium consists of two interpenetrating fluids: a *normal component*, density ρ_n, behaving like a conventional viscous fluid and carrying all the entropy of the system; and a *superfluid component*, density ρ_s, behaving like an ideal classical inviscid liquid and carrying no entropy. In the simplest cases the two fluids can move relative to one another without frictional interaction. The proportion of superfluid, ρ_s/ρ, decreases from unity at T=0 to zero at the λ-transition (Fig. 3).

In terms of the two-fluid model heat flow in superfluid helium takes place by counterflow of the two fluids; as long as the relative velocity of flow is less than a critical value, which we shall discuss later, the only dissipative process opposing this counterflow arises from the viscosity of the normal fluid and is very weak. The apparently conflicting values of the viscosity obtained from flow through a narrow channel and from the damping of an oscillating disc are easily understood: flow through the channel involves only the superfluid component; the oscillating disc is damped by the normal fluid. Two types of wave motion are possible in the system: one in which the two fluids oscillate in phase (ordinary or *first sound*); and one in which they oscillate in antiphase (*second sound* or temperature waves). A measurement of the speed of second sound provides a good way of obtaining from experiment the normal and superfluid densities as functions of temperature (Fig. 3). They can also be obtained from the *Andronikashvili experiment,* in which a measurement is made of the period of oscillation of a pile of closely spaced discs suspended by a torsion fibre in the

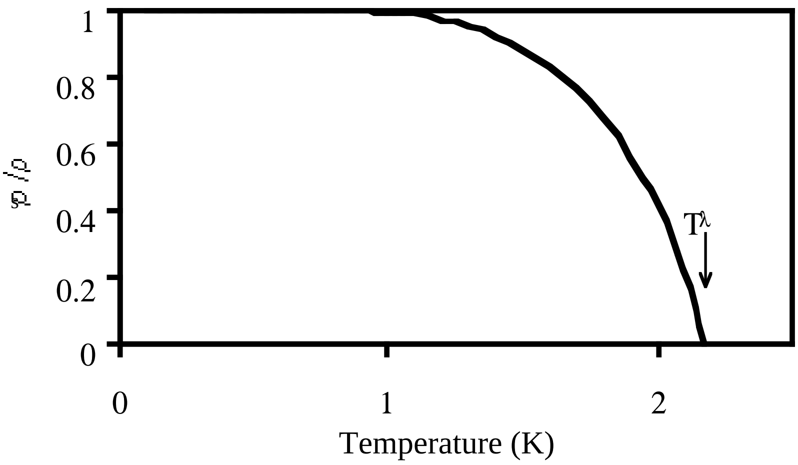

Fig. 3 The density of the superfluid component plotted against temperature

liquid. Only the normal fluid is dragged with the discs, so only the normal fluid contributes to the moment of inertia of the disc system. Explanations of the thermomechanical and mechanocaloric effects are fairly obvious and are left to the reader to consider.

When the two-fluid model was first proposed it was thought that it might be associated with Bose condensation, the normal fluid component being identified with the excited atoms and the superfluid component with the condensed atoms. We know now that this view is an oversimplification, as we shall see in the next section.

6. BOSE CONDENSATION IN LIQUID HELIUM

That a form of Bose condensation does indeed occur in liquid helium below the λ-transition was probably first demonstrated theoretically by Penrose & Onsager [2] (perhaps also by Bogolubov [3]), and confirmation has been provided by much subsequent theoretical study. Direct experimental evidence has been hard to find, but reasonably convincing evidence has now come from experiments on deep inelastic neutron scattering, which measures in principle the momentum distribution of the bare helium atoms.

However, Bose condensation in a liquid differs from that in the ideal gas in that the fraction of atoms in the lowest momentum state (which can be formally defined in terms of the single-particle density matrix) is much smaller and, in liquid helium at low pressures, reaches a value of only about 0.1 even at T=0. This means that this *condensate fraction* cannot be identified with the superfluid fraction ρ_s/ρ. We shall return to the question of what determines the superfluid fraction a little later.

6.1 Bose condensation and superfluidity

Let us imagine that the superfluid phase of liquid helium is contained in a long tube, and that we suddenly set the whole liquid into motion along the tube with velocity **V** by means of an impulsive pressure gradient. Since the whole liquid is set in motion the condensate must also be moving with the same velocity, so that the wave function of the state into which condensation has occurred takes the form

$$\Psi = \Psi_0 \exp(i\mathbf{k}\cdot\mathbf{r}), \tag{1}$$

where $\hbar\mathbf{k} = m_4\mathbf{V}$ and m_4 is the mass of a helium atom.

The condensate contains a macroscopic number of atoms in a single quantum state (this is still true even when the condensate *fraction* is relatively small). We can therefore regard Ψ, not as a single particle wave function, but rather as a *coherent particle wave* (the *condensate wave function*), analogous to the coherent electromagnetic wave produced by a laser.

As the liquid helium moves along the tube the helium atoms interact with the walls of the tube; scattering of atoms will occur, and some atoms will be scattered out of the condensate. This scattering out of the condensate will reduce the *amplitude* of condensate wave function, but it may not change the phase. Let us suppose that the phase is indeed maintained. The scattering will lead to equilibration of the helium with the walls, but the equilibration takes place in such a way that the phase of the condensate wave function is constrained to remain unchanged. This is a case of *broken symmetry*, and can be compared with the situation in a ferromagnetic material in which the overall magnetization is constrained to point in a certain direction, although the individual atomic magnetic moments are free to point in, for example, either of two opposing directions. Given the broken symmetry the equilibrium state of the helium may involve a non-zero mass current density, which is likely to be of the form

$$\mathbf{J}_s = \rho_s \mathbf{V}, \tag{2}$$

at low velocities. We have therefore described a persistent (frictionless) flow of the *superfluid component*, density ρ_s, moving with velocity $\mathbf{v}_s = \mathbf{V}$.

We emphasize that the equilibrium associated with the broken symmetry can be only metastable; the state of the helium with no supercurrent density has a lower energy. The metastable state must have a finite lifetime. The fact that superflow in helium can take place with no observable friction shows that this lifetime can be very long. Later we shall discuss what determines this lifetime, a discussion that will make clearer why the phase of the condensate wave function tends to be constrained to remain unchanged.

The broken symmetry represents an ordering in the liquid helium, associated with the Bose condensation. The condensate wave function Ψ can be regarded as the *order parameter* in the superfluid phase. It is analogous to the Ginsburg-Landau order parameter in superconductivity.

Although we have developed a satisfying description of superfluidity, it tells us nothing about the magnitude of the superfluid density ρ_s. Indeed we have so far presented no argument for supposing that ρ_s is non-vanishing at temperatures below the Bose condensation temperature.

7. THE NORMAL FLUID FRACTION: PHONONS AND ROTONS

Curiously enough some understanding of what determines the densities ρ_s and ρ_n was provided by Landau [4, 5] before the concepts of Section 6 had been developed. Landau introduced the idea that the low-lying excited states of helium (those that can be thermally excited at low temperatures) can be described in terms of a low-density gas of weakly interacting excitations. Nowadays we are familiar with this type of idea in connection with crystalline solids, where the excited states can be described in terms of quantized normal modes (lattice vibrations), which we call *phonons*. Landau suggested that the lowest-lying excitations in liquid helium are also phonons (longitudinal only), but that there are other excitations at somewhat higher energy, which he called *rotons*. We now know that the phonons and rotons are not clearly distinguishable, and that they form a continuous spectrum as shown in Fig. 4.

Direct experimental evidence for this form of the excitation spectrum has been provided by inelastic neutron scattering.

The thermally excited states of helium can be described in terms of a weakly interacting gas of phonons and rotons only as long as the density of these excitations is small. This means that the temperature must not be too large; in practice less than about 1.8 K. At higher temperatures the situation becomes more complicated.

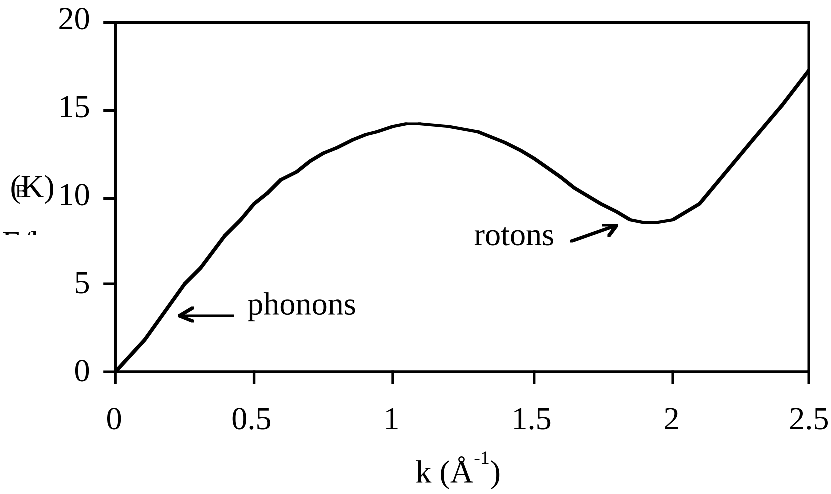

Fig. 4 The excitation spectrum in superfluid ^{4}He

We shall now argue that, at the low temperatures, the normal fluid can be identified with the gas of excitations, and that we can use this picture to calculate the normal and superfluid fractions.

Let us think again of the liquid helium as having been suddenly set in motion along a tube with velocity **V**. The equilibration process (subject to the constraint of an unchanged phase of the order parameter), which we described in general terms earlier, can now be seen to be the process in which the excitation gas comes into equilibrium with the walls of the channel, the background fluid continuing to move with the velocity **V**. Since the excitations are effectively an ideal gas, the equilibration is easily described; one has simply to calculate the distribution function for the excitations when they are in equilibrium with the channel walls, in the presence of the background velocity **V**. Having obtained the excitation distribution function, we can easily use it to calculate the momentum density, $\mathbf{P}_{ext}$, associated with the excitations. To obtain the new total momentum density in the liquid we must add to $\mathbf{P}_{ext}$ the original momentum density, $\rho\mathbf{V}$, giving

$$\mathbf{P}_s = \mathbf{P}_{ext} + \rho\mathbf{V}. \qquad (3)$$

For a sufficiently low temperature we find that $\mathbf{P}_s > 0$, $\mathbf{P}_s$ being proportional to **V** for small **V**, and we identify this residual momentum density with that of the moving superfluid

$$\mathbf{P}_s = \rho_s\mathbf{V}. \qquad (4)$$

Hence we have shown that ρ_s is non-zero, and we have a method for calculating its value at low temperatures. The results are in very good agreement with experiment, when use is made of the detailed form of the excitation spectrum obtained from inelastic neutron scattering.

7.1 Superfluidity and the form of the excitation spectrum

It must be emphasized that ρ_s is found to be non-zero only if the excitation spectrum has the appropriate form. The existence of only phonons at the lowest energies is a *sufficient* condition. Otherwise the liquid is not a superfluid, even though there may be a Bose condensate. The ideal Bose gas provides an example of a system for which the excitation spectrum does *not* have the right form and it is not a superfluid.

It seems therefore that superfluidity requires both a condensate and the appropriate form of excitation spectrum. Particle interactions are required in order to produce the required form of excitation spectrum.

Space does not allow us to discuss exactly why the excitation spectrum has the form observed. Much theoretical work has been devoted to this question; perhaps the most elegant discussion was provided by Feynman [6].

8. MACROSCOPIC QUANTUM EFFECTS

Superfluidity is clearly a quantum phenomenon. But most properties of condensed matter depend ultimately on quantum effects. What is unique to a superfluid is the appearance of quantum effects on a macroscopic scale, as we now describe. These quantum effects are related to the *quantization of superfluid circulation*, which arises from the long-range phase coherence in the condensate wave function.

8.1 Rotation of the superfluid, and the quantization of superfluid circulation

As is clear from Eq. (1), the velocity, $\mathbf{v}_s$, of the superfluid component is linked to the phase, *S*, of the condensate wave function. In general

$$\mathbf{v}_s = \frac{\hbar}{m_4}\nabla S \,. \tag{5}$$

It follows that $\mathrm{curl}\,\mathbf{v}_s = 0$, which suggests that the superfluid cannot rotate. (We note in passing that this irrotation condition is the analogue of the London equation in superconductivity.)

The irrotation condition forbids rotation only in a simply-connected volume. In a multiply-connected volume it allows a non-zero circulation round any circuit that cannot be continuously deformed into a loop of zero size while remaining in the liquid. The circulation in the superfluid component is defined as

$$\kappa = \oint \mathbf{v}_s \cdot d\mathbf{r} \,, \tag{6}$$

where the integral is taken round the circuit concerned (Fig. 5). However, in a superfluid the circulation cannot take any value. The condensate wave function must presumably be single-valued, and it is easy to see that this leads to the condition

$$\kappa = n\frac{h}{m_4}, \qquad (7)$$

where n is an integer. The quantum of circulation, h/m_4, is macroscopically large (roughly 10^{-7} m^2s^{-1}). The condition (7) has been verified experimentally by measuring the circulation round a stretched wire running through the liquid; the Magnus (lift) force that arises from the circulation modifies the modes of transverse vibration of the wire in a way that can be easily observed and measured. The quantization of circulation in a superfluid is analogous to the quantization of flux in a superconductor.

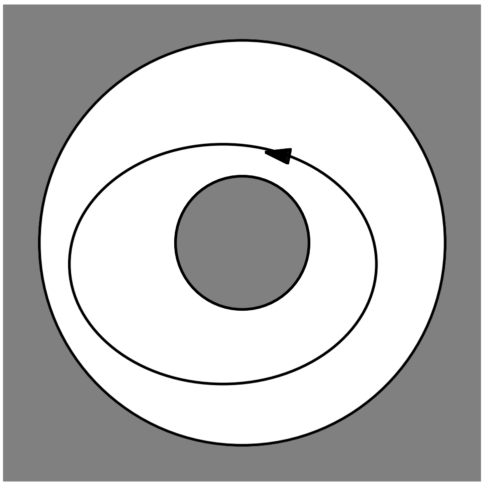

Fig. 5 A path round a torus along which the circulation does not necessarily vanish

8.2 Free quantized vortex lines

So far we have implicitly assumed that a volume of helium can be multiply-connected only if it is contained in a vessel with suitable geometry. However, a volume of liquid can also become multiply-connected if a small hole runs through it. We can establish a single quantum of superfluid circulation round this hole, and we have created a free quantized vortex line. Left to itself the hole acquires a size comparable with the interatomic spacing, so the circulation exists round what is virtually a line through the liquid. This type of quantized vortex line is the analogue of a flux line in a type II superconductor.

If the superfluid phase of liquid helium is situated in a rotating vessel of reasonable size, it is observed that the whole liquid appears to rotate with the vessel; the observed shape of the meniscus of the liquid indicates this quite clearly. The irrotation condition implies that the superfluid cannot be rotating like an ordinary classical liquid. What happens is that the superfluid becomes filled with an array of quantized vortex lines, all parallel to the axis of rotation (Fig. 6). For an angular velocity of 1 rad s^{-1}, the spacing between the lines is about 0.2 mm, and the spatially averaged flow velocity in the superfluid is then closely similar to that associated with solid-body rotation. The discrete vortex structure will give rise to dimples on the free liquid surface, but they turn out to be too small to see.

The presence of the free vortex lines changes some of the properties of the superfluid phase. The excitations that constitute the normal fluid are scattered by the cores of the vortices, and therefore any relative motion of the superfluid and the normal fluid results in a frictional force between them: a force of *mutual friction*. An observation and study of this force in uniformly rotating liquid helium provided the first experimental evidence for the existence of quantized vortex lines and indeed for the quantization condition on the circulation.

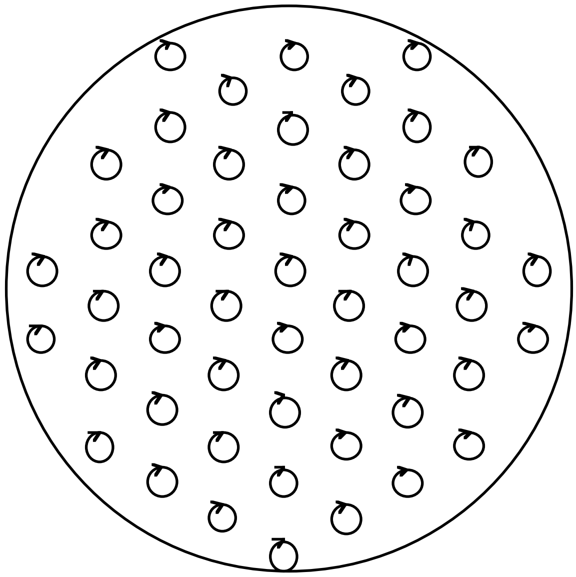

Fig. 6 Vortices in uniformly rotating superfluid helium

8.3 The breakdown of ideal frictionless superflow

As we mentioned earlier, frictionless superflow is observed only at velocities that are smaller than some critical value, which is typically of order 1 cm s^{-1}. Flow at supercritical velocities is found to be accompanied by a mutual friction between the two fluids, similar to that found in the uniformly rotating liquid. Detailed study showed that this "supercritical mutual friction" was due to the generation within the superfluid component of an irregular array of quantized vortices. The process is the quantum analogue of the production of turbulence in a classical fluid. An understanding of this quantum turbulence is necessary for a full understanding of heat conduction in superfluid helium, and it may therefore be important if superfluid helium is to be used as a cooling agent.

In connection with the use of helium as a cooling agent it should be added that, when heat flows from a solid body into liquid helium, there is a large thermal resistance (of order 2 K W^{-1} cm^2) associated with the boundary (the *Kapitza resistance)*. Heat flow through the boundary involves the transfer of energy between the excitations in the liquid and those in the solid, and the Kapitza resistance arises from a kind of acoustic mismatch across the boundary.

8.4 The metastability of superflow

We emphasized in Section 7 that ideal frictionless superflow can be only metastable, even at low velocities. If, for example, superflow is established round a toroidal channel, the supercurrent cannot persist for ever; it must ultimately decay, although the time associated with this decay may be very long.

We have seen in the preceding section that the generation of quantized vortices gives rise to a frictional force on the superfluid, and it must be the generation of such lines that causes the ultimate decay of a supercurrent. Let us look at the fundamental nature of this process in more detail.

Consider superflow round the toroidal channel shown in Fig. 7. The velocity of flow, $\mathbf{v}_s$, must be such that an integral number, *N*, of quanta of circulation are associated with the hole in the centre of the toroid. Decay of the supercurrent will involve the loss of one or more of these quanta. Such a decay can occur only if a free vortex is created in the helium and then moves across the channel. Such a process changes the phase of the order parameter within the helium, and it is often referred to as phase slip. In practice the phase cannot change in any other way.

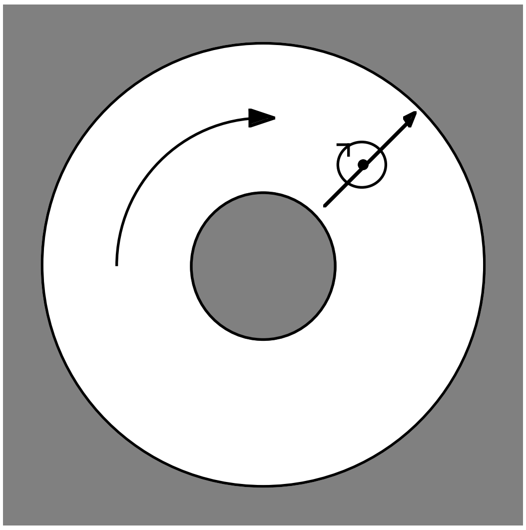

Fig. 7 Decay of a persistent supercurrent by vortex motion

It is easily seen that the creation and movement of the vortex across the channel is opposed by a potential barrier. This barrier has its origin in an attraction that exists between a vortex and a neighbouring wall (or equivalently between a vortex and its image in the wall). *The metastability of the superflow depends on the existence of this barrier*; the presence of a barrier of this type is characteristic of all metastable states. The lifetime of the state is determined by the rate at which the barrier can be surmounted.

The barrier height for the vortex motion in various geometries has been calculated. Its height depends on the velocity $\mathbf{v}_s$; for small velocities, of order 1 cm s^{-1}, and for temperatures not too close to the λ-line, the barrier turns out to be so high that the lifetime ought to be astronomical. In reality the lifetime is often much shorter; frictionless superflow seems to break down much more easily than is predicted. As is now widely accepted, this easy breakdown is associated with the almost inevitable presence in any real volume of helium of a few free vortices that are probably pinned to protuberances on the walls of the containing vessel and are remnants of a large concentration of vortices that is known to be created as the helium is cooled through the λ-transition [7]. These remanent vortices act to nucleate new vortices, in much the same way as remanent dislocations act to generate new dislocations in a solid crystal and hence reduce its intrinsic strength. In the past few years experimental conditions have been found in which the effect of these remanent vortices has been minimized or eliminated, and the vortex nucleation barrier has then been shown to agree with that predicted from the image forces. The barrier can still be overcome at sufficiently high flow rates, and recent experimental and theoretical studies (see, for example, references [1, 8, 9]) have demonstrated the existence of processes involving either thermal excitation over the barrier or quantum tunnelling through it, depending on the temperature.

9. SUMMARY AND CONCLUSIONS

In this paper we have tried to describe some of the remarkable phenomena associated with superfluidity in liquid ^{4}He, and to give an indication of the way these phenomena can be understood in terms of effects associated primarily with a form of Bose condensation in the liquid. One might wonder why such a wealth of strange phenomena has not led to more practical applications.

ACKNOWLEDGEMENT

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