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## Scalar Glueball–Quarkonium Mixing and the Structure of the QCD Vacuum

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### Abstract

We use Ward identities of broken scale invariance to infer the amount of scalar glueball– $\bar{q}q$  meson mixing from the ratio of quark and gluon condensates in the QCD vacuum. Assuming dominance by a single scalar state, as suggested by a phase-shift analysis, we find a mixing angle  $\gamma \sim 36^\circ$ , corresponding to near-maximal mixing of the glueball and  $\bar{s}s$  components.

Many scalar mesons are predicted in non-perturbative QCD, including  $\bar{q}q$  bound states, glueballs [1],  $\bar{q}q\bar{q}q$  molecules, radial excitations, and hybrids. Experimentally, there have also been many reported sightings of scalar states, including the  $\sigma(400 - 700)$ ,  $f_0(980)$ ,  $f_0(1300)$ ,  $f_0(1500)$ ,  $f_0(1700)$ , ... [2]. The theoretical identifications of these states are still largely open, in particular the identification of the lightest scalar glueball. This quest is complicated by the expectation that the various different scalar states could mix with each other [3], sharing out any characteristic glueball signatures and polluting even the strongest candidates [4] with, e.g.,  $\bar{q}q$  features [5].

One approach to the scalar-meson problem offered by non-perturbative field theory is based on the consideration of Green functions of composite operators such as  $\bar{q}(x)q(x)$  or  $G_{\mu\nu}^a(x)G^{a\mu\nu}(x) \equiv G^2$ , constrained by the (approximate and/or anomalous) Ward identities of non-perturbative symmetries of QCD. The highly successful prototype for this approach has been chiral symmetry, and it has also often been applied to broken scale invariance [6, 7, 8, 9], with some success.

Unlike chiral symmetry, where the Ward identities are dominated by low-mass pseudoscalar mesons, there is no guarantee that the Ward identities of broken scale invariance should be dominated by any single scalar-meson state. Under these circumstances, the best formulation of the approach may be phenomenological, setting up a number of sum rules [10] based on the Ward identities of broken scale invariance [11], and substituting experimental data into them, with the aim of exploring empirically which collection of states may saturate them.

We have recently implemented such a programme for Green functions involving the trace of the anomalous pure QCD energy-momentum tensor  $\theta(x) \equiv \frac{\beta}{4\alpha_s} G_{\mu\nu}^a(x) G^{a\mu\nu}(x)$  [11], evaluating the sum rules with data on  $\pi\pi$  and  $\bar{K}K$  phase shifts and phenomenological parametrizations of observed scalar mesons [12, 13]. We have found empirically that the sum rules are probably dominated by the  $f_0(980)$  state, with contributions from the lighter  $\sigma$  and heavier  $f_0$  mesons each contributing around the 10% level.

Does this mean that one should identify the  $f_0(980)$  as the lightest scalar glueball? Certainly not until one has analyzed the pattern of mixing with  $\bar{q}q$  states, a complicated issue which we broach in this paper.

We study simple Ward identities for the two-point Green functions of  $\theta$  and  $\bar{q}q$  operators. Assuming that the latter are dominated by the  $f_0(980)$ , as in the  $\theta\theta$  case, we find that the mixing of this “glueball” with an  $\bar{q}q$  meson must be large. For a nominal choice of vacuum parameters:  $\langle 0|\bar{s}s|0\rangle = 0.8\langle 0|\bar{q}q|0\rangle$ , where  $\bar{q}q$  represents either  $\bar{u}u$  or  $\bar{d}d$ ,  $\langle 0|\bar{q}q|0\rangle = 0.016 \text{ GeV}^3$  and  $\langle 0|\frac{\beta}{4\alpha_s}G^2|0\rangle = 0.013 \text{ GeV}^4$ , we find near-maximal mixing between glueball and  $\bar{s}s$  components:  $\gamma \sim 36^\circ$ , so that the  $f_0(980)$  contains an almost equal mixture of glueball and  $\bar{s}s$  states.

We start by considering the low-energy theorem [10]

$$\lim_{q \rightarrow 0} i \int dx e^{iqx} \langle 0|T \left\{ \frac{\beta(\alpha_s)}{4\alpha_s} G^2(x), \mathcal{O}(0) \right\} |0\rangle = (-d)\langle \mathcal{O} \rangle + O(m_q), \quad (1)$$

where  $d$  is the canonical dimension of the operator  $\mathcal{O}$ ,  $\beta(\alpha_s)$  is the Gell-Mann–Low function:  $\beta(\alpha_s) \equiv -b\alpha_s^2/2\pi + O(\alpha_s^3)$ ,  $b = (11N_c - 2N_f)/3$ , and  $O(m_q)$  stands for the terms linear in light quark masses. Here and subsequently, we work only with renormalization-group invariant quantities.

Next we use a spectral representation for the theorem (1), assuming that  $\mathcal{O}$  is a scalar Hermitian operator:

$$\begin{aligned} & \langle 0|T \left\{ \frac{\beta(\alpha_s)}{4\alpha_s} G^2(x), \mathcal{O}(0) \right\} |0\rangle \\ &= \int d^4k (2\pi)^{-3} \frac{1}{\pi} A(k) e^{-ikx} \theta(x^0) + \int d^4k (2\pi)^{-3} \frac{1}{\pi} B(k) e^{ikx} \theta(-x^0), \end{aligned} \quad (2)$$

where

$$\frac{1}{\pi} A(k) \equiv (2\pi)^3 \sum_n \delta^4(p_n - k) \langle 0|\frac{\beta}{4\pi} G^2(0)|n\rangle \langle n|\mathcal{O}(0)|0\rangle, \quad (3)$$

$$\frac{1}{\pi} B(k) \equiv (2\pi)^3 \sum_n \delta^4(p_n - k) \langle 0|\mathcal{O}(0)|n\rangle \langle n|\frac{\beta}{4\pi} G^2(0)|0\rangle. \quad (4)$$

Specializing to the relevant case where  $\mathcal{O}(x)$  is scalar, it is clear that  $A(k)$  and  $B(k)$  are the scalar functions of the form,

$$A(k) \equiv A(k^2)\theta(k^0), \quad B(k) \equiv B(k^2)\theta(k^0), \quad (5)$$

where the support of the spectral condition requires the factors  $\theta(k^0)$ . Assuming also that  $\mathcal{O}$  is CP even, as in the cases of  $\theta$  and scalar  $\bar{q}q$  densities, we may use time-reversal invariance to infer that

$$A(k^2) = B(k^2) \equiv \rho_{\mathcal{O}}(k^2). \quad (6)$$

Similarly, it can be shown that  $A(k)$  is real.

Inserting the resulting spectral representation

$$\begin{aligned}
& \langle 0|T\{\frac{\beta(\alpha_s)}{4\alpha_s}G^2(x), \mathcal{O}(0)\}|0\rangle \\
&= \frac{1}{\pi} \int \frac{d^4k}{(2\pi)^3} \rho_{\mathcal{O}}(k^2) \theta(k^0) \{e^{-ikx} \theta(x^0) + e^{ikx} \theta(-x^0)\} \\
&= \frac{1}{\pi} \int_0^\infty dm^2 \rho_{\mathcal{O}}(m^2) \int \frac{d^4k}{(2\pi)^3} \delta(k^2 - m^2) \theta(k^0) \{e^{-ikx} \theta(x^0) + e^{ikx} \theta(-x^0)\} \\
&= \frac{1}{\pi} \int_0^\infty dm^2 \rho_{\mathcal{O}}(m^2) \Delta_F(x; m^2)
\end{aligned} \tag{7}$$

into the theorem (1), we find the simple relation

$$\frac{1}{\pi} \int_0^\infty \frac{dm^2}{m^2} \rho_{\mathcal{O}}(m^2) = (-d) \langle \mathcal{O} \rangle + O(m_q), \tag{8}$$

whose physical properties we now discuss in more detail.

In general, the intermediate states created by the operator  $\mathcal{O}$  may include multi-particle states as well as single-particle states. If one approximates the intermediate states by a sum over single-particle states, one finds that the matrix elements here are scalar and can depend only on  $k^2 = m_\sigma^2$  in this case. Then the three-momentum integral is trivially done:

$$\frac{1}{\pi} \rho_{\mathcal{O}}(k^2) \theta(k^0) = \sum_\sigma \delta(m_\sigma^2 - k^2) \theta(k^0) \langle 0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^2 | k; \sigma \rangle \langle k; \sigma | \mathcal{O} | 0 \rangle, \tag{9}$$

where the index  $\sigma$  specifies the species of scalar meson, and  $|k; \sigma\rangle$  stands for a state of momentum  $k$ . The theorem (1) therefore becomes

$$\sum_\sigma \frac{1}{m_\sigma^2} \langle 0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^2 | k; \sigma \rangle \langle k; \sigma | \mathcal{O} | 0 \rangle = (-d) \langle \mathcal{O} \rangle, \tag{10}$$

which may further be simplified to

$$\frac{1}{m_\sigma^2} \langle 0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^2 | k \rangle \langle k | \mathcal{O} | 0 \rangle = (-d) \langle \mathcal{O} \rangle \tag{11}$$

in the case of the single sharp resonance.

We now show how the relation (1) can be used to fix the mixing between the scalar quark–antiquark and glueball states. We first choose  $\mathcal{O}(x) = \sum_i m_i \bar{q}_i q_i(x)$ , for which the spectral representation (1) becomes <sup>1</sup>:

$$\frac{1}{\pi} \int ds \frac{\tilde{\rho}(s)}{s} \simeq -3 \langle \sum_i m_i \bar{q}_i q_i \rangle, \tag{12}$$

where

$$\frac{1}{\pi} \tilde{\rho}(s) = (2\pi)^3 \sum_n \delta^4(p_n - k) \langle 0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^2 | n \rangle \langle n | \sum_i m_i \bar{q}_i q_i | 0 \rangle, \tag{13}$$

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<sup>1</sup>We recall that the canonical quantum operator dimension of  $\bar{q}q$  is 3, whereas the renormalization-group invariant combination  $m\bar{q}q$  has mass scaling dimension 4 [10]. We ignore operator mixing between  $G^2$  and  $\bar{q}q$ , assuming a mass-independent renormalization scheme such as  $\overline{MS}$ .

These relations demonstrate that the spontaneous breaking of chiral symmetry, reflected in a non-zero value of the quark condensate  $\langle 0|\bar{q}q|0\rangle$  necessarily implies mixing between the “glueball” and “quarkonium” components of physical scalar resonances  $\sigma$ . In the case of the single sharp resonance  $\sigma$ , the theorem (13) implies that

$$\frac{1}{m_\sigma^2}\langle 0|\frac{\beta(\alpha_s)}{4\alpha_s}G^2|k\rangle\langle k|\sum_i m_i\bar{q}_i q_i|0\rangle = -3\langle\sum_i m_i\bar{q}_i q_i\rangle. \quad (14)$$

Choosing instead  $\mathcal{O}(x) = \frac{\beta(\alpha_s)}{4\alpha_s}G^2(x)$  leads to another sum rule [10]:

$$\frac{1}{\pi}\int ds \frac{\rho(s)}{s} \simeq (-4)\langle\frac{\beta(\alpha_s)}{4\alpha_s}G^2\rangle, \quad (15)$$

where

$$\frac{1}{\pi}\rho(k^2)\theta(k^0) = (2\pi)^3\sum_n\delta^4(p_n - k)|\langle n|\frac{\beta(\alpha_s)}{4\alpha_s}G^2|0\rangle|^2, \quad (16)$$

which we now analyze together with (14).

Combining the relations (15) and (12) and discarding possible multi-particle intermediate states, we find the following general relation:

$$\frac{\sum_\sigma\langle 0|\frac{\beta(\alpha_s)}{4\alpha_s}G^2|\sigma\rangle\langle\sigma|\frac{\beta(\alpha_s)}{4\alpha_s}G^2|0\rangle/m_\sigma^2}{\sum_\sigma\langle 0|\frac{\beta(\alpha_s)}{4\alpha_s}G^2|\sigma\rangle\langle\sigma|\sum_i m_i\bar{q}_i q_i|0\rangle/m_\sigma^2} = \frac{4\langle\frac{\beta(\alpha_s)}{4\alpha_s}G^2\rangle}{3\langle\sum_i m_i\bar{q}_i q_i\rangle}. \quad (17)$$

In the case of two flavors and neglecting the breaking of isospin symmetry, (17) leads to

$$\frac{\sum_\sigma\langle 0|\frac{\beta(\alpha_s)}{4\alpha_s}G^2|\sigma\rangle\langle\sigma|\frac{\beta(\alpha_s)}{4\alpha_s}G^2|0\rangle/m_\sigma^2}{\sum_\sigma\langle 0|\frac{\beta(\alpha_s)}{4\alpha_s}G^2|\sigma\rangle\langle\sigma|\bar{u}u + \bar{d}d|0\rangle/m_\sigma^2} = \frac{4\langle\frac{\beta(\alpha_s)}{4\alpha_s}G^2\rangle}{3\langle\bar{u}u + \bar{d}d\rangle}. \quad (18)$$

In the case of three flavors, we know that  $m_s \gg m_u, m_d$ , and the  $\bar{s}s$  condensate is comparable to that of  $\bar{u}u, \bar{d}d$  [14]. Hence the denominator of the right-hand side of (17) must be dominated by the strange-quark contribution. If we further assume that the scalar strange contents of scalar mesons are of the same order of magnitude as those involving  $u$  and  $d$  quarks<sup>2</sup>, then the denominator on the left-hand side of (17) will also be dominated by the strange-quark contributions, and (18) can be re-written as

$$\frac{\sum_\sigma\langle 0|\frac{\beta(\alpha_s)}{4\alpha_s}G^2|\sigma\rangle\langle\sigma|\frac{\beta(\alpha_s)}{4\alpha_s}G^2|0\rangle/m_\sigma^2}{\sum_\sigma\langle 0|\frac{\beta(\alpha_s)}{4\alpha_s}G^2|\sigma\rangle\langle\sigma|\bar{s}s|0\rangle/m_\sigma^2} = \frac{4\langle\frac{\beta(\alpha_s)}{4\alpha_s}G^2\rangle}{3\langle\bar{s}s\rangle}. \quad (19)$$

The relations (17), (18), (19) demonstrate that the ratio of the glueball and scalar  $\bar{q}q$  components in the physical scalar resonances is determined by the ratio of the gluon and quark condensates in the vacuum, which is the key theoretical foundation of this paper.

We now turn to the phenomenological analysis of the above sum rules. Here, the key observation coming from the evaluation of the sum rule (15) using the available experimental data on  $\pi\pi$  and  $\bar{K}K$  phase shifts is that the dominant contribution to the spectral integral in (12) is due to the  $f_0(980)$  resonance [12, 13]. It therefore makes sense to consider approximate relations which follow from (17) in the case when both sum rules (16)

<sup>2</sup>A naive analysis of the  $\pi$ -nucleon  $\sigma$  term indicates that  $\langle N|\bar{s}s|N\rangle$  is not much smaller than  $\langle N|\bar{q}q|N\rangle$ , but there is no comparable information concerning scalar mesons.

and (12) are saturated by a single resonance, namely the  $f_0(980)$ <sup>3</sup>. Since the  $f_0(980)$ , with a width of  $\Gamma = 40 \div 100$  MeV [2], is relatively narrow compared with its mass and its separation from other scalar mesons, it is a reasonable approximation to approximate its spectral shape by a delta function. The sum rule (15) then can be written as

$$\frac{1}{m_{f_0}^2} |\langle f_0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^2 | 0 \rangle|^2 \simeq -4 \langle \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \rangle. \quad (20)$$

Since the quantity on the right-hand side, namely the gluon condensate, has been estimated from QCD sum rule analyses [15], (20) fixes the coupling of the  $f_0(980)$  resonance to the scalar glueball current.

Because  $m_s \gg m_u, m_d$ , it is reasonable to neglect the  $\bar{u}u + \bar{d}d$  contribution to the left-hand side of (17), and use the three-flavor relation (19) to determine the coupling of this resonance to the scalar quark-antiquark current  $\bar{s}s(x)$ :<sup>4</sup>

$$\frac{\langle f_0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^2 | 0 \rangle}{\langle f_0 | \bar{s}s | 0 \rangle} \simeq \frac{4}{3} \frac{\langle \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \rangle}{\langle \bar{s}s \rangle}, \quad (21)$$

which we now use to quantify glueball-quarkonium mixing in this state.

We assume that the  $f_0$  wave function is a superposition of glueball  $|G\rangle$  and quark-antiquark  $|\bar{Q}Q\rangle$  components<sup>5</sup>:

$$|f_0\rangle = \cos \gamma |G\rangle + \sin \gamma |\bar{Q}Q\rangle. \quad (22)$$

We next assume that the quark-antiquark component  $|\bar{Q}Q\rangle$  is mainly  $|\bar{S}S\rangle$ : if there is a substantial  $|\bar{U}U + \bar{D}D\rangle$  component, this would (barring a cancellation) tend to increase the estimate of the mixing angle  $\gamma$  given below. We further assume that the glueball component  $|G\rangle$  has the dominant coupling to the scalar gluon current,  $\frac{\beta(\alpha_s)}{4\alpha_s} G^2$ , and that the quark-antiquark component  $|\bar{S}S\rangle$  has the dominant coupling to the  $\bar{s}s$  current. If this were not the case, there would be additional mechanisms for large glueball-quarkonium mixing that would be difficult to quantify. Extracting a simple dimensional factor, these approximations imply that

$$\frac{\langle f_0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^2 | 0 \rangle}{\langle f_0 | \bar{s}s | 0 \rangle} = m_{f_0} \cotan \gamma, \quad (23)$$

and (21) can now be used to determine the magnitude of the mixing angle in (22):

$$\tan \gamma = m_{f_0} \frac{3}{4} \frac{\langle \bar{s}s \rangle}{\langle \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \rangle}. \quad (24)$$

Numerically, using  $\langle \bar{s}s \rangle = (0.8 \pm 0.1) \langle \bar{q}q \rangle$  [14],  $\langle \bar{q}q \rangle \simeq 0.016$  GeV<sup>3</sup> and  $\langle \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \rangle \simeq 0.013$  GeV<sup>4</sup>, we estimate on the basis of (24) that  $\gamma \simeq 36^\circ$ , i.e., the mixing between the quark-antiquark and the glueball components is strong, even close to maximal.

<sup>3</sup>Similar conclusions on large glueball-quarkonium mixing would hold if any other single meson state dominated the sum rules.

<sup>4</sup>We assume that the matrix elements in (13) and (16) are real, which must be the case for non-degenerate single-particle states.

<sup>5</sup>Analogous arguments leading to large mixing could also be made if the  $f_0(980)$  state turned out to be  $\bar{q}q\bar{q}q$  molecule, as sometimes argued.

To summarize, we have found that the mixing between the glueball and  $\bar{q}q$  components in the lightest scalar state dominating sum rules for  $\theta = \frac{\beta(\alpha_s)}{4\alpha_s}G^2$ , i.e., the best candidate for the lightest scalar “glueball”, is required by the ratio of the quark and gluon condensates to be very strong. We have evaluated this mixing for the  $f_0(980)$  state that has been found empirically in a phase-shift analysis [12, 13] to dominate the sum rule for  $\theta$ , and found it to be near-maximal. We note that this conclusion would only be strengthened if the sum rules were to be saturated by a state heavier than the  $f_0(980)$ .

Our analysis has, admittedly, been rather crude. However, we feel that it has demonstrated the power of sum rules derived from broken scale invariance to contribute to the debate concerning the identification of the lightest scalar glueball, indicating, in particular, that its mixing with a quark-antiquark state may not be neglected.

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