LOW EMITTANCE LATTICES

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Abstract
The new generation of synchrotron light sources is characterized by an extremely high brilliance synchrotron radiation achieved in low emittance storage rings specially designed to accommodate insertion devices. The basic principles guiding the minimization of the emittance are recalled. The magnet lattice configurations that are currently studied as candidates for these storage rings (DFA, TBA, FODO...) are described and the problems associated with these low emittance lattices and the required strong focusing discussed.

1. INTRODUCTION

High brilliance synchrotron radiation can be obtained through the utilization of insertion devices in low emittance storage rings. A small beam emittance is also very important for FEL experimentation. In that case additional requirements must also be fulfilled. In this lecture, emphasis will be placed on storage ring lattices specially designed to maximize the brilliance and achieve the full potential from insertion devices.

The photon flux is an important source characteristic. Nevertheless, for a number of experiments in which the beam is focused at the sample, the true figure of merit is the spectral brilliance B (called in short brilliance) which is defined as the photon flux per unit solid angle and per unit source area, emitted in a relative bandwidth:

$$B = \frac{d^4N}{dt\; d\Omega\; dS\,(d\lambda/\lambda)} \qquad (1)$$

with N = number of photons
t = time
λ = wavelength
Ω = solid angle
S = source size

Brilliance is generally expressed in units of:

$$\frac{\text{photons per second}}{(\text{mrad})^2\,(\text{mm})^2\;0.1\%\text{ bandwidth}}$$

Brilliance of X-rays has increased by many orders of magnitude since the advent of synchrotron radiation sources as shown in Fig. 1.

Apart from diffraction effects, we have $dS\,d\Omega \sim \varepsilon_x\,\varepsilon_y$, where ε_x and ε_y are the transverse particle beam emittances. Therefore the smaller the emittance of the electron beam, the higher the brilliance of the produced radiation. Most existing rings have emittances of 100 nanometer-radians. The new generation of rings under construction has design values between 5 and 10 nanometer-radians. Yet the maximum level of brilliance is reached only at the so-called diffraction limit when the beam emittance is about equal to a fraction of the wavelength of the radiation. As an example, a diffracted limited beam capable of producing 10 keV X-rays would have an emittance of the order of 0.01 nanometer-radians. This sets the scale for the ideally desirable beam emittance.

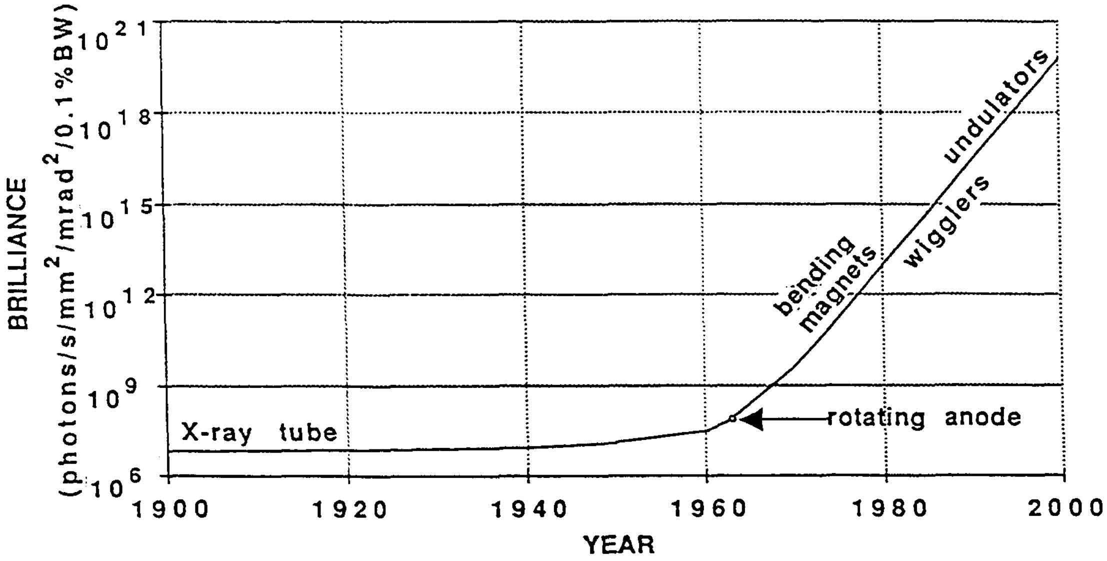

Fig. 1 Increase of brilliance over the last decades

The brilliance can be increased even more if insertion devices are used. Many undulations can be produced along the trajectory, thereby increasing the flux compared to an ordinary bending magnet. For wigglers, the brilliance is enhanced by approximately the number of poles of the device. In an undulator, waves radiated by each electron can mutually reinforce or suppress one another to enhance the radiation of certain wavelengths, resulting in extremely high brilliance.

2. LOW EMITTANCE LATTICES

2.1 Equilibrium emittance

The particle beam emittance in an electron storage ring is determined by an equilibrium between quantum excitation that causes individual particles to oscillate transversally and damping of the betatron oscillations.

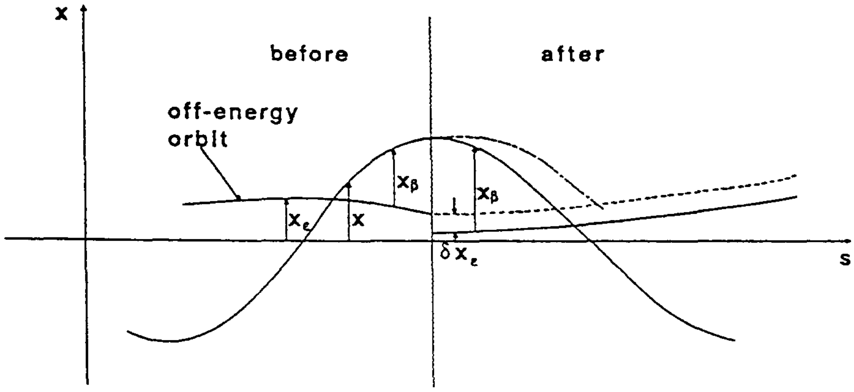

Fig. 2 Effect of energy loss on betatron oscillations

In an undistorted machine, radiation is produced mainly in the normal bending magnets. As a particle emits a photon, it loses energy and this results in a sudden change of its equilibrium orbit which causes a corresponding increase of the betatron oscillation amplitude about the new equilibrium orbit (Fig. 2). This process of continuous increase of the beam emittance is compensated by the damping provided by the RF accelerating system which restores the energy lost by synchrotron radiation only in the longitudinal direction.

The resulting horizontal equilibrium emittance is given by the expression [1]:

$$\varepsilon_{x_0} = \frac{C_q \gamma^2 < H / \rho^3 >}{J_x < 1 / \rho^2 >} \tag{2}$$

where < > means the average around the storage ring.

ρ = bending radius
γ = total energy in mc^2 units
J_x is the horizontal damping partition number
$C_q = \frac{55}{32\sqrt{3}} \frac{h}{2\pi m c} = 3.84\ 10^{-13}$ m

H is the Courant-Snyder dispersion invariant which is specified by the properties of the guide field. It is given by:

$$H = \gamma_x \eta^2 + 2\, \alpha_x\, \eta\, \eta' + \beta_x\, \eta'^2 \tag{3}$$

where α_x, β_x, γ_x are the Twiss coefficients and η, η' are the dispersion function and first derivative respectively.

For an isomagnetic guide field ($\rho = \rho_0$ = Constant in magnets, $\rho = \infty$ elsewhere), Eq. (2) becomes:

$$\varepsilon_{x_0} = \frac{C_q\, \gamma^2 < H >_{mag}}{J_x\, \rho_0} \tag{4}$$

where $<H>_{mag}$ is the average of H taken only in the magnets. That is:

$$< H >_{\mathrm{mag}} = \frac{1}{2\, \pi \rho_0} \int [\, \gamma_x \eta^2 + 2\, \alpha_x \eta\, \eta' + \beta_x \eta'^2\,]\, ds \tag{5}$$

In the vertical plane, there is radiation damping but ideally no dispersion and hence no quantum excitation, so that in a perfect machine the vertical emittance is zero. The vertical emittance is determined only by coupling between the horizontal and vertical planes due to magnet imperfections and misalignments. The coupling is generally expressed by a constant κ which describes the sharing of the natural emittance ε_{x_0} between the two planes:

$$\varepsilon_x = \frac{1}{1+\kappa}\varepsilon_{x0} \tag{6}$$

$$\varepsilon_y = \frac{\kappa}{1+\kappa}\varepsilon_{x0}$$

where ε_x and ε_y are the effective emittances. Typically the coupling is of the order of a few %. It can be controlled by means of skew quadrupoles.

2.2 Minimization of emittance

From Eq. (4), it is clear that minimization of emittance, assuming in a first approach J_x to be constant, is achieved by the minimization of the dispersion invariant function. This means that in the bending magnets, where the photons are emitted, the dispersion function η must be low and the β function optimized.

For a single magnet with length L and bending radius ρ, the integral for deriving the average value of H is given by:

$$I = \int_0^L (\gamma\eta^2 + 2\,\alpha\,\eta\,\eta' + \beta\,\eta'^2)\,ds \tag{7}$$

$$= (\gamma_0\,\eta_0^2 + 2\,\alpha_0\,\eta_0\,\eta'_0 + \beta_0\,\eta'^2_0) + 2\,(\alpha_0\,\eta_0 + \beta_0\,\eta'_0)\,\rho\,(L - \cos\frac{L}{\rho})$$

$$= (\gamma_0\,\eta_0^2 + 2\,\alpha_0\,\eta_0\,\eta'_0 + \beta_0\,\eta'^2_0) + 2\,(\alpha_0\,\eta_0 + \beta_0\,\eta'_0)\,\rho\,(L - \cos\frac{L}{\rho})$$

$$-2\,(\gamma_0\eta_0 + \alpha_0\,\eta'_0)\,\rho\,(L - \rho\,\sin\frac{L}{\rho}) + \frac{1}{2}\beta_0\left[L - \frac{\rho}{2}\sin\left(2\frac{L}{\rho}\right)\right]$$

$$-2\,\alpha_0\,\rho^2\left[\frac{3}{4} - \cos\frac{L}{\rho} + \frac{1}{4}\cos\left(2\frac{L}{\rho}\right)\right]$$

$$+\gamma_0\rho^2\left[\frac{3}{2}L - 2\rho\,\sin\frac{L}{\rho} + \frac{\rho}{4}\sin\left(2\frac{L}{\rho}\right)\right]$$

where the index 0 refers to the entrance of the bending magnet. A simplified expression can be obtained by approximating the resulting expression to second order in L/ρ. One gets:

$$I \approx (\gamma_0\eta_0^2 + 2\,\alpha_0\,\eta_0\eta'_0 + \eta'^2_0)\,L + (\alpha_0\eta_0 + \beta_0\eta'_0)\,\frac{L^2}{\rho} \tag{8}$$

$$-(\gamma_0 \eta_0 + \alpha_0 \eta'_0)\frac{L^3}{3\rho} + \left[\frac{\beta_0 L}{3} - \frac{\alpha_0 L^2}{4} + \frac{\gamma_0 L^3}{20}\right]\frac{L^2}{\rho^2}$$

Optimum values of the optical functions at the beginning of the magnet can then be derived from Eq. (8), in order to achieve the minimum emittance. Figure 3 shows how the emittance changes with variations of α_0 and β_0 away from their optimum in the particular case of zero dispersion at one end of the magnet.

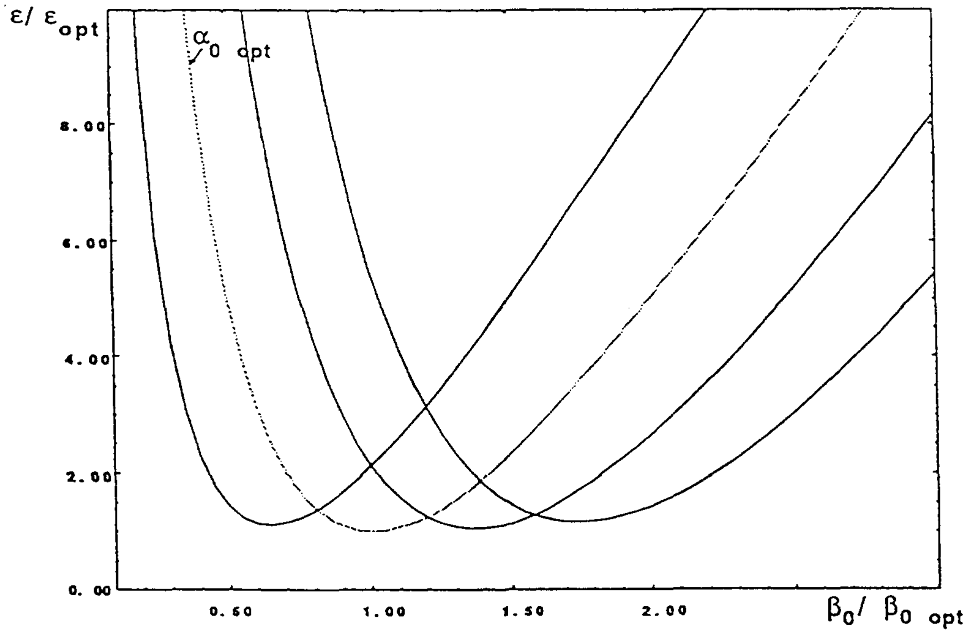

Fig. 3 Beam emittance as a function of initial betatron values

To minimize ε_{x0}, one can also take into account the variation of J_x. The horizontal partition number is given by [2]:

$$J_x = 1 - \frac{1}{2\pi\rho^2}\int_{mag} \eta\,(1 - 2n)\,ds \tag{9}$$

where the magnetic field index is defined by: $n = -\frac{\rho}{B}\frac{dB}{dx}$

In a machine with separated functions (i.e. zero field gradient in the dipoles), the quantity J_x is approximately equal to 1. J_x can be made larger by introducing vertical focusing in the dipoles but its maximum value is limited by the rule:

$$\Sigma J_i = J_x + J_y + J_\varepsilon = 4 \tag{10}$$

which means, in the particular case of no vertical bend, $J_x + J_\varepsilon = 3$, hence $J_x \leq 3$. In practice the maximum tolerable value is $J_x \leq 2$, which results potentially in an emittance reduction of a factor two.

Another possible way to achieve very low emittances is to install damping wigglers in dispersion-free straight sections. For instance, this method would reduce the emittance in PEP by almost one order of magnitude [3] with a total length of installed wiggler magnets of 200 m. This method is applicable to any storage ring but since the damping effect scales proportionally with the bending radius of the ring magnets, it is more efficient in large machines.

3. LATTICE TYPES

3.1 Generalities

The basic structure of low emittance lattices consists of straight sections designed for the installation of insertion devices separated by arc sections. To avoid emittance blow-up by insertion devices, it is usual to adjust the dispersion function in the insertions to zero. In the arcs, the focusing is chosen so as to minimize the chromatic invariant. Minima for H can be found for various types of lattices, involving proper choices for η_x, β_x, and their derivatives in the bending magnet.

A variety of types of lattices are available to design low emittance lattices. They can be divided into two classes:

i) structures without bending magnets in the dispersive section: double focusing achromat (DFA) commonly known as the Chasman-Green lattice [4], expanded Chasman-Green, empty FODO, triplet achromat lattice (TAL).

The double focusing achromat lattice has been used for the NSLS rings in Brookhaven [5]. An expanded Chasman-Green structure is the basis of the conceptual designs of several synchrotron radiation sources under construction: ESRF [6], APS [7], ELETTRA [8]. SUPERACO [9] is already operated with an expanded Chasman-Green structure. The triplet achromat lattice was used in ACO at Orsay [10]; it has the advantage of requiring the shortest circumference.

ii) structures with additional bending magnets in the dispersive region: triplet bend achromat (TBA), FODO.

Triple bend structures are utilized in both the ALADDIN [11] and BESSY [12] synchrotron radiation sources. ALADDIN is a machine optimized primarily to use radiation from the dipole magnet and does not provide zero dispersion in the long straight sections. In the BESSY structure, significant focusing in the vertical plane is provided by the edge focusing of the dipoles. This focusing is necessary in order to limit the amplitude of the vertical beta function.

The triple bend achromat lattice is the logical extension of the DFA. Adding a third bending magnet within the arc section allows extra flexibility. This lattice is proposed for the ALS in Berkeley [13], BESSY 2 [14], SRRC in Taiwan [15].

The FODO lattice is the most commonly used lattice for high energy physics storage rings because of its very compact structure. In contrast to the other types of lattices, the FODO does not naturally provide a space for insertion devices; special lattice sections must be introduced to achieve dispersion-free insertions. The damping rings at Stanford [16] and the SXRL storage ring [17] are good examples of the use of this structure.

3.2 The double focusing achromat

The double focusing achromat lattice or basic Chasman-Green represents the most compact of the structures used in low emittance storage rings. The basic scheme uses two dipole magnets with a focusing quadrupole between them (Fig. 4). The strength of the quadrupole is adjusted so that the dispersion generated by the first dipole is cancelled by passing through the second dipole.

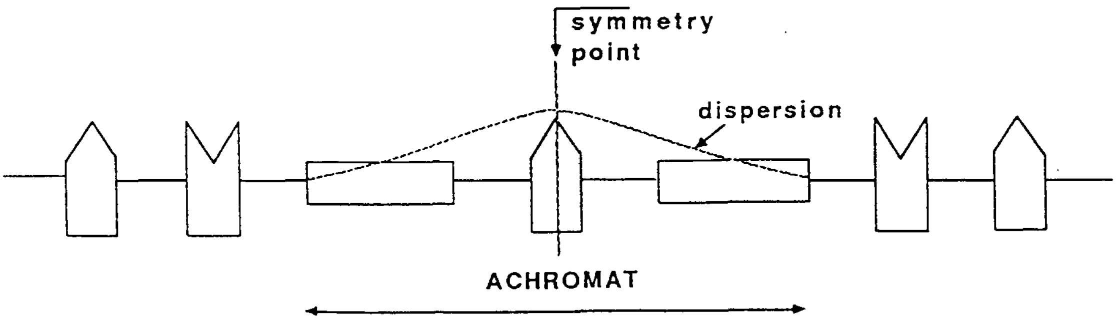

Fig. 4 Basic DFA structure

In this form, the structure is rather inflexible since the quadrupole does not provide focusing in both planes. Therefore, defocusing quadrupoles can be added upstream and downstream of the focusing quadrupole to restore focusing in both planes and to provide more flexibility in the adjustment of transverse dimensions (i.e. the ESRF with four quadrupoles or ELETTRA with three quadrupoles). This is the so-called expanded Chasman-Green achromat. The optical functions of these two lattices are shown in Figs. 5 and 6.

The emittance of a lattice built up of such cells has been computed by various authors [18, 19]. Since the dispersion and its derivative are zero at the entrance of the bending magnet, the emittance is given by:

$$\varepsilon_x = \frac{C_q\, \gamma^2}{J_x}\, \theta^3 \left(\gamma_0 \frac{L}{20} + \frac{\beta_0}{3L} - \alpha_0 \frac{L}{4} \right) \tag{11}$$

where β_0, γ_0, α_0 are the Twiss parameters at the entrance of the bending magnet, L is the length of the magnet and θ is the bending angle.

The emittance can be minimized by a proper adjustment of the optical parameters at the entrance of the bending magnet. The minimum value is achieved when the minimum of the horizontal beta function within the dipole occurs at a distance $s = 3/8\ L$ from the beginning of the magnet and the value of the minimum betatron function is :

$$\beta = L \frac{\sqrt{3}}{8\sqrt{5}} \tag{12}$$

It yields an emittance of:

$$\varepsilon_x = \frac{C_q\, \gamma^2}{J_x}\, \theta^3 \frac{1}{4\sqrt{15}} \tag{13}$$

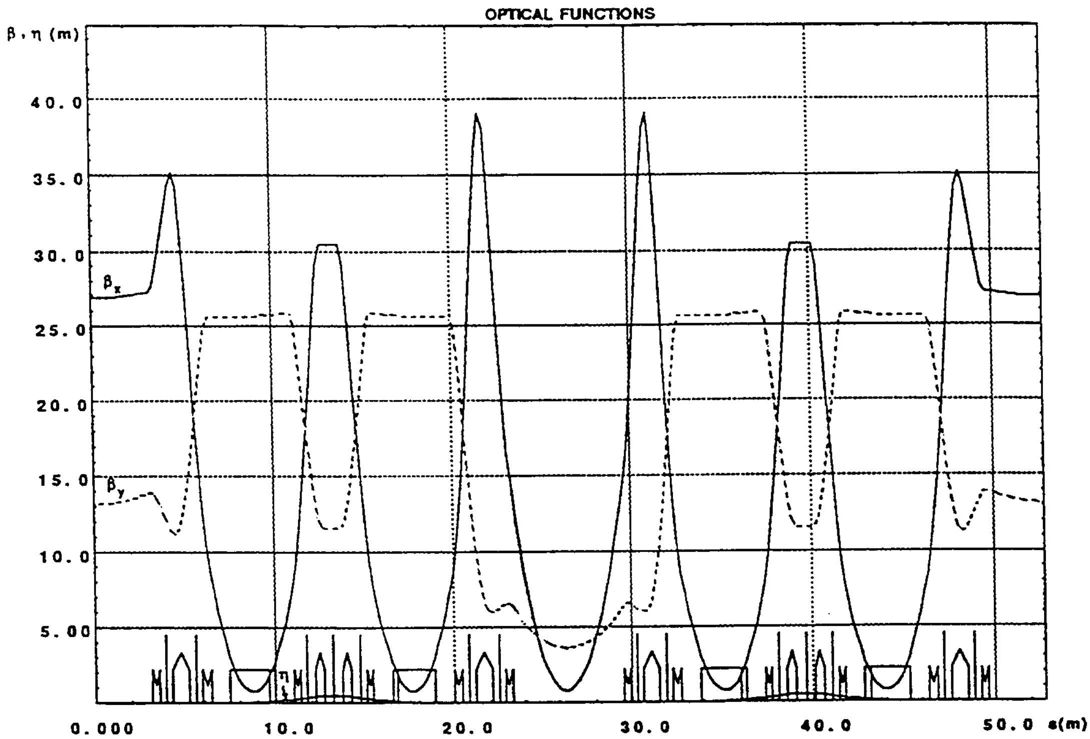

Fig. 5 ESRF lattice functions

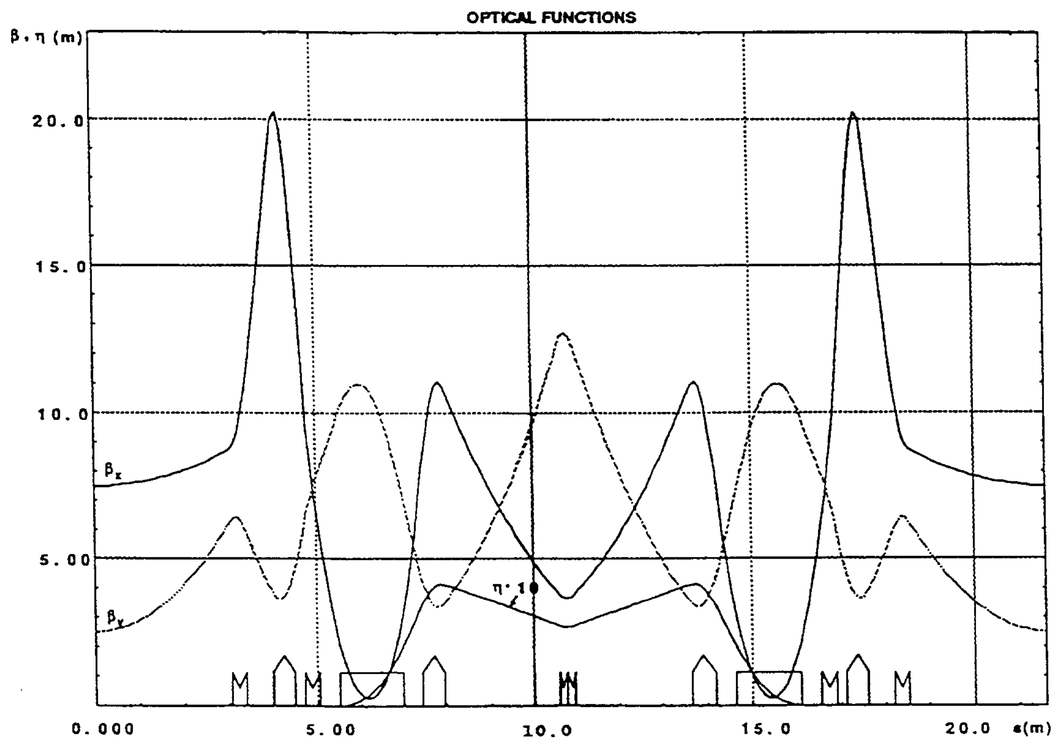

Fig. 6 ELETTRA lattice functions

It is difficult to reach the minimum emittance because the very low β value in the bending magnet (β_{min} = 19 cm for L = 2 m) generates unacceptably high β values in the focusing structure outside the bending magnet and leads to large chromaticities. Therefore, actual lattices result from a compromise with the needs of chromaticity correction and a more realistic estimate would be to increase the emittance by a factor two to three over the ideal minimum value.

The advantage of the DFA structure is to provide a large dispersion between the two bending magnets. As a consequence, sextupole strengths required for chromaticity correction, which are inversely proportional to the dispersion, are reduced. This is the reason why the expanded Chasman-Green is more suitable for large rings where the dispersion in the achromat becomes small. On the other hand, a clear disadvantage of such lattices is the fixed phase advance across the achromat which is approximately π. This places severe constraints on the horizontal betatron tune and limits the flexibility of the lattice since the β values in the insertions are coupled to the choice of tune.

3.3 Triplet achromat lattice

This lattice can be made very compact since there are no quadrupoles in the insertion straight sections, as shown in Fig. 7.

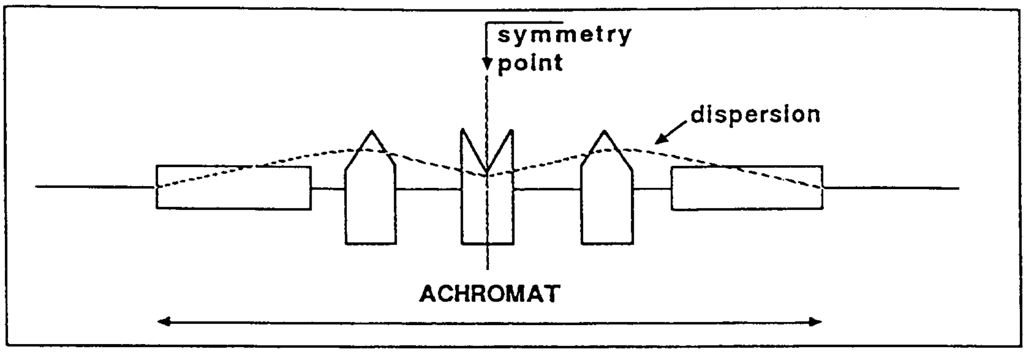

Fig. 7 Basic TAL structure

Using the same derivation as in the previous section, one can express the minimum emittance of the lattice in the form:

$$\varepsilon_x = \frac{C_q \gamma^2}{J_x} \theta^3 \frac{2}{3} \left(\frac{\beta}{L}\right)_{opt} \tag{14}$$

using the optimum value of β in the middle of an insertion section of length 2 L_i given by:

$$\left(\frac{\beta}{L}\right)^2_{opt} = \frac{3}{4}\left[\frac{1}{5} + \frac{L_i}{L} + \frac{4}{3}\left(\frac{L_i}{L}\right)^2\right] \tag{15}$$

The main disadvantage of the lattice is that the emittance depends on the value of the horizontal betatron function in the insertion region. Figure 8 shows the ACO lattice functions as an example of a triplet achromat structure.

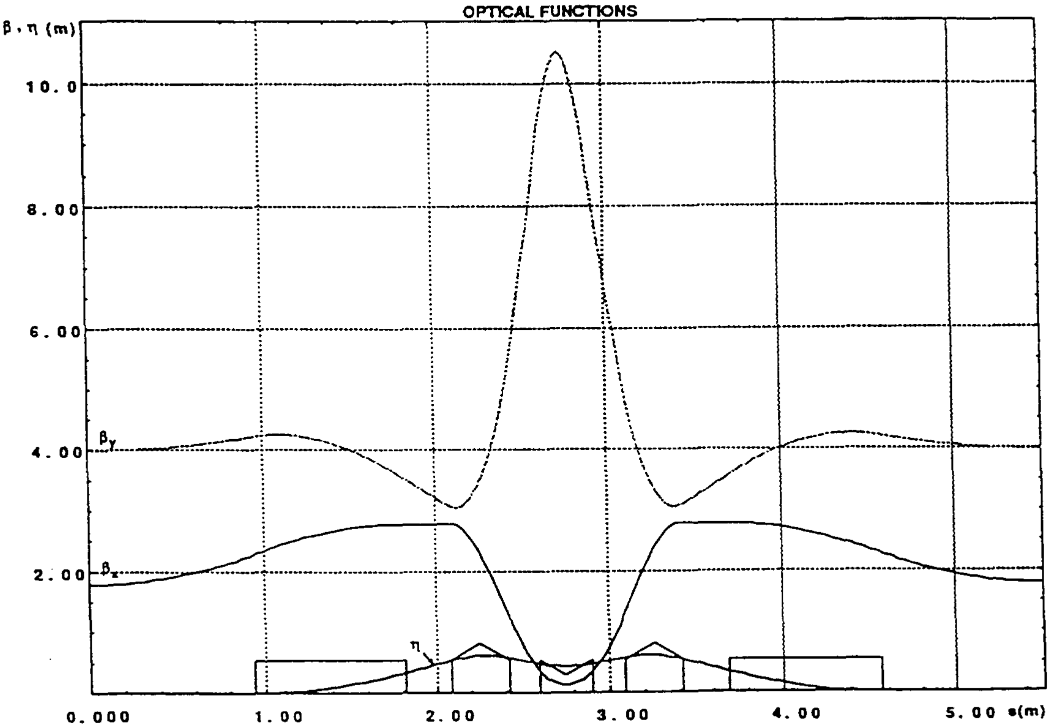

Fig. 8 ACO lattice functions

3.4 Triple bend achromat

A unit cell of the basic TBA lattice with one horizontally focusing quadrupole in-between the bending magnets is shown in Fig. 9. The addition of an extra bending magnet within the achromat gives more flexibility for adjusting the phase advance across the achromat. The phase advance can be varied from π to 2π by changing the positions of the achromat quadrupoles and/or the length of the central magnet compared to the outer magnets [20]. A more practical way to obtain a "movable" quadrupole with fixed magnet geometry is to include a second family of quadrupoles in the achromat.

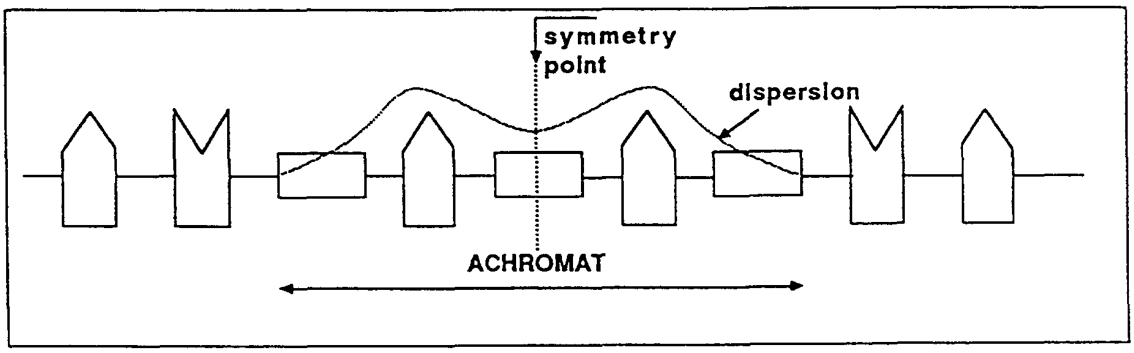

Fig. 9 Basic TBA structure

The emittance of the TBA lattice is obtained by summing the values of the driving integral $I = \int H\, ds$ over the two types of magnets. It can be expressed in the following form:

$$\varepsilon_x = \frac{C_q \gamma^2}{J_x} \frac{\frac{l_1}{\rho_1^3} \frac{2 l_0}{\rho_0^3}}{\frac{\theta_1}{\rho_1} + \frac{2\theta_0}{\rho_0}} \tag{16}$$

where the index 0 refers to the outer magnet (radius ρ_0 and deflection angle θ_0) and the index 1 to the inner magnet.

Assuming equal bending magnet deflection, the minimum emittance is given by:

$$\varepsilon_x = \frac{C_q \gamma^2}{J_x} \theta^3 \frac{7}{36 \sqrt{15}} \tag{17}$$

It is obtained for β values in the outer bending magnets identical to those of the double focusing achromat structure and for $\beta_x = L/\sqrt{15}$ in the inner magnet. Even lower emittance values can be obtained by optimizing the bending angle distribution. The optimum yields a structure in which the centre deflection angle is 1.5 times larger than the outer deflection angle.

To control the vertical beta function in the bending magnets, vertical focusing can be introduced either in the form of lumped quadrupoles adjacent to the bending magnet or by introducing a gradient into the bending magnet fields, as proposed by Vignola [21]. This also changes the damping partition numbers in such a way as to reduce the emittance. Such a solution with combined function magnets has been adopted in the ALS design (Fig. 10).

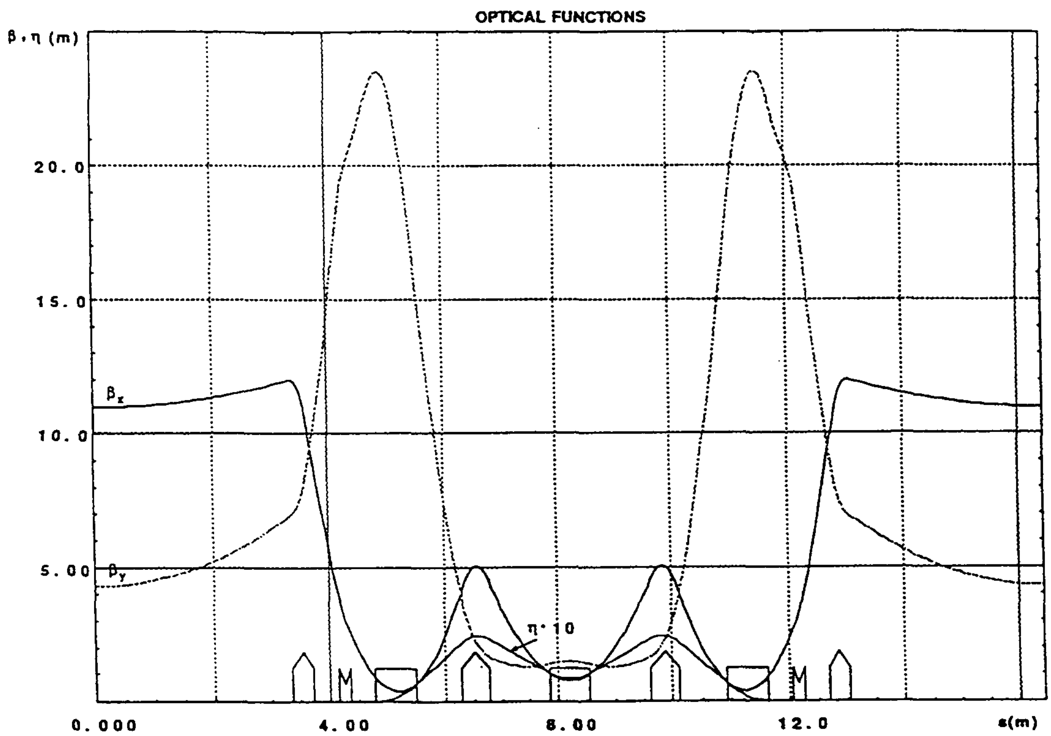

Fig. 10 ALS lattice functions

However, to keep the emittance small, the dispersion in the inner magnet must be reduced to small values which leads to large correcting sextupoles. When compared to the case of a double focusing achromat lattice with comparable emittance, these sextupoles need to be about two or three times stronger. Such structures are more suitable for small rings where the deflection angle per bending magnet, and correspondingly the dispersion, is larger.

3.5 FODO structures

As used in storage rings constructed for high energy physics, a FODO lattice consists of a sequence of alternating focusing and defocusing quadrupoles separated by bending magnets. The emittance of a regular FODO structure scales as [22]:

$$\varepsilon_x = 4\frac{C_q \gamma^2}{J_x}\theta^3 F(\mu_c) \qquad (18)$$

where $F(\mu_c)$ is a function of the betatron phase advance per cell μ_c.

The variation of $F(\mu_c)$ is plotted in Fig. 11. The minimum is reached at about $\mu_c = 135°$ and is rather flat in the range $100° \leq \mu_c \leq 160°$. The large phase advance required to achieve a small emittance implies very strong focusing.

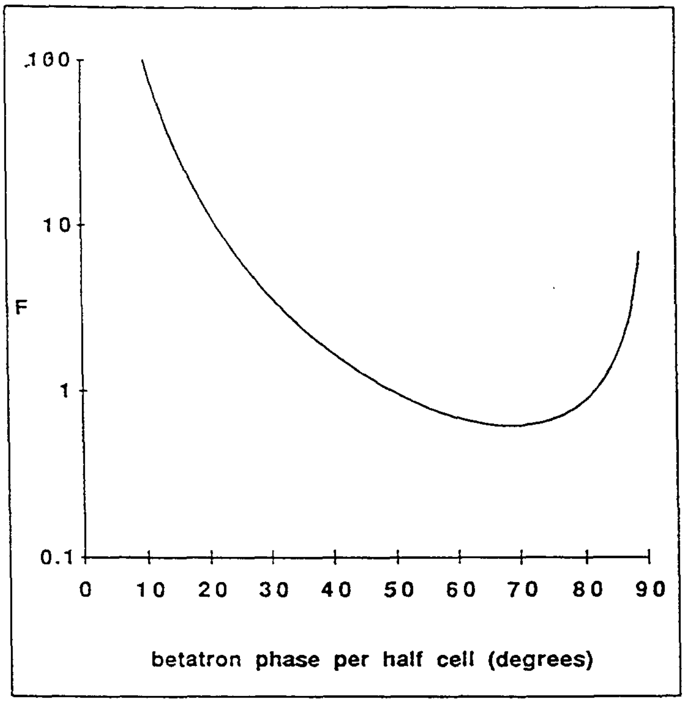

Fig. 11 Emittance behaviour as a function of the betatron phase per half cell

The FODO achromat is composed of a regular FODO structure followed by a dispersion suppressor, as shown in Fig. 12.

The overall emittance is produced by the contribution from the nominal FODO cell bending magnets and the dispersion suppressor bending magnet. It can be expressed as [23]:

$$\varepsilon_x = \frac{C_q\,\gamma^2}{J_x}\,\frac{\dfrac{\eta_0\, l_0}{\rho_0^3} + \dfrac{2\,l_1}{\rho^3}}{\dfrac{\eta_0\,\theta_0}{\rho_0} + \dfrac{2\,\theta_1}{\rho}} \qquad (19)$$

where n_0 is the number of FODO half cells per achromat and $\theta = L / \rho$.

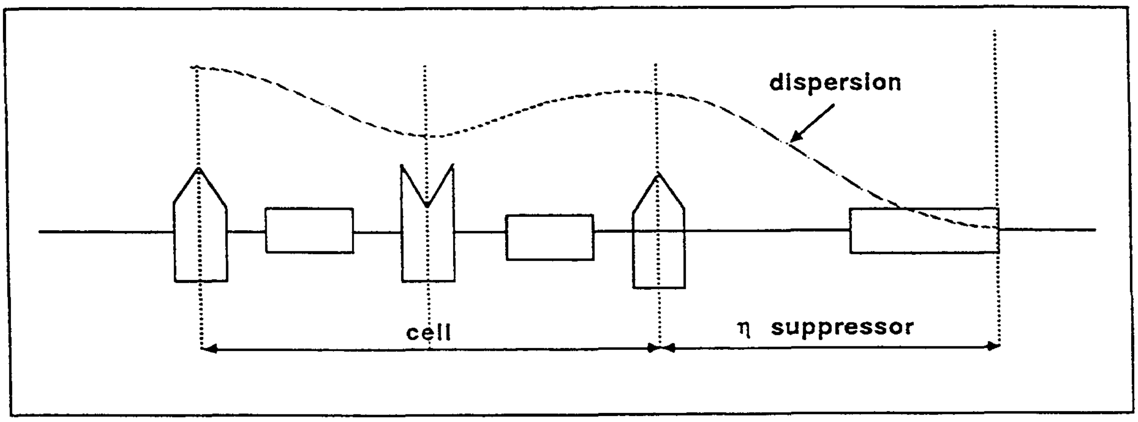

Fig. 12 Basic FODO achromat

Once the cell geometry and the phase advance are chosen, the emittance can be minimized by optimizing the length of the dispersion suppressor magnet and the bending angle distribution. Figure 13 (taken from [23]) shows the variation of emittance with these parameters.

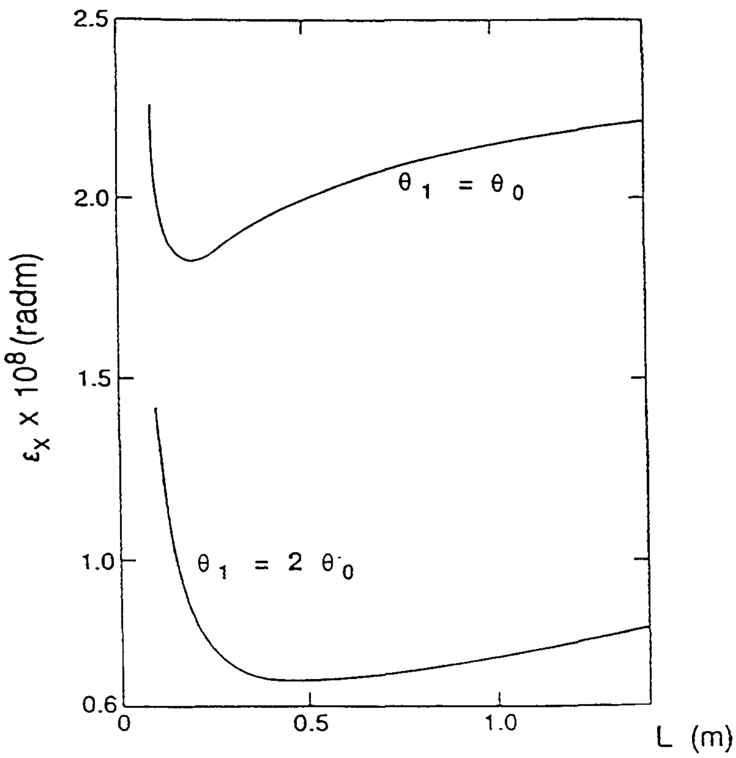

Fig. 13 Variation of emittance with length of dispersion suppressor

The FODO has the advantage of large flexibility. Also, since the average betatron function is reduced, the vacuum requirements can be relaxed. On the other hand, it is characterized by an extremely dense packing of magnetic elements which could lead to layout and engineering problems. Also it suffers from intrinsic problems similar to those of the triplet bend achromat, because of the small dispersion which is required to achieve a low emittance and leads to large sextupole strengths. The lattice functions of a FODO type structure are given in Fig. 14.

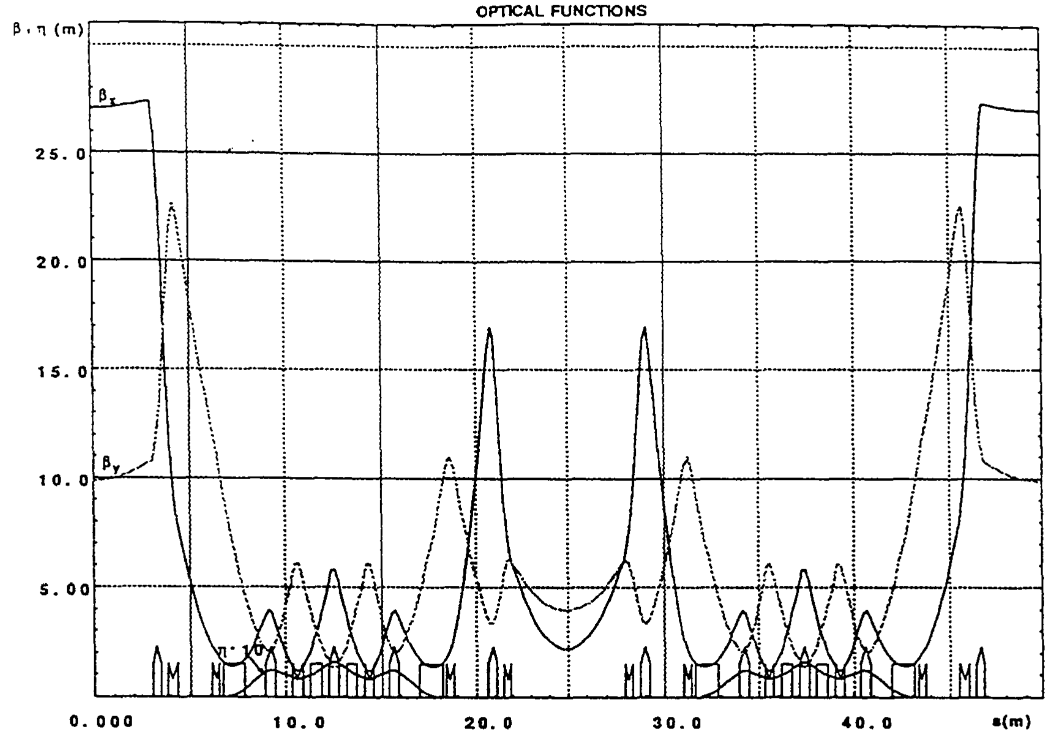

Fig. 14 Lattice functions of a FODO type structure

3.6 Comparison of the lattice performance

It can be seen from the previous sections that the beam emittance of all these lattices scales as $E^2\,\theta^3$ where E is the beam energy and θ the bending angle. This means that, in order to achieve a low emittance, the bending angle must be decreased and a large number of cells is needed. In order to make a comparison of the various types of lattices, it is useful to rewrite the emittance in the following form:

$$\mathcal{E}_x = \frac{E^2}{J_x N_d^3} F\,(\mu_x\,,\,lattice) \tag{20}$$

where N_d is the number of bending magnets and E the beam energy (expressed in GeV).

Table 1 lists the minimum values of F which are achievable for the various lattices and gives an example of each lattice case. F is varying over a range of roughly a factor ten from the FODO lattice down to the DFA lattice. But because of the cubic dependence on N_d, even a factor ten can be compensated by a factor of about two in N_d. Thus, to get the same emittance with a FODO, one has to double the number of bending magnets in the ring. Design considerations imply in most cases that the emittance is larger than the minimum value.

Table 1
Minimum values of F

	F_{min}	Examples	Energy (GeV)	N_d	F
DFA	2.35 10^{-5}	ESRF	6	64	5.02 10^{-5}
TAL	2.45 10^{-5}(*)	ACO	0.536	8	
TBA	1.83 10^{-5}	ALS (**)	1.5	36	1.14 10^{-4}
FODO(***)	4.52 10^{-4}				

(*) to be multiplied by $(\beta/L)_{opt}$
(**) with gradient in the dipoles
(***) regular cells without dispersion suppressor

The emittance is an important issue when choosing a lattice. However criteria for evaluation of lattices include other considerations, such as chromatic correction and dynamic aperture, sensitivity to errors, flexibility, beam lifetime.... Some of these problems will be discussed in Section 3.8.

3.7 New ideas for ultra-low emittance lattices

The lattices considered at present achieve low emittance by minimizing the dispersion function and the horizontal betatron function in the bending magnets. This design philosophy encounters its natural limits in the increasing difficulty to correct the chromatic errors due to the high focusing, especially as more than half of the chromaticity is created by quadrupoles in the dispersion-free sections.

To avoid these limitations, a new model has been recently proposed [24]. The design strategy consists in compensating the chromatic errors where they are generated. This is possible by maintaining dispersion throughout the lattice and integrating sextupole fields into the focusing elements. Nevertheless, in order to achieve a small emittance, the dispersion is minimized by stretching the bending over practically the total circumference and applying an extremely high focusing.

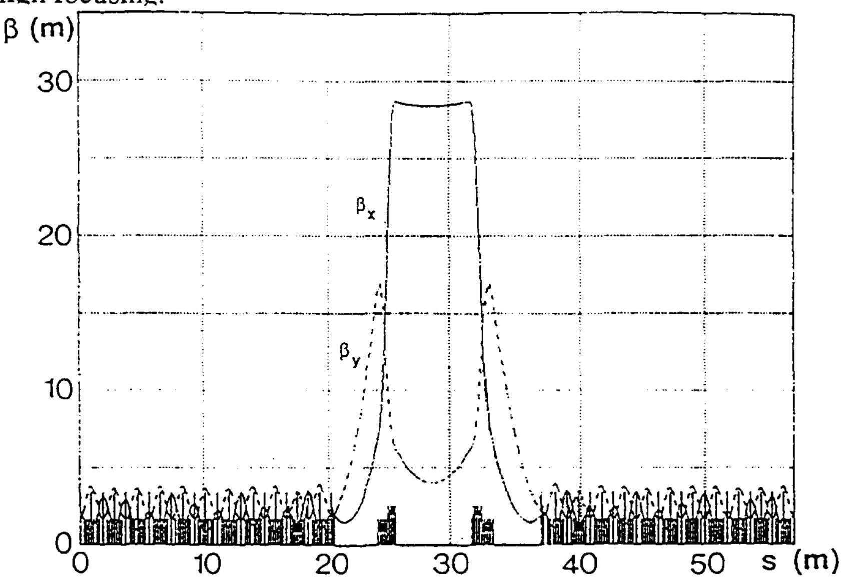

Fig. 15 Betatron functions of the ultra low emittance model

Obviously, this design philosophy is partly spoilt by the requirement of dispersion-free straight sections for insertion devices. Figure 15 shows the optical functions of a lattice composed of regular cells and a dispersion suppressor region. The structure is based on the use of multipole magnets with strong quadrupole fields integrated and with a low dipole field superimposed. An emittance of 0.7 nanometer-radians is achieved for a 6 GeV ring. The structure also provides one-order-of-magnitude larger momentum acceptance than present designs. This kind of lattice might be very attractive for synchrotrons and damping rings.

3.8 Problems associated with low emittance lattices

3.8.1 Chromaticity correction and dynamic aperture

Achieving such low emittances means that the storage ring will be highly sensitive to non-linear effects. One of the main problems raised is the correction of chromaticity. The chromaticity is the tune shift experienced by individual particles due to momentum errors $\Delta p/p$. It is defined as:

$$\xi_{x,y} = \frac{\Delta \upsilon_{x,y}}{(\Delta p/p)} = -\frac{1}{4\pi}\int \beta_{x,y}(s)\, K_{x,y}(s)\, ds \tag{21}$$

where $K_{x,y}$ is the quadrupole gradient.

The strong focusing required to obtain a low emittance implies strong gradients and also large values of betatron functions so that the chromaticity tends to be large. As a consequence, off-momentum particles will cross resonances and may be lost. In addition, to prevent instabilities and the so-called "head-tail" effect, the chromaticity must be corrected or adjusted to a slightly positive value. This is performed by means of sextupoles located in the dispersive region. The momentum error produces an orbit shift which generates a quadrupole field at each sextupole location so that the correcting effect is given by:

$$\xi_s = \frac{1}{4\pi}\int \beta(s)\, \eta(s)\, m(s) ds \tag{22}$$

where $m(s)$ is the sextupole gradient.

As a matter of fact, these sextupoles need to be very strong since the dispersion generated by the bending magnets is rather small in all lattices. Unfortunately, these sextupoles introduce various kinds of chromatic and geometric aberrations, such as betatron amplitude-dependent and momentum-dependent tune shifts, change of the betatron functions and dispersion with momentum. These aberrations limit the maximum stable amplitudes of oscillations. The dynamic aperture, defined as the boundary of stable motion in the x-y plane, may be considerably smaller than the physical aperture. Adequate dynamic aperture is an important figure of merit. The transverse aperture must be large enough to accommodate the oscillations of the injected beam and Coulomb scattered particles. Without this, the injection efficiency would be very poor and the beam lifetime very short. A large chromatic aperture is required to improve the lifetime due to Touschek and bremsstrahlung effects.

In order to obtain reasonably large dynamic apertures and minimize all detrimental effects, complicated arrangements of sextupoles have to be found for each lattice. In particular, additional sextupoles located in the dispersion-free straight sections are used to enlarge the dynamic aperture and reduce the tune shifts with amplitude. With regard to dynamic aperture, all lattices can be brought to comparable performance as shown in Fig. 16 for the ESRF.

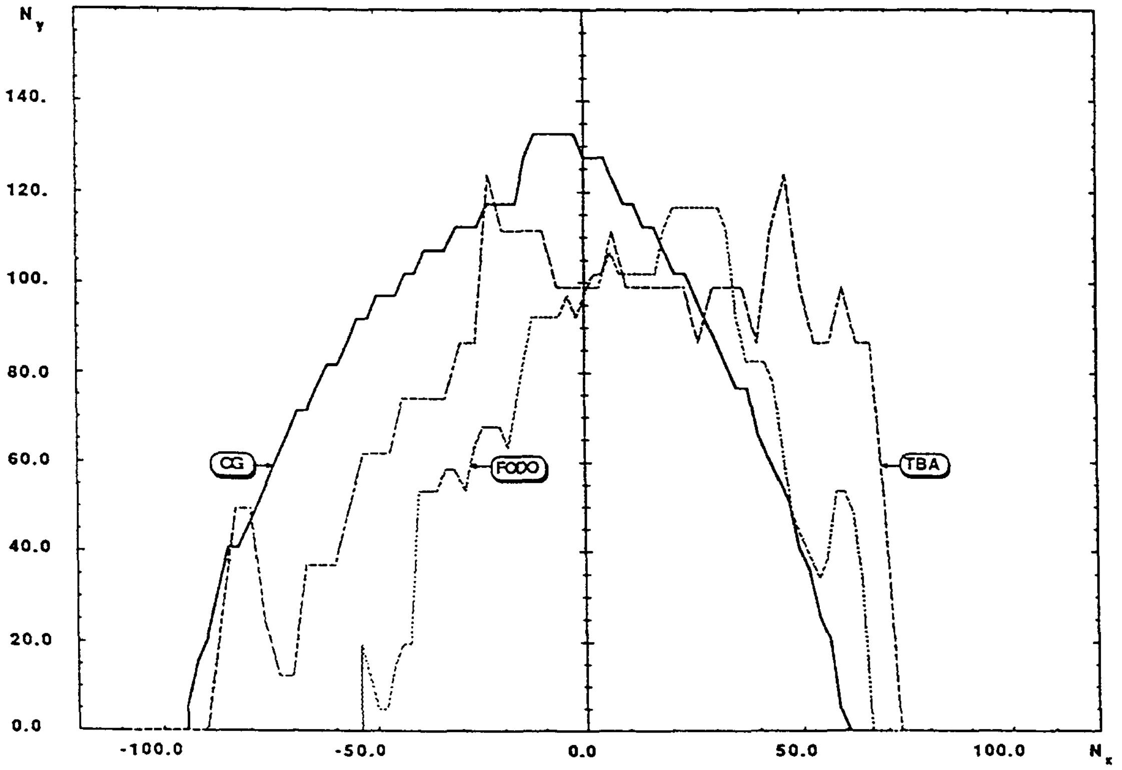

Fig. 16 Dynamic aperture of CG, TBA, FODO lattices for the ESRF

3.8.2 Sensitivity to errors

Errors of all kinds (magnet misalignments, systematic and random field errors) will significantly reduce the dynamic aperture. Because of the strong focusing, the amplification of quadrupole and sextupole misalignments is very large and generates unacceptable distortions of the linear optics, reduction of dynamic aperture, beam emittance blow-up or even unstable machines. The severity of this problem is illustrated in Fig. 17 which shows that a significant reduction of dynamic aperture is induced by very small quadrupole positioning errors (in that case the effects of a misalignment of 0.025 mm have been computed for different sample machines).

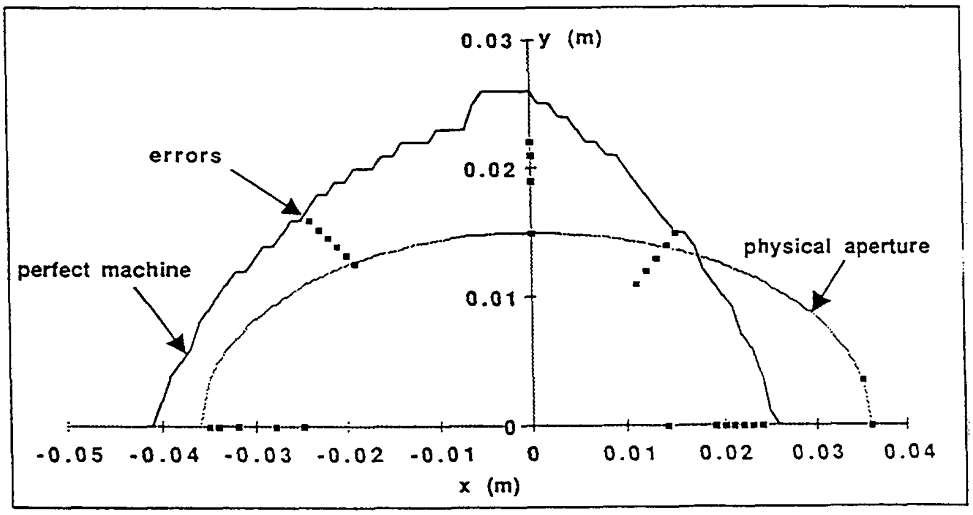

Fig. 17 Effects of quadrupole positioning errors on dynamic aperture

As a consequence, state-of-the-art alignment and construction capabilities (1/10 mm positioning errors for quadrupoles and sextupoles, tilts of magnetic elements of a few 10^{-4} rad) are required. Since a quite perfect closed orbit (i.e. 0.1 to 0.2 mm rms residual distortions) is necessary to achieve correct performance, sophisticated closed orbit correction schemes, including dynamic feedback systems to control beam stability, must be implemented.

3.8.3 Orbit stability

Orbit stability has more severe requirements than for other accelerator applications because of the very small vertical beam size at the source point (a typical figure is $\sigma_y < 100$ µm at undulator locations). Any change of the beam position with time will lead to an enlargement of the source and a macroscopic emittance growth. To avoid spoiling the source emittance, the beam centre of mass must be kept within a few µm and a few 1/10 µrad all around the machine.

The tolerances on the stability of the beam position concern both the long-term stability, therefore deal with ground settlement, and the short-term motion induced by vibrations transmitted through the ground.

As already pointed out, the closed orbit is extremely sensitive to quadrupole centre displacements. Therefore, the buildings, infrastructure, slab and magnet supports must be carefully designed to minimize transmission of vibrations to the magnetic elements. Figure 18 shows an example of tolerances on vibration amplitudes measured at the location of magnetic elements

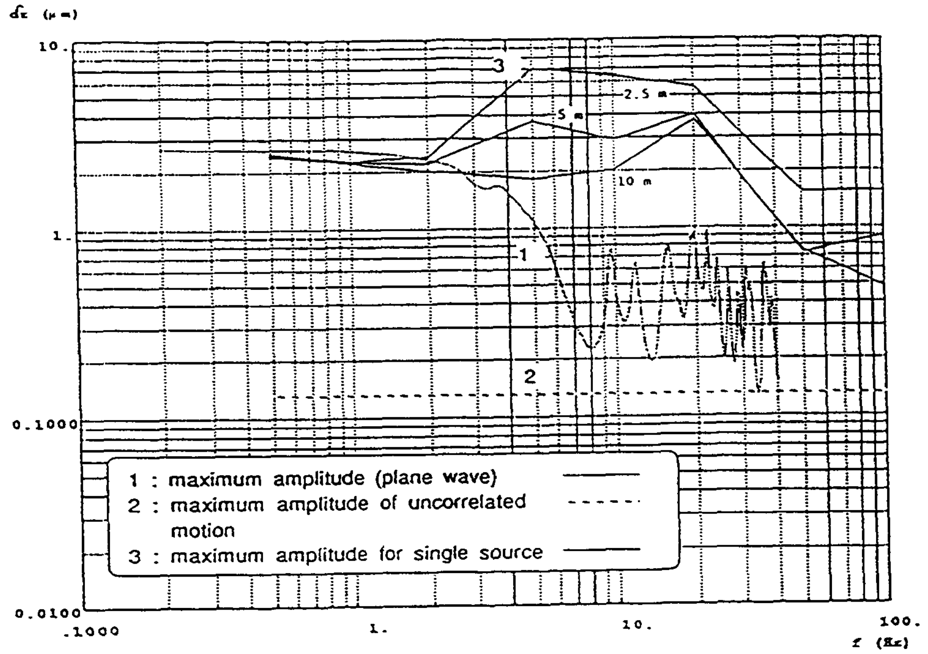

Fig. 18 Allowable amplitude of vibrations

The standard closed orbit position monitors are not able to guarantee the requested stability. Therefore, it is necessary to use feedback systems based on the detection of the photon beam and local steering magnets.

3.8.4 Small momentum compaction

Low emittance lattices tend to have a very low momentum compaction factor a, because they usually have low dispersion. Typical values are :

$\alpha \sim 3.0\ 10^{-4}$	for APS, ESRF
$\alpha \sim 1.5\ 10^{-3}$	for ALS, ELETTRA

When α becomes very small, non-linear contributions become important and can lead to longitudinal instabilities. The low value of the momentum compaction also affects bunch length and very short bunches can be obtained.

* * *

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