THE SOURCE SIZE DEPENDENCE ON THE M_{hadron} APPLYING FERMI AND BOSE STATISTICS AND I-SPIN INVARIANCE[a]

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The emission volume sizes of pions and Kaons, $r_{\pi^{\pm}\pi^{\pm}}$ and $r_{K^{\pm}K^{\pm}}$, measured in the hadronic Z^0 decays via the Bose-Einstein Correlations (BEC), and the recent measurements of $r_{\Lambda\Lambda}$ obtained by through the Pauli exclusion principle are used to study the r dependence on the hadron mass. A clear $r_{\pi^{\pm}\pi^{\pm}} > r_{K^{\pm}K^{\pm}} > r_{\Lambda\Lambda}$ hierarchy is observed which seems to disagree with the basic string (LUND) model expectation. An adequate description of $r(m)$ is obtained via the Heisenberg uncertainty relations and also by Local Parton Hadron Duality approach using a general QCD potential. These lead to a relation of the type $r(m) \simeq Constant/\sqrt{m}$. The present lack of knowledge on the $f_o(980)$ decay rate to the $K^0\overline{K}^0$ channel prohibits the use of the $r_{K^0_S K^0_S}$ in the $r(m)$ analysis. The use of a generalised BEC and I-spin invariance, which predicts an BEC enhancement also in the $K^{\pm}K^0$ and $\pi^{\pm}\pi^0$ systems, should in the future help to include the $r_{K^0_S K^0_S}$ in the $r(m)$ analysis.

1 Introduction

For more than three decades the Bose Einstein Correlations (BEC) of identical bosons, mainly charged pions, were utilised to estimate the dimension of their emission source. In nucleus-nucleus reactions a clear dependence of $r_{\pi\pi}$ on the size of the nucleus is observed. In e^+e^- annihilations leading to hadronic final states no clear evidence is seen for a change in $r_{\pi\pi}$, the emission volume size, as the $\sqrt{s_{ee}}$ increases from 3.1 GeV (the J/ψ mass) to the Z^0 mass (see Fig. 1). In particular if one remembers the large variety of experimental procedure adopted by the different experiments. Nevertheless, in order to be sure to isolate the effect of the hadron mass we concentrated our study to e^+e^- annihilation at one energy only namely, at the Z^0 mass. At this energy the very large statistics of hadronic Z^0 decays of high average charged multiplicity, accumulated by the four LEP experiments, ALEPH, DELPHI, L3 and OPAL, affords an excellent opportunity to investigate whether the dimension r of two-particle emitter size does depend on their mass.

In our analysis we rely on the 1-dimensional BEC studies where the emission source

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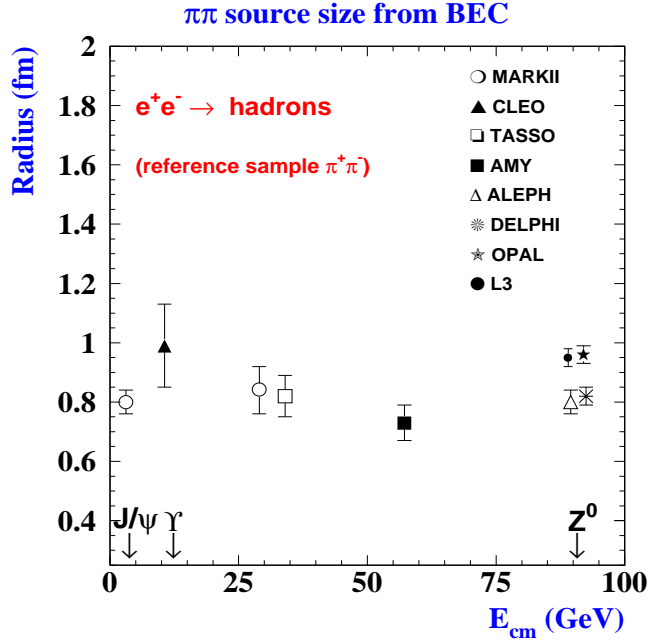
Figure 1: A compilation of $r_{\pi^\pm\pi^\pm}$ measured in e^+e^- annihilations.

is taken to be a sphere with a Gaussian density distribution. For the analysis the Lorentz invariant variable $Q = \sqrt{-(q_1 - q_2)^2}$ is used together with the Goldhaber correlation parametrisation $C(Q) = 1 + \lambda \exp(-r^2 Q^2)$. Here q_i are the 4-momenta of the two identical bosons, λ is the strength of the BEC effect and r is the emission volume size.

2 Measurements of the source size r

2.1 The $r_{\pi^\pm\pi^\pm}$ and $r_{K^\pm K^\pm}$

In Table 1 we present the weighted average value of the $r_{\pi^\pm\pi^\pm}$ measured in the Z^0 decays where the large systematic error reflects the spread of the individual r values due to the choices of the reference sample. In the same table are also listed the $r_{K^\pm K^\pm}$ derived by DELPHI from the BEC of charged Kaons and the preliminary value of OPAL[1].

2.2 The $r_{K^0_S K^0_S}$ and the Generalised BEC

The three LEP experiments, OPAL, DELPHI and ALEPH, have also measured the BEC of the $K^0_S K^0_S$ pairs [2] obtaining the $r_{K^0_S K^0_S}$ values listed in Table 1. The $K^0_S K^0_S$ pairs have three sources where two of them, the $K^0 K^0$ and $\overline{K}^0 \overline{K}^0$, are identical di-bosons. As for the boson-antiboson pair $K^0 \overline{K}^0$, no BEC effect is expected if one sums up all their possible decay pairs $K^0_S K^0_S, K^0_L K^0_S, K^0_S K^0_L$ and $K^0_L K^0_L$. However by selecting only the $K^0_S K^0_S$ or the $K^0_L K^0_L$ pairs BEC enhancement is expected with a matching decrease of the $K^0_S K^0_L$ pairs. Thus the $r_{K^0_S K^0_S}$ could have been used for our $r(m)$ investigation was it not for the fact that the upper side of the width of the enigmatic $f_o(980)$ resonance is above the $K\overline{K}$ threshold. Recently it has been shown [3] that by invoking the generalised Bose-Einstein statistics and using I-spin invariance one can evaluate the minimum BEC contribution of $K^0_S K^0_S$ pairs to the low mass enhancement. If the over-all hadronic final state of the Z^0 decay is in a pure I=0, here denoted as ψ_o, then one has the relation:

$\sum_X P[\psi_o \to K^0_S(\vec{p})K^0_S(-\vec{p})X] = (1/2) \cdot \sum_X P[\psi_o \to K^\pm(\vec{p})K^\pm(-\vec{p})X] + \sum_X P[\psi_o \to K^0 \overline{K}^0 X \to K^0_S(\vec{p})K^0_S(-\vec{p})X]$ where X represents the hadrons accompanying the Kaon pair. From this relation follows that there will exist a BEC enhancement in the $K^0_S K^0_S$ pairs the height of which will at least be half of that present in the charged identical Kaon pairs. Since this analysis has not been done so far, the current quoted $r_{K^0_S K^0_S}$ values cannot serve our analysis. Here is to note that the generalised Bose statistics predicts that BEC enhancement should also occur in the $\pi^\pm \pi^0$ (not studied so far) system in a similar strength to that seen in the $\pi^\pm \pi^\pm$ pairs.

The r values obtained from the $\pi^\pm \pi^\pm$ and $K^\pm K^\pm$ BEC analyses of DELPHI and the average values obtained from existing LEP experiments are shown in Fig 3. A first indication for the decrease of r with the increase of the hadron mass is seen but the data within the errors are still compatible with a constant r value, or even with a slight rise, as m increases. This situation calls for a BEC analysis of higher mass hadrons like e.g. the $\eta'(958)$ meson. However with its very low production rate of about 0.14 per Z^0 decay, this avenue is closed to us.

2.3 Estimate of $r_{\Lambda\Lambda}$

It was pointed out some time ago by G. Alexander and H.J. Lipkin [4] that r values for pairs of baryons can be measured by observing the onset of the Pauli exclusion principle as the two identical baryons approach their threshold in their centre of mass system. The ratio ε, defined as $\varepsilon = (S = 1)/[(S = 1) + (S = 0)]$, measures the relative contribution of the S=1 state to the two Λ system. Experimentally this can be measured through the angular distribution of the angle between the decay protons of the two identical hyperons defined as $y^* = \cos(\mathbf{p_1 p_2})$. Here $\mathbf{p_1}$ and $\mathbf{p_2}$ are the momenta of the protons coming from the two Λ's decay defined after two transformations: first the Λ's are transformed to their centre of mass system

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Table 1: BEC results obtained at LEP1. (a) From spin composition analyses where no need exists for reference sample and Coulomb correction.

	λ	r (fm)	Experiment
$\pi^\pm\pi^\pm$	$-$	$0.78 \pm 0.01 \pm 0.16$	LEP Average
$K^\pm K^\pm$	$0.82 \pm 0.22 \; ^{+0.17}_{-0.12}$	$0.56 \pm 0.08 \; ^{+0.07}_{-0.06}$	OPAL(99)
	$0.82 \pm 0.11 \pm 0.25$	$0.48 \pm 0.04 \pm 0.07$	DELPHI(96)
$K^0_S K^0_S$	$1.14 \pm 0.23 \pm 0.32$	$0.76 \pm 0.10 \pm 0.11$	OPAL(95)
	$0.96 \pm 0.21 \pm 0.40$	$0.65 \pm 0.07 \pm 0.15$	ALEPH(94)
	$0.61 \pm 0.16 \pm 0.16$	$0.55 \pm 0.08 \pm 0.12$	DELPHI(96)
$\Lambda\Lambda$	$-$	$0.11 \pm 0.02 \pm 0.01$	ALEPH(99)
$\Lambda\Lambda^{(a)}$	$-$	$0.19 \; ^{+0.37}_{-0.07} \pm 0.02$	OPAL(96)
	$-$	$0.11 \; ^{+0.05}_{-0.03} \pm 0.01$	DELPHI(98)
	$-$	$0.17 \pm 0.13 \pm 0.04$	ALEPH(99)

and in the second each proton is transformed to the centre of mass of its parent Λ. Using this method OPAL, DELPHI and ALEPH have estimated the $r_{\Lambda\Lambda}$ value which are listed in Table 1. In order to determine the combined $r_{\Lambda\Lambda}$ value of these three experiments we have considered the values of $\varepsilon(Q)$ shown in Fig. 2a. These values were fitted to the expression $\varepsilon(Q) = 0.75 \, [1 - e^{(-r^2_{\Lambda\Lambda}Q^2)}]$ with the result shown by the solid line in Fig. 2a corresponding to

$$r_{\Lambda\Lambda} = 0.15^{+0.07}_{-0.04} \; fm.$$

In Fig. 2b we show the distribution $\Delta\chi^2 = \chi^2 - \chi^2_{min}$ with $\chi^2_{min} = 2.9$.
The DELPHI values (circles) and the LEP averaged values (triangles) of $r_{\pi^\pm\pi^\pm}$, $r_{K^\pm K^\pm}$ and $r_{\Lambda\Lambda}$ are plotted in Fig. 3 where a clear hierarchy is seen, namely

$$r_{\pi^\pm\pi^\pm} \; > \; r_{K^\pm K^\pm} \; > \; r_{\Lambda\Lambda}.$$

The error bars represent the statistical and systematic errors added in quadrature. The ALEPH collaboration has also attempted to measure $r_{\Lambda\Lambda}$ by considering, similar to the BEC studies, the ratio between the frequency of data Λ pairs to that of Monte Carlo generated sample as a function of Q. The observed depletion of pairs at low Q was used to estimate $r_{\Lambda\Lambda} = 0.11 \pm 0.02 \pm 0.01$ fm (see Table 1). This method, which has a relative small statistical error in comparison to the spin composition measurement, rests however on the assumption that the depletion of
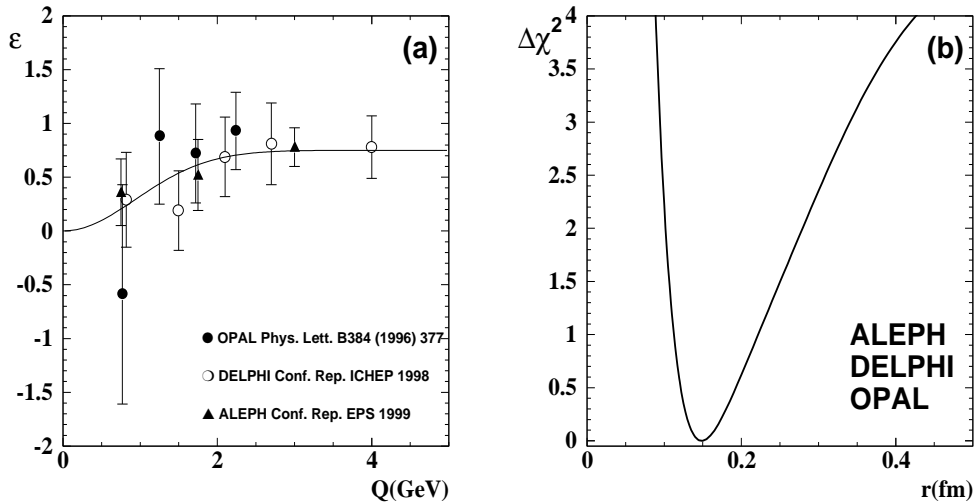
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Figure 2: (a) The S = 1 fraction, ε, of the $\Lambda\Lambda(\bar{\Lambda}\bar{\Lambda})$ pairs measured as a function of Q by the LEP collaborations. The solid line represents the fit results as described in the text. (b) The $\Delta\chi^2 = \chi^2 - \chi^2_{min}$ dependence on $r_{\Lambda\Lambda}$ obtained in the fit to the combined LEP data.

events at low Q is indeed due to the Pauli exclusion principle and not for any other reason. This method, furthermore, has to rely on a reference sample.

3   Comparison between r(m) and theoretical models

The fact that $r(m)$ decreases with the mass of the particle can be checked against the predictions of multi-particle production models of e^+e^- annihilations. In particular this $r(m)$ behaviour seems to contradict the expectation of the string (LUND) model where $r(m)$ should increase with the particle mass.

On the other hand the $r(m)$ behaviour can be described in terms of the Heisenberg uncertainty relations [6] i.e., $\Delta p \Delta r = 2\,\mu\,v\,r = m\,v\,r = \hbar c$ and $\Delta E \Delta t = 2[p^2/(2m)]\Delta t = \hbar$ where m is the mass of the hadron, v and p its velocity and momentum and r is spatial separation between the two hadrons. In ΔE, possible contributions from a potential energy are neglected. From these equations one obtains

$$r(m) = c\sqrt{\hbar \Delta t}/\sqrt{m} \simeq 0.243/\sqrt{m(GeV)}\ \ fm$$

when considering only the kinetic energy, equating $\Delta r = r$ and setting Δt to 10^{-24} seconds representing the strong interaction time scale. As can be seen in Fig. 3 the thin solid line, which represents the expectation from the uncertainty relations,
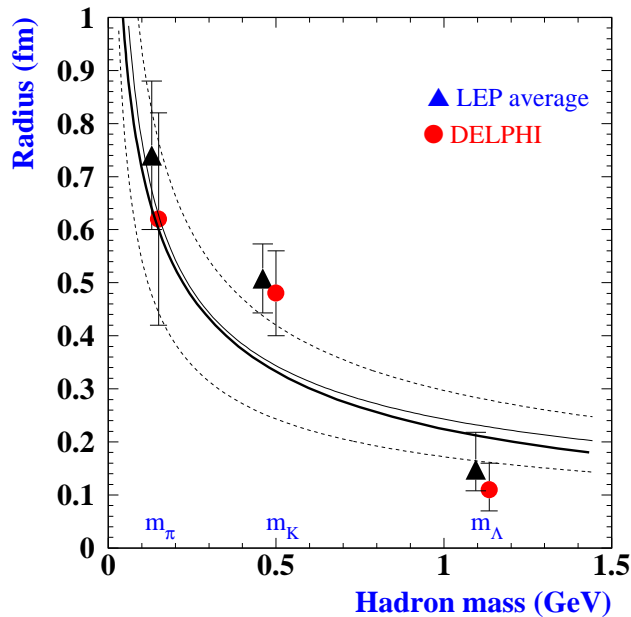
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Figure 3: The measured emitter radius r as a function of the hadron mass m compared to some theoretical predictions (see text).

describes fairly well the data both in magnitude and in shape. To illustrate the sensitivity to our particular choice of Δt we also show by the dashed lines the $r(m)$ expectations for $\Delta t = 1.5 \times 10^{-24}$ and 0.5×10^{-24} seconds. Rearranging the expression for $r(m)$ it is amusing to find out that $mr^2(m) = \text{Const.} \simeq 0.06$ GeV fm^2 i.e., the identical hadron pairs seem to emerge with the same moment of inertia irrespective of their mass value.

Another approach to describe the $r(m)$ dependence has been tried out[6]. Namely, by using the virial theorem in the frame work of the Local Hadron Parton Duality hypothesis. In this context the general QCD potential given by

$$V(r) = \kappa\, r - \frac{4}{3}\frac{\alpha_S \hbar c}{r}$$

was used with the parameters set of $\kappa = 0.14$ GeV$^2 = 0.70$ GeV/fm with $\alpha_s = 2\pi/9 \ln(\delta + \gamma/r)$, $\delta = 2$ and $\gamma = 1.87$ GeV$^{-1} = 0.37$ fm as obtained from the hadron wave functions and decay constants. The result of this QCD based approach is shown in Fig. 3 by the solid thick line differs only very slightly from the uncertainty relations outcome.

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4 Conclusion and summary

The source dimension r is found to change from ≈ 0.75 fm for charged pions to ≈ 0.15 fm for Λ's. This trend is opposite in direction to that expected in the basic string (LUND) model where $\partial r(m)/\partial m > 0$. It can however be described in terms of the Heisenberg uncertainty relations as well as in the frame work of the Local Parton Hadron Duality assumption. Both expect that $r(m) \simeq Constant/\sqrt{m}$. This relation leads to the conclusion that pairs of hadrons are produced with the same moment of inertia irrespective of their mass value, which presents a challenge for future attempts to formulate multi-particle production models. More precise r measurements for the source dimension will be beneficial for the comparison with models. Of particular interest will be the result from a BEC analysis of the $\eta\eta$ system which has a mass near the K-meson but it is strange-less and its production rate in the Z^0 hadronic decays is relatively high. At the higher mass end there is little chance to further explore the $r(m)$ behaviour. The production rate of the $\eta'(958)$ is too low and so are also the rates for the hyperons above the Λ baryon. Presently one cannot utilise for the $r(m)$ analysis the measured $r_{K_S^0 K_S^0}$ values due to the fact that they may be influenced by the enigmatic $f_o(985)$ resonance decay into the $K^0\overline{K}^0$ channel. In the frame work of the generalised BEC and I-spin invariance there is a possibility to separate the two sources of the $K_S^0 K_S^0$ system. This generalised BEC extension can be verified by observing BEC in the $\pi^\pm\pi^0$ pairs coming e.g. from hadronic Z^0 decays.

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