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About the helix structure of the Lund string

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Abstract

The helix structure of the Lund string, first derived from studies devoted to the emission of soft gluons at the end of the parton cascade, may be at the origin of a part of the 'correlations' observed in multidimensional analyses of the Bose-Einstein effect. It is found that a helix structure of the string corresponding to an emission of soft gluons from a regularly spinning source is supported by the data, more precisely by the inclusive single-particle spectra measured in the hadronic decay of Z^0.

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1 Introduction

The Lund string fragmentation model [1, 2] uses the concept of a string to model the confining colour field between partons carrying complementary colour charge. The string is viewed as an object composed of straight pieces stretched between individual partons according to the colour flow. Each piece of string has a uniform energy/momentum density along its axis. The fragmentation of such an object via the tunneling effect (creation of a pair of a quark and an antiquark with compensating transverse momenta) with a probability given by the fragmentation function provides the set of hadrons, each built from a $q\bar{q}$ pair (ev.diquark in case of baryons) and a piece of string between the two neighbouring string break-ups.

The model gives a fair description of the available high-energy hadronic data and is readily used for the simulation of hadronization at LEP. The parameters of the model are tunable and allow to adjust the simulation to the experimental data to some extent, sometimes at the risk of obscuring the physical origin of observed discrepancies.

The development of the model follows several lines : an improved description of a parton shower cascade (perturbative QCD) which defines the topology of the string, and an improved description of the string itself (nonperturbative QCD). The question of how the string is actually built is rarely evoked, even if the answer may be crucial both for string fragmentation and for parton shower development; the formation of the string should be a process parallel to parton showering, if the memory of the colour flow is to be preserved in the form of a homogenous colour field of the string.

A very interesting work devoted to the study of the properties of the emission of soft gluons was published by Andersson et al.[3]. The soft gluons emitted continuously from a current are shown to tend to form a helix-like field, due to the constraint imposed on the emission angle by helicity conservation, and using the assumption that the colour-connection between gluons tends to be as short as possible. Quantum-mechanical arguments further imply that gluons with transverse momentum lower than ~ 1.6 GeV with respect to the string axis cannot change the overall string topology. The helix structure of the field should therefore appear in the internal structure of the string rather than in gluonic excitations of the string.

2 Helix structure of the string

The implementation of a string with a helix structure radically changes the way hadrons acquire their transverse momentum. In the conventional ('standard') Lund model, the transverse momentum of the hadron is the (vectorial) sum of the transverse momenta of the (di)quarks which were created via a tunneling process during the breakup of the string. In practice, these transverse momenta are randomly picked up from a gaussian distribution in a random direction.

In the helix-like string, hadrons obtain their transverse momentum from the soft gluons forming the string (so there is no explicit need to create the transverse momentum via the tunneling effect), see Fig.1:

$$\vec{p}_{\rm t}(hadron) = \int_{\Phi_i}^{\Phi_j} \frac{\vec{p}_{\rm t}(gluon)}{{\rm d}\Phi} {\rm d}\Phi$$

where $\Phi_{i(j)}$ is the 'phase' of the helix (azimuthal angle) in the break-up point $i(j)$.

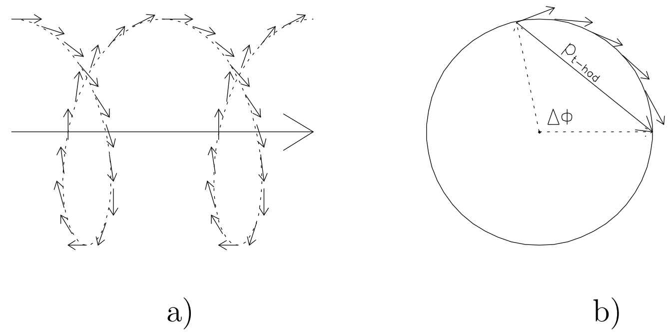

Figure 1: *a) The helix structure of the string due to the emission of soft gluons from the spinning source. b) After fragmentation, the transverse momentum of a direct hadron is the integral of the transverse momenta of the emitted gluons, integrated over the corresponding string piece.*

The most important consequence of the implementation of the helix structure is that the longitudinal and transverse momentum components of the string fragments (the hadrons) are no longer independent.

3 Different models for the helix string

The purpose of this section is to discuss the definition of the parameters of the helix string.

In [3], the phase difference of the helix winding was related to the rapidity difference of the emitting current. The current itself was parameterized in terms of rapidity and its differential linked to the emitted field quantum (a soft gluon). Requiring the emission of a real gluon, the parameterization of the helix structure of the resulting colour field was derived:

$$\Delta\Phi = \frac{\Delta y}{\tau}, \tag{1}$$

where τ is a parameter, and Δy is the rapidity difference.

The definition of the rapidity of the current was not explicitly given in [3], however for the practical purpose of the implementation of the helix structure into the Lund model, it was associated with the standard Lund definition of the rapidity

$$y = 0.5\ \ln(\frac{k^+p^+}{k^-p^-}), \tag{2}$$

where $p^{+,-}$ are the light-cone momenta of the quarks forming the string and $k^{+,-}$ their fractions defining a point along the string, see Fig.2.

The rapidity difference between two points along the string is

$$\Delta y = 0.5\ \ln(\frac{k_i^+k_j^-}{k_i^-k_j^+}) \tag{3}$$

and it is related to the *angular* difference of points in the string diagram (Fig.2).

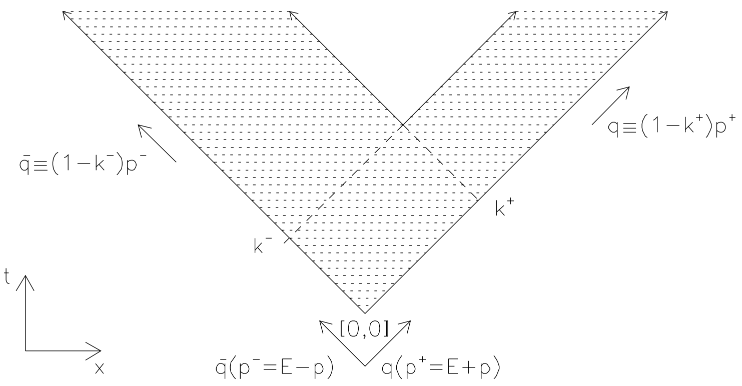

Figure 2: *Evolution of the string in the rest frame of the $q\bar{q}$ pair, including the first string break-up. The partons lose their momentum as they separate and the string - the confining field - is created. The space-time coordinates can be obtained from the relation [t,x]=(k^+p^++k^-p^-)/κ ($\kappa \sim$ 1 GeV/fm). The x direction is parallel to the string axis.*

The emitted soft gluon, one of the stream of gluons forming the field, represents a small part of the string with a length given by the energy of the gluon ($dl \sim \epsilon/\kappa$, where ϵ is the gluon energy and κ the string constant). Since such a piece of string cannot be associated with a rapidity difference defined by Eq.3 in a unique way, the form of the helix structure will be non-homogenous along the string and will depend on the actual position along the string. This is indeed a picture consistent with a fast emission of harder gluons by quarks and subsequent emission of softer gluons diluting the energy density of the string. However, the standard Lund fragmentation model is based on the assumption of the uniformity of the string field during the lifetime of the string, so in this sense there is a clash between the helix model and the fragmentation model.

The evolution of the phase of the helix string defined according to (1) is illustrated for a simple $q\bar{q}$ string in the string diagram Fig.3. The phase is fixed by a random choice at one point of the diagram (in our example Φ=0 at [k^+,k^-]=[1,1]) and is calculated for the rest of the diagram from Eq.1 with help of Eq.2. The parameter τ is set to 0.3 for definiteness, its value is irrelevant for discussion of the qualitative features of the model.

As stated above, the density of the Lund helix winding is non-homogenous along the string axis. Near the middle of the string ($k^+ = k^-$), the phase evolution is quite slow with the time, but it becomes faster near the ends of the string. The form of the helix along the whole string is defined independently from the string energy so that the change of the phase of the helix per unit of length of the string would be faster for strings with lower invariant mass.

The above features of the Lund helix model, as well as further reasons given below, led the author to consider an alternative "modified" model of the helix structure of the string field, based on the uniformity of the colour field required by the Lund fragmentation model.

The main arguments which lie behind the idea of the helix structure of the string as such are following:

– because of the large effective coupling α_s, there is a tendency to emit as many soft gluons as possible, while
– the helicity conservation imposes a certain minimal emission angle

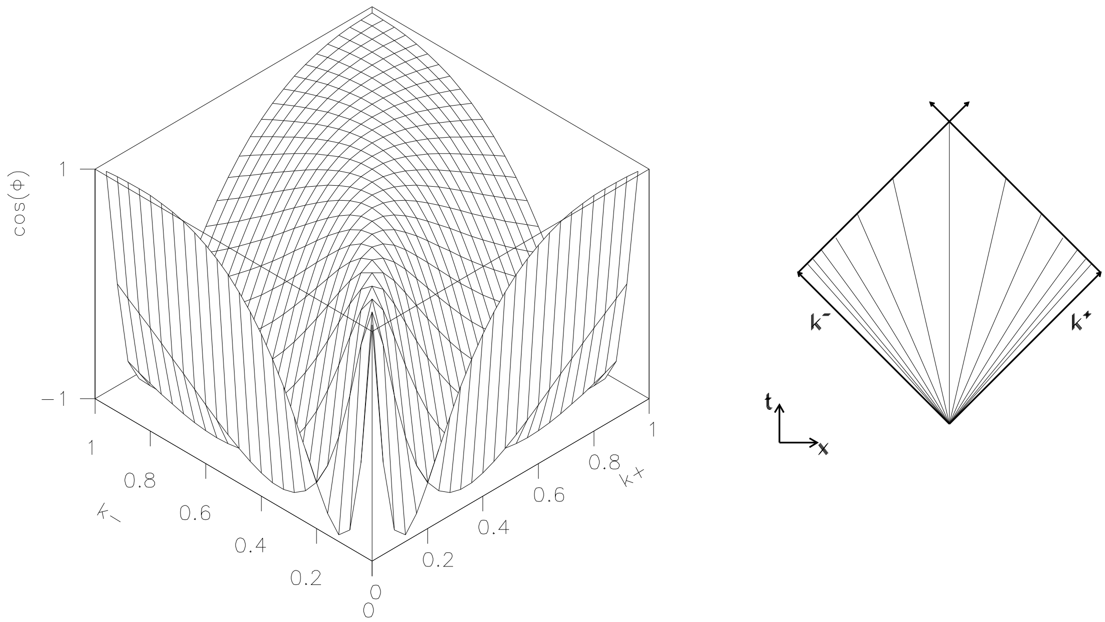

Figure 3: *On the left: the phase of the helix winding along the string according to the Lund prescription (Eq.1) for parameter τ=0.3. The phase was fixed by a random choice at one point. On the right: for better illustration of the phase evolution in the space-time coordinates, the points with equal phases are connected by lines in equidistant phase intervales.*

These arguments are valid both for $q \to qg$ and $g \to gg$ emissions. We assume a constant energy/momentum loss of the parent quarks, in agreement with the uniform longitudinal energy density of the created colour field. Since we know already that the favoured configuration of emitted soft gluons is helix-like, and the emitting quark has to absorb the recoil, we parameterize the quark momentum by

$$p_q(t) = [E_0 - \frac{dE}{dt}t, p_{l0} - \frac{dp_l}{dt}t, p_t\ \cos\Phi, p_t\ \sin\Phi] \tag{4}$$

The movement of the quark in the transverse plane generates an effective mass p_t for massless quarks, or $\sqrt{m_0^2 + p_t^2}$ for massive ones.

We require dE/dt=const, dp_l/dt=const together with constant longitudinal velocity of the quark in order to assure the uniform energy density:

$$\frac{p_{l_0} - \frac{dp_l}{dt}t}{E_0 - \frac{dE}{dt}t} = \frac{p_{l_0}}{E_0}\frac{1 - \frac{dp_l}{p_{l_0}}\frac{t}{dt}}{1 - \frac{dE}{E_0}\frac{t}{dt}} = \text{const} \tag{5}$$

which implies

$$\frac{dp_l/dt}{dE/dt} = \frac{dp_l}{dE} = \frac{p_{l_0}}{E_0} = \beta. \tag{6}$$

It should be noted that the rapidity of the quark is constant in this case:

$$y_q(t) = 0.5\ \ln\frac{E(t) + p_l(t)}{E(t) - p_l(t)} = 0.5\ \ln\frac{E_0 + p_{l0}}{E_0 - p_{l0}} \tag{7}$$

as well as the rapidity of the emitted gluons:

$$y_g = 0.5 \ln \frac{dE + dp_l}{dE - dp_l} = 0.5 \ln \frac{1 + \beta}{1 - \beta} \tag{8}$$

and that they are equal

$$y_q = y_g \tag{9}$$

independently of the energy of the emitted gluons.

We further set the difference in the helix phase to be proportional to the emitted energy

$$\Delta\Phi = \mathcal{S}\Delta E = \mathcal{S}\ \frac{\Delta k^+ + \Delta k^-}{2}\ M_0, \tag{10}$$

where M_0 stands for the invariant mass of the string and $\mathcal{S}$ is a parameter.

We consider the field quanta bound by the colour forces so that their longitudinal movement along the string is effectively compensated by the self-interaction of the field, and we suppose that the helix structure is preserved during such an interaction.

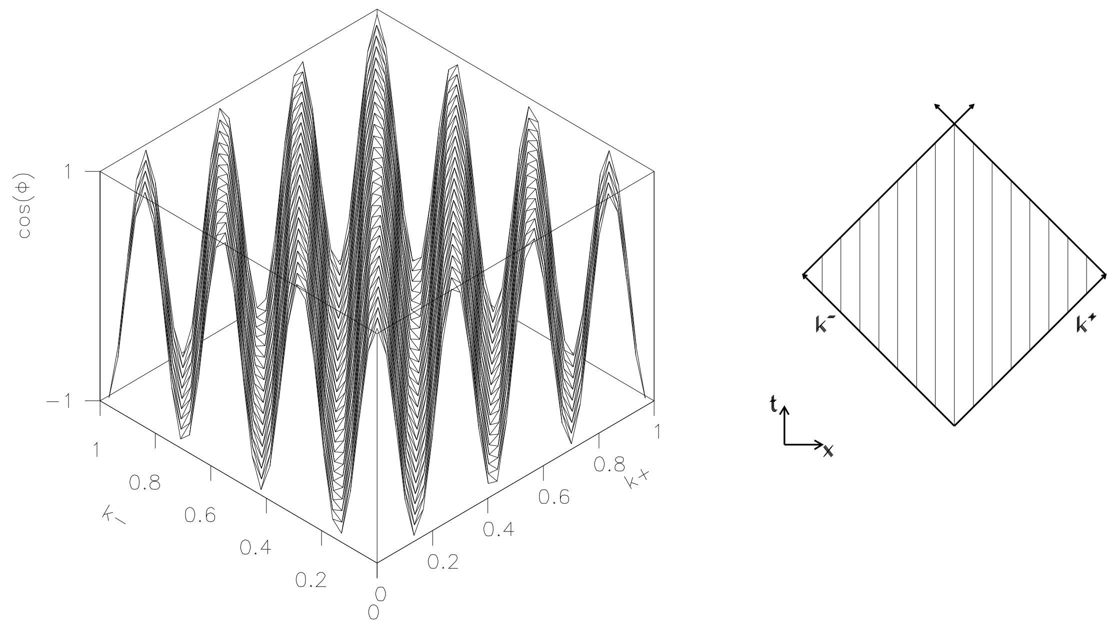

Figure 4: *On the left: the phase of the modified helix (Eq.10) for parameter S=0.5 rad/GeV and invariant mass of the string M_0=91.22 GeV. The phase was fixed by a random choice at one point. On the right: for better illustration of the phase evolution in the space-time coordinates, the points with equal phases are connected by lines in equidistant phase intervales.*

As shown in the string diagram of Fig.4, the definition Eq.10 corresponds to a helix with a constant pitch along the string (proportional to the energy density of the string). The phase in the modified helix scenario is constant in time for a given point along the string axis, forming a stationary wave (similar to the interference pattern due to an emission of waves from two sources).

The two alternative models of the helix string imply quite different observable effects.

In the Lund helix model, the phase difference is directly related to the rapidity difference. The fragmentation of the Lund string produces roughly one hadron per unit of rapidity. The hadrons, therefore, obtain – on average – a transverse momentum of about the same size, i.e. roughly independent of y (its fluctuation is however related to the fluctuation in rapidity), and the helix-like structure should be visible in the azimuthal angle as a function of the rapidity ordering.

The modified helix, on the other hand, introduces a strong correlation between the energy of the hadron and the size of its transverse momentum. In the rest frame of the string,

$$p_t(\text{hadron}) = 2\ r\ \sin\frac{\Delta\Phi}{2} = 2\ r\ \sin\frac{\mathcal{S}E_{had}}{2}. \tag{11}$$

where r (the 'radius' of the helix) has the physical meaning of the size of the transverse momentum of the mother parton (using the assumption that this quantity doesn't fluctuate significantly along the string piece), and $\Delta\Phi$ is the helix phase difference between the two break-ups which created the hadron.

It should be noted (as pointed out in [3]) that in either case the helix structure of string 'eats up' one degree of freedom in fragmentation. In the conventional Lund model, the size and the direction of the transverse momentum of the hadron are chosen independently, and the possible kinematical solutions lie along a hyperbola in the [k$^+$,k$^-$] plane (see [1, 3]):

$$\Delta k^+ \Delta k^- M_0^2 = m^2 + p_t^2. \tag{12}$$

The helix structure defines the transverse momentum as a function of $[\Delta k^+, \Delta k^-]$, so that the next break-up has to be found as an intersection of the hyperbola with the line corresponding to a given phase difference.

4 Observable effects related to the helix structure of the string

The search for an observable effect that could test the helix model was launched in [3] with the introduction of a variable called screwiness:

$$S(\omega) = \sum_e P_e |\sum_j \exp(i(\omega y_j - \Phi_j))|^2, \tag{13}$$

where y_j, Φ_j stand for the rapidity and the azimuthal angle of a final hadron, P_e is a normalization factor and ω a parameter related to the characteristic frequency of the helix rotation. The first sum goes over hadronic events (in our case, hadronic Z^0 decays), the second one over hadrons in a single event. The variable is sensitive to the rapidity-azimuthal angle correlations, as shown in Fig.5a, where it is plotted for final hadrons from fragmentation of a simple $q\bar{q}$ string [1)]. A peak appears around $\omega \sim 1/\tau$, but its significance is reduced as the τ parameter decreases. No peak structure is observed in the modified helix model (Fig.5b) for $\mathcal{S}$=0.5 rad/GeV.

[1)] Here and elsewhere in this paper, the radius of the helix string is picked from a gaussian distribution with parameters tuned in order to restore the mean multiplicity and the mean transverse momentum of the final particles, see Appendix.

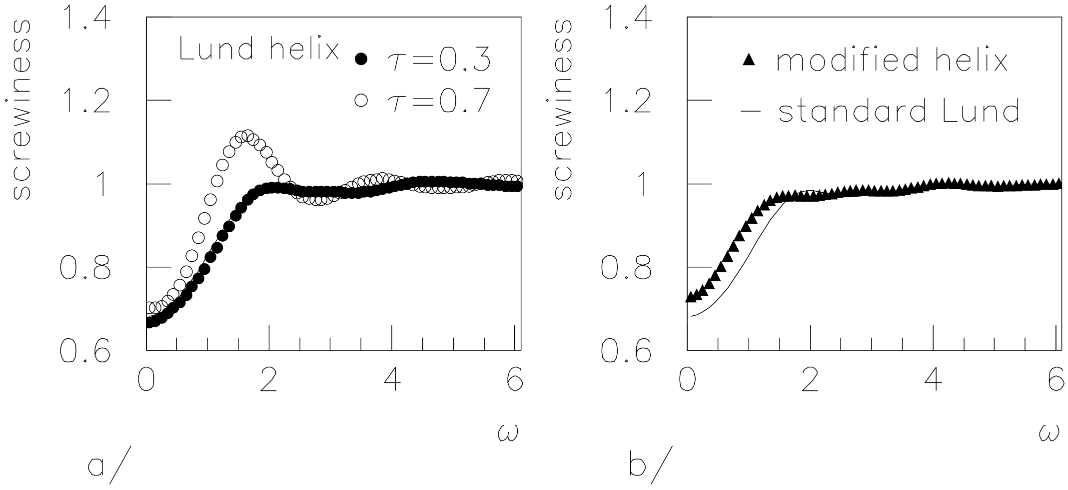

Figure 5: *Screwiness in the fragmentation of a simple $q\bar{q}$ string: a) Lund helix, b) modified helix. The conventional Lund model is marked by the full line in both plots.*

The signal is strongly reduced when parton showering is included in the simulation. To distinguish the signal, events with low emission of hard gluons have to be selected (with help of a cut on the Thrust value of the event) and the signal has to be searched in the low-rapidity region (it should be noted, however, that the rapidity of the hadron differs from the rapidity difference in Eq.1).

The preliminary search for a signal of screwiness in the LEP data [4] gave a negative result (setting an upper limit on the range of the τ parameter). In this paper, the attention will be concentrated on the energy-transverse momentum correlations introduced by the modified helix model (Eq.10).

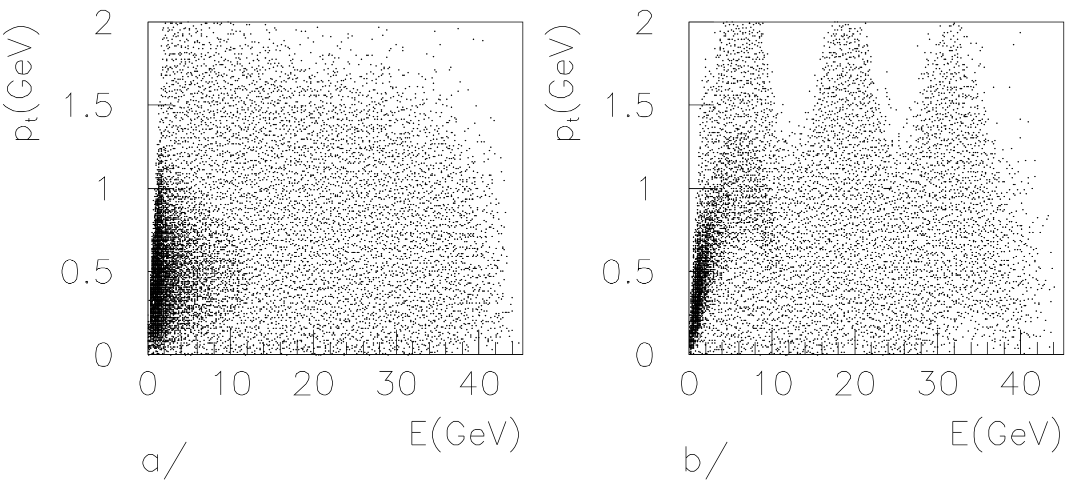

Figure 6: *Transverse momentum versus the energy of direct hadrons from the fragmentation of a simple $q\bar{q}$ string in a) conventional Lund model (JETSET) and b) model with modified helix structure.*

Fig.6 shows the transverse momentum versus energy distribution for the direct hadrons from the fragmentation of a simple $q\bar{q}$ string in the conventional Lund model (Fig.6a) and in the model with modified helix structure (Fig.6b). In the latter, a sinusoidal structure is clearly visible, despite the fluctuations. (The Lund helix model does not show any visible differences with respect to the conventional Lund model, and the figure is not included).

4.1 Helix string and single-particle spectra

Somewhat surprisingly, it is the modified helix structure of the string (Eq.10) that seems to be supported by the LEP data. The structure shown in Fig.6 is smeared out by the parton showering, but the remaining features are favoured by the single-particle spectra. In this section, the available indirect evidence will be gathered and discussed.

Let us recollect that, in general, hadronization models fail to describe the transverse momentum spectra observed in hadronic Z^0 decay, especially at high values of the transverse momenta p_{t-in} and p_{t-out} and of the scaled momentum x_p, even after the available model parameters have been tuned to fit the data (see Fig.7a,8a taken from [5]). In Fig.7a and 8a it can be seen that the relative number of hadrons with very high transverse momentum, especially at very high momentum, is underestimated by all of the models. A characteristic discrepancy is observed in the shape of the transverse-momentum distribution which points to a problem in the understanding of this physical quantity.

These spectra, however, turn out to be sensitive to the helix structure of the string, as seen in a comparison of the simulation with the conventional Lund model and with the helix string model (Fig.7b,c and 8b,c). There is a striking similarity between the shape of the experimental data and that of the model with modified helix structure. The simultaneous change in the scaled and transverse momentum distributions is a non-trivial consequence of the energy-transverse momentum relation imposed by the helix structure of the string.

The comparison is less favourable for the original Lund helix, even if it modestly improves the data/MC agreement, too (see tail of the p_{t-out} and scaled momentum distribution).

Several comments can be made at this point, concerning the modified helix model. First of all, there is a clearly observable effect in the basic distributions related to the transverse and scaled momentum. More important still is that the effect concerns variables which are not well described by the currently used hadronization models, and even more so that the modified helix string seems to be able to actually cure the problems. Whether this fact implies the correctness of the physical picture of the model is a matter of further investigation. It is interesting, however, to see the non-perturbative effects on that part of the spectrum supposed until now to be dominated by hard gluon emission (the tail of the transverse-momentum distribution).

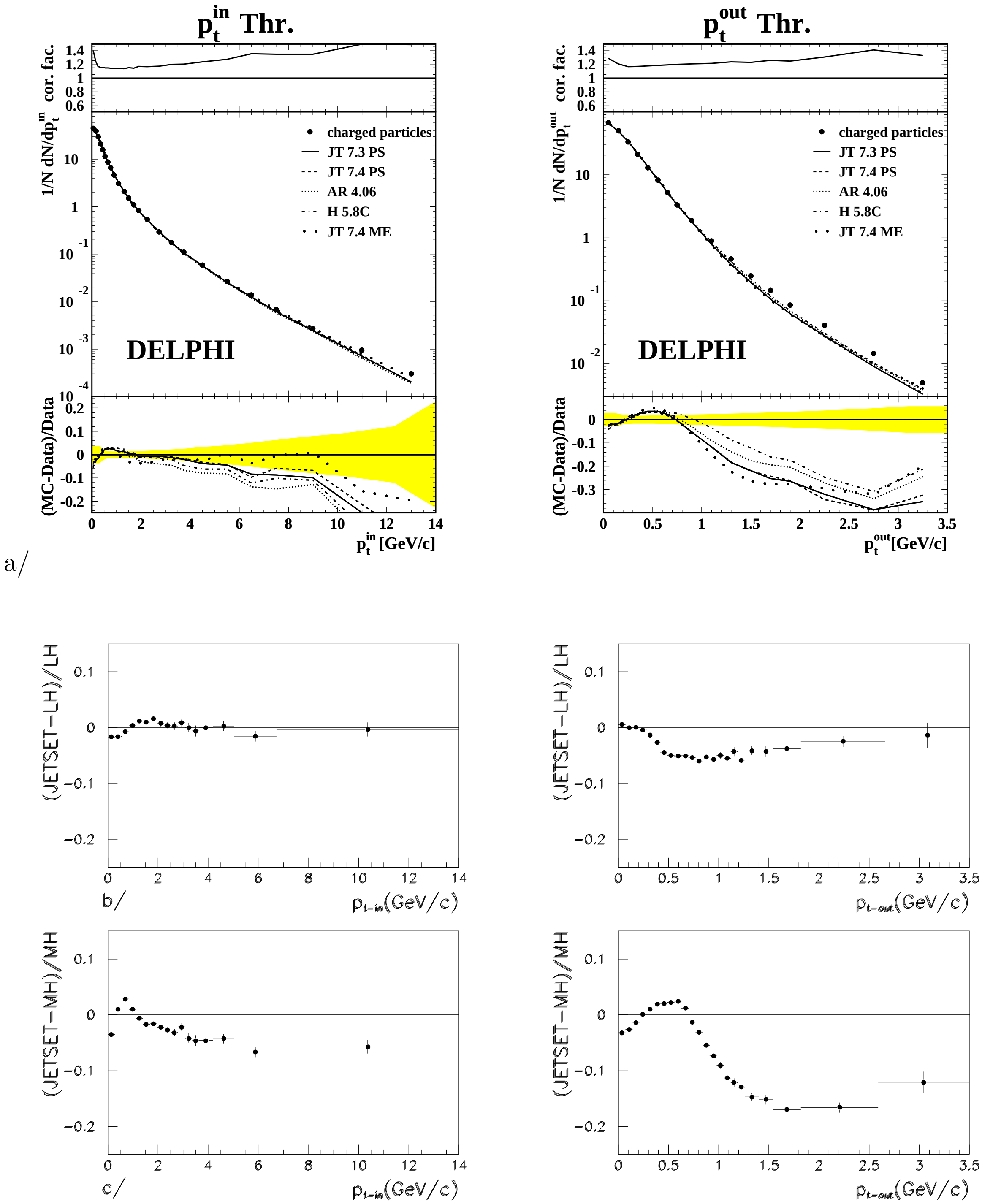

Figure 7: a) Comparison of the Z0 data and MC in the transverse-momentum distributions [5], $p_{\mathrm{t-in(out)}}$ being the projection of the particle momentum onto the major(minor) axis of the event. b)-c) Effect of the implementation of a string with helix structure on these distributions: b/ Lund helix (LH) compared to JETSET 7.4 PS(τ=0.7), c/ modified helix (MH) compared to JETSET 7.4 PS ($\mathcal{S}$=0.5 rad/GeV).

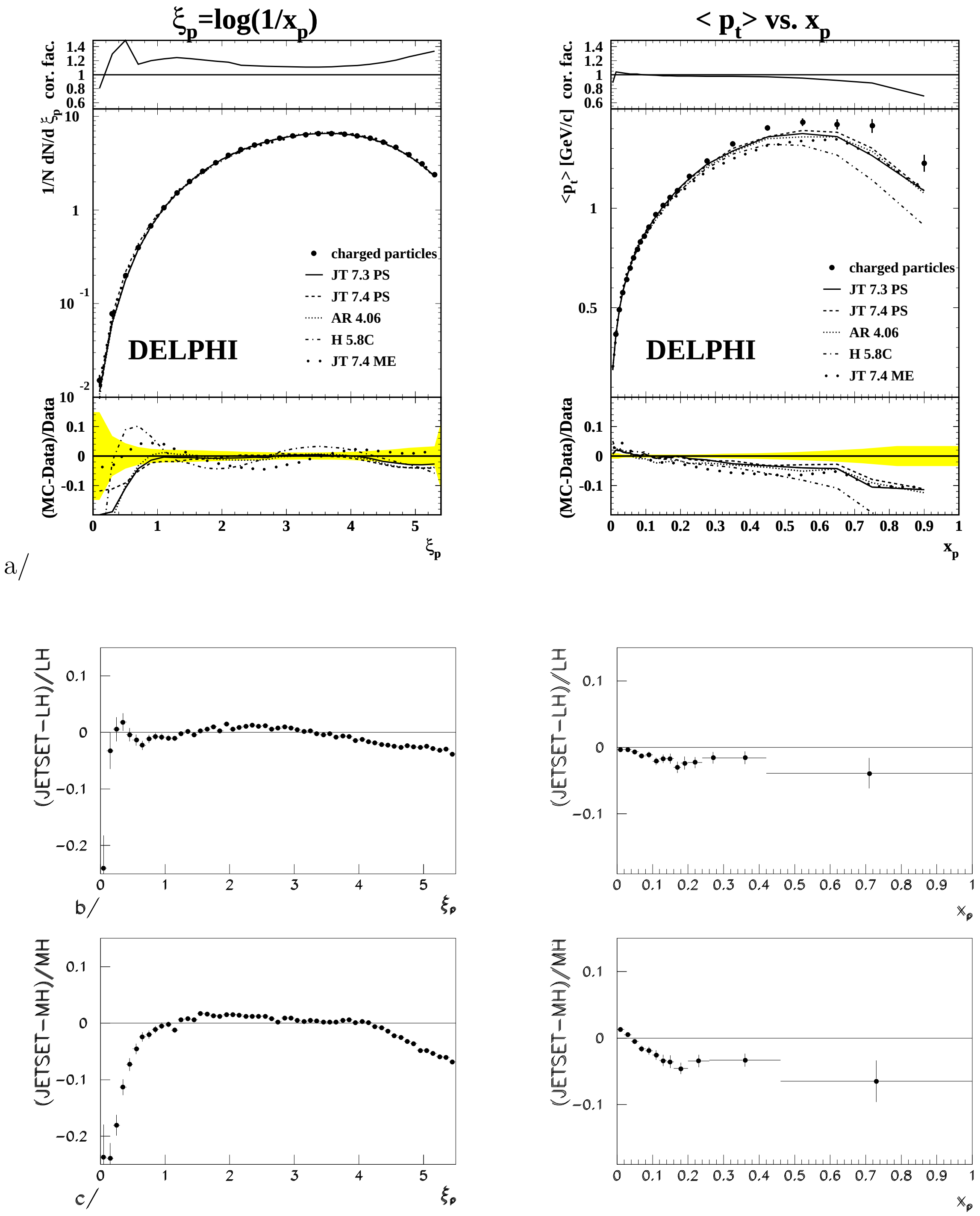

Figure 8: a) Comparison of the Z0 data and MC in the scaled-momentum distribution and mean p_t versus scaled momentum [5]. b)-c) Effect of the implementation of a string with helix structure on these distributions: b/ Lund helix (LH) compared to JETSET 7.4 PS (τ=0.7), c/ modified helix (MH) compared to JETSET 7.4 PS ($\mathcal{S}$=0.5 rad/GeV).

4.2 Helix string and two-particle correlations

The study of particle correlations, in particular those associated with Bose-Einstein (BE) interference (also called the Hanbury-Brown-Twiss effect), is in fact the main reason why the modified helix structure was introduced. Without going into details irrelevant for the present discussion, let me remind the reader that the Lund string fragmentation has a quantum-mechanical formulation (due to the work published in [6]) which allows the full symmetrization of the matrix element associated with fragmentation and results in interference terms describing the BE correlations in the longitudinal direction with respect to the string axis. Yet, recent multidimensional analyses of the BE effect [8] show clearly that correlations exist also in the transverse direction.

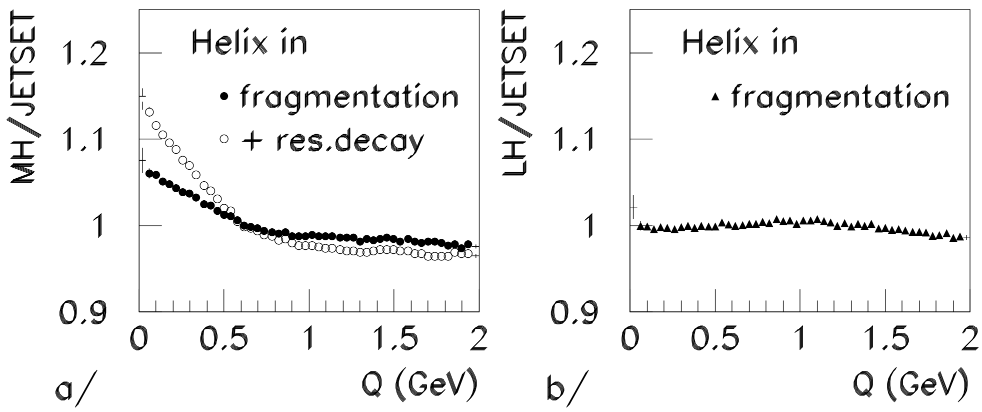

Figure 9: *Comparison of the momentum-transfer distributions for like-sign pairs from hadronic Z^0 decay: a) the ratio (modified helix model(MH)/conventional Lund model(JETSET)), b) the ratio (Lund helix model(LH)/conventional Lund model(JETSET)).*

There are indeed several ways how to include transverse correlations in the model (one of them, based on the tunneling mechanism, was described in [7]), but the helix string seems to be a particularly elegant one: if a tight relation between the transverse and longitudinal components of the momentum is installed, correlations should appear simultaneously in both directions. That was the original idea behind the modified helix scheme, but the realization has surpassed the expectations. Besides the effect on the single-particle spectra discussed in the previous section, also the two-particle spectra are significantly influenced by the helix structure of the string, even in the absence of genuine BE correlations.

In Fig.9, the 2-particle momentum transfer $Q = \sqrt{-(p_1 - p_2)^2}$ is compared for the helix-like string and the conventional Lund string; the ratio of the two distributions is a correction to the 2-particle correlation function due to the helix structure of the string. The modified helix model, contrary to the Lund helix model, visibly enhances the two-particle density function, and therefore mimics, to some extent, genuine BE correlations.

This behaviour is largely due to the modification of the single particle momentum spectrum. As can be seen in Fig.10, the effect comes mostly from the transverse component of the correlation function and is visibly enhanced when the helix structure of the string is maintained through the decay of the shortly living resonances (open circles).

As long as the helix structure is regular all along the string, the effect is similar

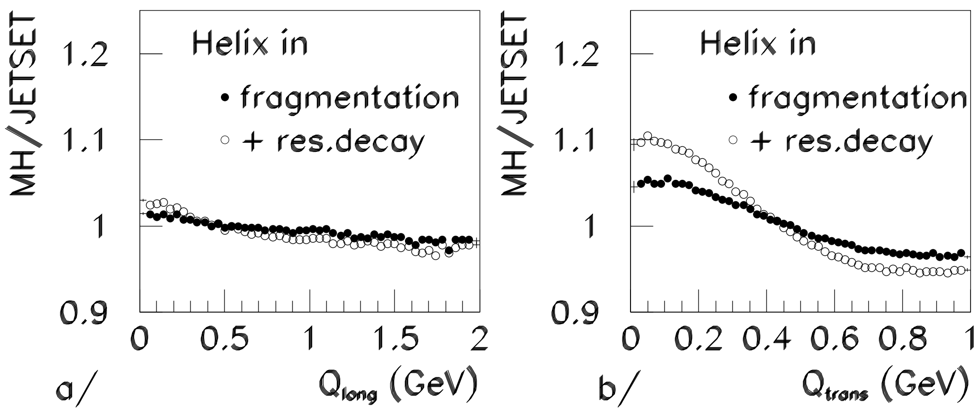

Figure 10: *Comparison of the string with the modified helix(MH) structure and the conventional Lund string. The momentum-transfer distribution in longitudinal and transverse projection (calculated in the LCMS system) and their ratio in Z^0 decay are shown for like-sign pairs of particles.*

for both like-sign and unlike-sign pairs of particles, see the shift of the mass spectrum of the unlike-sign pairs in Fig.11a and in particular the enhancement of the unlike-sign two-particle density function in Fig.11b, to be compared with Fig.9a.

5 Conclusions

The properties of fragmentation of a string with helix-like structure are discussed. Two different helix structures are studied, the original Lund helix model, where the pitch is related to the rapidity difference, and an alternative model with the helix structure regular with respect to the energy density of the string.

In both models, the helix structure imposes correlations between the transverse and longitudinal momentum of the final hadrons.

The two models differ however at the level of the underlying physical picture of the emission of soft gluons; the Lund helix model considers emission of soft gluons from the colour current, while the modified helix scenario supposes the helix structure is created by the emission of soft gluons from the quark.

The difference between the models translates into different types of observable effects. In the Lund helix model, the direction of the transverse momentum becomes correlated with the rapidity of the hadron, while in the modified helix scheme, the size of the transverse momentum depends on the energy of the hadron.

A comparison with the LEP data indicates that there is abundant indirect experimental evidence for longitudinal-transverse momentum correlations similar to those originating from regular helix-like structure of the string.

It would therefore be reasonable to review the available experimental data in terms of the helix string model, and to extend the analysis to multidimensional studies of event shapes. The effects related to the helix structure are smeared by hard-gluon emission.

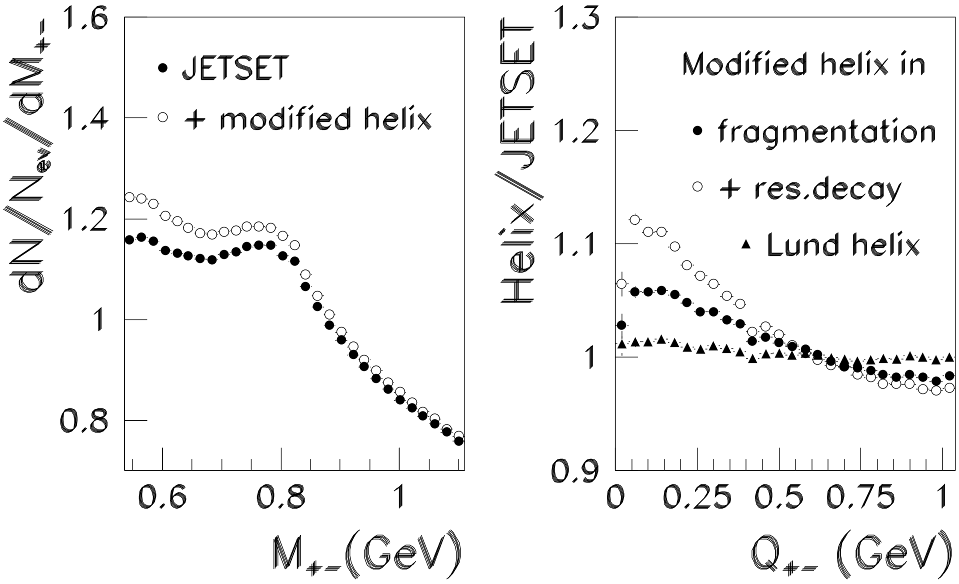

Figure 11: *On the left: Comparison of the string with modified helix structure to the conventional Lund string. The mass distribution for unlike-sign pairs of particles from the Z^0 decay in the region of ρ^0(770) resonance. On the right: The comparison of the momentum-transfer distributions for unlike-sign pairs from hadronic Z^0 decay. The ratio (helix model/JETSET) is shown for different helix models.*

Therefore, a dependence on the thrust value, as well as on the jet structure of the events, should be observed.

If the helix string proves to be a viable model, the consequences for the study of the BE effect are rather serious. Part of the correlations observed in the data can be eventually attributed to the helix structure rather than to genuine BE correlations, and problems may arise around the construction of the reference (uncorrelated) sample.

In summary, the detailed study of hadronic data may provide unexpected insights into the mechanism of confinement and the structure of the confining field, if only the right questions are formulated and incorporated into hadronization models.

Acknowledgements

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Appendix

The set of relevant JETSET parameters used in the present study is shown in Table 1. The mean multiplicity of final state particles with momentum above 0.1 GeV is shown, as well.

parameter	Λ[GeV]	Q_0[GeV]	Lund **a**	Lund **b**[GeV^{-2}]	σ_q[GeV]	$< N >$
value	0.297	1.56	0.417	0.844	0.408	39.21±0.02

Table 1: *Set of JETSET parameters.*

When the conventional Lund string is replaced by the string with helix structure, the width of the transverse momentum distribution σ_q becomes obsolete, and it is replaced by the parameters defining the mean 'radius' r of the helix (i.e. the size of the transverse momentum of the mother quark), its spread and the pitch of the helix.

For the Lund helix, the value of these parameters was taken from [9], see Table 2 (the radius of the helix is $r = \tau \cdot m$).

parameter	τ	σ_τ	m [GeV]	$< N >$
value	0.3	0.2	1.1	39.22±0.08
value	0.7	0.35	0.68	39.62±0.02

Table 2: *Set of parameters for the Lund helix model.*

The set of parameters used in the present study for the modified helix string is shown in Table 3. A rough tuning of these parameters was performed, reducing to $\sim 1\%$ the mean multiplicity difference with respect to the standard JETSET.

parameter	$\mathcal{S}$ [rad/GeV]	r [GeV]	σ_r [GeV]	$< N >$
value	0.5	0.5	0.25	39.71±0.02

Table 3: *Set of parameters for the modified helix model.*

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