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Parametric Resonance For Complex Fields

Rouzbeh Allahverdi and R. H. A. David Shaw

Department of Physics, University of Alberta

Edmonton, Alberta, Canada T6G 2J1

Bruce Campbell

Theory Division, CERN

CH1211, Geneve 23, Switzerland

Abstract

Recently, there have been studies of parametric resonance decay of oscillating real homogeneous cosmological scalar fields, in both the narrow-band and broad-band case, primarily within the context of inflaton decay and (p)reheating. However, many realistic models of particle cosmology, such as supersymmetric ones, inherently involve complex scalar fields. In the oscillations of complex scalars, a relative phase between the oscillations in the real and imaginary components may prevent the violations of adiabaticity that have been argued to underly broad-band parametric resonance. In this paper, we give a treatment of parametric resonance for the decay of homogeneous complex scalar fields, analyzing properties of the resonance in the presence of out of phase oscillations of the real and imaginary components. For phase-invariant coupling of the driving parameter field to the decay field, and Mathieu type resonance, we give an explicit mapping from the complex resonance case to an equivalent real case with shifted resonance parameters. In addition, we consider the consequences of the complex field case as they apply to “instant preheating,” the explosive decay of non-convex potentials, and resonance in an expanding FRW universe. Applications of our considerations to supersymmetric cosmological models will be presented elsewhere.

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1 Introduction

With the advent of inflationary theories of the early universe, it has been argued that the present stage of hot FRW “big-bang” cosmology was preceded by an epoch of cosmological evolution dominated by the dynamics of scalar fields [1]. The success of inflationary models in providing explanations for flat large-scale geometry (as suggested by the location of the acoustic peaks in the CMBR anisotropies), and for the origin of approximately scale-free adiabatic density perturbations (which can be used to simultaneously fit both the CMBR anisotropies and observations of cosmic structure formation), lends support to the idea of an early scalar-field dominated epoch. A crucial question in this picture is the nature of the transition from the scalar field dominated epoch, to the hot FRW epoch, which is referred to as reheating. The nature of this transition also relates to other aspects of early universe dynamics necessary for a successful cosmology, such as mechanisms of baryogenesis, the resolution of cosmological moduli problems, and possible sources for non-thermal dark matter.

The standard approach to reheating, which applies to sufficiently weakly coupled inflaton fields [2], is to treat quanta of the inflaton field as particles, which undergo independent single particle decay; this treatment, if adequate, has the advantage that the post-inflation reheat temperature is determined by the microphysics of the model. For inflaton fields with mass as suggested by the simplest chaotic or supersymmetric models, and decaying by gravitational strength interactions, this treatment is adequate, leading to moderate reheat temperatures ($\lesssim O(10^{10})$ GeV) which are consistent with the absence of GUT-scale defects such as monopoles, which are capable of incorporating a variety of (s)leptogenesis or electroweak mechanisms for generation of the BAU, and which avoid, in the supersymmetric case, cosmological problems with thermal overproduction of gravitinos after reheating [3].

Recently it has been realized that the standard treatment of reheat in terms of single particle inflaton decay may be seriously misleading in circumstances where there is coherent enhancement of the transition to bosonic decay products [4–16]. For large mode occupation numbers of the decay product field we may treat its dynamics as being essentially classical. Mode by mode for the decay product field its coupling to the oscillating inflaton field induces a periodic time dependence in the mode mass (modulated by cosmic expansion which “sweeps” the time dependence of each comoving mode through the bands of the stability chart of the mode equations.) This periodic modulation of the parameters of the oscillator associated with each mode of the decay product field, can induce

parametric resonance in bands of the mode parameters, leading to exponential growth in the decay mode amplitude. The resonant decay of the inflaton may have important cosmological implications like non-thermal symmetry restoration and subsequent formation of topological defects [17, 18], revival of GUT baryogenesis scenarios [19, 20], supersymmetry breaking [21], superheavy particle production [16, 22], and gravitino production [23, 24].

The exponential growth in the mode occupation number may be modified or regulated by a number of physical processes. These include the decay of produced quanta to other particles [20, 25] or the rescattering of final state particles [9]. Another possibility, occurring in models with gauge-strength self-interactions between the produced final state particles, is the regulation of the parametric resonance by the self-interaction induced effective masses of the produced quanta, which can move these quanta out of the available resonance bands; in this case, resonance only proceeds as thermalization and Hubble dilution reduce the plasma masses of the final state quanta, resulting in a quasi-steady-state resonance conversion of inflaton oscillation energy to decay products [26]. This general scenario for the physically realistic case of decay products with gauge charge has been verified in explicit calculations in the narrow-band resonance case [26], and in numerical simulations in the broad-band case [10].

While to date analytical and numerical treatments of parametric resonance have considered oscillations of a single real field decaying to a single real decay product field, in realistic models the field content is often more extensive. In the case of supersymmetric theories this is unavoidably the case, as the physical scalars of simple ($N=1$) supersymmetry come as components of chiral multiplets and are complex. So for these types of theories, we should at the very least consider the nature of coherent decays when the fields involved are complex, though non-supersymmetric models with multiple real scalar fields may share some of the features of the simplest complex case.

Within supersymmetric models of particle physics, there are several different circumstances under which the decay of a homogeneous complex scalar condensate may occur in the early universe. At the end of inflation one expects to have a spatially homogeneous inflaton scalar condensate, whose decay energy will ultimately be responsible for cosmic reheating and the initiation of hot big-bang cosmology. As well, in the supersymmetric standard model there are directions in the scalar field space of squarks and sleptons which are F-flat and D-flat, and which only gain a potential from supersymmetry breaking. In the early universe these directions may be populated with (very) large

vev's after the end of the inflationary epoch; these vev's may carry enormous vev to mass ratios (Mathieu resonance parameter q) and couple to other directions in scalar field space with couplings capable of inducing resonant decay. Finally, supersymmetric models are generically plagued with gauge singlet scalar moduli, whose homogeneous oscillation poses grave cosmological difficulties which might be ameliorated by coherent decay of their oscillation amplitude.

For self-interactions of complex scalars of the general form dictated by the F-term and D-term couplings arising in globally supersymmetric theories, the fields generally appear in complex conjugate pairs for the F-terms and diagonal D-terms. For example, let us consider a complex scalar field Φ in a chiral supermultiplet whose decay will be induced by a trilinear (renormalizable) coupling in a superpotential W to a chiral multiplet labelled by its scalar Ξ , where $W \simeq \Phi \Xi \Xi$. The resulting F-term coupling inducing the decay is then of the form $\Phi^* \Phi \Xi^* \Xi$, and is invariant under global phase redefinitions of either the Φ or the Ξ . We will see in the next section that in cases such as this the phase invariance of the resulting couplings implies that the equations for modes of the real and imaginary components of Ξ are decoupled and independent, and will allow us to simply analyze the resonant decay of a Φ condensate with out of phase oscillations for the real and imaginary components of Φ , into real and imaginary components of the decay product field Ξ .

We can always phase rotate our scalar field Φ to a basis such that its initial vev lies along the real axis. If there is no component of force along the direction of the imaginary axis (i.e. the scalar potential is phase invariant), the trajectory of the motion of Φ is limited to the real axis and the field hits the origin as it oscillates back and forth. In this case, provided that the coupling of the oscillating field to the final state field is also phase invariant, the situation is exactly that of a real oscillating field, and the same arguments apply for parametric resonance particle production. However, if the scalar potential is not phase invariant, i.e. depends on the phase of the oscillating field as well, a torque is exerted on the field. This leads to the deflection of the trajectory from a straight line and results in changing the trajectory into something that finally resembles an ellipsoid, after the torque in field space has effectively ceased its action. In this case the field no longer passes through the origin but rather has a finite distance of closest approach to it. This will have important implications for broad-band parametric resonance as we discuss below.

In supersymmetric models not only are complex scalar fields inherently involved, but also a phase dependent part of the scalar potential can arise naturally from supersymmetry breaking.

Let us consider the simplest case with the following terms only involving the inflaton in the superpotential $W = \frac{1}{2}m\Phi^2 + \frac{1}{3}\lambda\Phi^3$. In supergravity models with broken supersymmetry, there is a corresponding phase dependent term (the ‘‘A-term’’) $Am_\Delta \frac{\partial W}{\partial \Phi} + \text{h.c.}$ in the scalar potential, where A is a dimensionless model-dependent constant and m_Δ is the scale of supersymmetry breaking in the sector in which Φ lies. There is also a phase dependent term $m\lambda^*\Phi\Phi^{*2} + \text{h.c.}$ in the F-term part of the scalar potential. This generically occurs in minimal supergravity models for inflation where the superpotential contains a series of $\lambda_n \frac{\Phi^n}{M^{n-3}}$ terms [27], and occurs even in ‘‘no scale’’ supergravity models after the inclusion of radiative corections to the effective potential [28].

For $V \supset \Phi^m(\Phi^*)^n$ the potential along the angular direction is periodic with $m - n$ minima. In general, during inflation Φ rolls down to its minimum both along the radial and angular directions. In order to have a torque to deflect the trajectory, Φ must not be at the minimum along the angular direction at the onset of radial motion. This can happen in two ways: either there are several phase dependent parts of the potential with a non-adiabatic transition from the minimum of one to another, or Φ does not settle at the minimum along the angular direction. The first possibility happens when other supersymmetry breaking sources in the early universe (e.g. non-zero energy density of the universe or finite temperature effects) are dominant over the low energy one. In this case the minimum in the angular direction at early times is different from that at late times (due to independent phases for the coefficients of different A-terms). If the transition from one minimum to another one is non-adiabatic, Φ will not be at the minimum at late times regardless of its start at the minimum at early times. The second possibility happens when the potential along the angular direction is flat enough during inflation. In this case Φ will not roll down to its minimum and can start at any position at the onset of radial motion. In both cases, the further Φ is away from the minimum, the larger the deflection of its trajectory and the wider the ellipsoidal shape will be.

2 Complex Mathieu Resonance

As described in the previous section, the potential for complex scalar oscillations usually includes (as well as the scalar mass-squared terms) Hubble induced A-terms which are mainly effective during the first few cycles of oscillation. The A-terms provide a ‘‘torque’’ to the complex oscillation during the first few cycles, resulting in a net ‘‘elliptic’’ motion in the mass-term induced potential, after

the A-terms have effectively ceased to be active. The resulting elliptic oscillation in the mass term potential will be damped by the Hubble drag, resulting in the ellipse shrinking over time.

In order to introduce new considerations characteristic of resonance with complex fields, without getting involved in model dependent details, in most of this paper we shall simply ignore the damping and consider complex, elliptic, constant amplitude oscillations. In particular, this means we do not need to specify which particular field is considered (e.g. inflaton versus susy standard model flat direction), nor do we need to determine the cosmological details involved in determining expansion and damping at the time that the oscillations of the field in question occur. In addition to presenting a tractable and interesting mathematical problem, consideration of the undamped oscillation should also provide the essential features of the full cosmological case including the effects of expansion, at least in the generic case of broad-band resonance. This follows from the key observation of [4, 5] that in the broad-band case the resonant excitation of the decay product field occurs over a tiny fraction of the cycle of the driving field, when the latter passes near the origin, as only here do the decay product field dynamics depart from the adiabatic regime. So mode number excitation proceeds by a series of abrupt jumps, and the dynamics of a given jump may be considered with the instantaneous value of the oscillator parameters, resulting in the picture of “stochastic resonance” analyzed in [5]. The present paper discusses the changes in the dynamics of decay mode excitation which arise from the complex nature of the driving field oscillation—the differences in question arise from suppression of the adiabaticity violations that induce the jump in mode occupation numbers of the decay product field, so we expect that considerations using the instantaneous value of the driving oscillation amplitude should give insight in the complex case, much as such considerations did in the real stochastic resonance case.

In any case, for the purposes of our present calculations we shall consider the parametric resonance production of decay product field modes Ξ , from phase-invariant coupling to constant-amplitude out of phase (“elliptic”) oscillations of a driving field Φ . Detailed cosmological studies of applications to inflaton or moduli oscillations will be considered elsewhere.

With the couplings discussed in the previous section, after the A-terms cease to be effective, the equation of motion for the Ξ field is of the form:

$$\ddot{\Xi}_k + \left(\frac{k^2}{a^2} + m_{\Xi}^2 + g^2 |\Phi|^2 \right) \Xi_k = 0, \quad (1)$$

where Ξ_k is the decay product field mode with comoving wavenumber k , a is the FRW scale factor, m_Ξ the mechanical mass of the Ξ , and the superpotential coupling is as above. We note that both the real and imaginary piece of the Ξ field will separately obey this equation, and hereafter we use χ to denote either the real or imaginary piece of Ξ .

In our analysis, we will treat the physical momentum of the decay field mode and the relative phase and amplitude of the driving field oscillation as fixed parameters, and attempt to map out the regions of instability in their parameter ranges. As noted above, for the case of stochastic broad-band resonance, where the amplification occurs in small intervals while the field passes close to the origin, one should be able to approximate the instantaneous behaviour within each interval by the corresponding behaviour of a system of the type we analyze here.

We decompose the driving field Φ into real and imaginary pieces as follows:

$$\Phi = \phi_R + i\phi_I. \quad (2)$$

By a phase rotation we put the largest amplitude component of oscillation into the real piece, and so we write:

$$\phi_R = \phi \sin(m_\phi t) \quad (3)$$

$$\phi_I = f\phi \cos(m_\phi t), \quad (4)$$

where now ϕ is the constant amplitude of the real component of oscillation and $f \in [0, 1]$ is the “out of phase” fractional component giving elliptic oscillation in the complex Φ plane; we will be particularly interested in the case where $f \ll 1$.

We wish to cast this into the canonical form of the (real) Mathieu equation:

$$y'' + (A - 2q \cos(2z))y = 0, \quad (5)$$

where $'$ denotes derivative with respect to the independent variable z . We begin by substituting our definition of the Φ field into (1) above, giving

$$\ddot{\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2 + g^2 \phi^2 (\sin^2(m_\phi t) + f^2 \cos^2(m_\phi t)) \right) \chi_k = 0. \quad (6)$$

We replace $\cos^2(m_\phi t)$ with $1 - \sin^2(m_\phi t)$ and collect terms, giving

$$\ddot{\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2 + f^2 g^2 \phi^2 + (1 - f^2) g^2 \phi^2 \sin^2(m_\phi t) \right) \chi_k = 0. \quad (7)$$

Using the half-angle formula $\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$ we obtain the form

$$\ddot{\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2 + f^2 g^2 \phi^2 + \frac{1}{2} (1 - f^2) g^2 \phi^2 (1 - \cos(2m_\phi t)) \right) \chi_k = 0. \quad (8)$$

which may be rewritten in the form of the Mathieu equation

$$\chi_k'' + (A_k(f) - 2q(f) \cos 2z) \chi_k = 0 \quad (9)$$

with the following new identifications:

$$z = m_\phi t \quad (10)$$

$$A_k(f) = \frac{\frac{k^2}{a^2} + m_\chi^2 + f^2 g^2 \phi^2}{m_\phi^2} + 2q(f) \quad (11)$$

$$q(f) = \frac{(1 - f^2)g^2 \phi^2}{4m_\phi^2}. \quad (12)$$

Notice that the coefficients of the Mathieu equation are now functions of the imaginary fraction f . This is an important feature, as it means that the characteristic behaviour of the parametric resonance as described by the Mathieu equation takes the same form in the complex case as in the real case; however, *the relationship between the physical parameters of the process and the Mathieu coefficients is redefined.*

So we see that there is a mapping that takes the Mathieu equation for the complex modes of the decay field when driven by the complex parametric field with out of phase real and imaginary pieces in its oscillation amplitude, and maps it to a real Mathieu equation for the oscillations of the real and imaginary pieces of the decay product field with shifted parameters. There are several features of this mapping that simply encapsulate physical features of the original problem.

First we note that for the case where $f = 0$, where the oscillation of the parametric driving field is along the real axis, the coefficients A_k and q reduce to those previously considered in the literature for the case of purely real parametric oscillation [4, 5]. In the other extreme limit, when $f = 1$, we have $q(1) = 0$, meaning that one is restricted to be along the A_k axis on the Mathieu equation stability diagram, allowing only non-resonant particle production. This corresponds to the physical observation that because the decay couplings were phase invariant, our original equation for the complex oscillations involved only the magnitude of Φ in the oscillation equation. In the

case that the real and imaginary pieces of the Φ oscillation have the same amplitude ($f = 1$), then the coefficients in the χ equation of motion become time independent and there can be no parametric amplification. We also note that the imaginary fraction f of the oscillations enters into the coefficients only as f^2 , meaning that the effect is the same regardless of direction traveled around the ellipse (i.e. whether f is positive or negative). This reflects that fact that the original equation for the complex oscillations is second order and symmetric under time inversion, which corresponds to having the parametric field circulate about the oval in the Φ plane in the opposite sense, or reversing the sign of f .

Finally, in the intermediate regime $f \in (0, 1)$, $q(f)$ is always lessened, while A_k is always increased (compared to $f = 0$), so increasing f never causes the system to leave the physical regime. As noted above, increasing f means moving “inland” on the stability diagram for the Mathieu equation. In general, this causes suppression of the resonant growth of the χ modes; however, it also allows one to explain the counterintuitive observation that in certain cases the resonant band exponential growth parameter μ_k may actually increase as one turns on the out of phase component f . To understand this, imagine that for oscillations with no imaginary fraction f one is sitting in parameter space at the lower border of one of the instability bands (where μ_k is zero). Now slowly increase f . The parameter mapping derived above implies that you start to move in a “northwest” direction on the band chart into the instability band. As such, your μ_k begins to increase. As you continue to increase f , you will eventually hit the maximum possible μ_k along your trajectory, after which your μ_k begins to drop again. Eventually you will leave the instability band altogether for sufficiently large f . For very high order instability bands, it should be possible to encounter many bands along one trajectory of increasing f .

From a different point of view, were one to look at the instability diagram as a function of the the A_k and q of standard real Mathieu parametric resonance (ie. as a function of our $A_k(0)$ and $q(0)$, for different values of f), the effect of turning on f would be seen to manifest itself as both a narrowing and a downwards shift of the instability bands as f increased. In addition, isocontours of μ_k would be seen to “flow out” of the bands as f increases. For $f = 1$ each instability band collapses to a line with $\mu_k = 0$, as with a phase independent coupling of the parameter field to the mode field there would be no time dependence in the mode field equation of motion, and the system would be stable for all A_k and q . Figure 1 illustrates in detail both the bending of the bands and the

decrease of μ_k for the first resonance band as we turn on the imaginary fraction f of our oscillations. We expect that the suppression of resonance for fixed non-zero imaginary fraction f is stronger in higher resonance bands, as at larger q the resonance proceeds by violation of adiabaticity in the decay mode evolution, and for large q at fixed f the induced decay mode field mass is always large. This is illustrated in Figure 2 where we show the quenching of resonance for the first 7 resonance bands, as f is turned on. We see that there is more severe suppression for the higher bands, in accord with our physical intuition; in the next section we will analytically estimate the extent of the domain of surviving resonant bands, for fixed imaginary fraction f .

3 Parameter Domain For Broad-Band Resonance

Let us first briefly discuss the effect of mixing in the narrow-band regime. The narrow-band resonance in the case of a real oscillating field is efficient for $(\frac{m}{g})^{\frac{4}{3}} \leq \phi \leq \frac{m}{g}$ [6]. In the case of a complex field with mixing parameter f this reads as

$$\left(\frac{m^2 + f^2 g^2 \phi^2}{(1 - f^2)g^2} \right)^{\frac{2}{3}} \leq \phi \leq \left(\frac{m^2 + f^2 g^2 \phi^2}{(1 - f^2)g^2} \right)^{\frac{1}{2}}, \quad (13)$$

where m and g are replaced with $\sqrt{m^2 + f^2 g^2 \phi^2}$ and $\sqrt{1 - f^2}g$, respectively. It is easily seen that the condition for an efficient narrow-band resonance remains almost unchanged in the complex case, unless f is close to 1. Therefore, in physically interesting situations the narrow-band resonance will not be substantially affected by the mixing. Of course, in the extreme case with $f = 1$, there is no time variation in the Mathieu equation and, hence, no narrow-band resonance.

In the case of real broad-band parametric resonance, Kofman, Linde, and Starobinski [4, 5] argue that the requirement of adiabaticity violation for broad-band parametric amplification implies that it only occurs with significant μ_k for $A_k - 2q \lesssim \sqrt{q}$. Their argument presupposes that the decay terminates after it has entered an “explosive” phase, where the effective mass of the decaying ϕ is dominated by the coupling to the plasma of decay product χ modes which has been built up by parametric resonance decay. The effective physical 3-momentum of the quasi-relativistic decay χ modes is of order their energy, which is no more than of order the induced mass of the decaying ϕ ; this is of order $g\chi_{\text{end}}$ which in turn is of order $g\phi_{\text{end}}$ which can also be written $\sqrt{gm_{\phi}^{\text{eff}}\phi_{\text{end}}}$. So

$(k_{\text{phys}}^2/m_\phi^2) \lesssim (g\phi_{\text{end}}/m_\phi^{\text{eff}})$, which can be rewritten as $A_k - 2q \lesssim \sqrt{q}$. For a detailed discussion we refer to the treatment in [5].

We have seen in the preceding section that the case of complex oscillation with imaginary fraction f can be mapped onto a Mathieu equation with shifted resonance parameters. By substituting the “shifted” parameters induced by the non-zero imaginary component of oscillation, we should thus be able to determine what range of q supports broad-band resonance for oscillation with a given fraction of out of phase imaginary component for the oscillation of the driving parameter.

Recall the expressions for the equivalent shifted $A_k(f)$ and $q(f)$ from equations (11) and (12) respectively. Substituting these expressions into the relation $A_k - 2q \lesssim \sqrt{q}$ allows us to write it as:

$$\frac{\frac{k^2}{a^2} + m_\chi^2 + f^2 g^2 \phi^2}{m_\phi^2} \lesssim \frac{\sqrt{(1-f^2)}g\phi}{2m_\phi}. \quad (14)$$

This leads us to the relation

$$A_k(0) - 2q(0) + 4f^2 q(0) \lesssim \sqrt{1-f^2} \sqrt{q(0)}, \quad (15)$$

or, defining $E_k \equiv A_k(0) - 2q(0)$, we have

$$E_k + 4f^2 q(0) \lesssim \sqrt{1-f^2} \sqrt{q(0)}. \quad (16)$$

If we recall that physical values of E_k are positive semi-definite, we find that for a fixed non-zero imaginary fraction f there is an upper bound on the parameter $q(0)$ for which resonance occurs, and the allowed range of resonant $q(0)$ is bounded above as $\frac{1-f^2}{16f^4}$. (For the small imaginary fractions f of physical interest, however, the weaker approximate bound of $\frac{1}{16}f^{-4}$ will suffice). So instead of an ever widening resonance region above the $A_k = 2q$ line, with thickness of order \sqrt{q} , as one has in the real case, in the complex case with fixed non-zero imaginary fraction f , one instead has a region above the $A_k = 2q$ line of finite extent, with an upper bound on the values of the q parameter which can result in resonance. This is qualitatively reasonable, as a fixed imaginary fraction f for the oscillation means that as we scale up q the ellipse of Φ broadens as it lengthens, preserving its shape; so throughout the Φ oscillation $|\Phi|$ has a large value, inducing a large mass for the modes of the decay field Ξ , which in turn leads to adiabatic evolution of the Ξ , and suppression of broad-band parametric resonance production of the Ξ .

4 Complex Resonance in “Instant Preheat”

Recently, a simpler method of efficient scalar field decay has been proposed, called “instant preheat” [16]. Within models of this type the decaying field rolls once through the origin, at which point the mass of the decay product field to which it is coupled passes through zero, and modes of the decay product field experience non-adiabatic excitation. As the decaying field rolls away from zero (perhaps monotonically) the modes of the decay product field grow in mass; they drain energy from the decaying field through their mass. As the mass of the modes of the decay product field grows, so does their decay width; their subsequent decay, after their mass and decay width have grown sufficiently, then releases the energy they have taken from the original decay field, and dumps it into their final decay products, which thermalize the resulting energy.

It is interesting to note that while final state effects such as rescattering, backreaction, or plasma masses can prevent preheating from occurring, they are unimportant in the instant preheating scenario. The reason is that for these effects to become important, (at least) several oscillations are needed to build up a large enough occupation number for the final state field. In the instant preheating, on the other hand, the energy drain from the oscillating field occurs during each half of an oscillation. In fact, instant preheating can be efficient even if the adiabaticity condition is violated only during the first half of the first oscillation. Therefore, instant preheating is essentially unaffected by the final state effects. The mixing of the real and imaginary parts of the oscillating field, on the other hand, has the same effect in the instant preheating case as in the standard preheating scenario. We recall that the torque from A-terms deflects the trajectory of the oscillating field from that of a straight line. The Hubble induced A-terms have their largest value at the beginning of the oscillations, and rapidly decrease with Hubble expansion. This means that the deflection is largest during the initial oscillations. Thus, a large enough f to restore adiabaticity in the preheating case could do the same for the instant preheating case.

5 Non-Convex Potentials

Another possibility to achieve rapid decay of a homogeneous condensate occurs in the case that the potential governing the evolution of the condensate scalar has non-convex behaviour over some region of field space [12, 29]. In this circumstance, it becomes energetically favorable for a scalar

condensate in the non-convex region to decompose into inhomogeneous modes; provided the inhomogeneity occurs over long enough wavelengths, the price one pays in kinetic energy for the inhomogeneity is more than compensated by the decreased average potential energy of the regions of field excess and deficit compared to the average field value. This produces a wavenumber band for exponential growth of the mode amplitudes. This has been considered in both the case of inflaton decay [12], and in the case of the growth of inhomogeneities in scalar condensates corresponding to F-flat and D-flat directions of the standard model with non-convex potentials of the type arising from gauge-mediated supersymmetry breaking [29]. It is clear that this is one type of instability which is not vitiated by having the scalar order parameter complex or involving multiple scalar fields. If there is a region in field space with respect to which the potential is non-convex in some direction, then fluctuations corresponding to modes of the field variation in that field direction, of sufficiently long wavelength, will win on the potential versus kinetic energy budget, and grow exponentially. Indeed the treatment of (complex) flat directions in the supersymmetric standard model in [29] explicitly analyzes the conditions for instability of a complex field with a potential which is a non-convex function of the field modulus, and exhibits the resulting instability bands.

6 Other Couplings

Here, we briefly comment on the situation for another type of coupling between Φ and Ξ fields which is also of interest and application. This is the coupling $g^2(\phi_R\chi_R + \phi_I\chi_I)^2$, where its simplest manifestation is for the potential $V(\Phi) = \frac{1}{4}\lambda|\Phi|^4$, with Φ and Ξ being the same field. It also arises in supersymmetric models from the D-term part of the scalar potential. This type of coupling leads to the mixing of χ_R and χ_I mode equations:

$$\ddot{\chi}_{R,k} + \left(\frac{k^2}{a^2} + m_\chi^2 g^2 \phi_R^2 \right) \chi_{R,k} + g^2 \phi_R \phi_I \chi_{I,k} = 0 \quad (17)$$

$$\ddot{\chi}_{I,k} + \left(\frac{k^2}{a^2} + m_\chi^2 g^2 \phi_I^2 \right) \chi_{I,k} + g^2 \phi_R \phi_I \chi_{R,k} = 0. \quad (18)$$

In this case the mass eigenstates are $\frac{\phi_R\chi_R + \phi_I\chi_I}{\phi}$ and $\frac{\phi_I\chi_R - \phi_R\chi_I}{\phi}$ instead of ϕ_R and ϕ_I themselves. For oscillatory motion of ϕ_R and ϕ_I with a phase difference, there are two periodic changes that may lead to resonance: change in the mass eigenvalues (the usual parametric resonance) and change in

the mass eigenstates. They can't be simply superimposed and it is not very easy to give rough arguments for the instability bands and the respective value of μ_k 's. The important point is that for such a coupling, even in the $f = 1$ case there is still time variation in mode equations. This variation is present in both the mass eigenstates and mass eigenvalues.

7 Cosmic Expansion and Complex Resonance

So far, we have considered modifications to parametric resonance decay which arise in complex field oscillations in the absence of effects of Hubble expansion. As we noted above, since broad-band resonance is induced by non-adiabaticity of the χ evolution during small intervals of the ϕ oscillation, instantaneous approximation of the χ excitation should be a useful guide during each of the jumps in mode number. Cosmic expansion then functions to shift the parameters of the oscillator between episodes of mode excitation as ϕ passes near zero. We now examine the implications of this in both the narrow- and broad-band cases.

Implications for the narrow-band case are simple; as we have seen, the introduction of a phase difference between real and imaginary components of our complex inflaton field Φ only kills narrow-band resonance for phase differences approaching $\frac{\pi}{2}$, or, in the language of this paper, for $f \cong 1$. Therefore, the resonance should be qualitatively the same in the static approximation and with the Hubble expansion included.

For broad-band resonance the situation is completely different. According to equation (16), the broad-band resonance is shut off for $q > \frac{1}{16}f^{-4}$. In the static limit f and q are both constant and resonance is either suppressed, or viable. In an expanding universe, f eventually becomes approximately constant as the Hubble induced A-terms turn off (indeed, as pointed out earlier, after several Hubble times the motions along the real and imaginary directions are decoupled and free), while, on the other hand, $q(t) = \left(\frac{g\phi(t)}{2m}\right)^2$ is redshifted as a^{-3} . This implies that even if the resonance is suppressed initially, it may be initiated after a sufficient time such that $q(t) < \frac{1}{16}f^{-4}$. The right-hand side is less than 1 (or very close to it) for $f \gtrsim \frac{1}{2}$. Therefore, in the case of large out of phase components of oscillation, the broad-band resonance is killed and resonance may resume only in the narrow-band regime at a later time. In most physically interesting cases, however, $f < \frac{1}{2}$ and the right-hand side is considerably greater than 1. In such cases, broad-band resonance

is not eliminated in an expanding universe, but rather delayed. The parameter f is determined by the action of the scalar potential (including A-terms) for the oscillating field, and after the initial oscillations it often becomes effectively time-independent. Depending on the dimensionality of the A-term, it may be a function of q_i , the value of q at the start of oscillations. If $q_i < \frac{1}{16}f^{-4}$, the onset of broad-band resonance will be unaffected. For $q_i > \frac{1}{16}f^{-4}$, resonance is delayed initially, but will resume after sufficient expansion such that $q < q_{\text{eq}} = \frac{1}{16}f^{-4}$. A larger f leads to a smaller q_{eq} , for $f \gtrsim \frac{1}{2}$ we have $q_{\text{eq}} < 1$ and resonance can only occur in the narrow-band regime. For $f = 1$ resonance is eliminated.

Even though the broad-band resonance (for interesting cases) is only delayed in an expanding universe, the mixing still has important consequences. Perhaps the most notable example relates to the production of superheavy particles during resonance. In the standard preheating, Ξ 's with a mass up to $q^{\frac{1}{4}}m_\phi$ can be produced. A reduction in q at the onset of resonance implies a reduction in the maximum mass of produced particles. This is even more pronounced in the instant preheating case. Here Ξ decay products Ψ with masses up to $q^{\frac{1}{2}}m_\phi$ and which have a large enough coupling h to Ξ , can be produced [16]. A smaller q_{eq} has a two-fold effect in this case. First, the allowable masses are smaller, and second, the decay rate $\Gamma_d = \frac{h^2}{8\pi}g\phi$ may not be large enough (compared to the frequency of oscillations m_ϕ) for efficient production of Ψ 's. It is easily seen that $\Gamma_d \leq m_\phi$ for $h \lesssim 4\pi^{\frac{1}{2}}f$. Therefore, Ψ production is not efficient if $h \lesssim 4\pi^{\frac{1}{2}}f$. Even for $h \gg 4\pi^{\frac{1}{2}}f$, only Ψ 's with a mass $m_\psi \lesssim \frac{1}{4f^2}m_\phi$ can be produced.

8 Conclusions

In this paper, we have considered the changes to the standard picture of parametric resonance decay of a real homogeneous cosmological scalar field which arise if the field is instead complex, with out of phase oscillation of its real and imaginary components and a phase invariant decay coupling. For the case of complex Mathieu type resonance, we give an explicit mapping to a corresponding real Mathieu resonance with shifted parameters that encode the effects of the out of phase components of the oscillating decay field. We showed the resulting effects on the instability bands, demonstrating how they shift and shrink with increasing out of phase (“elliptic”) component of the driving field motion, limiting the swath of instability to a finite area on the A_k - q chart, and eliminating broad-

band resonance in the higher modes. We argued that similar effects may be present in the case of complex field models of “instant preheat,” but that instabilities due to regions in field space with non-convex potentials are qualitatively the same in the complex case. Finally, in the context of an expanding FRW universe, we noted that the presence of a fraction of out of phase oscillation would usually delay the onset of parametric resonance, but not eliminate it entirely.

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Figure Captions

Figure 1: The Mathieu equation stability diagram for the first resonance band. The physical region lies above and to the left of the diagonal line $A_k = 2q$. Contour lines represent isocontours of μ starting from $\mu = 0$ (band boundary) and increasing by units of 0.1 as one moves from left to right in the band. Different panels represent different imaginary fractions f : (a) $f = 0$; (b) $f = 0.2$; (c) $f = 0.4$; (d) $f = 0.6$.

Figure 2: The Mathieu equation stability diagram for the first 7 resonance bands. The physical region lies above and to the left of the diagonal line $A_k = 2q$. Contour lines represent isocontours of μ starting from $\mu = 0$ (band boundary) and increasing by units of 0.1 as one moves from left to right in the bands. Different panels represent different imaginary fractions f : (a) $f = 0$; (b) $f = 0.2$; (c) $f = 0.4$; (d) $f = 0.6$.







