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## Topology Dependence of Event Properties in Hadronic Z Decays

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### **Abstract**

Three-jet events are studied for different event topologies. Experimental evidence is presented that the multiplicities of quark and gluon jets depend both on the jet energy and on the angles between the jets.

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# 1 Introduction

Much work has been invested in the study of differences between quark and gluon jets, showing that the two types of jets differ qualitatively as expected from QCD. Gluon jets for example are found to be wider than quark jets, have a higher multiplicity and correspondingly softer fragmentation function. At LEP most of the studies [1] are based on comparisons between quark and gluon jets in symmetric Y-shaped three jet events. In this kinematical configuration, jets of fixed energy are compared while varying the gluon/quark purity by means of flavour tagging.

The present study tries to go one step further and infer something about the properties of jets, and particularly charged multiplicities, in arbitrary three-jet kinematic configurations. Whereas symmetric events are completely specified by only one variable, general three jet events are described by two independent variables. Consequently the jet properties can be functions of those two variables. The two independent variables used in this analysis are, for a randomly selected jet, the jet energy and the difference of the opening angles to the other two jets. The variation of this angle at fixed jet energy will be referred to as variation of the event topology.

The analysis presented here is based on a sample of around 1 million hadronic events at  $\sqrt{s} = M_Z$  measured with the ALEPH detector at LEP in 1992 and 1993, giving approximately 300,000 three jet events, and about 2 million Monte Carlo events for detector corrections and investigation of systematic errors. The variations of the jet and event charged particle multiplicities for fixed jet energies are studied as function of the event topology. As the analysis is insensitive to the normalization of the measurements most systematic uncertainties cancel.

## 2 Data Analysis

The ALEPH detector, which provides both tracking and calorimetric information over almost the full solid angle, is described in detail elsewhere [2]. The momentum of charged particles is given by a fit to the information provided by the three tracking devices: a double-sided silicon microvertex detector (VDET), an eight-layer axial-wire chamber (ITC) and a large time projection chamber (TPC). The momentum resolution achieved in the combined fit is  $\Delta p/p = 0.0006 p/\text{GeV}/c \oplus 0.005$ , where the two contributions are to be added in quadrature. This measurement is combined with calorimeter and muon chamber informations in the reconstruction of the energy flow which allows reliable reconstruction of jet energy and direction in hadronic final states. The energy resolution obtained is  $\Delta E = 0.59\sqrt{E/\text{GeV}} + 0.6 \text{ GeV}$ . Parton directions are reconstructed with an angular resolution between 25 mrad for 40 GeV jets and 40 mrad for 10 GeV jets. A detailed description of the performance of the energy flow algorithm can be found in [3].

Two independent analyses of data taken in 1992 and 1993 at the peak of the Z-resonance are performed. The nominal analysis follows the method presented in [4]. An event is accepted if it has at least 5 good charged tracks, a total charged energy in excess of 15 GeV, and if the polar angle of the sphericity axis with respect to the beam is in the range  $35^\circ \leq \theta_{sph} \leq 145^\circ$ . A good charged track is required to have at least 4 coordinates in the TPC and to originate from a cylindrical region with radius  $d_0 = 2 \text{ cm}$  and length  $z_0 = 10 \text{ cm}$  around the interaction point. The transverse momentum  $p_T$  with respect to the beam axis has to be larger than 200 MeV/c and the polar angle between  $20^\circ$  and  $160^\circ$ . A “loose-cut” analysis was performed as a cross check to the nominal one, where the cuts on the track- $p_T$  and the polar angle of the sphericity axis were dropped and the requirement for the total charged energy relaxed to 9.2 GeV.

Accepted events are clustered with the Durham jet finding algorithm [5] with a resolution parameter of  $y_{cut}^D = 0.01$  in the E-recombination scheme, applied to the good charged tracks

plus neutral energy flow objects with energies above 0.4 GeV. Including a pseudo particle carrying the missing momentum during the clustering ensures planar three jet events after the clustering. For the “loose-cuts” analysis the cut on the energy of neutral energy-flow objects was dropped and the clustering done without imposing momentum balance on the event. Then the event plane was determined from the normalized jet momentum tensor and the jet 4-vectors projected onto it. Finally the jet energies are recomputed from the jet-jet angles in the event plane, assuming massless planar kinematics.

The observables studied in this analysis are the mean charged particle multiplicities in individual jets and in entire events in bins of different three-jet topologies. The raw measurements are corrected bin-by-bin using a hadronic event generator based on DYMU3 [6] and JETSET 7.3 [7] with parameters adjusted to describe the ALEPH data [4] for the effects of geometrical acceptance, detector efficiency and resolution. The variation of the charged jet or event multiplicities over the available phase space from both analyses are found to agree within the statistical errors and thus were combined for further studies.

Systematic effects on the charged multiplicities were estimated by doing an alternative correction, either by weighting each track with a momentum dependent weight or by replacing the full detector simulation by simple fiducial cuts. The bin-to-bin systematic errors obtained by the two methods are of approximately the same size. For each bin the larger of the two was taken as the systematic error. For all results shown below the statistical and systematic bin-to-bin uncertainties were added in quadrature.

## 3 Qualitative Evidence for Topology Dependence

### 3.1 Gluon Jet Purities

The corrected mean charged multiplicities are studied, jet by jet, as a function of the energy of the jet,  $E_{\text{jet}}$ , and the difference of the opening angles,  $\Delta\theta$ , to the other two jets of the event. This choice of variables is motivated by the observation, that the jet-energy  $E_{\text{jet}}$  essentially fixes the gluon purity  $p^g$  (see below) of the jet under consideration, while  $\Delta\theta$  allows one to vary the event topology. Symmetric events have  $\Delta\theta=0$ . The variables are independent, spanning a rectangular phase space  $0 < E_{\text{jet}} < E_{\text{beam}}$  and  $0 < \Delta\theta < 180^\circ$ , although the experimental requirement  $y_{\text{cut}}^D = 0.01$  restricts the analysis to a subspace with curved boundaries.

The actually accessible phase space is shown in Fig.1. The almost vertical lines show the contours of constant gluon purity. The dotted lines are the results of a Monte Carlo study, which takes into account higher order QCD effects, the continuous lines being the expectation from the leading order QCD matrix element for the probability (gluon purity) that a given jet  $i$  originated from a gluon [8]:

$$p_i^g \sim \frac{x_j^2 + x_k^2}{(1-x_j)(1-x_k)}. \quad (1)$$

Here  $x_i$  ( $i, j, k \in 1, 2, 3$ ) are the three scaled energies,  $x_i = 2E_i^{\text{jet}}/E_{\text{cm}}$ . The agreement between leading order QCD and MC calculations is rather good, indicating that higher order QCD effects are small.

The Monte Carlo results shown here require a precise definition of a gluon-jet. The procedure used to determine the gluon or quark nature of a reconstructed MC jet is the following: for each Monte-Carlo event, having three jets according to the above definition after the full simulation (detector level) has, the jet clustering algorithm is applied to the partons at the end of the parton shower process (parton level) and to the hadrons before they go through the detector (hadron level). In both cases a three jets topology is required with a value  $y_3$ , the resolution

parameter where the event makes the transition from three-jet to a two-jet event, within 25% of the  $y_3$  obtained using the reconstructed objects. If the relative variation is larger than 25% the event is rejected. For accepted events, an angular matching is performed first between reconstructed and hadron jets and then between hadron and parton jets. This gives a one to one correspondance between jets at each level. The matching is an angular matching in the event plane, where first the closest of the nine possible pairs is matched. The remaining two jet pairs are matched by choosing the combination which minimizes the sum of the angular differences. Finally the quark or gluon “flavour” of a jet of partons is defined as the sum over all the final state partons in the jet with +1 for  $q$ , -1 for  $\bar{q}$  and 0 for  $g$ . Events with a flavour pattern different from  $(-1, 0, 1)$  ( $\approx 2\%$ ) are rejected, for the remaining ones the reconstructed jet type is given by the type of the parton jet it is matched to.

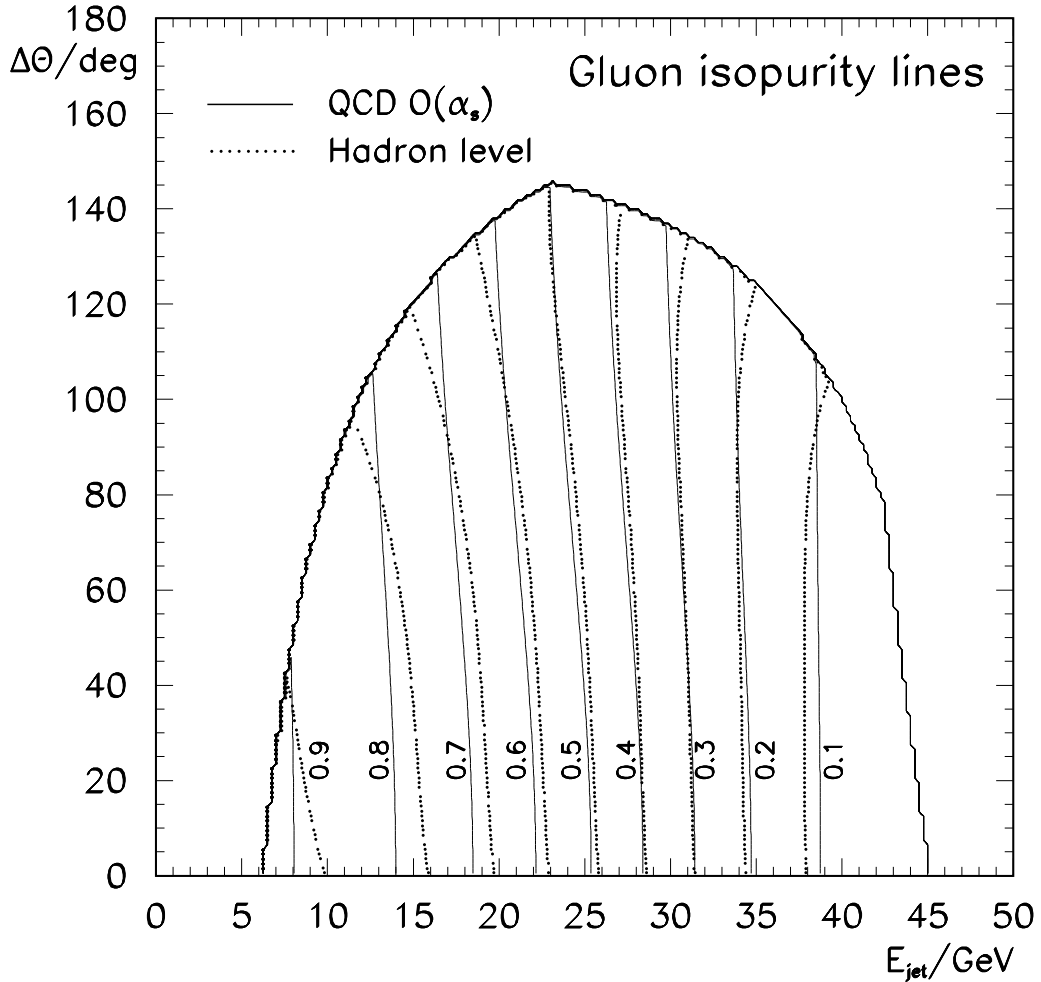


Figure 1: Lines of constant gluon purity as function of  $E_{\text{jet}}$  and  $\Delta\theta$ . The leading order matrix element prediction is compared to a Monte Carlo study including perturbative higher orders and hadronization effects. The outer curve corresponds to the phase space boundary for  $y_{\text{cut}}^D=0.01$ .

### 3.2 Jet Charged Particle Multiplicities

The corrected charged particle multiplicities for individual jets as function of the difference in opening angle  $\Delta\theta$  to the other two jets are displayed in Fig.2 for 8 equal size bins of jet energy

between  $10 \text{ GeV} \leq E_{\text{jet}} \leq 42 \text{ GeV}$ . If jet properties would only depend on the jet energy, then the jet multiplicities should be constant as function of  $\Delta\theta$ . Since the gluon purity is essentially fixed by the jet energy, even a difference between quarks and gluons would not lead to a  $\Delta\theta$ -dependence of the jet multiplicity. The data, however, show a marked dependence on the asymmetry variable  $\Delta\theta$ , especially for intermediate jet energies between 20 GeV and 30 GeV. This effect cannot be caused by a  $\Delta\theta$ -dependence of the mean jet energy inside a given  $E_{\text{jet}}$ -bin, because the change observed by varying  $\Delta\theta$  from  $\Delta\theta = 0^\circ$  to  $\Delta\theta = 160^\circ$  is much larger than the change between adjacent  $E_{\text{jet}}$ -bins for constant  $\Delta\theta$ .

Still, it is conceivable that the observed topology dependence of the jet multiplicity does not reflect properties of isolated partons but rather a feature of the jet algorithm. If that were the case, the observed  $\Delta\theta$ -dependence would disappear at higher energies, where the assignment of a particle to a jet becomes less ambiguous. To distinguish between those alternatives a variable is needed, which does not rely on the assignment of particles to jets. The obvious candidate is the total charged multiplicity of the event, which, although in a more diluted way, still reflects the multiplicities created by the primary hard partons. It is a much simpler quantity and thus can also be expected to be much more amenable to a description by simple phenomenological models.

### 3.3 Event Charged Multiplicities

That the charged particle multiplicity of the whole event contains information about topology dependence of jet multiplicities can be illustrated by means of a simple numerical example.

At the jet energy  $E_{\text{jet}}=E_1=28 \text{ GeV}$  the charged particle multiplicity of the event measured in this analysis varies by  $\Delta n = 4.2 \pm 0.5$  between  $\Delta\theta = 10^\circ$  and  $\Delta\theta = 130^\circ$ . Thus, if the event multiplicity is the linear sum of each jet's multiplicity (which is the assumption underlying any jet algorithm) and if the jet multiplicity is supposed to vary only with  $E_{\text{jet}}$  for a given jet flavour, then this difference has to come from the multiplicities of the other two jets in these events. The calculation of the kinematics and the determination of the gluon purities from Fig.1 gives  $E_2 = 30.2 \text{ GeV}$  and  $E_3 = 33 \text{ GeV}$  with  $p_2^g \approx p_3^g \approx 30\%$  for the (almost) symmetrical case  $\Delta\theta = 10^\circ$ , and  $E_2 = 18.7 \text{ GeV}$  and  $E_3 = 44.5 \text{ GeV}$  with  $p_2^g \approx 65\%$  and  $p_3^g \approx 0\%$  for the highly asymmetrical case  $\Delta\theta = 130^\circ$ .

The total event multiplicity can then be computed for each case from a parametrization of the quark jet multiplicity as function of the jet energy,  $n_q(E_{\text{jet}})$ , and an assumption for the ratio  $f_g$  between the gluon jet and the quark jet multiplicity  $f_g = n_g/n_q$ . With the simple phenomenological parametrization discussed below in section 4,  $n_q(E_{\text{jet}}) = 10.475(2E_{\text{jet}}/M_Z)^{0.439}$ , and the asymptotic QCD expectation that  $f_g$  is given by the ratio of the colour charge of a quark and a gluon,  $f_g = C_A/C_F = 9/4$ , one calculates a difference  $\Delta n(\text{calc}) = 2.0$ . This is a much smaller difference than the one actually observed. Using a value  $f_g = 1.3$ , which is close to the measured multiplicity ratio between quark and gluon jets [9], the difference is reduced further to  $\Delta n(\text{calc}) = 0.6$ , giving clear evidence for topology dependence of jet properties.

## 4 Quantitative analysis: Comparison with Models

In order to quantify the effect described above, two models are considered in the following. In both cases the total charged particle multiplicity of a three jet event is written as the sum of three contributions, from a quark, an antiquark and a gluon:

$$\langle n_{\text{evt}}^{\text{ch}} \rangle = n_q + n_{\bar{q}} + n_g \quad (2)$$

## Jet Multiplicities

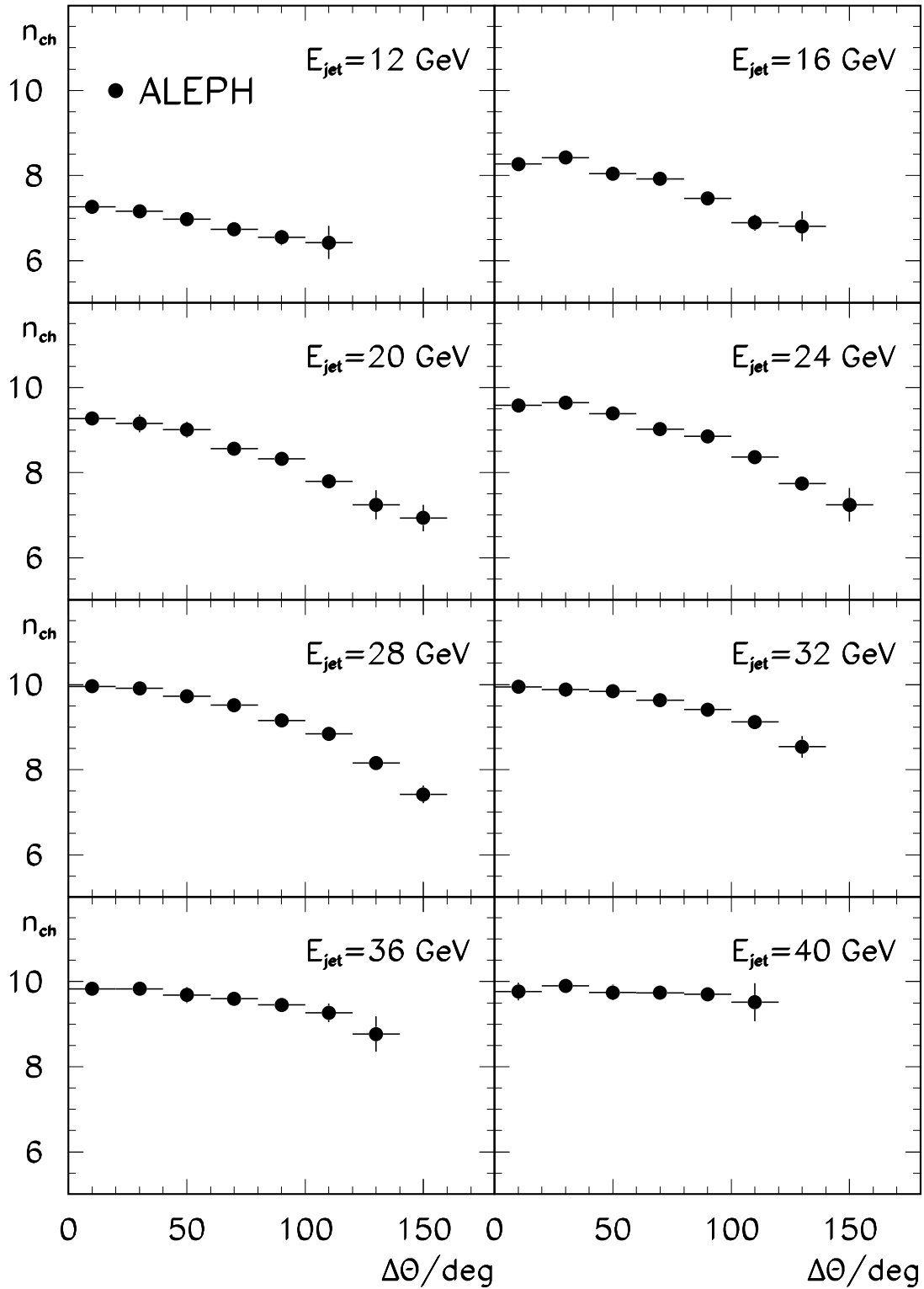


Figure 2: Charged particle multiplicity for individual jets of fixed energy as function of the event topology as parametrized by  $\Delta\theta$ .

The two models differ in the functional form chosen for  $n_q$ ,  $n_{\bar{q}}$  and  $n_g$ . If no flavour identification of the final state partons is performed, as is the case for this analysis, the ansatz Eq.(2) has to be averaged over all possible flavour assignments. This is done with the relative weights given by the leading order QCD matrix element (Eq.(1)).

The first model, “Model 1”, based on the arguments given in the preceding section, assumes that the properties of a jet originating from a parton of type  $p$  are a function of only one scale  $Q_p$  proportional to the parton energy:

$$Q_p = 2E_p \quad (3)$$

This model does not exhibit “topology dependence”. The second one, “Model 2”, does. Here the jet properties are assumed to depend on the jet energy and the opening angle between the original parton and its colour connected partner(s). For three jet events this implies that the properties of an (anti)quark jet depend on the energy of the (anti)quark and the opening angle between the (anti)quark and the gluon and that the properties of a gluon jet are a function of the gluon energy and the opening angles to the quark and the antiquark. A more formal consideration [10] based on the dipole picture of gluon radiation suggests the following scale as the relevant variable determining the properties of the jet from a parton  $p$  which is colour connected to another parton  $q$ :

$$Q_{pq} = 2E_p \sin \frac{\theta_{pq}}{2} \quad (4)$$

This scale is similar to the evolution variable used in the HERWIG [11] model. For small opening angles it is the relative transverse momentum of two partons, in the limiting case of two back-to-back quarks it becomes the total centre-of-mass energy.

For the following it will be assumed that the multiplicity generated by a QCD process at a certain scale  $Q$  is proportional to one universal function  $F(Q)$ . Given this function, the two models are:

Model	$n_q$	$n_{\bar{q}}$	$n_g$
Model 1	$NF(Q_q)$	$NF(Q_{\bar{q}})$	$f_g NF(Q_g)$
Model 2	$NF(Q_{qq})$	$NF(Q_{\bar{q}\bar{q}})$	$f_g N[F(Q_{gq}) + F(Q_{g\bar{q}})]/2$

The scale variables for Model 1 and Model 2 are defined according to Eq.(3) and Eq.(4), respectively. In both cases  $N$  is an unknown normalization constant and  $f_g$  the relative gluon-to-quark-jet multiplicity for symmetric (“Mercedes-star”) 3-jet events.

The function  $F(Q)$  can be extracted from measurements of the charged particle multiplicities in  $e^+e^-$ -annihilations into hadronic final states for various centre-of-mass energies, which is dominated by two jet events. The data taken from the compilation given in [12] are shown in Fig.3, together with two parametrizations  $F(E_{cm})$  of the type

$$\langle n_{ch}(E_{cm}) \rangle = 2N_0 F(E_{cm}). \quad (5)$$

QCD, in the framework of the modified-leading-log approximation (MLLA) and local parton-hadron duality (LPHD), suggests a functional form for  $F(E_{cm}) = F_\alpha(E_{cm})$  [13]

$$F_\alpha(E_{cm}) = \alpha_s^a(E_{cm}) \exp\left(b/\sqrt{\alpha_s(E_{cm})}\right) \quad (6)$$

with the coefficients  $a \approx 0.492$  and  $b \approx 2.265$ . Here the energy dependence is a consequence of the running of the strong coupling constant  $\alpha_s(E_{cm})$ . A good fit is obtained by using the

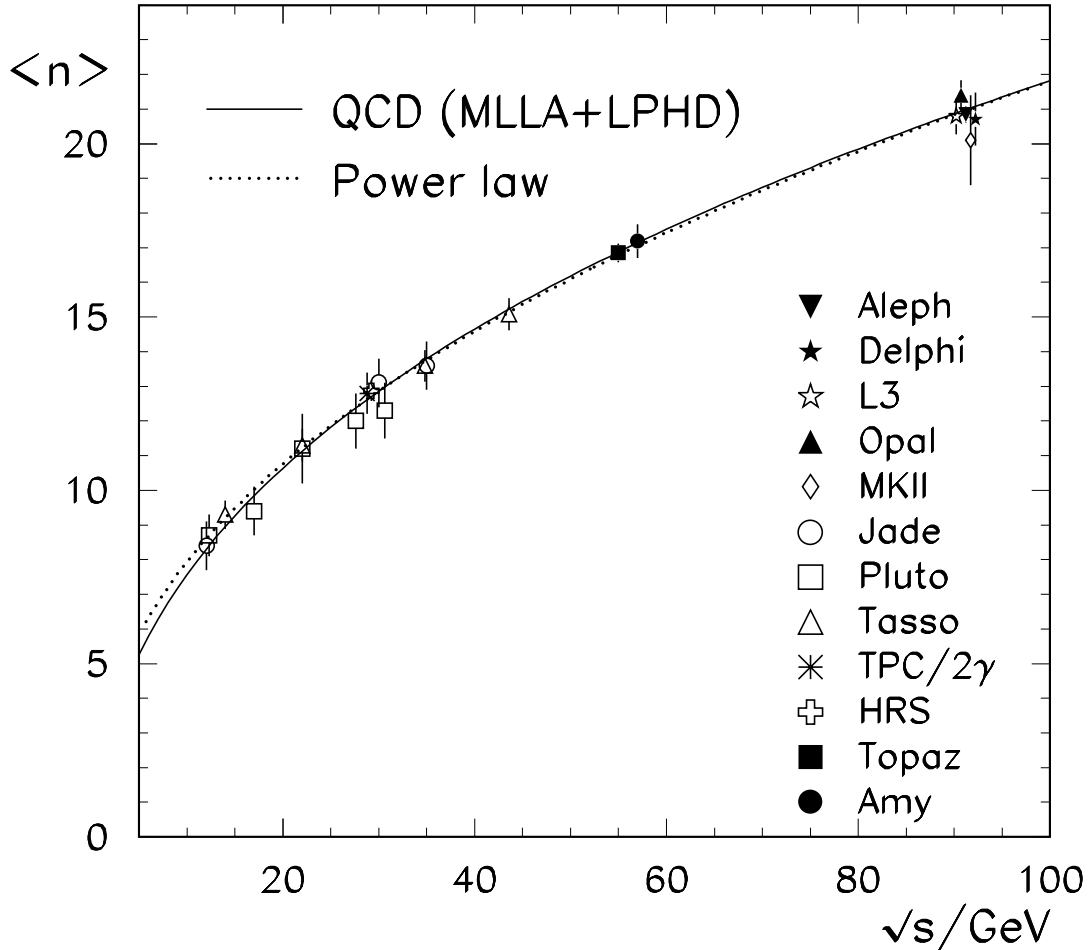


Figure 3: Energy evolution of the total charged particle multiplicity in  $e^+e^-$ -annihilation into hadronic final states.

two-loop formula for  $\alpha_s(E_{cm})$  with  $\alpha_s(M_Z) = 0.118$  [12], adjusting the normalization in Eq.(5) to  $2N_0 = 0.0821$ . An alternative, purely phenomenological fit of equal quality is obtained by a simple power law  $F(E_{cm}) = F_\gamma(E_{cm})$

$$F_\gamma(E_{cm}) = \left(\frac{E_{cm}}{M_Z}\right)^\gamma \quad (7)$$

with  $2N_0 = 20.95$  and  $\gamma = 0.439$ .

Results of fitting Model 1 and Model 2 with  $F = F_\alpha$  to the experimental data are displayed in Fig.4. The free parameters are the overall normalization  $N$  and the relative gluon- to quark-jet multiplicity  $f_g$ . The scale-dependence of  $F(Q)$  is determined by the choice  $\alpha_s(M_Z) = 0.118$ . It is evident that Model 1, i.e. jet properties depending only on the jet energies, cannot reproduce the experimental results. The best fit has a  $\chi^2/df = 692/55$ . Model 2, on the other hand, gives a good description of the data with  $\chi^2/df = 46.3/55$ . The parameter  $f_g$  is determined as  $f_g = 1.48$ . As a cross check, it is interesting to use this model with the fitted parameters to calculate the ratio of the quark- to gluon-jet multiplicity for symmetric Y-shaped configurations with an opening angle of  $150^\circ$  between the high energy jet and the two low energy ones. One finds a ratio  $\langle n_{ch}(\text{gluon}) \rangle / \langle n_{ch}(\text{quark}) \rangle = 1.27$ , close to the experimental values [1].



## Event Multiplicities

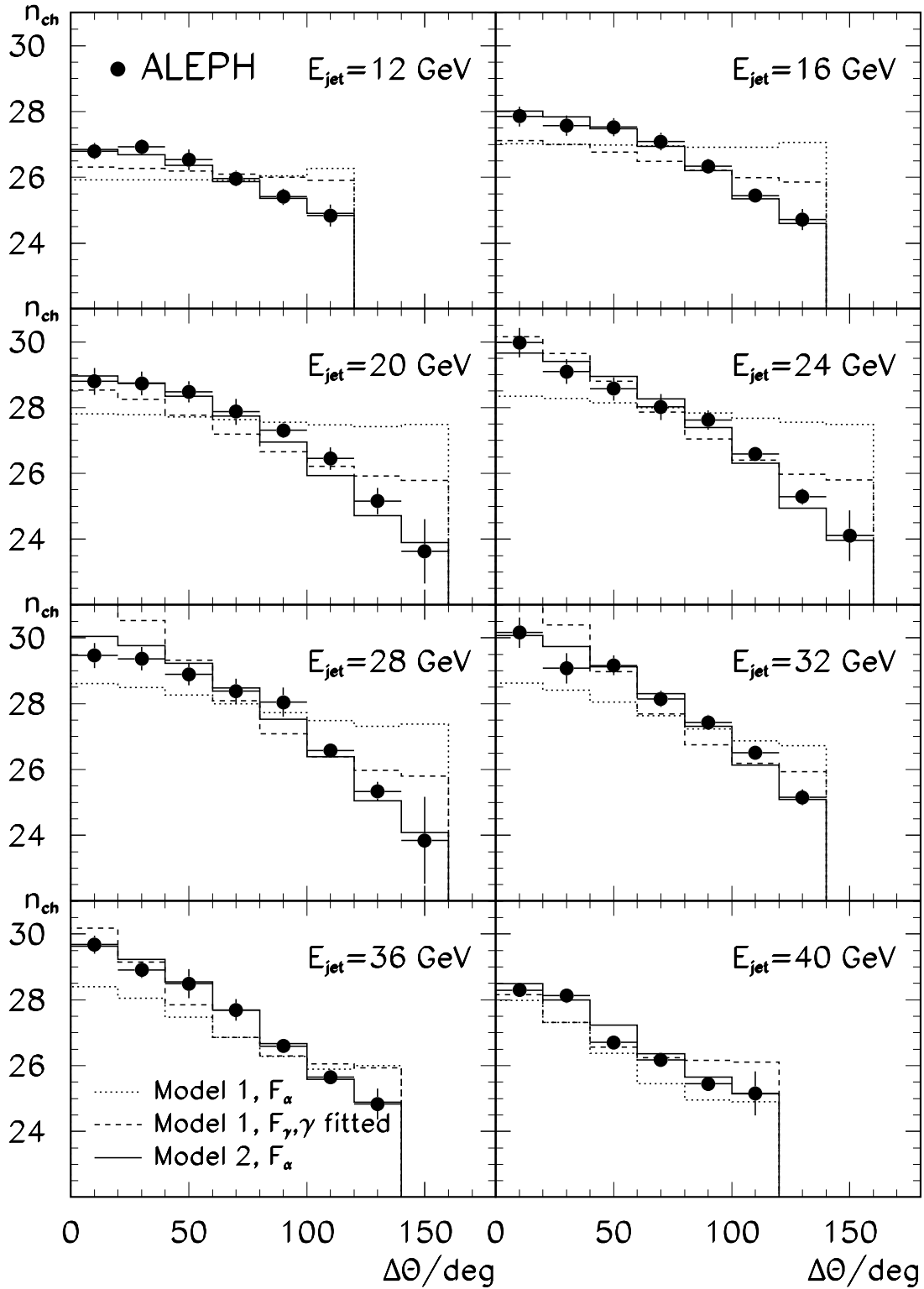


Figure 4: Event charged multiplicity for bins of fixed jet energy as function of the event topology. The curves are best fit results for the models described in the text.

That the failure of Model 1 to describe the data is not a consequence of fixing the value of the strong coupling constant to  $\alpha_s(M_Z) = 0.118$  can be checked by repeating the analysis with  $F = F_\gamma$  adjusting simultaneously  $N$ ,  $f - G$  and the parameter  $\gamma$ , i.e. allowing also the shape of  $F(Q)$  to vary. The choice of  $F_\gamma$  for this test is motivated by the fact, that the simple power law has more flexibility for the shape of  $F(Q)$  than the MLLA-formula with a variable parameter  $\alpha_s(M_Z)$ . The result is superimposed as well on the data in Fig.4. Although the  $\chi^2/df = 261/54$  is much lower than before, the fit is still bad. Furthermore, the best fit value  $\gamma = 2.93$  corresponds to an energy variation that is in disagreement with the measurements shown in Fig.3. In addition, the fitted  $f_g = 5.6$  is much larger than the experimental results and even the asymptotic QCD expectation.

## 5 Summary and Conclusions

It has been shown that charged particle multiplicities in three-jet events cannot be understood quantitatively if jet properties are a function of only the jet energy. The data are, however, consistent with a simple model assuming that the properties of a jet scale with the  $Q = E_{\text{jet}} \sin \theta/2$ , where  $\theta$  is the opening angle to its colour connected partner in the overall centre-of-mass system. The scale dependence can be parametrized by the same functional form that also describes the total charged particle multiplicity in  $e^+e^-$ -annihilations into hadrons as function of the centre-of-mass energy. The value found for the ratio of quark- and gluon jet multiplicities is consistent with results from the study of symmetric Y-shaped events.

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