OPTICS LIMITS FOR THE HIGHEST ENERGY

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Abstract

The maximum energy allowed by the LEP RF voltage is around 100GeV after upgrade. The power converters allow about the same maximum energy with the present 102/90 optics. With an RF voltage limited to a lower value, a maximum of 0.1GeV per beam could be gained using a 108/90 optics. This is probably not worthwhile. A gain of about 0.8GeV per beam is possible using a 131/90 optics. However this needs new power converters for at least the QF quadrupoles and the SD1 sextupoles, as well as a solution to the present injection problem. Therefore such an optics can only be considered for the very last LEP weeks of operation to obtain the last 10pb^{-1} at the ultimate energy.

As a conclusion it is clear that the 102/90 optics will be kept as the operation optics until the end of LEP. Gaining about 0.8GeV per beam at fixed RF voltage is possible with some investment.

1 INTRODUCTION

The maximum LEP energy is strongly constrained by the RF voltage. A little gain can be obtained by reducing the momentum compaction factor for a given RF voltage.

At first the optics limitations imposed by the present power converters are recalled. Then more basic optics limitations are examined.

The ultimate energy can only be obtained with a minimum momentum compaction. This is why a strong focusing optics has been tested. The future of such an optics is discussed eventually.

2 OPTICS LIMITATIONS DUE TO THE PRESENT POWER CONVERTERS

For a good performance at high energy, the optics must have a small momentum compaction factor. In the range where LEP is working presently, the horizontal anharmonicity is positive and increases very steeply when the momentum compaction factor is decreased, i.e. when the horizontal phase advance is increased. This is why it is not worth developing an optics with an horizontal phase advance smaller than 120° (for which the anharmonicity is infinite) since :

- it has been observed that already with a phase advance of 108° the horizontal anharmonicity is too large.
- the gain in emittance (for beam-beam tune-shift) and bucket width (to increase the energy at fixed RF voltage) are too small to be attractive.

Under these conditions the limitation of the quadrupole gradients due to the present power converters is not critical. With those foreseen for 1999, the maximum energy is limited to about 103GeV. This has been checked by M. Lamont and D. Brandt, using the actual calibration curves of the LEP quadrupoles.

The variation of the limit due to the maximum QF gradient with the horizontal phase advance in the arc cells is shown on figure 1. It appears clearly that the 102 optics is a beautiful optimisation of LEP for a maximum working energy of about 100GeV. With an RF cavity voltage limited to that of 1998, only 0.1GeV could be gained with the 108 lattice. This gain is probably too small to justify the development efforts and the associated loss of physics time.

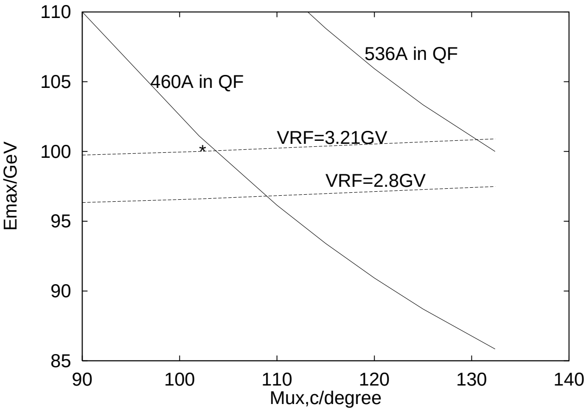

Figure 1: Variation of the maximum energy with the horizontal phase advance for two values of the maximum current in the QF quadrupoles and for different RF voltages. The star $*$ corresponds to the present 102/90 lattice. The voltage used in 98 with its upgrade corresponds to a total of 2.8GV

A larger energy gain can be obtained by increasing the QF current a little above its design value of 525A. This is discussed in section 4.

The skew quadrupoles used to compensate the solenoid coupling should not introduce any limitation as the solenoid strengths decrease with energy. Typically what is needed is a constant gradient for a given insertion optics. However if the insertion optics have to be changed for any reason, the phase advances in the low-β insertions is available to make the coupling compensation possible since the bunch train scheme is no longer used.

3 FUNDAMENTAL OPTICS LIMITATIONS

3.1 Non-linear chromaticity correction

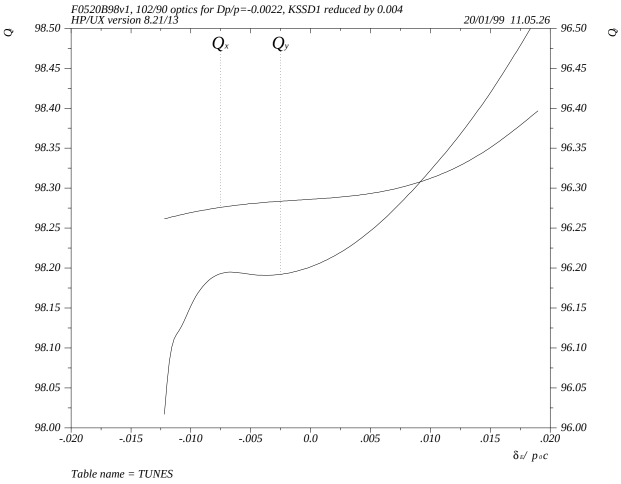

Figure 2: Variation of the tunes with momentum deviation for the present 102/90 lattice with β_y^*=0.04m and strength of SD1 family increased by 0.004. The nominal tunes and chromaticities of +1 are set for Δp/p=-0.0022 (120Hz). The vertical instability occurs for Δp/p=-0.0132.

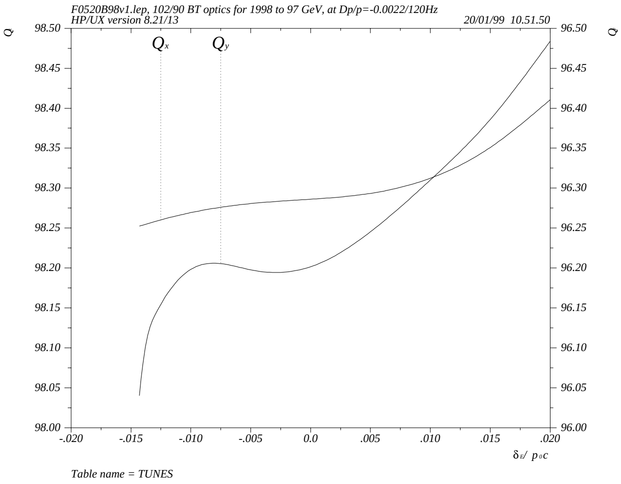

Figure 3: Variation of the tunes with momentum deviation for the present 102/90 lattice with β_y^*=0.05m. The nominal tunes and chromaticities of +1 are set for Δp/p=-0.0022 (120Hz). The vertical instability occurs for Δp/p=-0.0143.

The requested momentum aperture scales in the same way as the relative r.m.s. beam energy spread, i.e. linearly with energy. Jorg tested in an end-of-run experiment that when the difference between the K' of the SD families is reduced by 0.008m^{-3}, the beam life-time decreases to 3h (this sort of check has been done repeatedly since the beginning of LEP [3]. Ghislain did it in 1996 and 1997 and found the same sort of results).

This test shows that there is presently a margin for the correction of the non-linear chromaticity. The variation of the tunes with momentum for Jorg's experiment is shown on figure 2. The stability limit occurs for a negative relative momentum deviation k of -0.0122, i.e., for this fixed value of the momentum deviation, the transfer matrix of the machine has a trace larger than 2. Jorg's measurement was done at 94.5GeV with an RF frequency shift of 120Hz, i.e. a relative momentum deviation of -0.0022. This leaves an available relative momentum aperture of 0.01 to accommodate the synchrotron oscillations and this is the very minimum needed to guarantee the beam life-time. Note that it represents about 5.5σ_e (σ_e=0.001795 for Δp/p=-0.0022 at 94.5GeV). Scaling the needed relative momentum aperture to 100GeV gives a value of 0.0106, which can be easily achieved as shown by the variation of the tunes with relative momentum deviation for the 102/90 optics with β_y^*=0.05m on figure 3. If β_y^* is reduced to 4cm, the margin still exists as shown on figure 4. For this case the momentum aperture is limited for the negative momentum deviations to 0.0131. The margin is 0.0109, i.e. just sufficient. If β_y^* is reduced further, the phases of the experimental IR's have to be changed to improve the non-linear chromaticity correction. The vertical phase advance in the arcs can also be decreased a little to this end.

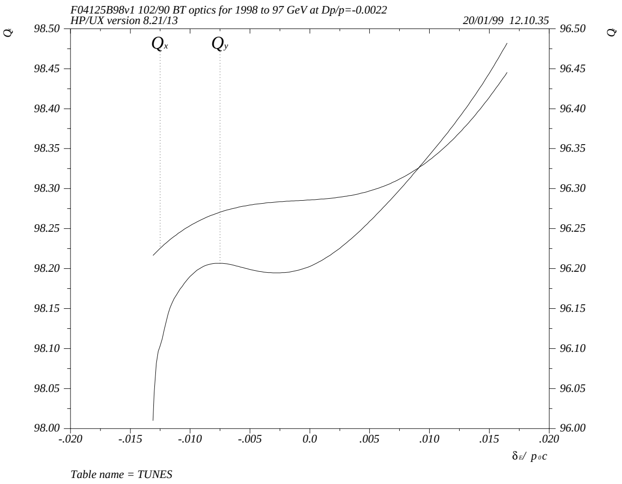

Figure 4: Variation of the tunes with momentum deviation for the present 102/90 lattice with β_y^*=0.04m. The nominal tunes and chromaticities of +1 are set for Δp/p=-0.0022 (120Hz). The vertical instability occurs for Δp/p=-0.0131.

However if the available momentum aperture is of fundamental importance, it is not enough to guarantee a good performance. Even for a perfect chromaticity correction, the dynamic aperture is eventually limited by the radiation loss in the low-beta quadrupole (RBSC [4]). This is examined in the next section. In the case of a non perfect correction of the non-linear chromaticity, too large a value of Q” excite second order synchro-betatron resonances. This is examined in a further section.

3.2 Radiative synchro-betatron coupling (RBSC)

This phenomenon limits ultimately the dynamic aperture, whatever optics perfection is reached. For the 98 optics it limits precisely he dynamic aperture [5], which shows the quality of this optics. The particles with a large betatron amplitude have unstable trajectories because of the energy they lose in the low-β quadrupoles, which in turns increases their momentum deviation outside the RF bucket [4]. So, even for a perfect chromatically corrected optics, i.e. an optics which does not change with momentum deviation, the particles are eventually lost on the vacuum chamber in a place where the dispersion is non zero.

For the 98 optics the maximum horizontal action associated with this phenomemon is larger than 9×10^{-6}m, taking into account possible optics imperfections [5], with a value of β_x^* of 1.5m and for a vertical action up to 1×10^{-6}m (remember the vertical emittance is about 0.2nm, so this represents more than $20\sigma_y$). At 100 GeV the horizontal beam emittance is 26.3nm (with the RF shift of 120Hz), i.e. the horizontal acceptance associated with RBSC is more than $18\sigma_x$ (together with a vertical acceptance of more than $20\sigma_y$). Note that on the central orbit, the horizontal beam emittance is 44.3nm and the the horizontal acceptance associated with RBSC is still $10\sigma_x$.

The synchrotron energy loss scales with the inverse of the radius of curvature of the trajectory. In the low-β quadrupoles the radius of curvature scales with the inverse of the value of the β-function. Then the synchrotron loss in the low-β quadrupoles scales with $(\beta^*)^{-1/2}$. This means that reducing β_x^* to 1.2m reduces in turn the dynamic acceptance due to RBSC to 8×10^{-6}m, which represents still $16\sigma_x$ (for values of σ_y up to 20). So the RBSC’s will not create any problem for the highest LEP energy.

3.3 Synchro-betatron resonances

Their effect can be shown with the following heuristic calculation.

We start from the transverse equation of motion in normalised coordinates (the independent variable θ is equal to the phase advance divided by the tune Q, it varies from 0 to 2π over one turn) :

$$X'' + Q^2 X = 0 \tag{1}$$

The exact solution of equation (1) is :

$$X = Asin(Q\theta + \phi)$$

where A and ϕ are determined by the initial values of X and X'. We forget the initial phase ϕ which will not play any role and keep as “general solution” :

$$X = AsinQ\theta$$

The modulation of the tune due to synchrotron oscillations of amplitude δ and frequency Q_s is introduced as :

$$Q = Q_0 + Q'\delta sinQ_s\theta + Q''\frac{\delta^2}{2}sin^2Q_s\theta$$

in equation (1), Q' and Q'' being the first and the second derivatives of the tunes with momentum.

We seek a perturbed solution of the form :

$$X = AsinQ\theta + \epsilon$$

where ϵ is a function small compared with A. In order to obtain the differential equation for the function ϵ, the next steps are :

- put this expression in equation (1),
- expand and keep the first order in ϵ on the left
- keep the second order in δ on the right

This is not mathematically correct in that sense that terms are missing. The correct procedure would consist of computing ϵ as a power series of δ. Not doing this makes certain terms proportional to δ^2 disappear, which is not fundamental for the present heuristic approach and alleviate the calculation. Keeping this in mind, we obtain :

$$\epsilon'' + Q^2\epsilon = AsinQ\theta(Q'^2 + QQ'')\delta^2 sin^2Q_s\theta$$

This is the equation of a driven linear oscillator as the product of sine and cosine can be expanded as a sum of sine. Keeping only one term to alleviate the formulae, we obtain eventually :

$$\epsilon'' + Q^2\epsilon = \frac{A}{4}(Q'^2 + QQ'')\delta^2 sin(2Q_s - Q)\theta$$

the solution of which is :

$$\epsilon = \frac{A(Q'^2 + QQ'')}{16Q_s(Q - Q_s)}\delta^2 sin(2Q_s - Q)\theta \tag{2}$$

Formula 2 contains all the features of the resonance theory. The amplitude of the perturbation ϵ becomes infinite for the tune $Q = Q_s$. We know experimentally that such tunes have to be avoided. Far from the resonance defined by $Q = Q_s$ the frequency of the perturbed motion is made from combinations of the betatron and synchrotron tunes, e.g. $2Q_s - Q$ in equation 2. Here we have to remember that several terms have been forgotten in the expression of ϵ. The complete calculation shows that the frequencies $Q_s + Q, 2Q_s + Q, 2Q_s - Q$ are involved.

The relative amplitude of the perturbing oscillation extracted from equation 2 is :

$$\frac{(Q'^2 + QQ'')}{16Q_s(Q - Q_s)}\delta^2 \tag{3}$$

In order to have an idea of whether the perturbation affects the core of the beam, δ has to be taken equal to σ_e, i.e.

0.0018 at 100GeV. The fractional part of the vertical tune is 0.19 and Q_s is about 0.11 in physics. Putting these numbers in formula 3, the perturbation is a fraction equal to $2.310^{-5}(Q'^2 + QQ'')$ of the unperturbed oscillation amplitude. As Q'' is usually below 1000 this perturbation is of the order of a per-mill and should not intervene in the emittance buildup. However, as the perturbation scales with δ^2, it becomes sizable in the tail of the transverse particle distribution and can contribute to the "non-Gaussian tails" which were observed at the end of 1997 with the 102/90 optics.

This was the reason to propose a re-cabling of the SF sextupoles. This was indeed useful as it was observed in 1998 that the horizontal tails of the transverse particle distribution were less populated than they were at the end of 1997 for the test of the 102 optics where there was only a single SF family. For this last case, Q''_x had a value of about 600. The re-cabling made it decrease to some tenths so that their effect become comparable to that of the linear chromaticity. This experimental fact shows that it is probably worth decreasing Q''_y with is about 2000 with a β^*_y of 0.04m. This has to be tried by changing slightly the vertical phase advance between the even IP's and the arcs and possibly the vertical phase advance in the arcs cells.

4 A VERY LOW EMITTANCE OPTICS

The ultimate LEP energy can only been achieved with an optics with a small momentum compaction factor as already discussed last year [1]. The momentum compaction factors of three typical optics are given in table 1.

Table 1. Momentum compaction factor η and ultimate energy for three LEP optics. The ultimate energy is obtained when the RF voltage makes a beam life time of 24h (computation with WIGWAM [2]). The circumferential voltage of 3098.3MV is associated with 100GeV for the 131/90 lattice.

Optics	90/60	102/90	131/90
$\eta/10^{-4}$	1.855	1.560	1.076
Ult. energy/GeV	98.83	99.11	100

The drawback of a very low emittance optics is its large negative horizontal anharmonicity $\partial Q_x/\partial E_x$ which makes the horizontal tune extremely sensitive to horizontal closed orbit distortions.

4.1 Present status

An optics with an horizontal phase advance per cell of 132.8° (Qh=118.28 and Qv=92.18) and $\partial Q_x/\partial E_x$=-5.24$\times 10^4 m^{-1}$ was made and tested in the machine [6] in 1998. Apart of an injection problem which was present in all optics in 1998, an upper limit of the dynamic aperture, estimated from the beam life-time with emittance wigglers on, was about 0.8 $10^{-3} m^{1/2}$. As the natural beam emittance at 100GeV is 29nm, a dynamic aperture of at least $5.5\sqrt{0.029}$ = 0.94 $10^{-3} m^{1/2}$ is needed to guarantee the life-time. This means that such an optics can probably not work or only with perfect optics corrections, i.e. not in routine operation (this has been shown clearly in J. Jowett's simulations [7]).

This bad dynamic aperture is in fact associated with the "small" horizontal anharmonicity. Accepting an increase of this parameter to -1.3$\times 10^5 m^{-1}$ by setting the SF1 family to its present maximum at 100GeV, increases the dynamic aperture by about 50%. This is enough for the beam life-time but then we face a problem similar to that encountered with the 108/90 lattice. Such a large negative horizontal anharmonicity puts the horizontal tune on a fourth order resonance for an amplitude of 3σ at 100GeV. This could possibly increase the emittance, which is probably not the most serious problem since the emittance is already very small. As the fourth order resonances are probably less important than the third order ones, it is not clear whether this problem is really serious. Only an experiment can answer this. In any case as this optics is not dedicated to mass production,

4.2 Power converters

The limitations due to the power converters are more serious than for the 102 optics. The current of the QF quadrupoles has to be increased to 536A for 100GeV whilst their design current is 525A. The current in the SD1 sextupole has to be increased to 320A (their design current is 350A). The same problem as for the 102 optics occurs both for QS18 and QL18 (only QL18 made a problem with the 102/90 optics) and can probably be solved by re-matching.

4.3 Non-linear chromaticity

The non-linear chromaticity can be corrected as well as for the 102/90 optics. The variation of the tunes with momentum is shown on figure 5.

4.4 Prospects

It is clear that the 131 optics can only be used for dedicated runs and not for standard production.

The first problem to solve is that of the LEP injection. If this can be solved at the startup, then it is worth testing again the maximum intensity which can be stored with the optics with the large anharmonicity and to measure its dynamic aperture.

If a satisfactory answer can be obtained before summer, then it can be envisaged to upgrade the power converters as mentioned above. This action needs at least six months and costs some hundredth of KSF.

The time needed to obtain 10pb^{-1} can be estimated as follows. We assume a current per bunch of 300μA, an horizontal emittance of 32nm, a vertical emittance of

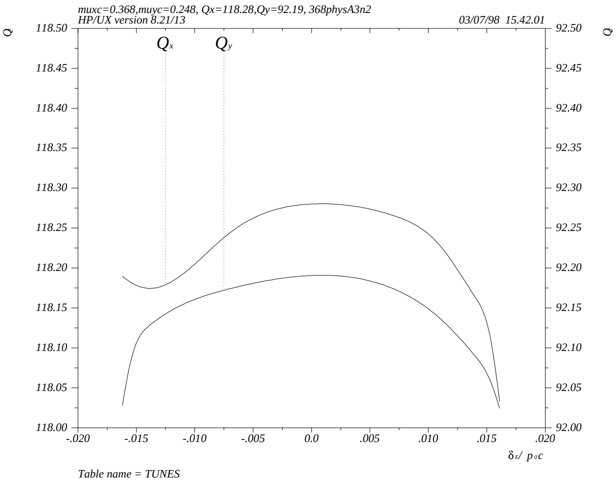

Figure 5: Variation of the tunes with momentum deviation for the 132/90 lattice with β_y^*=0.05m. The phase advances per cell are 0.386$\times 2\pi$ in the horizontal plane and 0.248$\times 2\pi$ in the vertical plane. It is not exactly $\pi/2$ in this plane to maximise the momentum acceptance.

0.15nm. This makes beam-beam tune-shifts of 0.012 in horizontal and 0.027 in vertical, which is extremely comfortable for operation. The maximum luminosity is 1.4$\times 10^{31} cm^{-2} s^{-1}$. Assuming an efficiency of 0.15, the luminosity integrated over one day is 0.18^{-1}. Thus one month and a half is needed to obtain 10pb^{-1}. Adding half a month to put the optics in operation, this means that two months should be reserved at the end of the LEP life to operate this optics if the decision is taken to use it.

5 CONCLUSION

It appears clearly that the 102/90 optics is the only one which can be used for physics production up to the end of LEP. The gain obtained with any optics with an horizontal phase advance smaller than 120° is too marginal to justify any further development in this direction.

In order to get to the ultimate energy, 0.8GeV more than with the 102 optics can be gained, thanks to a very low momentum compaction factor. To this end an optics with an horizontal phase per cell of about 131° could be used. However this needs the investment of new power converters for both the QF quadrupoles and the SD1 sextupoles. Such an optics is only useful to produce the last 10pb^{-1} needed for a fast search at the very maximum energy [8].

6 REFERENCES

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