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VAPOUR BUBBLE DYNAMICS IN HYDROGEN BUBBLE CHAMBERS

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SUMMARY *)

Theoretical considerations of vapour bubble dynamics in a hydrogen bubble chamber are given. The system of equations takes into account the heat and mass exchange between a bubble and liquid. The vapour in the bubble is described with the real gas equation of state. Computer solutions are in agreement with the corresponding experiment. A set of temperature distributions in the surrounding liquid during the cycle of bubble chamber is presented. The important role of the temperature distribution formed during the bubble growth is shown for the correct description of the bubble collapse. Some predictions are made for the bubble behaviour in fast-cycling chambers.

The results of numerical calculations for the bubble growth and collapse in the static pressure of the bubble chamber are discussed. The stability of self-similar solutions describing bubble growth is proved. Conditions for self-excitation oscillations in the bubble collapse take place. These natural frequency pulsations lead to the lifetime increasing. In this connection the possibility of the existence of stable bubbles pulsating in the overpressed liquid is considered using both analytical and numerical methods. It is shown that the assumption of the existence of such bubbles leads to a contradiction in the system of equations.

*) The above summary in English was provided by the authors, L.G. Tkachev and V.D. Shestakov.

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From a practical standpoint, the technical development of bubble chambers has led to the necessity of examining the dynamics of a vapor bubble in a liquid. A significant number of theoretical[1)-6)] and experimental [7), 8)] works are devoted to the examination of this thermo-hydrodynamic problem. Due to the complexity of the problem, it is impossible to obtain an exact analytical solution of the system of equations describing the behavior of a bubble. In the present work numerical calculations are used to examine the growth and the collapse of a vapor bubble in a hydrogen bubble chamber, taking into account rectified heat diffusion [9), 12)].

1. Let us limit ourselves to an examination of the behavior of a spherical vapor bubble in an incompressible liquid. The temperature field around the bubble is assumed to be isotropic. The bubble is thought to be homogeneous. The vapor is in thermodynamic equilibrium with the surface layer of the liquid, which means that

$$P'(t) = P(R,t) + 2\sigma/R, \quad T'(t) = T(R,t), \tag{1}$$

where R is the radius of the bubble, t -- time, σ -- surface tension of the liquid, P', T' and P,T -- pressure and temperature of the vapor and liquid respectively.

The equations governing the behavior of a bubble, follow from the laws of the conservation of momentum, mass and energy. The continuity equation in an incompressible liquid

$$\dot{r} r^2 = U_R R^2 \tag{2}$$

allows us to express the law of the conservation of momentum during the movement of the liquid as

$$R \dot{U}_R + 2U_R \dot{R} - \frac{1}{2} U_R^2 = \frac{P' - \frac{2\sigma}{R} - P_\infty}{\rho}, \tag{3}$$

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where r is the point coordinate in the liquid, $\dot{r}$ and U_R -- speeds at the points r and R, respectively, ρ -- density, and P_∞ -- pressure in the liquid. The speeds $\dot{R}$ and U_R are correlated, following from the law of the conservation of mass during evaporation (condensation)

$$\dot{R} = \frac{U_R\,\rho + \frac{R}{3}\frac{dP'}{dt}}{\rho - \rho'}, \tag{4}$$

where ρ' -- density of the vapor. $P'(t)$ is determined by the law of the conservation of energy during evaporation/condensation

$$\frac{dP'}{dt} = \frac{3}{R}\cdot\frac{k\frac{\partial T}{\partial R} - \rho' L\dot{R} + T'\frac{\partial\sigma}{\partial T'}\frac{2\dot{R}}{R}}{L\frac{d\rho'}{dP'} + C_s\,\rho'\frac{dT'}{dP'}}, \tag{5}$$

where k is the thermal conductivity of the liquid, L -- the heat of vaporization, C_s -- heat capacity of vapor along the curve of phase equilibrium $\partial T/\partial R \equiv \partial T(r,t)/\partial r|_{r=R}$. The derivations $d\sigma/dT'$, $d\rho'/dP'$, dT'/dP' are calculated along the curve of phase equilibrium.

It is known that the equilibrium vapor pressure for a given temperature depends upon the radius of the bubble

$$P' = P_s + \frac{2\sigma/R}{1-\rho/\rho'}, \tag{6}$$

where P_s is the equilibrium pressure at the plane surface of phase separation, connected to the equilibrium temperature by the dependence $T'_s = f(P_s)$ -- the equation of the curve of phase equilibrium (formula (6) is valid when $2\sigma/R \ll P_s$). Therefore, the pressure and temperature of the saturated vapor in a bubble are connected by the relation

$$T'(P') = f\left(P' - \frac{2\sigma/R}{1-\rho/\rho'}\right). \tag{7}$$

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Similar relations are used in the calculation of all thermodynamic values along the curve of phase equilibrium. Since the behavior of a bubble depends upon the temperature gradient, it is necessary to examine the equation of heat conduction in the liquid

$$\frac{\partial T}{\partial t} + \frac{U_R R^2}{r^2} \frac{\partial T}{\partial r} = \frac{D}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial T}{\partial r}), \qquad (8)$$

where D -- thermal diffusivity of the liquid. The sum of the equations (2) - (5) and (8) with corresponding initial and boundary conditions form a closed system.

2. Let us examine the growth and the collapse of asymptotic vapor bubbles in equations (3) and (5), when terms due to surface tension are unimportant. It is assumed that the pressure in the liquid is constant. Under this condition, the influence of the inertia of the liquid upon the dynamics of a bubble is negligibly small [1) - 5)], so that equation (3) degenerates into the equality $P' = P_\infty$. The change in the bubble radius is determined by the heat flow across its surface and is described by the equations

$$k \frac{\partial T}{\partial R} = \rho' L \dot{R} \qquad (9)$$

$$\frac{\partial T}{\partial t} + \frac{\dot{R} R^2}{r^2} (1 - \frac{\rho'}{\rho}) \frac{\partial T}{\partial r} = \frac{D}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial T}{\partial r}), \qquad (10)$$

which are a special case of equations (5) and (8). It is known[5)], that with additional conditions

$$T(R,t) = T', \ T(r=\infty, t) = T_\infty, \ T(r, t=0) = T_\infty, \qquad (11)$$

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$$T_\infty - T' > 0, \quad \dot{R}(0) = 0 \tag{12}$$

equations (9) and (10) have similar solutions, which describe the growth of a vapor bubble

$$R(t) = A_1 \sqrt{2Dt} \tag{13}$$

$$T(\nu) = T_\infty + (T' - T_\infty)\left[1 - \frac{A_1^2}{\sqrt{\pi} A_0} \int_\nu^1 \exp\left[A_1^2 \Phi(x)\right] dx\right], \tag{14}$$

where

$$\begin{aligned} A_0 &= (T_\infty - T')k/(L\rho' D) \\ \Phi(x) &= (x-1)\left[1 + x - 2x^2(1 - \rho'/\rho)\right]/2x^2 \\ \nu &= R/r, \end{aligned} \tag{15}$$

T_∞-- temperature of the liquid at infinity, and the parameter A_1 is calculated from the boundary condition $T(0) = T_\infty$.

When the collapse of a bubble is examined, the relations

$$T_\infty - T' < 0, \quad \dot{R}(0) = R_0 > 0 \quad , \tag{12'}$$

corresponding to (12), lead to the fact that initial and boundary conditions contradict the hypothesis of self-similarity. Analogous solutions, derived in works [3), 4)], similarly , describe only the growth of a bubble. The technique of successive approximations [1)] is also inapplicable in the case of bubble collapse, since the warm layer in the liquid adjoining the bubble is large. Nevertheless, the opinion that the processes of growth and collapse of a vapor bubble in an asymptotic stage are governed by identical laws [2), 4)] is

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widespread, that is, the change in the radius of a collapsing bubble satisfies the relation

$$R^2 = R_0^2 - A^2 D t \ , \tag{16}$$

where A is a dimensionless constant. In paper [6] it is shown that an asymptotic bubble should collapse like a Rayleigh cavity

$$t = R_0 \left(\frac{\rho}{6 (P_\infty - P')} \right)^{1/2} \int_{(R/R_0)^3}^{1} \frac{x^{-1/6}\, dx}{(1-x)^{1/2}} \ , \tag{17}$$

that is, the dynamics of the collapse, in contradiction to former assertions, are defined by equation (3), where $P_\infty - P' = \text{const.}$

In connection with the above, the derivation of numerical solutions of equations (2) - (5), (8) are of interest. In the case of bubble collapse, one can opine about the mechanism of the collapse according to the nature of the solution : whether it is determined by thermodynamic factors, that is, by relations (9), (1o) and (16), or by mechanical factors, that is, by equations (3) and (17); or, finally, whether there is perhaps some third possibility.

Diagram 1 shows results of numerical solutions of the equations describing the behavior of a bubble in liquid hydrogen. If the pressure of the saturated vapor P_S is greater than the pressure in the liquid P_∞ , then the curve of diagram 1 corresponds to the growth of the bubble, otherwise -- to its collapse. The initial temperature distribution in the liquid was selected either according to the formula of self-similarity solution (14) (solid lines) or according to that for homogeneity (dashed curves). The initial radius was selected to be equal to $5.1o^{-4}$ cm or $1.1o^{-3}$ cm.

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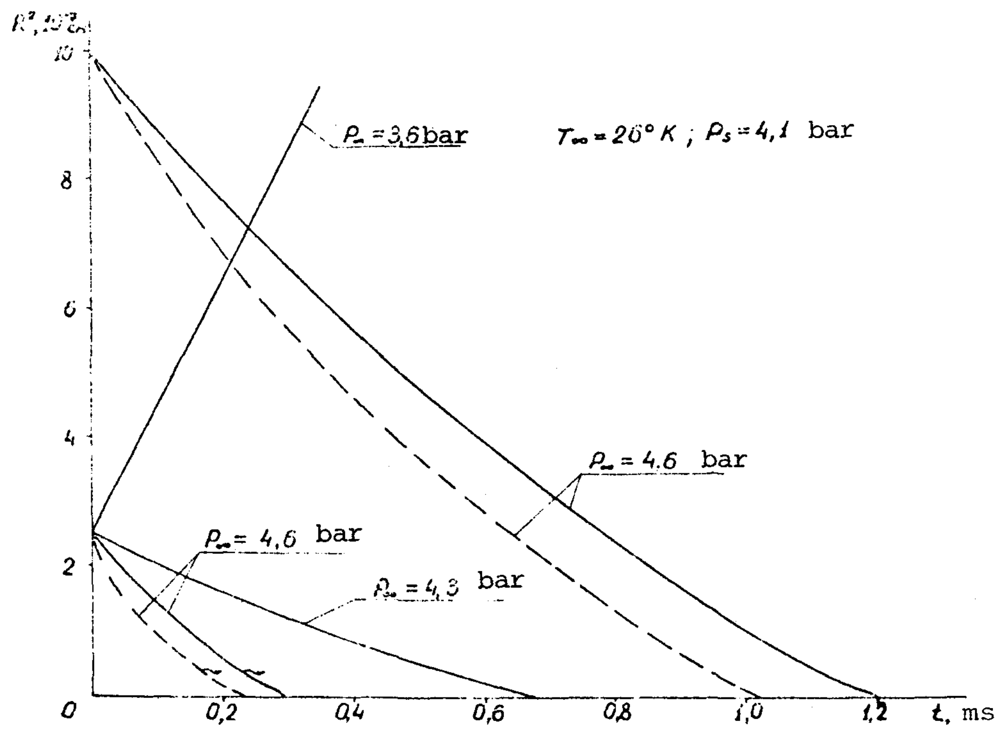


10
8
6
4
2
0
0,2
0,4
0,6
0,8
1,0
1,2
t, ms
P_∞ = 3,6 bar
T_∞ = 20° K ; P_s = 4,1 bar
P_∞ = 4.6 bar
P_∞ = 4,6 bar
P_∞ = 4,3 bar

Diag. 1

The curve of growth corresponds to the result of the self-similarity solution (13). The disturbances of the self-similarity solution, linked to the pulsations of the bubble on its own frequency, quickly disappear. It follows, then, that self-similarity solutions describing the growth of a bubble are stable.

Examining bubble collapse, we become first of all convinced that, as in the case of its growth, the pressure inside the bubble hardly differs from the pressure in the liquid. Therefore, in the description of this process equations (9) and (10) are most important. For a non-homogeneous initial temperature distribution the life spans of bubbles are proportional to R_0^2. The same result is derived from formula (16), although the change of the bubble radius with time is different in detail. It is also seen that, with given pressure, the smaller life span of a bubble corresponds to the homogeneous initial distribution.

3. Differing from the above, in bubble chambers the growth and the collapse occurs with pressure change. It is of interest to compare results of corresponding measurements[8] with calculations. Solving equations (2) - (5) and (8), we come to the conclusion that inertia factors are of little influence upon the behavior of an asymptotic bubble, that is, the equality $P' = P_\infty (t)$, can be substituted for equation (2) and equation (5) changes into:

$$L \frac{dM}{dt} + C_s M \frac{dT'}{dt} = 4\pi R^2 k \frac{\partial T}{\partial R}, \tag{18}$$

its special case being equation (9), where M -- vapor mass in the bubble. Since equations (9) and (18) differ, it follows that in examining the behavior of a bubble during variable pressure, it is meaningless to take into account in formula

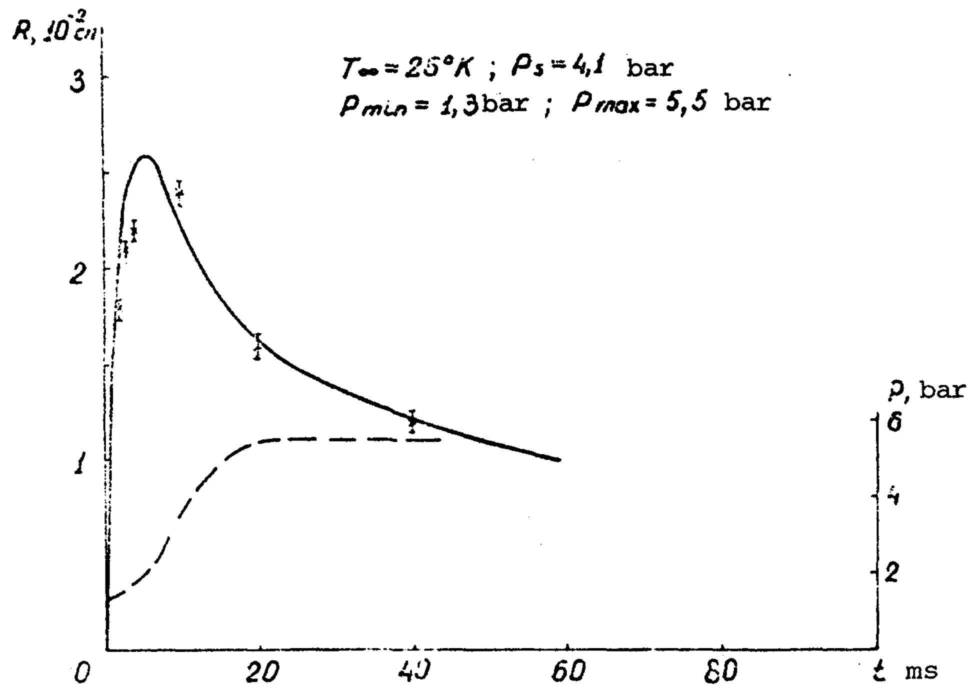
(13) the dependence of parameter A upon time. Diagrams 2 - 4 show the dependencies R(t) and $R_\infty(t)$ which correspond to experimental results [8]. On the basis of available facts concerning the conditions during measurements, the curve $P_\infty(t)$ was approximated in the following manner

$$P_\infty(t) = \begin{cases} P_{min} + at^2, & 0 < t < t^* \\ P_{max} - b(t - t_{max})^2, & t^* < t < t_{max} \\ P_{max}, & t_{max} < t, \end{cases} \qquad (19)$$

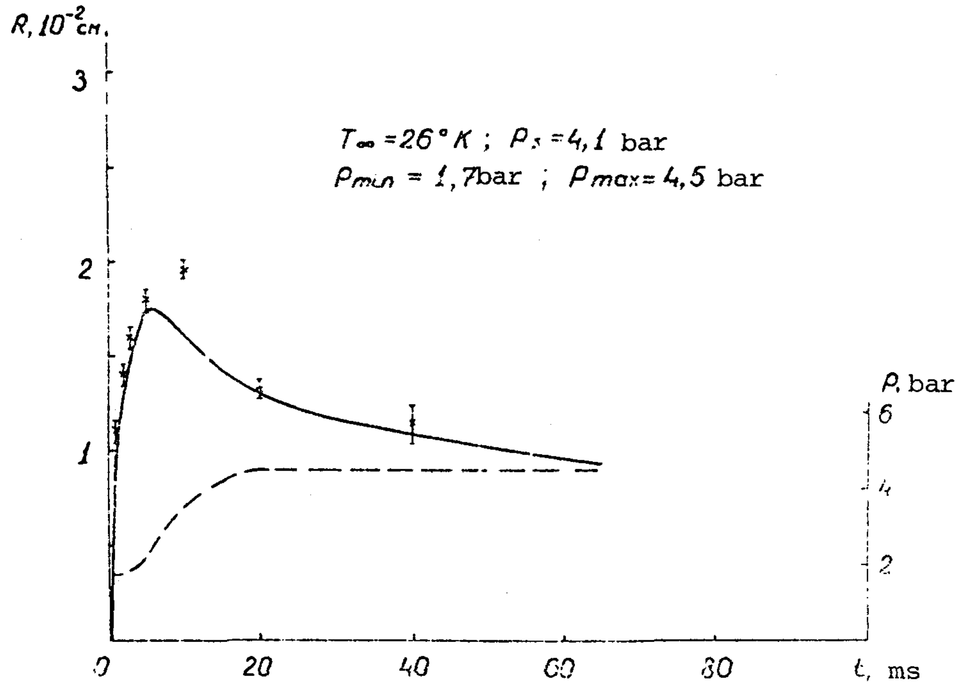
where P_{min} -- minimum pressure in the bubble chamber, during which the formation of embryonic bubbles by charged particles takes place, P_{max} -- maximum pressure reached during the time t_{max} during which the bubble collapses. Parameters a,b, and t* were calculated from the condition of continuity of function P_∞ (t) and its first derivative, here the condition $P(t_s) = P_s$ was considered, where t_s -- known time.

As seen from diagrams 2 - 4, theoretical curves describe satisfactorily the growth and the collapse of a bubble. The existing prior discrepancy [8] between the observed and the theoretically predicted speed of a bubble collapse can be explained, apparently, by the fact that bubble collapse is influenced by the nature of the temperature distribution which was formed in the overheated liquid during bubble growth. From diagram 5 can be seen that during the lengthy period after t_{max}, the character of the temperature distribution differs characteristically from the one generally used during calculations.

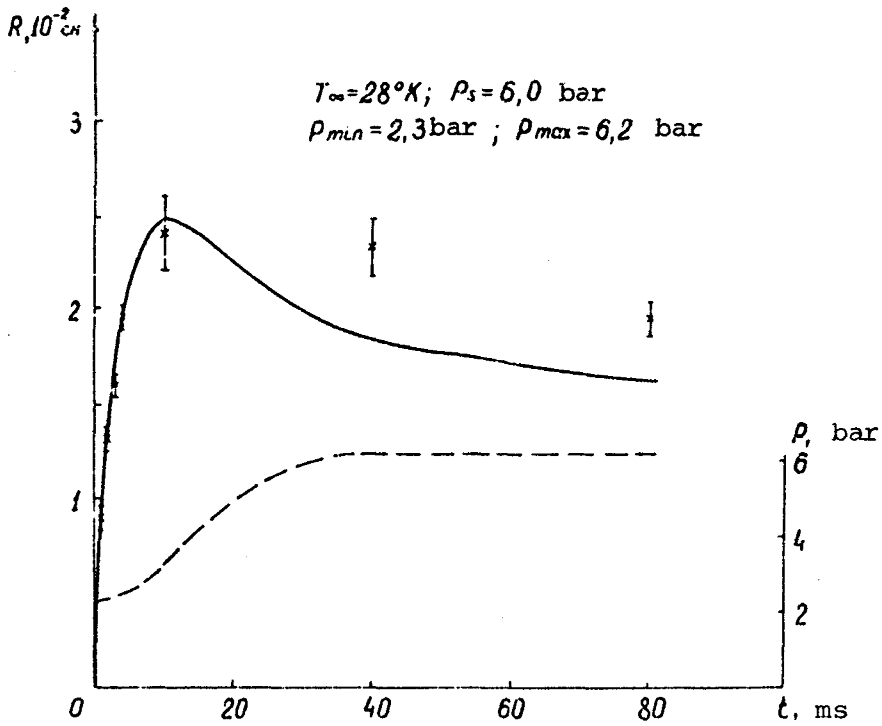
The speed of bubble collapse, corresponding to the asymptotic temperature distribution, is considerably


R, 10⁻² cm
T∞ = 25°K ; Ps = 4,1 bar
Pmin = 1,3 bar ; Pmax = 5,5 bar
3
2
1
0
20
40
60
80
t ms
P, bar
6
4
2

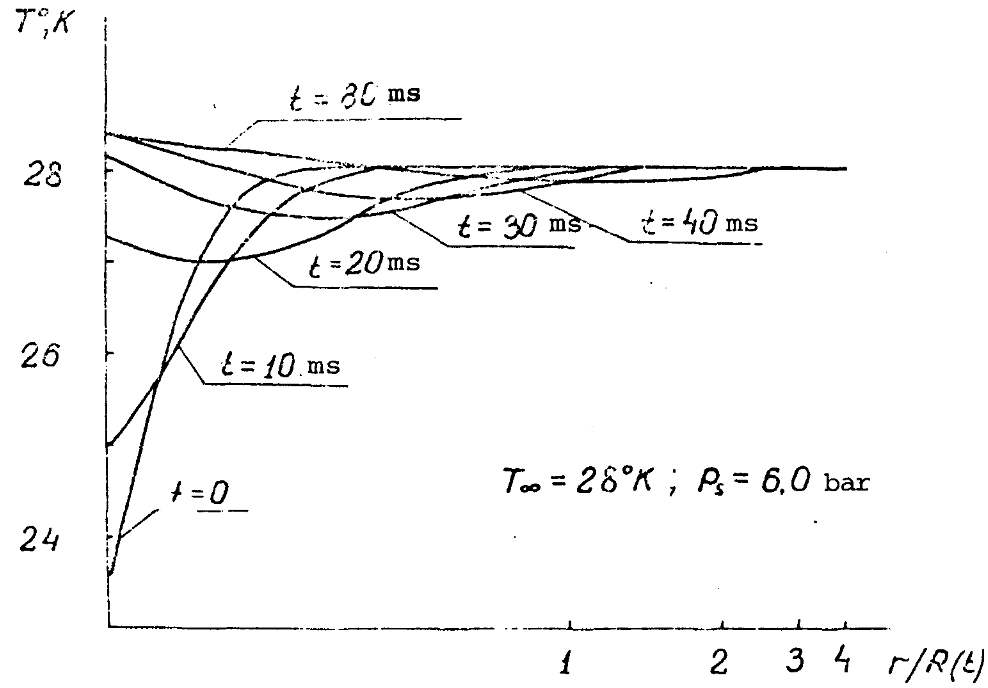
Diag. 2


R, 10⁻² cm.
3
2
1
T∞ = 26° K ; Ps = 4,1 bar
Pmin = 1,7 bar ; Pmax = 4,5 bar
P, bar
6
4
2
0
20
40
60
80
t, ms

Diag. 3


R, 10⁻² см
T_∞=28°K; P_s=6,0 bar
P_{min}=2,3bar ; P_{max}=6,2 bar
3
2
1
P, bar
6
4
2
0
20
40
60
80
t, ms

Diag. 4



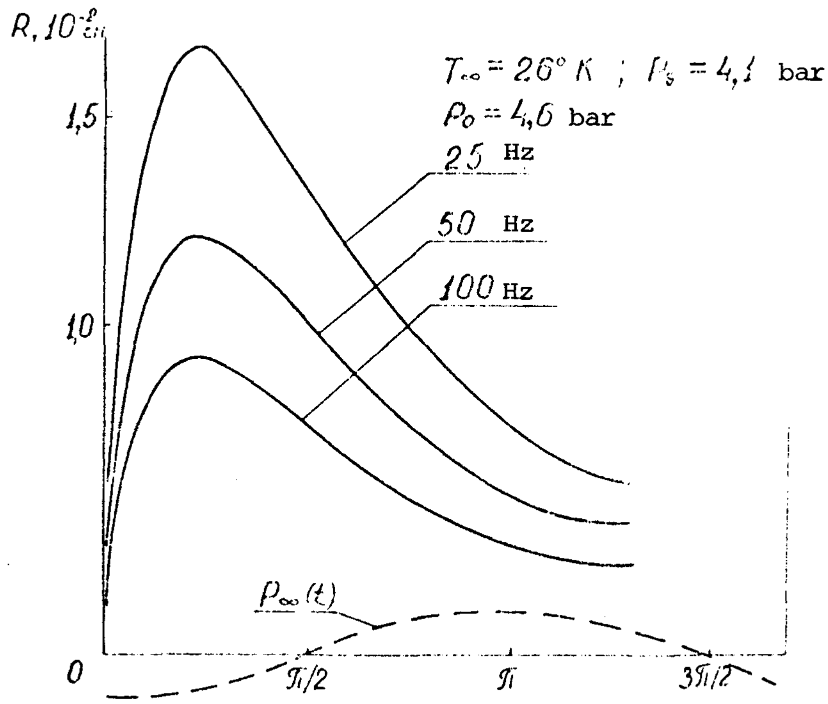
Diag. 5. Temperature distribution inside the liquid at variable time, corresponding to the behavior of the bubble, shown in diagram 4.

less than at time t_{max}. The observed difference in the calculated and the real behavior of a bubble is due, apparently, to the fact that formulas (14) and (19) correspond only approximately to the actual initial temperature distribution and to the change of pressure in the liquid. The lift of the bubble was also not considered.

4. In connection with resonance bubble chambers [15], let us examine the behavior of a vapor bubble during changing pressure in the liquid. Characteristic frequencies for such chambers are tens of Hz; moreover, their main parameter - the repetition rate - is determined to a considerable degree by the dynamics of bubble growth and collapse.

Diagram 6 shows the dependencies of R(t) for the frequencies 1oo, 5o and 25 Hz, calculated on the assumption that the pressure in the liquid changes according to $P_{\infty} = P_o - P_1 \sin \omega t$ and that the bubble is formed during the minimal pressure, when the phase is equal to $\pi/2$. Apparently, for a fixed amplitude of alternating pressure P_1, the minimum bubble radius reached during the end of its collapse, grows in proportion to the decrease in frequency. The maximum bubble radius behaves in like manner. In order that the bubble can reach visible size and collapse before the beginning of the next cycle, it is sufficient to increase the static pressure and the amplitude of the alternating pressure.

5. Examining the dynamics of a vapor bubble in an ultrasonic field, one becomes convinced that during the half-cycle of compression, when $P_{\infty} > P_s$, pulsations in the natural frequency arise and become extinguished during the half-cycle of expansion, when $P_{\infty} < P_s$ [11]. The cause



Diag. 6

of self-excitation of natural pulsations (self-oscillations) can be understood in terms of rectified and static heat diffusion [11),12)]. The point is that rectified heat diffusion is always directed from the liquid into the bubble, that is toward [i.e. against, G.H.] static diffusion as long as $P_\infty > P_s$. Therefore, under certain conditions the monotonous change of the bubble radius becomes unsteady and self-oscillations arise.

The behavior of a bubble during such oscillations depends essentially on the inertia of the liquid : it is necessary, therefore, to examine the entire set of equations (2) - (5) and (8). Analyzing numerical solutions, we can see that a bubble's own pulsations, caused in some way or other, disappear, if its radius is sufficiently large. The relative role of rectified diffusion grows in proportion to the decrease of the bubble, since it is determined by its surface curvature; therefore, during [bubble] collapse favorable conditions arise for the development of steady self-oscillations with small amplitude.

In diagram 1 the values of the bubble radius, during which self-generated self-oscillations take place are indicated by arrows. Diagram 7 shows that during the period of the heat-and-mass exchange with the surrounding liquid, the values of the bubble's natural frequencies differ from the corresponding values determined by formula (6)

$$f_0 = \frac{1}{2\pi R}\left(n\,\frac{3P_\infty}{\rho}\right)^{1/2}, \qquad (2o)$$

where n is a constant, equal to the ratio of specific heats of vapor $\gamma = C_p/C_v$ during adiabatic pulsations of the bubble and equal to 1 during its isothermic pulsations.

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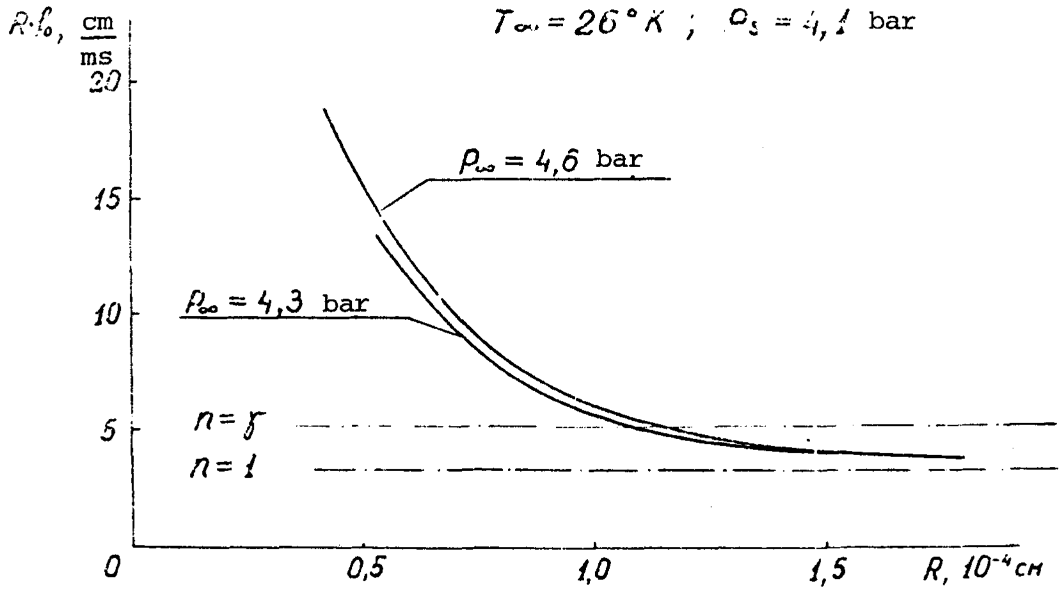

R·f₀, cm/ms
Tₒₒ = 26°K ; pₛ = 4,1 bar
20
15
10
5
0
pₒₒ = 4,6 bar
pₒₒ = 4,3 bar
n = γ
n = 1
0,5
1,0
1,5
R, 10⁻⁴ cm

Diag. 7

The smaller the bubble, the greater the deviation, and, consequently, the more essential, in comparison with volumetric factors, the relative role of the heat-and-mass exchange through its surface.

When $P_\infty > P_s$, self-oscillations arise which lead to rectified diffusion from the liquid into the bubble, and cause an increase in the life span of such bubble. A question arises, then, whether there exist such values for the thermodynamic parameters P_∞ and T_∞, during which the life span of a bubble becomes infinitely large, due to the fact that static and rectified diffusion counterbalance each other. The surface tension σ increases the pressure and the temperature of the vapor , leading to increased static heat diffusion; therefore, one must search for stable bubbles first of all in the neighborhood of the critical point, where σ is small. From the curves shown in diagram 8 one cannot conclude that such bubbles exist. If the existence of stable bubbles whose mean radius is $\bar{R}_\infty > R_o$ were possible during $P_\infty > P_s$, then the initial bubble, striving toward a stable condition, should have increased [in size] as a result of rectified diffusion. On the contrary, comparing the curves for $R_o = 1.10^{-4}$ cm and $R_o = 5.10^{-5}$ cm, we see that the smaller the bubble radius, the faster it collapses during $P_\infty > P_s$, or - the faster it grows during $P_\infty < P_s$.

The results obtained do not exclude the possibility of the existence of stable bubbles, if the range of the stability is sufficiently narrow : $0 < P - P_s < 0.01$ bar, or if there is a limitation for the value $(R_o - \bar{R}_\infty)$. Does the original system of equations allow in principle for the existence of stable, self-oscillating bubbles ? Let us suppose, that such bubbles do exist, that is ,

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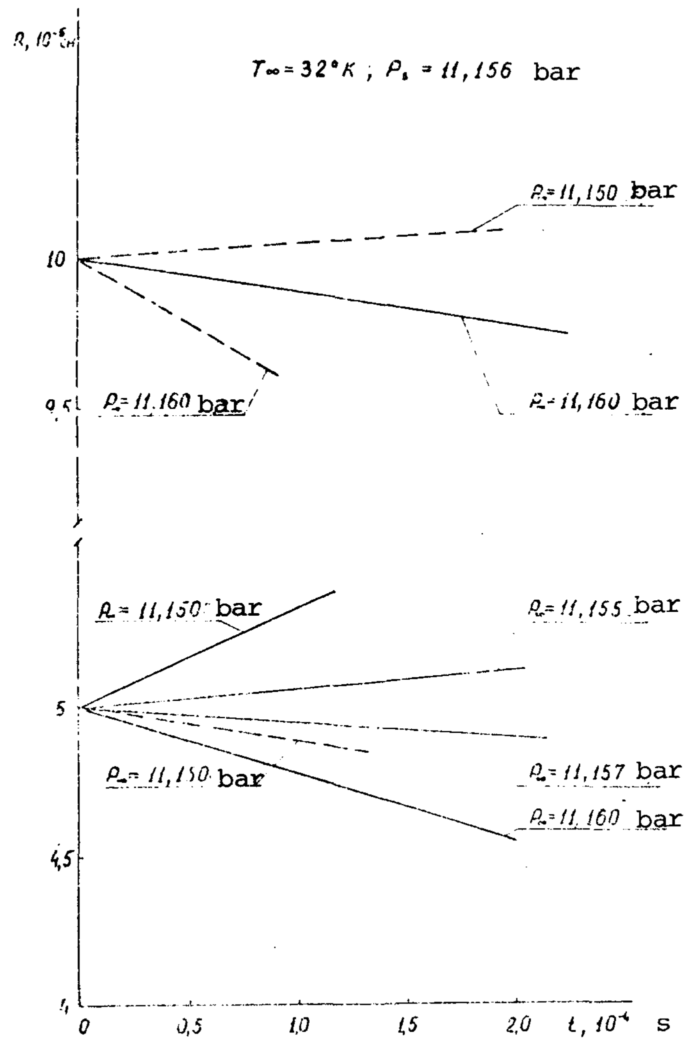

Diag. 8. The average dependency of the bubble radius upon time according to its own pulsations. The dashed curves correspond to the real values of σ, the solid ones -- to the idealized situation, when $\sigma \equiv o$.

$$R(t) = \bar{R}_{\infty}(1 + \delta \sin \omega_0 t), \tag{21}$$

where δ and ω_0 -- the amplitude and the frequency of the bubble's own pulsations. From numerical calculations it follows that $\delta \ll 1$. Here one can expand the separate terms of the equations into a series with respect to the powers of the small parameter δ. Let us look at the equations, derived in the zero and the first order approximation. We look for the dependency P'(t) in the form of

$$P'(t) = \overline{P}'[1 + \delta \Phi(\omega_0 t)], \tag{22}$$

where $\bar{P}'$-- is the mean vapor pressure in the bubble independent of time. From equations (3), (4) it follows that

$$\overline{P}' - P_{\infty} = 2\sigma/\bar{R}_{\infty}, \tag{23}$$

$$\Phi(\omega_0 t) = \frac{A}{\bar{P}'} \sin \omega_0 t, \tag{24}$$

$$U_R = -(A + 2\sigma/\bar{R}_{\infty})\delta \cos \omega_0 t/(\rho \bar{R}_{\infty}\omega_0), \tag{25}$$

where

$$A = \frac{\rho - \rho' + 2\sigma/(\bar{R}^3_{\infty}\omega_0^2)}{\frac{1}{3} d\rho'/dP' - 1/(\bar{R}^2_{\infty}\omega_0^2)}. \tag{26}$$

Substituting (21), (22) and (23) into equation (5) we obtain

$$\frac{\partial T}{\partial R} = \delta \omega_0 \bar{R}_{\infty} B \cos \omega_0 t, \tag{27}$$

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where parameter β is independent of time. Selecting as independent variables $\nu = R/r$ and $\tau = \omega_0 t$ for the equation of heat conduction (8), we have

$$\frac{\partial T}{\partial \tau} + \nu(1-a\nu^3)\,\delta\cos\tau\,\frac{\partial T}{\partial \nu} = \nu^4\beta(1-2\delta\sin\tau)\frac{\partial^2 T}{\partial \nu^2}, \tag{28}$$

where

$$a = -(A + 2\sigma/\bar{R}_\infty)/(\rho\,\bar{R}_\infty^2\omega_0^2), \quad \beta = \frac{D}{\bar{R}_\infty^2\,\omega_0}.$$

It is natural to look for the solution of this equation in the form of a series

$$T(\nu,\tau) = T_0(\nu,\tau) + \delta\,T_1(\nu,\tau). \tag{29}$$

The boundary conditions must also be expanded into a series. Here equation (28) breaks down into a system of 2 equations with corresponding boundary conditions

$$\frac{\partial T_0}{\partial \tau} = \beta\nu^4\frac{\partial^2 T_0}{\partial \nu^2} \tag{30}$$

$$T_0(\nu=1,\tau) = T'(\bar{P}'), \quad T_0(\nu=0,\tau) = T_\infty$$

$$\frac{\partial T_1}{\partial \tau} + \nu(1-a\nu^3)\cos\tau\,\frac{\partial T_0}{\partial \nu} = \nu^4\beta\left[\frac{\partial^2 T_1}{\partial \nu^2} - 2\sin\tau\,\frac{\partial^2 T_0}{\partial \nu^2}\right]$$

$$T_1(\nu=1,\tau) = \frac{\partial T'(\bar{P}')}{dP'}\,A\sin\tau, \quad T_1(\nu=0,\tau) = 0. \tag{31}$$

The asymptotic solution of equation (30) which satisfies the boundary conditions has the following form

$$T_0(\nu,\tau) = T_\infty + [\,T'(P') - T_\infty]\,\nu. \tag{32}$$

.../...

From here it follows that

$$\frac{\partial T_0(\nu,\tau)}{\partial \nu}\Big|_{\nu=1} = T'(\bar{P}') - T_\infty . \tag{33}$$

Expanding the left side of equation (27) with respect to powers of δ, we obtain

$$\frac{\partial T_0(\nu,\tau)}{\partial \nu}\Big|_{\nu=1} = 0. \tag{34}$$

It is easy to see that relations (33) and (34) exclude each other. Indeed, from relations (7) and (23) we obtain

$$T'(\bar{P}') = T'(P_\infty + \frac{2\sigma}{\bar{R}_\infty}) = f\,(P_\infty + \frac{2\sigma}{\bar{R}_\infty}\cdot\frac{\rho}{\rho-\rho'}), \tag{35}$$

therefore,

$$T'(\bar{P}') - T_\infty = \frac{df\,(P_s)}{dP}\,(P_\infty - P_s + \frac{2\sigma}{\bar{R}_\infty}\cdot\frac{\rho}{\rho-\rho'}) > 0. \tag{36}$$

The inequality results from the condition $P_\infty > P_s$. This conclusion will not be altered even during the idealized situation - when the surface tension of the liquid equals zero. The incompatibility of equations (33) and (34) means that within the limits of the present approach, the supposition that stable bubbles pulsating on their own frequency exist inside the liquid, is unjustifiable.

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