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MODEL-INDEPENDENT DETERMINATION OF THE REAL PARTS OF THE pp FORWARD SCATTERING AMPLITUDES NOT ALLOWING FOR SPIN EFFECT

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1. Introduction

When a real part of the pp and $\bar{p}p$ forward scattering amplitude is discussed, it is normally assumed that there is no dependence whatsoever on spin (viz., for example, [1]). The scattering amplitude (in the laboratory system) may then be represented in the form of an ordinary complex number, let us say,

$$f_{\pm}(\omega) = D_{\pm}(\omega) + iA(\omega) \ . \qquad (1)$$

Normalization was selected so that the optical theorem took the form $\sigma_{\pm}(\omega) = 4\pi A_{\pm}(\omega) / k$, where k and ω are the momentum and energy (hp) of the nucleon respectively.

The relationship $D_{\pm} / A_{\pm} \equiv a_{\pm}$ for this amplitude may be determined from its interference with the Coulomb amplitude, by studying scattering at very small angles (viz., for example,[2]). From previous papers, we managed to find 35 values "a" determined by this method for pp scattering (table 1, fig. 1)*. In general, the experimental situation is not at all satisfactory, as can be clearly seen from the wide range of values in fig. 1.

Attempts have been made to determine theoretically the real parts of these amplitudes on the basis of dispersion relations [13-20]. However, all these calculations call for certain assumptions concerning the unphysical and asymptotic regions [15-18] or just the asymptotic region[19-20]. This is undesirable, especially since the appearance of the Serpukhov data on the total cross-sections

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*For $\bar{p}p$ scattering, we know of only one experimental value a, at 11,9 GeV/c[7].

of interactions between various particles, as these have raised certain doubts as to the validity of the Pomeranchuk theorem. It is therefore advisable to take a fresh look at the calculation of the real parts of these amplitudes, taking into account the new experimental data on the total cross-sections[21] of $\bar{p}p$ scattering and on the real parts of pp forward scattering[9]*.

This paper uses a new model-independent method for determining the real parts of amplitudes. We have already used this method in a modified form[22] for $K^{\pm}$ p scattering.

2. Description of the method

The analytic structure of amplitude (1) and of the complex energy plane ω is shown in fig. 2, where $-m_p$ and m_p denote the beginning of the physical thresholds for pp and $\bar{p}p$ scattering respectively, $m_{2\pi}$ is the beginning of the unphysical region and π is the pole position. Little is known experimentally about $\bar{p}p$ scattering in the low-energy physical region between m_p and ω_0.

We now define the function:

$$\Delta(\omega) = D_-(\omega) - D_+(m_p) + \frac{\omega + m_p}{\pi}[I(\omega) + J(\omega)] , \qquad (2)$$

Where

$$I(\omega) = \frac{1}{4\pi} P \int_{m_p}^{W} \left[\frac{\sigma_+(\omega')}{(\omega' - m_p)(\omega' + \omega)} - \frac{\sigma_-(\omega')}{(\omega' + m_p)(\omega' - \omega)}\right] k' d\omega' , \qquad (3)$$

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*Viz paper [2] for a good survey of the present state of experimental and theoretical data in terms of a for pp scattering.

$$J(\omega) = -\mathrm{Im} \int_{S(W)} \frac{f_-(\omega')\, d\omega'}{(\omega' + m_p)(\omega' - \omega)} \quad . \tag{4}$$

W is the highest energy ($\approx$50 GeV) at which the experimental value for the total cross-section $\sigma_-(\omega)$ of $\bar{p}p$ scattering is known [21], S(W) signifies that integration is done in terms of a semicircle of radius $|\omega| = W$ in the upper half-plane.

Let us consider the closed contour in the ω-plane, consisting of the straight line joining the points $-W + i\varepsilon$ and $W + i\varepsilon$ ($\varepsilon \to 0^+$), and the semicircle S(W). By applying Cauchy's theorem to the function

$$F(\omega'; \omega) = \frac{f_-(\omega')}{(\omega' + m_p)(\omega' - \omega)} \tag{5}$$

around this contour and using the well-known analyticity and crossing properties of $f_\pm(\omega)$, it is easy to show that

$$\Delta(\omega) = \frac{\omega + m_p}{\pi} \left[\frac{\pi X(\pi)}{(\omega_\pi + m_p)(\omega_\pi - \omega)} + \int_{\omega_{2\pi}}^{m_p} \frac{A_-(\omega')\, d\omega'}{(\omega' + m_p)(\omega' - \omega)} \right], \tag{6}$$

where

$$\omega_\pi = \frac{m_\pi^2 - 2m_p^2}{2m_p} \quad ,$$

$$X(\pi) = \frac{g_\pi^2 [(m_\pi - m_p)^2 - m_p^2]}{4 m_p^2}$$

and g_π^2 is the $\bar{p}p\pi$ coupling constant.

If we assume the Pomeranchuk theorem to be valid, then in the limit $W \to \infty$ the relation obtained by equating expressions (2) and (6) reduces to a conventional dispersion relation for $f_-(\omega)$ with one subtraction at the pp threshold. However, the validity of the relation obtained by this method is entirely independent of the asymptotic behaviour of the scattering amplitude at finite energy W.

The total cross-sections $\sigma_+(\omega)$ for pp scattering are known experimentally in the range 0,06 GeV/c $\leq$ k $\leq$ 24 GeV/c. Below 0,06 GeV/c they can be adequately calculated for our purposes on the basis of an effective radius approximation with a = - 17$\pm$0,2 fm, r = 2,83$\pm$0,03 fm /25/. $\sigma_-(\omega)$ are known experimentally /21,23,24,26/ in the interval $\omega_0 \leq \omega \leq \omega_0 W$, where ω_0 = 0,996 GeV.

If we assume that $\sigma_+(\omega)$ does not vary substantially in the interval from 24 GeV/c to 50 GeV/c and that the value $D_+(mp)$ is known, and if we ignore the fact that the $\sigma_-(\omega)$ values are not known in the range $m_p \leq \omega \leq \omega_a$ and that J (ω) is also unknown, then the experimental values of function may be calculated from formula (2) at all points where the $D\pm$ values are known. On the other hand, equation (6) expresses $\Delta(\omega)$ as a sum of the contributions from the unphysical region.

It is clear from equation (6) that the function $\Delta(\omega)$ is relatively smooth in the energy region where it can be calculated from the experimental data. This is due to the fact that $\Delta(\omega)$ at moderate or high energies $|\omega|$ is expressed as a sum of the contributions from relatively distant energies. Therefore, we may expect that $\Delta(\omega)$ can be expressed by means of a small number of parameters. The essence of our method is the fact that, by virtue of equation (2), a fit to $\Delta(\omega)$ also signifies a fit to $D\pm$ (ω). A rapidly convergent expansion may be obtained for function $\Delta(\omega)$ by means of one of the variants of the conformal mapping technique. It can be seen from fig. 2 that the only singularities of $\Delta(\omega)$ in the ω - plane are the π-pole and the unphysical cut joining points $\omega_{3\pi}$ and mp.

We now introduce the new variable:

$$\xi = \frac{\sqrt{\omega - \omega_0} - \sqrt{\omega - \omega_{2\pi}}}{\sqrt{\omega - \omega_0} + \sqrt{\omega - \omega_{2\pi}}} \qquad (7)$$

With an appropriate choice of square roots, equation (7) represents a conformal mapping of $\xi(\omega)$, which transforms the entire ω - plane into the unit circle $|\xi| = 1$. ξ is the plane shown in fig. 3. We wish to point out that by mapping out the interval mp $\leq \omega \leq \omega_0$ on to the circumference, we include it in the unphysical region and thus avoid the need to know $\sigma_-(\omega)$ in this region. The function $\Delta(\xi)$ is analytic within the unit circle, except for the pole at $\xi = \xi_\pi$. If this pole is removed, by multiplying $\Delta(\xi)$ by $\xi - \xi_\pi$, the new function $H(\xi)$ may be expanded in a power series

$$H(\xi) = (\xi - \xi_\pi)\ \Delta(\xi) = \sum_{n=0}^{\infty} a_n \xi^n , \tag{8}$$

which is convergent for $|\xi| < 1$.

Moreover, by introducing the variable

$$\eta = \frac{\sqrt{W+\omega} - \sqrt{W-\omega}}{\sqrt{W+\omega} + \sqrt{W-\omega}} \tag{9}$$

the ω - plane may be mapped in a unit circle $|\eta| = 1$ so that the asymptotic region, i.e. $|\omega| > |W|$, falls on the circumference (fig. 4). Therefore, the asymptotic contribution $\omega + m_p/\pi\ J(\omega)$ to function $\Delta(\omega)$ may be expressed as a power series

$$\frac{\omega + m_p}{\pi} J(\omega) = \sum_{n=0}^{\infty} b_n \eta^n . \tag{10}$$

which is convergent for $|\eta| < 1$.

Thus, it can be said that our method involves a fit within the circumference of the experimentally measured function

$$G(\omega) = \Delta(\omega) - \frac{\omega + m_p}{\pi} J(\omega) \tag{11}$$

in the following form:

$$G(\omega) = \frac{1}{\xi - \xi_\pi} \sum_{n=0}^{\infty} a_n \xi^n - \sum_{n=0}^{\infty} b_n \eta^n . \qquad (12)$$

It should be noted that not one of the series in (12) can express $G(\omega)$ by itself. It can be seen from (4), (6) and (12) that $G(\omega)$ has both a low-energy singularity (π is the pole and unphysical cut) and a high-energy cut (with branch points at $\omega = \pm W$). In the energy range with which we are concerned and where $G(\omega)$ is known from experiment, the first series in (12) cannot alone express $G(\omega)$ due to the high-energy cut, whilst the second series cannot alone express $G(\omega)$ due to the low-energy singularities.

General considerations and also the experiment described in paper /22/ indicate clearly that the series in (12) converge more rapidly close to the circle centres. Only the real parts of pp scattering have been calculated in this paper, and therefore the situation is unfavourable (viz. figs. 3 and 4) as the region within which $G(\omega)$ is known experimentally is situated in both circles to the right (to the left) of the centre and use is not made of the whole region within which there is good convergence adjacent to the centre. Therefore, additional conformal mapping techniques were used

$$\zeta = \frac{\xi - r}{1 - r\xi} , \qquad (13)$$

$$\nu = \frac{\eta - q}{1 - q\eta} , \qquad (14)$$

where r and q are constants. These techniques enable the region where $G(\omega)$ is known from experiment to be shifted along the real axes at fixed end-points $\xi(\omega_0)$, $\xi(\omega_{2\pi})$, $\eta(-W)$ and $\eta(+W)$.

Thus, an optimal position can be found on the real axes for regions where $G(\omega)$ is known from experiment.

Moreover, the term $D_+(m_p)$ in (2) was omitted from the practical calculations as the analytic properties of $G(\omega)$ do not vary, and the error is reduced.

3. Calculation

During the calculations, and in accordance with conventional statistical criteria, we ascertained that the optimal number of parameters is 6 - $a_0 - a_2$ and $b_0 - b_2$. The total value of χ^2 equals 145,4 for 37 points. Of these points 6 - at p = 1,29; 1,39; 1,54; 10,11; 19,33 and 26,42 GeV/c - have abnormally large χ^2 values: 24,8; 42,7; 16,5; 10,1; 7,3 and 9,1 respectively. Therefore, these points were rejected and the calculations repeated. The value of χ^2 proved to be 13,9 for the same number of parameters (the optimal number as before). The χ^2 values for the separate points do not normally exceed one.

4. Results and conclusions

The results of the calculations are shown in table 2 and as a curve in fig. 1.

As regards the points which were rejected, the values of a at p = 10,11 GeV/c; 19,33 GeV/c and 26,42 GeV/c [10] obviously have reduced errors and the a values at p = 1,29 GeV/c; 1,39 GeV/c and 1,54 GeV/c [4] are either just wrong or, if the validity of dispersion relations is not rejected, then pp scattering in this energy range involves an abnormal dependence on spin. Therefore, there is a need for fresh measurements of the a value in the range p = 1-2 GeV/c.

In conclusion, it must be noted that it would be possible in principle to continue analytically this pp scattering amplitude into the $\bar{p}p$ scattering region and also to the π-pole, thus obtaining information on $D_-(\omega)$ and g_π^2 . However, the practical calculation showed that, as with $K^\pm p$ scattering /22/, this is impossible due to the great instability of such a procedure.

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Table 1

Values for $d_{pp} \equiv (\mathrm{Re}\ f_t/\mathrm{Im}\ f_t)$, obtained by the interference method

P_{lab} (GeV/c)	$d_{pp} \pm \Delta d_{pp}$	Reference	P_{lab} (GeV/c)	$d_{pp} \pm \Delta d_{pp}$	Reference
0,44	I,85± 0,20	3 x)	9,94	-0,302± 0,07I	7
0,54	I,55± 0,I2	3 x)	I0,00	-0,330± 0,035	I0
0,56	I,36± 0,I5	3 x)	I0,II	-0,430± 0,043	I0
0,85	0,7I± 0,I6	3 x)	I0,89	-0,26 ± 0,05	5
0,93	0,62± 0,15	3 x)	II,94	-0,290± 0,033	7
I,29	- 0,76± 0,I3	4	I2,I4	-0,258±0,067	7
I,39	- 0,58± 0,06	4	I4,03	-0,272± 0,033	7
I,54	- 0,32± 0,07	4	I7,82	-0,307± 0,065	7
I,69	0,I0± 0,I6	4	I9,22	-0,2I0± 0,024	9
I,70	- 0,007± 0,070	3	I9,33	-0,330± 0,033	I0
2,78	- 0,I2± 0,07	5	20,24	-0,205± 0,033	7
3,83	- 0,24± 0,08	6	24,00	-0,I9 ± 0,09	II
4,85	- 0,38± 0,I0	5	24,I2	-0,I57 ± 0,038	7
6,87	- 0,30± 0,07	5	26,I2	-0,I54 ± 0,045	7
7,8I	- 0,33I± 0,034	7	26,42	-0,320-± 0,033	I0
7,85	- 0,29± 0,03	8	27,50	-0,23 ± 0,I3	I2
7,92	- 0,247± 0,082	7	39,93	-0,I87± 0,024	9
8,89	- 0,33± 0,08	5	50,93	-0,I37 ±0,030	9
9,39	- 0,3I9± 0,040	9	56,93	-0,I35 ±0,040	9
9,86	- 0,345± 0,038	7	70,93	-0,II9 ± 0,0I7	9

x) value d_{pp} obtained from phase-shift analysis.

Table 2

Calculated values for d_{pp}

P_{lab} (GeV/c)	d_{pp}	P_{lab} (GeV/c)	d_{pp}
0,5	I,6I	4,0	−0,22
0,6	I,4I	5,0	−0,24
0,7	I,I3	6,0	−0,26
0,8	0,85	7,0	−0,28
0,9	0,67	8,0	−0,30
I,0	0,55	9,0	−0,3I
I,I	0,42	I0,0	−0,32
I,2	0,30	II,0	−0,3I
I,3	0,I8	I2,0	−0,3I
I,4	0,I0	I3,0	−0,29
I,5	0,08	I4,0	−0,28
I,6	0,03	I5,0	−0,26
I,7	−0,05	I6,0	−0,25
I,8	−0,06	I7,0	−0,24
I,9	−0,07	I8,0	−0,23
2,0	−0,09	I9,0	−0,2I
2,I	−0,I0	20,0	−0,2I
2,2	−0,II	2I,0	−0,20
2,3	−0,I3	22,0	−0,20
2,4	−0,I4	23,0	−0,I9
2,5	−0,I5	24,0	−0,I9
2,6	−0,I7	25,0	−0,I9
2,7	−0,I8	30,0	−0,I9
2,8	−0,I9	35,0	−0,I9
2,9	−0,I9	39,0	−0,I8
3,0	−0,20		

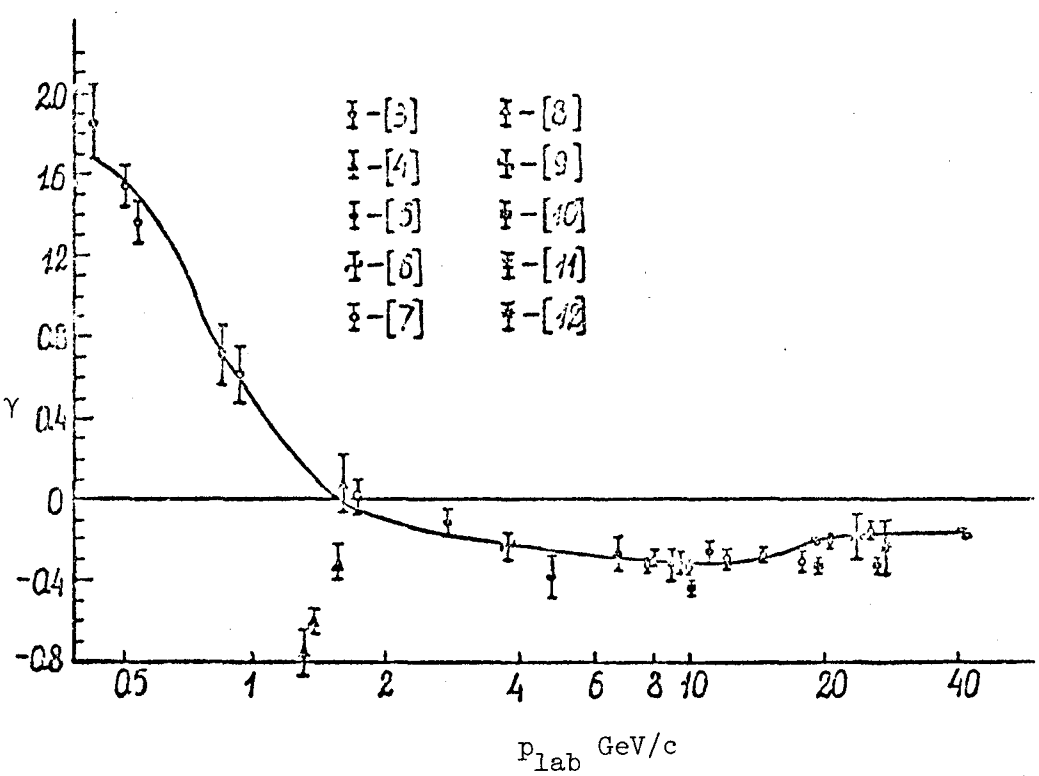

Fig. 1 Experimental values of a_{pp}. The curve represents the theoretical values for a_{pp} obtained in this report.

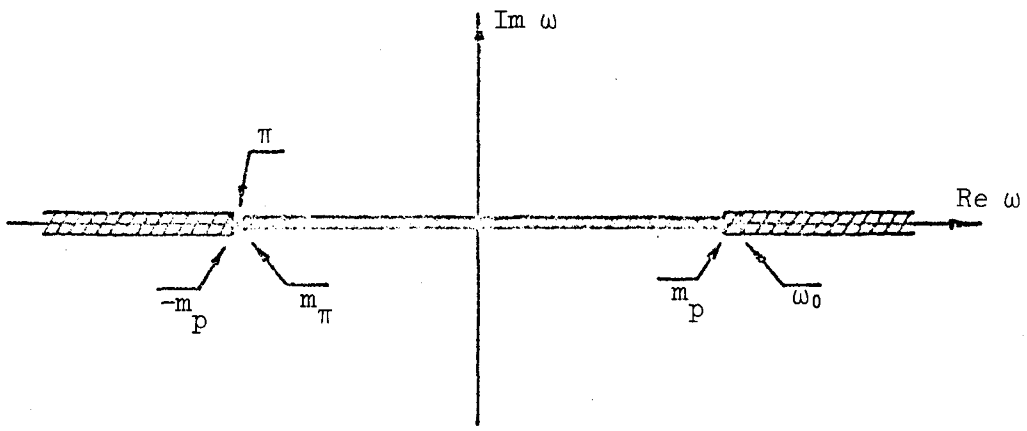

Fig. 2 Analytic structure of amplitude in a complex energy ω-plane. The black dot and strip signify the pole and unphysical cut respectively. The hatched areas correspond to the physical regions.

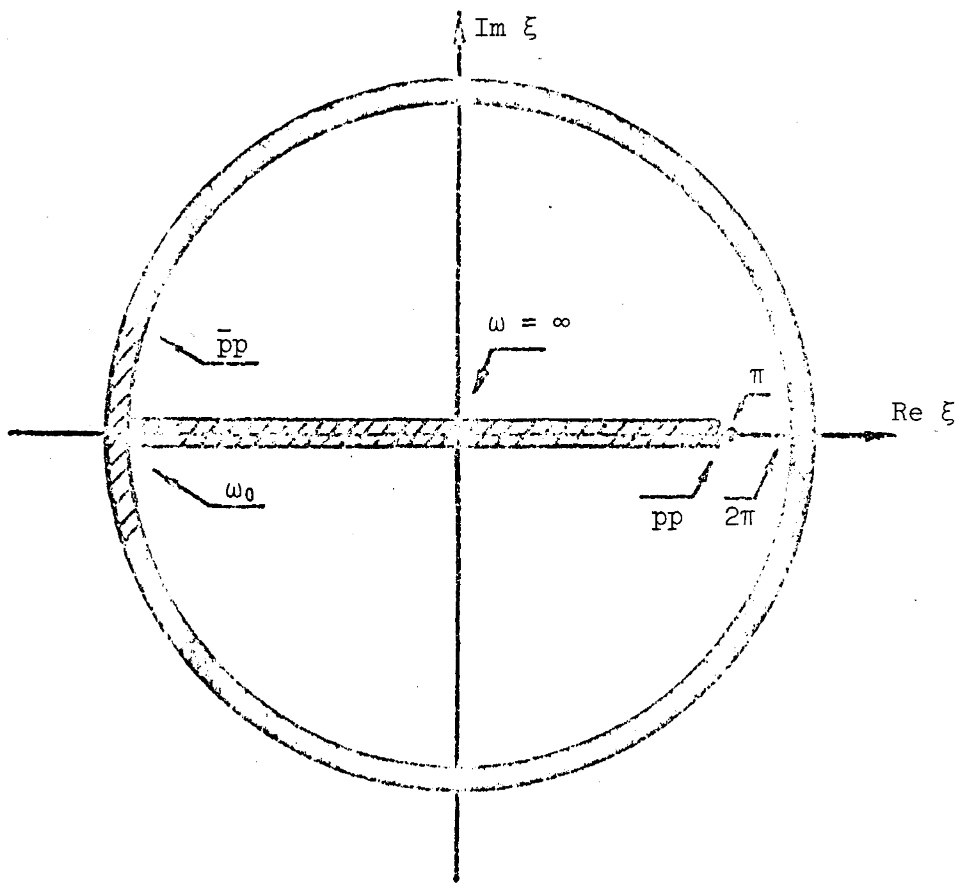

Fig. 3 Structure of the Δ function in the ξ-plane.

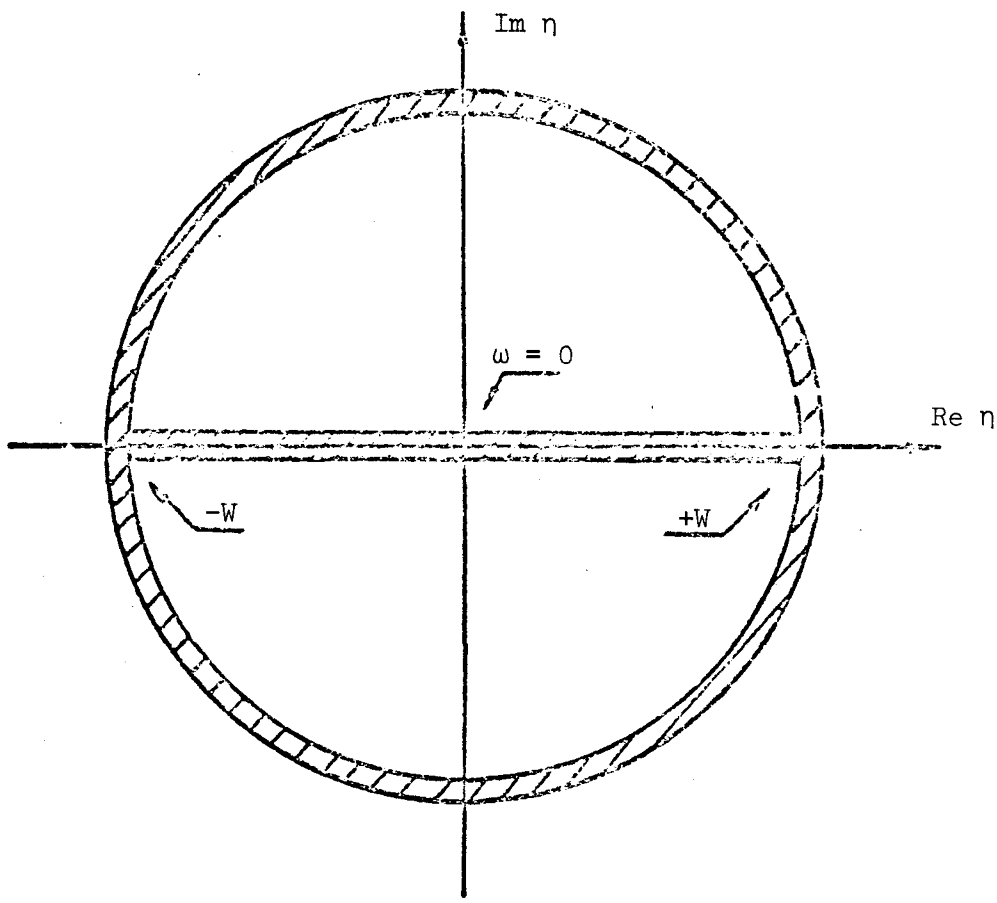

Fig. 4 Structure of the Δ function in the ξ-plane.