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RELATIVISTIC INCREASE OF IONIZATION  
AND SENSITIVITY OF NUCLEAR EMULSIONS

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The aim of this paper is to give a quantitative explanation of the value found by experiment and the energy dependence of the track density of relativistic particles in a nuclear emulsion, based on the peculiarities of the ionization process for silver bromide and on the physico-chemical structure of emulsion. An understanding of these points is important for the identification of high-energy particles in nuclear emulsions, and also for an explanation of the activation mechanism of AgBr microcrystals.

I. Misuse of the Bethe-Bloch formula to describe the dependence of track density on particle velocity in nuclear emulsion

Numerous experiments /1-15/ have been devoted to the study of the relativistic increase in track density of particles in nuclear emulsion. Those papers show that the increase in track density at the ionization plateau (relativistic rise)

$$R = g_p / g_{min} - 1 \quad (1)$$

is 10 - 15% above the minimum ionization density when the average track density of the relativistic particles is  $g_p = 18-25$  grains/

100 microns<sup>\*)</sup>. (A detailed summary of the experimental data is given in /15/).

An approximate value for the energy dependence of the track density of particles in nuclear emulsion is usually found by means of the Bethe-Bloch formula with a correction for the medium's polarization /16/, i.e. it is considered that the track density  $g$  is proportional to the specific energy losses along a particle track in silver bromide.

$$\left(-\frac{dE}{dx}\right)_{T_0} = \frac{2\pi n e^2 Z^2}{m c^2 \beta^2} \left[ \ln \frac{2 m c^2 \beta^2 T_0}{I^2 (1 - \beta^2)} - \beta^2 - \delta(\beta) \right], \quad (2)$$

where  $e$  and  $m$  are the electron's charge and mass;  $Z e$  is the particle's charge;  $\beta$  is the relationship of its speed to the speed of light  $C$ ;  $n$  is the density of electrons per  $1 \text{ cm}^3$ ;  $I$  is the mean ionizing potential of AgBr;  $\delta(\beta)$  is the correction for the density effect; and  $T_0$  - the highest recorded energy of  $\delta$  electrons - is selected from a (2-50) KeV range.

We will show that it is incorrect to use the Bethe-Bloch formula, allowing for polarization of the medium, for nuclear emulsions. In fact, formula (2) may only be used

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\*) The value  $R = 18\%$ , obtained in /7/, is wrong in so far as the authors only related the relativistic increase to collisions with energy transfers of  $W < 2 \text{ KeV}$ . The recalculation of  $R$  using this paper's data gives  $R = 12\%$ . According to the author's evidence, it seems that the value  $R = 10\%$  is underestimated due to the regression of electron tracks with ionization in the plateau region. The results of paper /8/, where a drop in the plateau level was found when  $\gamma \gtrsim 200$ , will also not be used here.

when  $T$  is significantly greater than the ionizing potential of all the atomic electron shells. At the same time, the ionizing potentials of the silver and bromine shells K and L equal

$$U_{Ag}^K = 25.53 \text{ KeV}, U_{Br}^K = 13.48 \text{ KeV}, U_{Ag}^L = 3.4 \text{ KeV}, U_{Br}^L = 1.55 \text{ KeV},$$

i.e. of the order of, or greater than  $T_0$ .

It is possible to calculate  $(-dE/dx)_{T_0}$  correctly for any  $T_0$  value by using a formula for specific energy losses by a particle in collisions where energy transfers are greater than the threshold value  $W_m$

$$\begin{aligned} \left(-\frac{dE}{dx}\right)_{W_m < W < T} = & \frac{2\pi n e^2 z^2}{mc^2 \beta^2} \left\{ \sum_{i=1}^k f_i \left[ \ln \frac{2mc^2 \beta^2 T}{I_i^2 (1-\beta^2)} - \beta^2 \right] + \right. \\ & \left. + \sum_{i=1}^k f_i \ln \frac{T}{W_m} - \delta(W_m, \beta) \right\}. \end{aligned} \quad (3)$$

Expression (3) is based on the collision theory for a polarized medium which was simulated by harmonic oscillators /17/. In this case,  $r_i$  and  $I_i$  - the force and energy of the  $i$  oscillator ( $i = 1, 2, \dots, k$ ) - are linked with the mean ionizing potential by the relationship

$$\ln I = \sum_{i=1}^k f_i \ln I_i,$$

and were calculated from the dependence of the photoabsorption cross-section  $\sigma_{AgBr}$  on the energy of the photon  $h\nu$  (fig.1),, as

shown in /18/ \*).  $T$  is the energy which is significantly greater than the ionizing potential of the atomic electron shells;  $S$  is the number of oscillators with  $I_i < W_m$  and  $\delta(W_m, \beta)$  is the correction for polarization of the medium. When  $W_m = 0$ , formula (3) becomes the Bethe-bloch formula (2) and  $\delta(W_m, \beta)$  coincides with  $\delta(\beta)$  as obtained by Sternheimer /16/.

It is clear that a particle's specific energy losses  $(-dE/dx)_{T_0}$ , where  $T_0$  is any energy less than  $T$ , may be expressed thus:

$$\left(-\frac{dE}{dx}\right)_{T_0} = \left(-\frac{dE}{dx}\right)_{0 < W < T} - \left(-\frac{dE}{dx}\right)_{T_0 < W < T}, \quad (4)$$

and  $T$  meets the following condition for nuclear emulsion

$$T \gg U_{K_2}^K = 25,53 \text{ KeV}.$$

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\*) In the range  $h\nu = 2-14$  eV,  $\sigma_{AgBr}$  was established according to the data in /19-21/. Above 100 eV, it was accepted that  $\sigma_{AgBr} = \sigma_{Ag} + \sigma_{Br}$ . The value  $\sigma_{AgBr}(h\nu)$  is known from experiments /22-26/ and the dependence  $\sigma_{Br}(h\nu)$  was re-calculated from the experimental data for the next element-krypton-using the empirical formula

$$\sigma_{Br}(h\nu) = (Z_{Br}/Z_{Kr}) \sigma_{Kr}(h\nu) Z_{Kr}^2 / Z_{Br}^2,$$

The results coincided closely with the measurements for  $\sigma_{Br}$  available in the range above 8 KeV /22/. The value  $\sigma_{AgBr}$  in the range  $h\nu = 14-100$  eV was found by interpolation, as shown in Fig. 1, and the values obtained for  $\sum_{i=1}^K f_i$  and  $I$  were close to those expected for  $\sum_{i=1}^K f_i = 1$  and  $I \approx 440$  eV /27/.

The results obtained by calculating the relativistic increase in energy losses in AgBr using formulae (3) and (4), for different  $T_0$  values

$$R_E = \left(-\frac{dE}{dx}\right)_{T_0, p} / \left(-\frac{dE}{dx}\right)_{T_0, min} - 1$$

differ sharply from the  $R_E$  calculations using the Bethe-Bloch formula with a correction for the density effect (table 1). As was to be expected,  $R_E$  rises (and does not drop in accordance with the Bethe-Bloch forecasts) with the growth in  $T_0$  - due to the increase in the relative contribution from collisions with the electrons of the Ag and Br inner atomic shells, where the density effect is weaker. In addition, table I shows that even when formula (3) is applied correctly, it is still impossible to reconcile the experimental values with the calculated values for the relativistic rise and grain density obtained when assuming that  $g$  is proportional to the specific energy losses  $(-dE/dx)_{T_0}^*$ .

Thus, it is wrong to assume that the grain density of nuclear emulsion is proportional to the specific energy losses  $(-dE/dx)_{T_0}$ .

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\*) Skirda's paper /28/ also mentions the misuse of the Bethe-Bloch formula, yet the author proposes to retain it for the description of ionizing effects in nuclear emulsion, having replaced  $n$ ,  $I$  and  $I_0$  by their "effective" values. This method will be disproved by the data in this paper.

2. Energy transfer distribution in collisions with  
Ag Br microcrystals

Attempts have already been made in several papers to explain the track density observed for relativistic particles in nuclear emulsions, bearing in mind the fact that only collisions accompanied by sufficiently high energy transfers may be photographed efficiently /29-33/. However, these papers hardly mention the dependence of track density on particle energy, and the energy transfer distribution in collisions with crystals in the emulsion is calculated by an extremely simple method.

From now on, as in /33/ the nuclear emulsion will be taken to include spherical crystals of silver bromide of diameter  $d$  which are distributed at random in the gelatin and are identical photographically /34/. As the results of the calculations do not depend on  $d$ , the size of the microcrystals may vary. When a charged particle collides with these crystals,  $\delta$  electrons, Auger electrons and also photoelectrons occur. The latter ionize the Ag and Br atoms, forming a latent image.

Let us assume that when a particle passes through the emulsion, it does not undergo more than one collision in each microcrystal of silver bromide (it is shown below that the collision multiplicity factor involves some slight corrections). Let  $f(E, W)$  be the differential distribution of the energy transfer  $W$  for particles of energy  $E$  in collisions with silver bromide; in that case the number of grains per 1 cm. of track is:

$$g(E) = \eta \int_0^{T_0} F(W) f(E, W) dW, \quad (5)$$

where  $F(W)$  is the probability of the appearance of a single activated crystal  $\text{AgBr}^{(*)}$ , and  $\eta \approx 0.5$  - that part of the volume occupied by  $\text{AgBr}$  crystals in the emulsion.

From now on it will be convenient for us to use expression (3) and the integral distribution  $(dN/dx)_{W_m < W < T}$ , the number of collisions per 1 cm of  $\text{AgBr}$  which are accompanied by energy transfers from  $W_m$  to  $T$  /17,18/,

$$\begin{aligned} \left(-\frac{dN}{dx}\right)_{W_m < W < T} = \frac{2\pi n e^2 z^2}{m c^2 \beta^2} \left\{ \sum_{i=1}^K \frac{f_i}{I_i} \left[ \ln \frac{2mc^2 \beta^2}{I_i(1-\beta^2)} - \beta^2 \right] + \right. \\ \left. + \sum_{i=1}^K f_i \left( \frac{1}{I_i} - \frac{1}{T} \right) + \sum_{i=1}^S f_i \left( \frac{1}{W_m} - \frac{1}{T} \right) - \Delta(W_m, \beta) \right\}, \quad (6) \end{aligned}$$

where  $\Delta(W_m, \beta)$  is the correction for the density effect.

Expressions (3) and (6) are derived as a Born approximation and do not forecast the effects which Tsitovich found /35/ by allowing for radiative corrections but without providing reliable experimental proof. Formula (6) was successfully used for the description of the relativistic increase of ionization in bubble chambers /17,36/ and for the calculation of primary ionization in gasses /18,37/.

The integral distribution of energy transfers in collisions between relativistic particles and  $\text{AgBr}$  crystals,

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\*)  $F(w)$  also takes into account the spatial non-homogeneity of the microcrystals' sensitivity /31/.



when the particles' Lorentz factor is  $\gamma = 10^3$ , is calculated using formula (6), as shown in fig. 2. This distribution drops sharply with a rise in  $W$  (in 90% of collisions the energy transferred is less than 250 eV and only in 0.08% of collisions ( $\sim 0.16/100$  microns) is the electron able to escape from the path or create an additional cluster, i.e. secondary ionization /7,30/. Therefore, as a rule, electron paths which are formed by a charged particle are significantly less than  $d$ , i.e. not more than one crystal is activated as a result of each collision. At the same time we normally consider that when  $W > T_0$  the electron escapes from the track and does not activate the microcrystal in which it was formed. In this paper  $T_0$  was chosen to be 20 KeV /7,40/, yet this choice is not significant as the results are virtually independent of  $T_0$  when it is greater than 10 KeV.

The energy transfer distribution in the case of collisions with light atoms in the emulsion also drops sharply when  $W$  rises, and the number of collisions is several times smaller than in AgBr (fig. 2). Therefore electron paths formed in the gelatin are considerably less than the mean distance between the microcrystals and they do not generally activate the silver bromide: the number of  $\delta$  electrons penetrating the AgBr from the gelatin does not exceed 0.4/100 microns.

3. The probability of the appearance of activated microcrystals and track density in nuclear emulsion

The dependence of track density on particle energy in nuclear emulsion is determined to a considerable extent by the probability of the appearance of microcrystals  $F(W)$ . If  $F(W)$  has a well-defined threshold, such as  $F(W \leq W_m) = 0$  and  $F(W > W_m) = 1$ , then

$$g(E) = \eta \int_{W_m}^{T_0} \chi(E, W) dW = \eta \left( \frac{dN}{dx} \right)_{W_m \leq W < T_0}, \quad (7)$$

i.e.  $g(E)$  is proportional to the specific number of collisions with energy transfer from  $W_m$  to  $T_0$ . The grain density is proportional to the local energy losses only if the development probability is proportional to the energy transferred in the collision,  $F(W) = CW$

$$g(E) = \eta \int_0^{T_0} CW \chi(E, W) dW = C \eta \left( -\frac{dE}{dx} \right)_{T_0}. \quad (8)$$

In so far as  $F(W) \leq 1$ , then assuming that  $F(T_0) \leq 1$ , we obtain  $C \leq \frac{1}{T_0}$  and

$$g(E) \leq \frac{1}{T_0} \left( -\frac{dE}{dx} \right)_{T_0}. \quad (9)$$

The latter expression enables the upper limit of grain density to be calculated for different  $T_0$  values.

The results given in table 1 show that the assumption concerning the proportionality  $g(E)$  and  $(-dE/dx)T_0$  contradict the experimental data:  $R \approx 13\%$  when  $g_p = 18-25$  grains/100 microns.

In fact, the dependence  $F(W)$ , has a non-linear form /38/. To calculate  $g(E)$ , we shall investigate the range of variation of  $W$  for  $n$  intervals in each of which  $F(W)$  may be approximated by the linear function

$$F_i = a_i + b_i W, \quad (i=1, 2, \dots, n). \quad (10)$$

After substituting (10) into (4), we obtain:

$$g(E) = h \left[ \sum_{i=1}^n \left[ \int_{w_{i-1}}^{w_i} a_i \chi(E, W) dW + \int_{w_{i-1}}^{w_i} b_i W \chi(E, W) dW \right] \right],$$

or

$$g(E) = h \left[ \sum_{i=1}^n \left[ a_i \left( \frac{dN}{dx} \right)_{w_{i-1} < W < w_i} + b_i \left( -\frac{dE}{dx} \right)_{w_{i-1} < W < w_i} \right] \right], \quad (11)$$

where  $\left( \frac{dN}{dx} \right)_{w_{i-1} < W < w_i}$  and  $\left( -\frac{dE}{dx} \right)_{w_{i-1} < W < w_i}$

are found from (3) and (6) in the same way as relationship (4).

Thus, for any form of function  $F(W)$  and with the aid of formulae (3), (6) and (11) it is possible to find  $g(E)$  in nuclear emulsion as an approximation of single collisions.

#### 4. Calculation of the multiplicity factor for collisions in an emulsion's microcrystals

In practice, a particle may undergo 2, 3 or more collisions in each microcrystal. The Poisson probability for

the x-multiple activation of a spherical crystal with a diameter d is

$$P_x(E) = \frac{2(x+1)}{M} \left[ 1 - e^{-M} \sum_{\alpha=0}^{x+1} \frac{M^\alpha}{\alpha!} \right], \quad (12)$$

where  $M = d(dN/dx)T_0$  - the average number of collisions with  $W < T_0$  over a length d in AgBr.  $P_x$  drops sharply when the collision multiplicity factor increases, so that in the case of relativistic particles in emulsions where  $d \leq 0.4$  microns, it is possible to keep to  $x = 8-10$  (table 2). The mean multiplicity factor does not exceed 2 (table 2) for the activation of an emulsion's microcrystal by particles with ionization at a plateau range where  $d = 0.1 - 0.4$  microns. In this case, the average distance between successive collisions inside the crystal is greater than 0.2 microns and, according to the data given by Kuks /34/, they should function independently. The probability of the development of a microcrystal in which x independent collisions has occurred is

$$F_x = F(W_1, W_2 \dots W_x) = \left\{ 1 - \prod_{j=1}^x [1 - F(W_j)] \right\}. \quad (13)$$

From the above relationship, it is clear that the mean probability for the development of an  $\bar{F}_x(E)$  crystal which has undergone x - multiple activation is

$$\bar{F}_x(E) = 1 - [1 - \bar{F}_1(E)]^x. \quad (14)$$

Here

$$\bar{F}_1(E) = \frac{g(E)}{\eta (dN/dx)_T} \quad (15)$$

the mean probability for the development of a once-activated crystal, as found from (6) and (11). Hence

$$g(E) = \sum_x g_x(E) = K \sum_x P_x(E) \bar{F}_x(E), \quad (16)$$

where  $g_x(E)$  is the density of the grains formed as a result of the development of x-multiple activated crystals, and  $K = 3\eta / 2d$  is the number of crystals intersected by a particle per 1 cm of emulsion.

$g(E)$  was calculated both as an approximation of single collisions with the emulsion's crystals (11) and also for multiple collisions (14) and (16). The results proved to be closely related (the difference did not exceed 6% for  $g$  and 0.008 for  $R$ ) and this may be explained by the low mean probability of the development of crystals affected by a relativistic particle  $\bar{F} = 0.05 - 0.08$ .

## 5. Results of the calculations and their assessment

### 5.1. The dependence of grain density on particle energy and the sensitivity threshold of nuclear emulsion

In calculating the dependence  $g(\gamma)$  (fig.3), we used those sensitivity distributions for the  $F(W)$  microcrystals which

had been measured by Zhdanov and Kuks for the NIKFI-P emulsion /38/. However, these data had been obtained by electron irradiation of the emulsion's microcrystals which were surrounded by a layer of adsorbed gelatin whilst the  $g(\chi)$  calculations must relate directly to silver bromide: therefore a correction had to be made for electron braking in the gelatin. Using the path-energy relationship for slow electrons in the gelatin /39/, we found that the values  $R \simeq 13\%$  and  $gp \simeq 20/100$  microns found by experiment could be obtained if the thickness of the adsorbed gelatine,  $\sim 10^{-7}$  g/cm<sup>2</sup>, was considered. By taking into account electron braking in the gelatin shell, the microcrystals' sensitivity threshold is shifted from 170 eV /38/ to  $\sim 20$  eV, thus corresponding to the bare AgBr crystal.

The results of the calculations are shown in fig. 3 where the solid curve corresponds to  $F(W)$  when  $g_{\min} = 26.5/100$  microns;  $F(W)$  is taken from /38/ and converted to the threshold  $W_m = 20$  eV. This curve is an adequate approximation of the experimental data. A large number of other non-decreasing  $F(W)$  functions were also tested. The calculations showed that, irrespective of the shape of the  $F(W)$  curves when the sensitivity threshold ranges from 0 to 60 eV, the  $g(\chi)$  dependences obtained hardly differ from the solid curve in fig. 3 when  $R = 13\%$  and  $gp \simeq 20/100$  microns.

Table 3 shows the  $g_{\min}$  values calculated for the  $F(W)$  functions in /38/ converted to the threshold  $W_m = 20$  eV. These values proved somewhat lower than in /38/; however, bearing in

mind the error in measuring  $F(W)$  for low electron energies, they should not be thought to contradict each other.

The maximum value for an AgBr crystal's sensitivity threshold, at which the dependence  $g(\chi)$  comprises  $R = 13\%$  and  $g_p \simeq 20/100$ , is  $\sim 90$  eV and corresponds to  $F(W)$  with a well-defined threshold, i.e. when  $g(\chi) = \text{const} \left( \frac{dN}{dx} \right)_{90} < \frac{?}{mk T}$ . These data show that the sensitivity threshold of an AgBr microcrystal is considerably less than the 170 eV value obtained in /38/ and is not more than 90 eV.

We shall define the point where  $g(\chi)$  reaches a plateau by the value  $\chi_p$  which is equivalent to a relativistic growth of  $0.9 R$ , i.e.

$$\left[ g(\chi_p) / g_{\text{min}} - 1 \right] = 0.9 R.$$

Calculations show (fig. 3) that when  $R = 13\%$  and  $g_p \simeq 20/100$  microns

$$\chi_p = 50 \div 60.$$

The  $\chi_p$  value decreases slowly as the emulsion's sensitivity increases. The lower limiting value  $\chi_p = 40$  corresponds to  $F(W)$  with a well-defined threshold  $W_m = 90$  eV. However, the corresponding broken curve  $g(\chi)$  in fig. 3 is not an adequate approximation of the experimental data. The upper limiting value  $\chi_p = 90$  is shown by a broken and dotted curve in fig. 3, for

which  $F(W)$  was selected so that  $g(\chi)$  should reach a plateau as slowly as possible when  $R \approx 13\%$ . However, this curve corresponds to a grain density of only 6.3/100 microns. Therefore, we conclude that the relativistic growth  $R = 13\%$  when  $Q_p \approx 20/100$  microns corresponds to a  $\chi_p$  value within the range  $40 < \chi_p < 90$ . The  $g(\chi)$  dependence which we calculated reaches a plateau more quickly than the curves calculated by the Bethe-Bloch formula for which  $\chi_p = 90 - 140$ .

The slight dependence of  $\chi_p$  on the emulsion's sensitivity enables the curve  $g(\chi)$ , as shown in fig. 3, to be used for the approximation of experimental data when  $R = 11 - 15\%$  where  $R$  is the measured relativistic increase of ionization. For this purpose it is necessary to extend the scale in both directions along the ordinates' axes from the unit in the relationship  $1.13/(1+R_{\text{exp}})$ .

## 5.2. Relativistic increase of ionization and sensitivity of nuclear emulsions

The relativistic increase of ionization drops with the growth in grain density, i.e. with the rise in sensitivity of nuclear emulsions. Thus, using the  $F(W)$  curves obtained in /38/, as  $Q_p$  increases from 20 to 30/100 microns,  $R$  decreases from 13% to 11.5% - 10.5%.

Calculations using various  $F(W)$  functions with a threshold from 0 to 60 eV show that when the nuclear emulsion's sensitivity



drops 1.5 times, R increases, but also by not more than 1.5 times and vice-versa. Herz and Stiller observed a decrease in R with an increase in  $g_p/11/$ , and they found that in the emulsion Gewaert-715 with a sensitivity  $g_p = 33/100$  microns, R was considerably less than in other emulsions where  $g_p \simeq 20/100$  microns.

The reason for the inverse dependence between the relativistic increase in a nuclear emulsion's ionization and its sensitivity is summed up as follows. The increase in the emulsion's sensitivity is conditioned by the reduction in the  $W_m$  sensitivity threshold and the increase in the distribution curve of the microcrystals' sensitivity  $F(W)$ , this being achieved by the sensitization of the emulsion or the intensification of the development process. In both cases there is an increase in the probability that slightly activated microcrystals will be developed, i.e. there is an increase in the contribution from distant collisions accompanied by small energy transfers whose relativistic increase is suppressed to a very large degree by the medium's polarization effect.

The maximum possible relativistic increase in a nuclear emulsion  $R \simeq 40\%$  corresponds to  $F(W)$  with a clearly-defined threshold  $W_m = 4$  KeV, when

$$g = \int (dN/dx)_{4, \mu < W < \tau}.$$

which leads, however, to a very low grain density  $\sim 1/100$  micron.

It is worth making a closer experimental investigation of the rise in the relativistic increase of ionization when the

sensitivity of the nuclear emulsion drops, as that rise aids the identification of relativistic particles. In this connection we notice that a considerable increase in  $R$  is observed for tracks of multiply charged particles in extremely underdeveloped emulsions. The dependence of the relativistic increase on the sensitivity of the emulsion calls for the careful use of calibration curves obtained on samples with a particular sensitivity.

### 5.3. The independence of $g$ and $R$ on the dimensions of the microcrystal

Calculations show that  $g$  and  $R$  are virtually independent of the dimensions of the emulsion's crystals (table 4). This is because  $g$  and  $R$  are hardly affected by the collision multiplicity factor: it is the relatively rare collisions involving high energy transfers which are photographically efficient, and not the frequent weak multiple collisions. The independence of  $R$  on the dimensions of the microcrystals when there is close sensitivity in the nuclear emulsion is confirmed by the experiment performed by Herz and Stiller /11/.

### 5.4. Dependence of $g$ and $R$ on $T_0$

Contrary to the forecasts of the Bethe-Bloch formula,  $g$ ,  $R$  and  $\chi_p$  increase when  $T_0$  rises due to the growth in the relative contribution of close collisions (fig.4). When  $T_0 \gtrsim 10$  KeV  $g$ ,  $R$  and  $\chi_p$  virtually cease to depend on  $T_0$  due to the small number of collisions with energy transfers of  $W > 10$  :

$(dN/dx)_{T_0} > 10 \text{ KeV} \approx 0.4/100 \text{ microns}$ . The constancy of  $g$ ,  $R$  and  $\delta_p$  when  $T_0$  is large makes the selection of  $T_0$  non-critical when  $T_0 > 10 \text{ KeV}$ .

#### 5.5. Relativistic increase of ionization in dilute emulsions

As the relativistic increase in the specific number of collisions with the light atoms of the emulsion forming the gelatin is considerably less than in AgBr, an increase cannot be expected in the relativistic rise of grain density in dilute emulsions. On the contrary, the penetration of a small number of energetic  $\delta$ -electrons from the gelatin into the AgBr and back again may lead to a slight reduction in  $R$ .

#### 5.6 Relativistic rise of grain density and energy losses in a nuclear emulsion.

An answer can now be found to the question why the measurements of the curve and value for the relativistic rise of grain density should generally agree with the calculations of specific energy losses made using the Bethe-Bloch formula; it has been shown above that it is wrong to include the density effect in that formula. The reason for this is that because both dependences have the same form and because of the low sensitivity threshold of the nuclear emulsion, the values for the relativistic increase in grain density and for the energy losses were similar; an arbitrary selection of the  $T_0$  parameter in the range 2-50 KeV leads to an even closer match between those 2 values. It is noted that in bubble chambers where the sensitivity threshold is considerably higher

( $W_m \simeq 0.1 - 0.5$  KeV and the "effective" threshold is close to 1 KeV), the relativistic rise in track density is always considerably greater than for  $(-dE/dx)_{T_0}/36/$ .

It should also be noted that our conclusions partly coincide with, and partly differ from Skirda's conclusions /28/. He investigated ionization effects in nuclear emulsions using the false assumption that  $g$  and  $(-dE/dx)_{T_0}$  were in proportion.

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Table 1

Relativistic rise of the specific energy losses of charged particles  
in silver bromide depending on  $T_0$

$T_0$ , KeV		1	2	5	10	15	20	40
Relativistic rise of R, %	Calculation using the Bethe-Bloch formula with an allowance for the density effect <sup>16)</sup>	21.6	19.8	17.8	16.4	16.7	15.1	13.9
	Calculation using for- mulae (2,3) in this paper	4.7	7.2	9.6	11.1	11.9	12.2	12.1
Calculation of grain density at a plateau using formula (8) in this paper, assuming that $g \sim (-dE/dx)T_0$ , grains/100 microns.		17.3	11.0	5.3	2.9	2.0	1.6	0.9

Table 2

Probability of multiple collisions of a charged particle  
in microcrystals of nuclear emulsion

Type of nuclear emulsion	Mean diameter of undeveloped microcrystal	Volume concentration of AgBr in nuclear emulsion	No. of microcrystals crossed by charged particles	Multiplicity factor x of collisions in microcrystals of nuclear emulsion with an energy transfer $W \leq 20$ KeV						Mean multiplicity factor for collision in crystal of nuclear emulsion	
	d, microns	$\eta$	K/100 microns	0	1	2	3	4	5	6	$\bar{x}$
Especially fine-grained	0.10	0.490	736	0.6549	0.2690	0.0636	0.0108	0.0015	0.0002	0.0000	0.44
Ilford-L4	0.14	0.489	525	0.5568	0.3121	0.1018	0.0240	0.0045	0.0007	0.0001	0.61
Ilford-K5	0.20	0.489	367	0.4400	0.3383	0.1537	0.0510	0.0134	0.0029	0.0006	0.87
Ilford-C5	0.27	0.489	272	0.3384	0.3339	0.1986	0.0870	0.0304	0.0086	0.0028	1.17
NIKFI-P	0.28	0.510	273	0.3145	0.3284	0.2080	0.0974	0.0365	0.0114	0.0039	1.21
Kodak NTB-4	0.40	0.490	184	0.2154	0.2841	0.2345	0.1458	0.0732	0.0309	0.0161	1.73

Table 3

Comparison of experimental and calculated data  
in terms of the sensitivity of nuclear emulsion NIKFI-P

Experimental data <sup>38)</sup>	Calculation using the sensitivity distributions of an AgBr microcrystal <sup>38)</sup> , converted to the threshold $W_m = 20$ eV
$g_{\min}$ (100 microns) <sup>-1</sup>	$g_{\min}$ (100 microns) <sup>-1</sup>
9.5 ± 0.5	8.5
14.5 ± 0.5	12.7
26.4 ± 2.4	18.8
42.5 ± 2.5	25.0
55 ± 5	31.3

Table 4

Dependence of grain density of particle tracks and of relativistic increase of ionization on the size of a silver bromide microcrystal

Type of nuclear emulsion	Mean diameter of an undeveloped crystal in an emulsion, d microns	Grain density at the plateau $g_p$	Relativistic rise in grain density R(%)
Especially fine-grained	0.10	23.2	13.4
Ilford L4	0.14	23.0	13.3
Ilford K5	0.20	22.8	13.2
Ilford G5	0.27	22.5	13.0
Kodak NTB4	0.40	22.0	12.7

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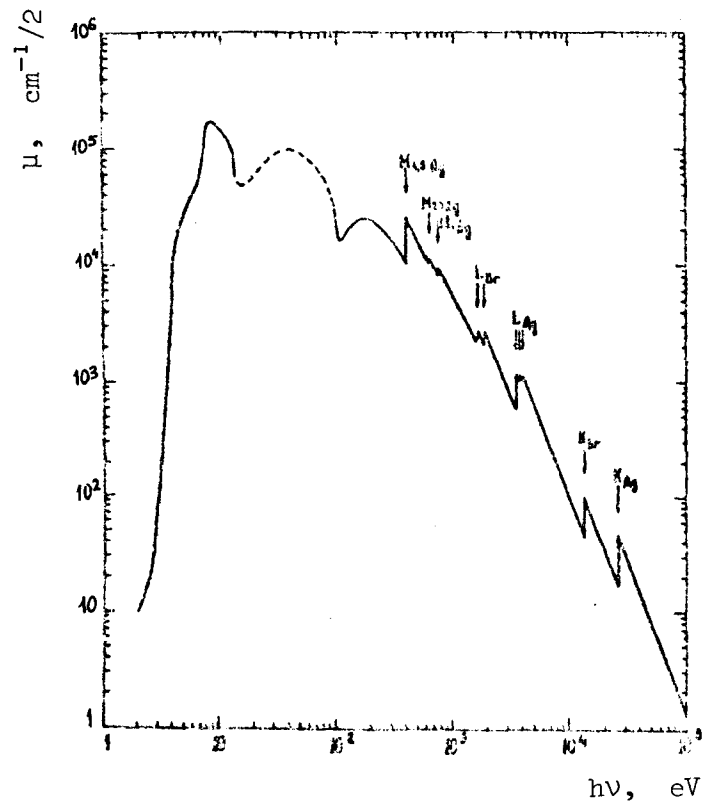


Fig. 1 Mass absorption coefficient of photons by silver bromide  $m_{\text{AgBr}}(h\nu) = 6.02 \times 10^{23} \sigma_{\text{AgBr}}(h\nu)/A$ . The arrows indicate the absorption limits of the K, L, and M shells of the Ag and Br. The dotted interpolation  $m_{\text{AgBr}}(h\nu)$  in the range  $h\nu = 14-100$  eV in accordance with the conditions  $\sum_{i=1}^K f_i = 1$ ,  $I = 440$  eV (3.5).

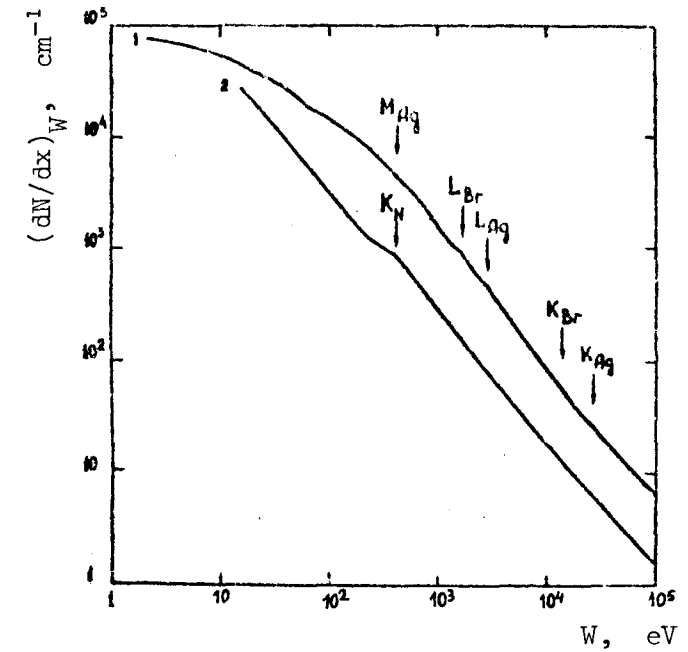


Fig. 2 Integral distribution of the number of collisions of a relativistic charged particle ( $\gamma = 10^3$ ) in nuclear emulsion: (1) silver bromide; (2) gelatin (calculation done for molecular nitrogen with density  $1.312$  g/cm<sup>2</sup>, equal to density of the gelatin). The irregular shape of the curves is linked with the absorption limits of AgBr and N<sub>2</sub>.

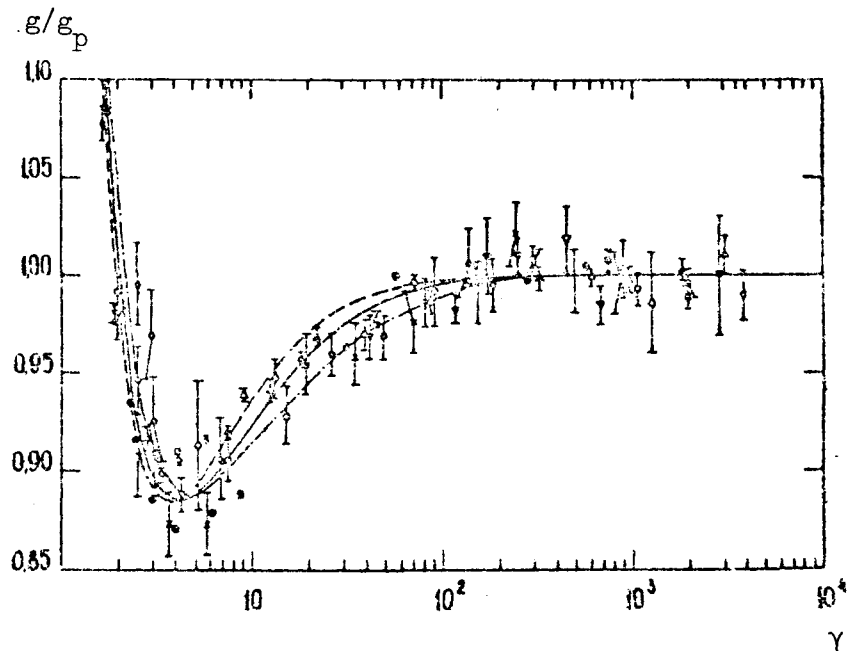


Fig. 3 Track density of relativistic particles in nuclear emulsion in relation to the Lorentz factor  $\gamma$ . The solid curve indicates a highly probable curve for the dependence  $g(\gamma)$  when  $R = 13\%$ ; the dotted curves and broken and dotted curves indicate the maximum possible deviations from that curve. The experimental points are taken from the following papers: X (Ref. 4);  $\nabla$  (Ref. 6);  $\diamond$  (Ref. 7);  $\Delta$  (Ref. 9);  $\bullet$  (Ref. 10);  $\square$  (Ref. 11);  $\circ$  (Ref. 12). The points obtained in Refs. 9 and 12 are also grouped together for close values of  $\gamma$ .

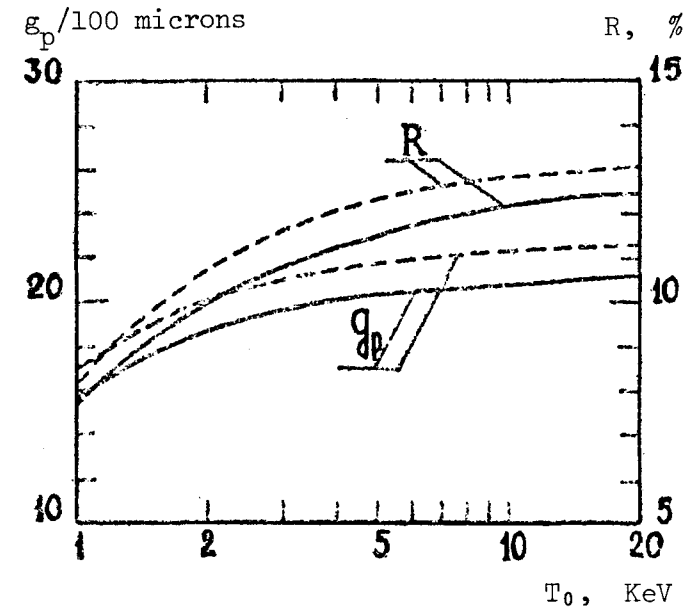


Fig. 4 Dependence of grain density on the plateau  $g_p$  and of the relativistic rise in  $R$  on  $T_0$ . Solid curves are obtained for the microcrystals' sensitivity distribution converted to the threshold  $W_m = 20$  eV and corresponding to  $g_{\min} = 26.5/100$  microns<sup>38</sup>). The dotted curves for  $F(W > W_m) = 1 - \exp[-(W - W_m/W_0)]$ , when  $W_m = 35$  eV and  $W_0 = 0.5$  KeV.