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INVESTIGATION OF THE LONGITUDINAL BEAM INSTABILITY IN A STORAGE RING RESULTING FROM SYNCHROTRON-OSCILLATION RESONANCES

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The present paper is devoted to the investigation of effects which arise from the simultaneous action of beam-induced voltages and of resonances of synchrotron oscillations. This problem has not been examined in the literature, although many papers are devoted to detailed investigation of longitudinal instabilities and resonances of synchrotron oscillations.

The measurements have been carried out on the 70 MeV electron storage ring of the Physico-Technical Institute of the Academy of Science, Ukrainian SSR[1]. The complete layout of the measuring apparatus is shown in Fig. 1. For the excitation of the instability, a special passive resonant cavity has been used, the characteristics of which have been carefully measured[*]. The same cavity has also been used for the registration of coherent synchrotron oscillations during the rise of the instability.

The results of the measurement of instability thresholds in the absence of resonance excitation are presented in Fig. 2. In the same figure the theoretical threshold values are presented for comparison; these were computed by means of the results of Karliner et al.[2]. It is evident from the graph that on the major part of the curves there is a good agreement between the theory and the experiments; however, for small detunings out of the resonance, a substantial deviation is observed. This deviation is particularly well visible in Fig. 3 where the contribution of the Landau damping is plotted versus the detuning. Here the experimental curve has a clearly expressed maximum which is absent in the theoretical curve. Apparently one has to reckon with the theory yielding incorrect values of the threshold currents for small detuning.

As the measurements have shown, during the excitation of resonances of synchrotron oscillations one observes a lowering of the longitudinal instability. The effective lowering occurs in a narrow frequency-band of resonance excitation, which is less than the resonance width. In the case of a parametric resonance this band is substantially narrower than in the case of a forced resonance. The results of measurements of the thresholds of instability as a function of the resonance excitation

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*) It has been shown in a special theoretical investigation that in the majority of practical cases the physical processes in the system beam-passive cavity and beam-active cavity are identical.

force and of the magnitude of the accelerating voltage are presented in Figs. 4 and 5 for the case of forced and of parametric resonance, respectively. One can see in the graphs that already for small magnitudes of the force of excitation, one observes an effective suppression of the instability (ten times the threshold). While this happens, the force of resonance is so small that the resonance oscillations of the beam are not seen.

The maximum value I_n/I_{n_0} in the curves Figs. 4 and 5 is approximately equal to 50. Experimentally a suppression of the instability has also been observed for high currents. However, for I_n, with I_{n_0} increasing a hundred times, the instability is not completely suppresseed. Meanwhile, one observes beat oscillations, for which no exact interpretation of their character has been given.

Experimentally one observed also a suppression of the longitudinal instability to one-tenth, by means of the synchro-betatron resonance of vertical betatron oscillations $\nu_z = 2/3$.

For the theoretical interpretation of the results obtained, we consider the equation of synchrotron oscillations with simultaneous action of induced voltages and parametric resonance of synchrotron oscillations. We restrict ourselves to the case in which the cavity oscillations are substantially attenuated during a period of synchrotron oscillations. In this case, as follows from the results quoted elsewhere[3], the equation of synchrotron oscillations in cubic approximation is:

$$\eta'' - 2(\xi - \zeta)\eta' + \Omega^2\eta = -\frac{\Omega^2}{2}\,\mathrm{ctg}\,\phi_s\,\eta^2 + \frac{\Omega^2}{6}\eta^3 + g\Omega^2\eta\cos(\nu\theta + \chi)\,, \tag{1}$$

where g = amplitude, ν = frequency of parametric excitation, ξ = constant of antidamping of the instability, ζ = constant of radiation damping, ϕ_s = synchronous phase, Ω = frequency of synchrotron oscillations; the differentiation is carried out with respect to the dimensionless time variable $\theta = \omega_s t$; ω_s = revolution frequency of a synchronous particle.

Substituting

$$\Delta = \nu - 2\Omega;\ |\Delta| \ll \nu;\ \eta = Ae^{i\Omega\theta} + A^*e^{-i\Omega\theta};\ A = ae^{i\psi};\ w = \theta\Delta + \chi - 2\psi \tag{2}$$

we obtain, as usual [e.g. Kolomenskij and Lebedev[4)]], the following abbreviated equation:

$$a' = (\xi - \zeta)a + \frac{g a \Omega}{4}\sin w; \quad w' = \Delta + \frac{\Omega}{2}a^2 + g\frac{\Omega}{2}\cos w. \tag{3}$$

It follows from formulae (3) that if the inequality $4|\xi-\zeta| < g\Omega$ is satisfied, there exists a stationary solution:

$$a_0 = const = \sqrt{-\frac{2\Delta}{\Omega} \pm g\sqrt{1-\frac{16(\xi-\zeta)^2}{g^2\Omega^2}}}; \quad \sin w_0 = const = -\frac{4(\xi-\zeta)}{g\Omega}, \tag{4}$$

where, in the case of stability, there appears the solution with $\cos \omega_0 < 0$, which has the (+) sign under the square root.

It follows from formulae (4) that for high instability we have $[4(\xi-\zeta) \geq g\Omega]$ or, oppositely, for high stability ($\xi-\zeta < 0$; $4|\xi-\zeta| \geq g\Omega$) resonance phenomena do not show up.

We now examine whether the opposite situation may arise, when the resonance suppresses the instability. For this purpose we analyse the equation of cubic approximation for small deviations from the stationary solution $b = a - a_0$, which is, as is easily shown,

$$b'' - \frac{g\Omega^2 a_0^2}{4}\cos w_0 b - 2(\xi-\zeta)b' = \frac{3g\Omega^2 a_0}{8}\cos w_0 b^2 + \frac{g\Omega^2}{8}\cos w_0 b^3. \tag{5}$$

From Eq. (5) it follows that there exists the following dependence of the frequency on the amplitude:

$$\Delta\omega = \frac{3}{16}\cdot\frac{b^2_{max}}{a_0^2}\omega; \quad \omega = \frac{\Omega a_0}{2}\sqrt{g|\cos w_0|} \quad . \tag{6}$$

If the phase-length of the bunch is $2\Delta\phi$, then the largest amplitude $b_{max} = \Delta\phi$. Consequently synchrotron oscillations during resonance consist of oscillations of the frequency $\nu/2$ and oscillations of the frequencies between $\nu/2 + \omega$ and $\nu/2 + \omega \pm \Delta\omega + \Delta\Omega$ [where $\Delta\Omega$ is the spread of proper frequencies in the beam, $\Delta\Omega = \Omega(\Delta\phi^2/16)$]. The full frequency spread of synchrotron oscillations, which determines the influence of Landau damping on the threshold of instability, is equal to $2\Delta\omega + \Delta\Omega$.

In the absence of a resonance this spread is simply $2\Delta\Omega$. It is easily seen that if the condition

$$a_0 \ll 3\sqrt{g \cos W_0} \tag{7}$$

is satisfied there will be $2\Delta\omega \gg \Delta\Omega$, i.e. during resonance there is a frequency spread, and consequently also the contribution of Landau damping strongly increases.

In this way the experimentally observed effect of diminution of the longitudinal instability is explained by the increase of the non-linearity of synchrotron oscillations during resonances, and, as a consequence of this, an increase of the frequency spread of the oscillations in the bunch and a growing influence of Landau damping. Since for small a_0 the derivative $da_0/d\Delta$ is large (for $a_0 = 0$, $da_0/d\Delta = \infty$), an efficient suppression of the longitudinal instability must occur within a narrow frequency band. This is also observed in experiments. In the case of forced resonance, one obtains analogous results, but in so far as $da_0/d\Delta$ has a substantially smaller value, the band of efficient suppression must be somewhat larger.

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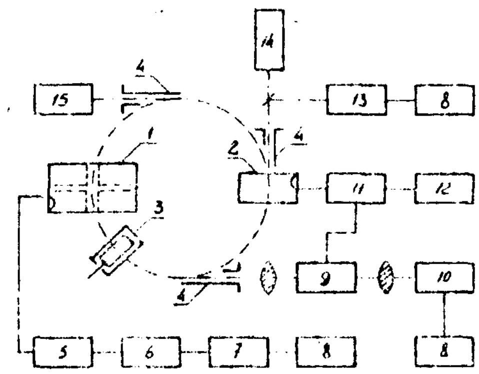


Fig. 1

Apparatus layout as used in the experiment on the investigation of the longitudinal instability:

1 = passive resonant cavity, tuned to second harmonic of revolution frequency ∿ 104.5 MHz;

2 = accelerating straight section;

3 = pick-up electrode for measurement of stored current;

4 = exit windows for synchrotron radiation;

5 = superhetero dyne receiver;

6 = frequency detector;

7 = selective amplifier;

8 = oscilloscope;

9 = electron optical converter;

10 = dissector tube for measurement of longitudinal distribution of the beam;

11 = RF generator of accelerating voltage (frequency 52.25 MHz);

12 = modulator for the excitation of resonances of synchrotron-oscillations;

13 = dissector for the measurement of the transverse distribution of the beam;

14 = TV set-up;

15 = photoelectric converter to measure stored current.

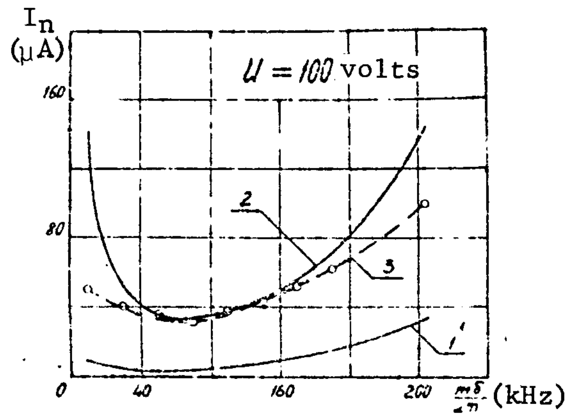


Fig. 2

Dependence of the threshold of the instability on the detuning from resonance with 100 V accel. voltage:

1 = theoretical curve disregarding Landau damping;

2 = theoretical curve including Landau damping;

3 = experimental curve (dotted).

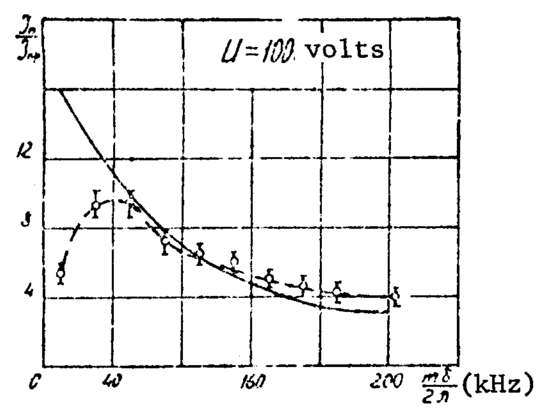
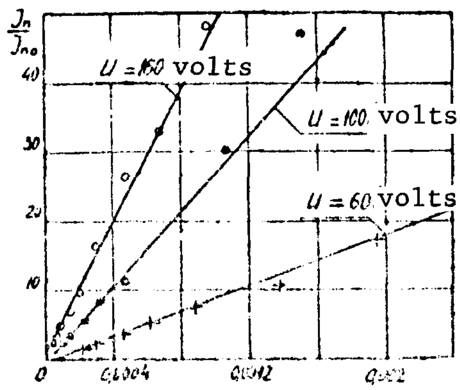


Fig. 3

Contribution of Landau damping (ratio of actual threshold current to its calculated value without Landau damping) versus detuning, the accel. voltage being 100 V:

Continuous curve = theoretical results;

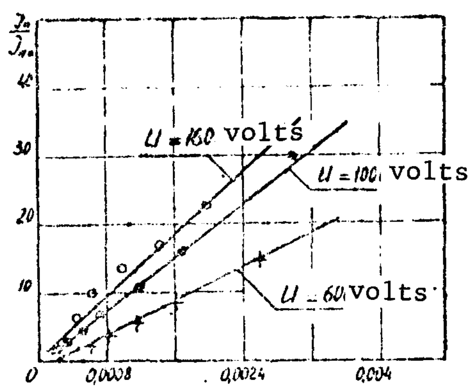
dotted = experimental results.



Index of frequency modulation

Fig. 4

Threshold of the instability versus force of excitation of forced synchrotron-oscillation resonance with an accel. voltage of 60 V (curve 1), 100 V (curve 2) and 160 V (curve 3). On the abscissa the index of frequency modulation is put down. On the ordinate the ratio of the threshold current I_n to its value I_{n_0} in absence of a resonance.



Degree of amplitude modulation

Fig. 5

Threshold of instability versus force of excitation of parametric resonance of synchrotron oscillations with an accel. voltage of 60 V (curve 1), 100 V (curve 2), 160 V (curve 3). On the abscissa the degree of amplitude modulation of the accelerating voltage is put down (which is equal to the force of resonance).