INSTITUTE OF HIGH-ENERGY PHYSICS, SERPUKHOV

Report IFVE 70-15

PHASE MOTION OF THE HIGH INTENSITY BEAM OF THE STRONG-FOCUSING SERPUKHOV PROTON SYNCHROTRON

P.T. Pashkov and I.A. Shukejlo

Serpukhov, 1971

Translated at CERN by O. Barbalat
(Original: Russian)
Not revised by the Translation Service

(CERN Trans. 71-3)

GENEVA
1971

1. INTRODUCTION

The intensity of strong focusing proton synchrotrons is limited in general by the transverse incoherent Coulomb effect. It excites betatron oscillations, when the working point crosses a strong resonance line under the influence of space-charge forces.

The intensity of the CERN and Brookhaven accelerators is limited for this reason. Apparently the IHEP Accelerator works now in a regime near the limit. The characteristic intensity level at which space-charge influences significantly the betatron motion is 10^{12} protons per pulse.

The largest unfavourable effects appears at the initial stage of the accelerating cycle. The maximum current depends on the energy of the particles (neglecting the effect of the vacuum chamber walls and of the magnet poles) as $\gamma\ (\gamma^2-1)$ (γ : relativistic factor). The natural way to overcome the difficulty connected with this transverse repulsion is to increase the injection energy. The task of the next years appears to be the increase of intensity of the large proton accelerators to the 10^{13} - 10^{14} proton/pulse level.

In the accelerators with a high enough injection energy, the intensity limitation factor may well be the longitudinal Coulomb repulsion of the particles /1/. The space-charge force may become comparable with the focusing forces of the accelerating electric field.

In this report, we study the longitudinal motion taking into account the proper field of the bunch. In order to solve the phase equations we have used the analog computer MN-7 which has been used earlier to study the phase motion without space-charge /2/. The results are obtained in a concrete aspect. The difficult solution of non-linear equations in the non-adiabatic region near transition is not called for. One succeeded in obtaining the essential results by integrating the equation in a sufficiently short interval with respect to the variation of the variable so that one can avoid errors occuring by long duration integration on analog computers. The numerical values correspond to the 70 GeV IHEP proton synchrotron.

2. ELECTRIC FIELD OF THE BUNCH

We calculate the self-field of the bunch in the system of coordinates which follows the centre of gravity of the bunch. We suppose that the bunch length is much larger than the transverse dimension of the vacuum chamber, but the relation between the line charge density and the azimuthal coordinate s is described by a general function $\lambda(s)$. The beginning of the calculation coincides with the position of the bunch centre of gravity.

The metallic vacuum chamber wall screens the electric field of the particles separated by a distance $\Delta s > h$ (h : chamber height). The longitudinal field is equal to :

$$\mathcal{E}_s^o = - \frac{dU(s)}{ds} \tag{1}$$

where U is the average potential in a transverse section of the bunch. The relation between the longitudinal charge density and the potential is determined by :

$$U(s) = \frac{1}{C} \lambda(s) \tag{2}$$

C is the electric capacity of a unit length of the beam-chamber system.

In this way, with the adopted hypothesis, it is sufficient in order to find the longitudinal field to calculate the capacity and to know the charge distribution along the bunch.

In the simple case, of a bunch with circular cross-section of radius r_1 , in a circular metallic tube of radius r_2 the capacity is equal to

$$C = \frac{1}{2\left(\ln \frac{r_2}{r_1} + \frac{1}{4}\right)} \tag{3}$$

The logarithmic dependence of the capacity with the ratio of chamber to beam size makes it possible to limit oneself to relation (3). The examination of other cases (elliptic beam in elliptic chamber, circular beam between parallel surface) leads to the conclusion that the capacity in all cases of practical interest does not differ from the value given by formula (3) by more than 20%. The capacity depends also very weakly on the charge distribution across the bunch cross-section. In particular if the charge is concentrated in a thin circular layer of radius r_1 , we get the well-known formula of the cylindrical condenser :

$$C = \frac{1}{2 \ln \frac{r_2}{r_1}} \quad .$$

The explicit form of the $\lambda(s)$ function is determined by the trapping conditions in the accelerating regime and the subsequent adiabatic damping. At very high intensities, the longitudinal charge distribution may change under the action of the self-fields. Usually it is assumed for the calculation that the longitudinal phase plane is uniformly filled. If the phase oscillations amplitude is small, the line density is an even function of s :

$$\lambda(s) \sim \sqrt{s_{max}^2 - s^2}$$ ($2\, s_{max}$: longitudinal length of the bunch)

Nevertheless, the differentiation of the equation set (1) and (2) leads to infinite values of the field intensity at the boundaries of the bunch. Indeed the field strength is everywhere limited and on the edge of the bunch it is simply the hypothesis that the solution is uniform which is violated. Consequently /3/, we shall start with a longitudinal distribution for which $\lambda(s) \sim s_{max}^2 - s^2$.

In this case, the phase plane density decreases slightly towards the bunch edge which seems natural.

Thus :

$$\lambda(s) = \frac{3}{4} \frac{N e}{h s^3_{max}} (s^2_{max} - s^2) \tag{4}$$

$$\mathcal{E}_s^{(o)} = \frac{3}{2} \frac{N e s}{C h s^3_{max}} \tag{5}$$

here N is the number of protons in the accelerator,
h the number of bunches (RF harmonic number).

When going to the laboratory coordinate system, the relativistic factor $\frac{1}{\gamma^2}$ appears in (5).

3. PHASE EQUATIONS

The phase equations with the canonic variables $\frac{\Delta E}{\omega_o}$, ϕ without taking into account the self-fields of the bunch may be written as :

$$\frac{d}{dt} \left(\frac{\Delta E}{\omega_o}\right) = \frac{e V}{2 \pi} (\cos \phi - \cos \phi_o) \tag{6}$$

where V is the amplitude of the accelerating voltage; ϕ_o the equilibrium phase; e V cos ϕ the energy gain per turn; ΔE the energy variation from equilibrium and ω_o the revolution frequency.

The action of the bunch self-fields on an arbitrary particle may be taken into account in the phase equation if one makes the substitution :

$$e V \cos \phi \rightarrow e V \cos \phi + e \mathcal{E}_s \cdot 2\pi \bar{R}$$

$$\mathcal{E}_s = \mathcal{E}_s^{(o)} / \gamma^2$$

$\bar{R}$: mean accelerator radius .

One can deduce a few general conclusions by considering the small amplitude phase equations.

Let us take : $\phi = \phi_o + \eta \qquad \eta \ll 1$

We get :

$$\frac{d}{dt}\left(\frac{E_o \;\dot{\eta}}{\omega_o^2 \; h \; K}\right) + \left(\frac{eV}{2\pi} \sin \phi_o + \frac{3}{2}\; \frac{N \; e^2 \; h}{C \; \bar{R} \; \gamma^2 \; \eta_{max}^3}\right) \eta = 0 \qquad (7)$$

$$\eta_{max} = - h \; \frac{s_{max}}{\bar{R}} \qquad (8)$$

$$K = \frac{\alpha \; \gamma^2 - 1}{\gamma^2 - 1}$$

For an adiabatic variation of the parameters, the factor $\frac{E_o}{\omega_o^2 \; h \; K}$ may be taken out of the derivation bracket so that :

$$\ddot{\eta} + \Omega^2 \; \eta = 0 \qquad (9)$$

$$\Omega^2 = \frac{\omega_o^2 h \; K \; e \; V \; \sin \phi_o}{2\pi \; E_o} \; (1 \mp \chi) \qquad (10)$$

$$\chi = \frac{3\pi \; h \; e \; N}{c \; \bar{R} \; V \; |\sin \phi_o|} \; \frac{1}{\gamma^2 \; \eta_{max}^3} \qquad (11)$$

Ω is the frequency of small amplitude oscillations which is constant in the adiabatic region along a given phase trajectory, but in the general case it has the meaning of instantaneous frequency.

The minus sign applies when $\gamma < \gamma_{tr}$ $(K<0, \; \sin \phi_o<0)$
The plus sign applies when $\gamma > \gamma_{tr}$ $(K>0, \; \sin \phi_o>0)$
γ_{tr} is the relative transition energy of the accelerator.

The space charge decreases the longitudinal focusing of the oscillating system for $\gamma<\gamma_{tr}$ and increases it for $\gamma>\gamma_{tr}$.

In a qualitative and rough evaluation of the intensity limit, one may take the condition : $\chi = 1$.

As shown by (11) the effect of the self-field on the longitudinal motion depends extremely strongly on the bunch length. On Fig. 1 the graph of the function :

$$f(\gamma) = \frac{1}{\gamma^2 \eta^3_{max}(\gamma)}$$

is given for the IHEP Accelerator. It is assumed that the relation $\eta_{max}(\gamma)$ follows the law of linear oscillations of particles of a low intensity beam, and V and sin ϕ_o are constant in the accelerating process.

It is clear that the relative effect of the Coulomb term in the phase motion is largest around transition.

4. SOLUTION OF THE EQUATION IN THE TRANSITION REGION

The time interval during which the effect of space charge is important, is determined before all by the beam intensity and the bunch length η_{max} . In the IHEP accelerator, this interval does not exceed 100 μsec (compared with an acceleration time of 2.5 sec.).

The integration of the phase equation was carried in an interval of 24 μsec before transition, to 24 μsec. after. The integration interval was split in 6 μsec steps up to the transition point and 2 μsec steps afterwards. Such a procedure is justified because after the transition point, the bunch dimensions change much more rapidly. Within each step, the Coulomb term is approximated by a linear function.

The results of the integration are represented on the phase plane in the canonic units $\Delta E/\omega_o$, ϕ by closed curves containing all the representative points at a given time instant. In the region where parameters vary adiabatically, these curves coincide with boundary phase trajectories. The area enclosed by the boundary curve must be preserved in agreement with Liouville's theorem. The area variation in the integration process characterizes the error of the computing machine.

The first results were obtained with the assumption that the bunches have a natural length determined by the usual trapping conditions and the subsequent adiabatic damping.

At transition $\eta_{max} = 8^{o}$. It so happens that at the 10^{12} protons/pulse intensity level, the space-charge effect is negligibly small, but by 10^{13} p/p after crossing transition energy, the radial bunch dimensions equal the vacuum chamber width.

Let us suppose that the bunch length in the transition region be increased for instance by means of noise excitation of phase oscillations. The bunch lengthening appears mostly as a simple means of decreasing the longitudinal Coulomb effect if there is no special requirement on the particle density in the longitudinal phase plane. The effective phase volume at the end of the accelerating cycle is smaller in the case where there has not been a preliminary excitation of oscillations. In the example described below, the initial beam size is three times larger than the "natural"; the intensity being 5.10^{13} p/p.

The particularities of the transition region crossing are illustrated by the boundary curves (curve 2) represented on Fig. 2. For comparison we have drawn the curves (curve 1) which correspond to zero intensity and enclose the same area.

The parameters entering in the phase equations and in formula (5) have in this example the following values :

$N = 5.10^{13}$	$h = 30$	$\gamma_{trans} = 9.46$
$c = 1/4.5$	$V = 380$ kV	$\omega_o = 1.25\ 10^{6}\ sec^{-1}$
$\bar{R} = 236$ m	$\cos\phi_o = 0.5$	$\alpha = 0.011$

The time τ is calculated from the moment of crossing the transition point. The boundary curve at the limit of the represented interval ($\tau = -24$ μsec) practically coincides with the phase trajectories. The shape of the trajectories differ slightly from an ellipse because the oscillations are not completely linear. The hypothesis on the symmetric charge distribution with respect to the equilibrium point (formula 4) may yet be fulfilled in as much as the Coulomb term in the right hand side of the equation does not exceed 0.1 of the basic term, for a bunch for which $\eta_{max} > 30^{o}$. If $\eta_{max} < 30^{o}$, the longitudinal asymmetry of the bunch lies in the limit of accuracy of the instruments which gives the results of the calculations. The non-linearity of the accelerating field is most important and must be taken into account in the integration.

Important shape differences between curves (1) and (2) appear suddenly after crossing transition. At this instant, the azimuthal bunch dimension diminishes and the self-field effect is largest. The space-charge force strength increases 6-8 times in 10 μsec. The period of phase oscillations in the vicinity of the transition point is large and the deformation of the area representing the beam occurs non-adiabatically; as a result of this bunch dimension,oscillations are created. The non-linear alteration of the bunch area is determined by the non-linear internal fields. The effective phase surface is increased by about 4 in the represented example.

When going away from the limits of the represented interval one can calculate that the phase oscillations vary adiabatically.

The authors express their thanks to V.I. Balhekov for his review of this work and his valuable remarks.

REFERENCES

1. A. Sørenssen. Proc. of the Sixth Intern. Conf. on High Energy Accelerators, Cambridge, USA, CEAL - 2000, (1967)p. 474.

2. E.A. Miae, P.T. Pashkov. Preprint IHEP 68-36 K. Serpukhov 1968.

3. H.G. Hereward. Estimates of bunch lengths and longitudinal space charge forces in the CPS, CERN, MPS/DL Int. 66-3, 1966.

4. A.A. Kolomenskii, A.N. Lebedev. Theory of cyclic accelerators. Moscow Fizmatgiz 1962. Chap. IV.

<u>Distribution</u> (open)
MST

Received by the publishing Group
February 10, 1970

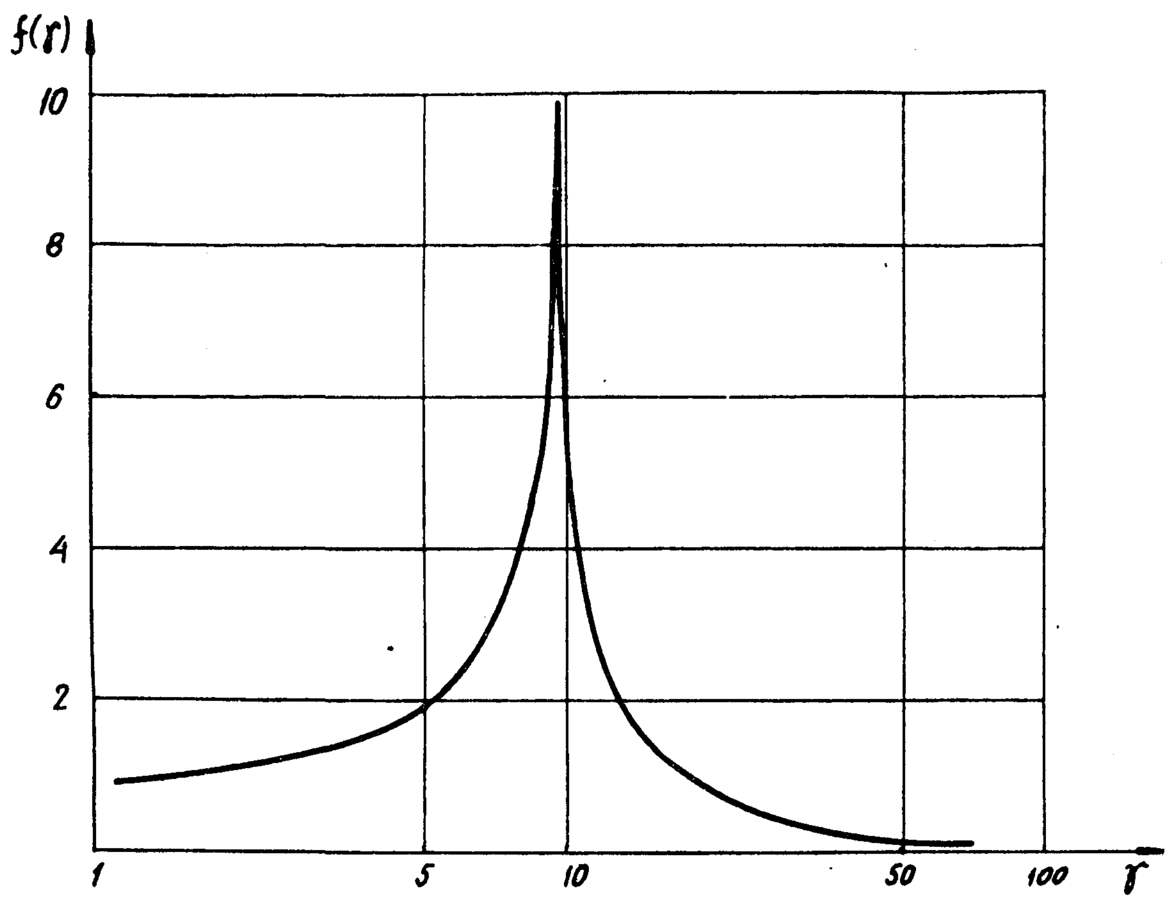

Fig. 1 : Function f(γ) for the IHEP accelerator.

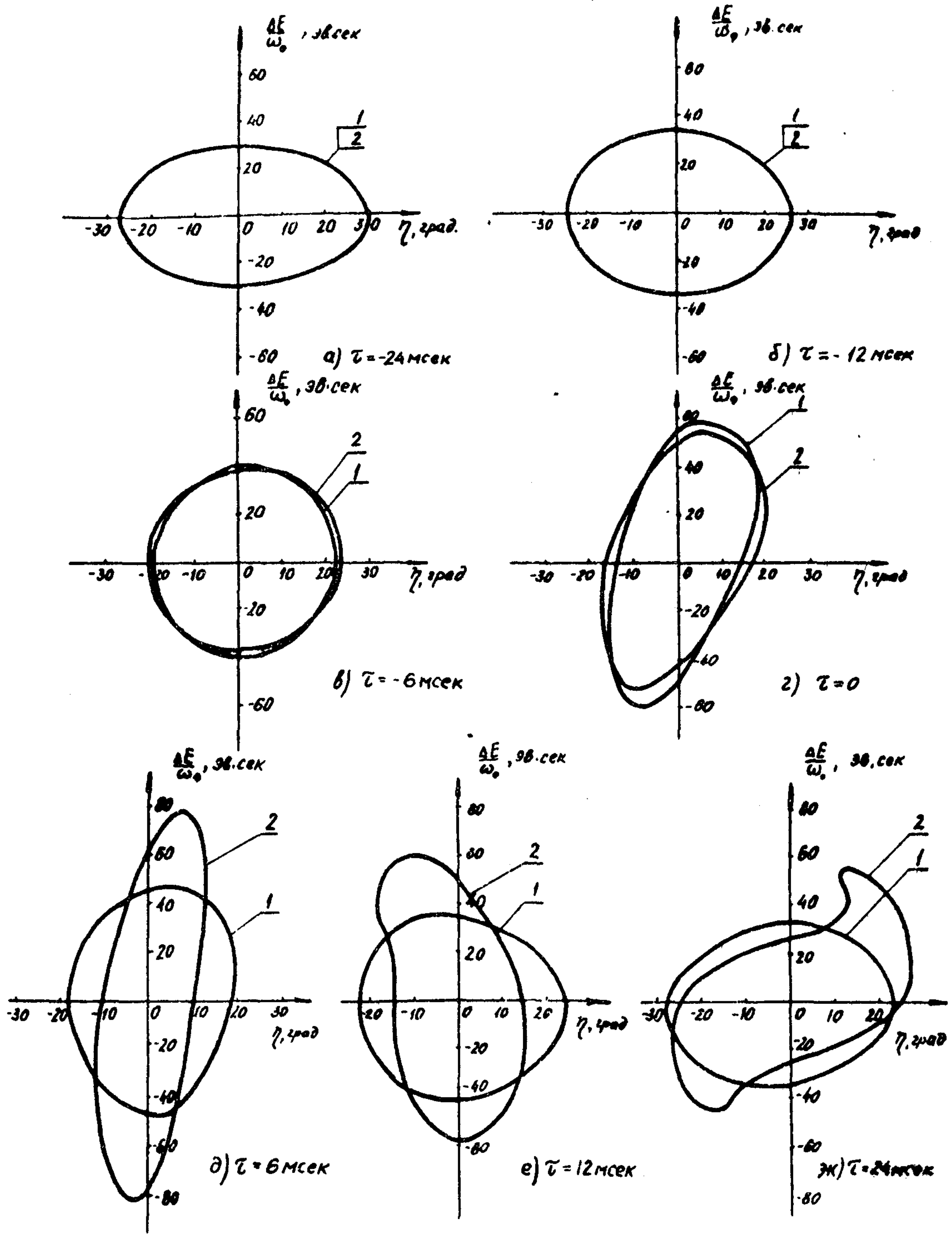

Fig. 2 : Solution of the phase oscillation equations in the transition region
i) without longitudinal space charge
ii) with longitudinal space charge