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ON THE DYNAMICS OF VAPOUR BUBBLES IN LIQUID HYDROGEN IN ULTRASONIC BUBBLE CHAMBERS

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INTRODUCTION

Experimental results of the study of the influence of ultrasonic field on the formation of tracks of charged particles in bubble chambers, derived from experiments with liquid helium[1] and liquid hydrogen[2],[3], allow us to conclude, that it is possible to construct ultrasonic bubble chambers (USBC). Their advantages over the usual bubble chambers consist of the possibility of an increase in the repetition rate and the achievement of the necessary, controlling, conditions.

The study of the characteristics of the dynamics of vapour bubbles in an ultrasonic field in bubble chambers constitutes an interesting problem. The present work examines theoretically the influence of ultrasound on the dynamics of vapour bubbles in liquid hydrogen, in which the study of the interaction of elementary particles is of even greater interest.
First of all we shall give a characteristic physical picture of the behaviour of a single vapour bubble in a liquid.

If the amplitude of the ultrasonic field P_m is equal to zero, and the pressure P_o is greater than the vapour pressure P_s, at a given temperature of the liquid T_∞, then the bubble collapses. When this happens the temperature of the vapour T' is greater than T_∞. The speed of the collapse of the bubble is determined by the heat conductivity of the liquid. This process, which we shall designate "static heat diffusion", is examined in detail in the works [4],[5],[6].

If the $P_m > 0$, then the changing pressure in the liquid brings forth

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pulsations of the bubble, which are accompanied by condensation and evaporation on its surface. During the phase of the decrease of pressure of the ultrasonic field, the vapour bubble grows and there is evaporation of the liquid, while during the phase of an increase of pressure, the bubble decreases in size and condensation of vapour takes place on its surface. Moreover, the change in the mass of vapour in the bubble at any given time is determined by the surface of the bubble and also by the temperature gradient in the spherical layer of the liquid surrounding the bubble. Since the thickness of this layer is larger during the decrease of the bubble diameter than during its growth, evaporation of the liquid into the bubble occurs from a greater surface with a higher temperature gradient; while condensation of the vapour occurs on a smaller surface with a smaller temperature gradient. Thus, for each period of ultrasonic field there is more evaporation of the liquid than condensation of the vapour, and the average quantity of mass of vapour in the bubble and its mean size increase. We shall call this phenomena "rectified heat diffusion" by analogy with rectified gas diffusion, examined in [7), 8)].

As the average radius of the bubble grows, the effect of the rectified diffusion, conditioned by the curvature of the surface of the phase separation, decreases, and with $P_o > P_\infty$ is balanced by the effect of static diffusion. A state of dynamic equilibrium is reached then, during which the average radius of the pulsating bubble attains an asymptotic value.

FORMULATION OF THE PROBLEM

Let us examine a spherical vapour bubble, making radial pulsations under the influence of ultrasonic field in the liquid, which is assumed to be incompressible. Then the change of the radius R of the bubble is described by the Rayleigh equation[9)]

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$$R\ddot{R} + \frac{3}{2} \dot{R}^2 + \frac{1}{\rho} \left[P_\infty (t) - P_R (t) \right] = 0. \qquad (1)$$

with

ρ — density of the liquid

$P_R(t)$ — pressure of the liquid on the surface of the bubble

$P_\infty(t)$ — pressure of the liquid at infinity

$$P_\infty(t) = P_o - P_m \sin (2\pi ft) \qquad (2)$$

with

f — frequency of the ultrasonic field.

To the equation (1) we shall add the following initial conditions :

$$R(0) = R_0, \ \dot{R} (0) = \dot{R}_0 \qquad (3)$$

where R_0 and $\dot{R}_0$ are the initial radius and initial speed of the bubble, correspondingly.

It is assumed that the vapour in the bubble is in thermodynamic equilibrium with the surface layer of the liquid, that is

$$P_R (t) = P' - \frac{2\sigma}{R} \qquad (4)$$

$$T_R (t) = T' \qquad (5)$$

where P'and T' are the pressure and temperature of the vapour in the bubble, T_R is the temperature of the liquid on the

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surface of the bubble, and σ is the surface tension of the liquid. The correlations (4) (5) are realized during a quasi-balanced process of evaporation (condensation) which occurs if [9]

$$\dot{R} < a \left(\frac{BT}{2\pi\mu} \right)^{\frac{1}{2}} \tag{6}$$

where a is the coefficient of accommodation, B universal gas constant, and μ gram mole . Moreover, the vapour in the bubble is saturated, P' and T' both are dependant upon the vapour pressure curve $P' = P'_S\ (T')$.

From the equations (1), (4) it follows that for the determination of R(t) we must know the dependence of P' on time, and this is determined by the conservation of energy during the evaporation and condensation processes on the surface of the bubble.

$$k \left(\frac{\partial T}{\partial r} \right)_R dt = \rho' \, L \, dR + \frac{R}{3} C_s \, \rho' \, dT' + \frac{R}{3} \left(\frac{\rho' L}{P'} - 1 \right) d \, P'. \tag{7}$$

with

- ρ' — density of the vapour
- C_s — specific heat capacity of the vapour along the curve of phase equilibrium[10]
- k — coefficient of heat conductivity in the liquid
- L — heat of vaporization
- $\left(\frac{\partial T}{\partial r}\right)_R$ — temperature gradient in the liquid at the surface of the bubble.

In the derivation of equation (7) it was assumed that the vapour

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is an ideal gas.

The temperature gradient in the liquid at the surface of the bubble is determined by the equation of heat conductivity

$$\frac{\partial T}{\partial t} + \dot{r}\ \frac{\partial T}{\partial r} = \frac{D}{r^2}\ \frac{\partial}{\partial r}\left(r^2\ \frac{dT}{dr}\right) \tag{8}$$

with

D — Diffusivity of the liquid

$\dot{r}$ — speed of the liquid at the point with the coordinate r, determined from the continuity equation in the incompressible liquid

$$\dot{r} = \dot{R}\ \frac{R^2}{r^2} \tag{9}$$

Let us supplement equation (8) with the initial condition

$$T\ (r,0) = T_0\ (r), \quad R_0 \leq r < \infty \tag{10}$$

and the boundary conditions

$$T\ (\infty,t) = T_\infty, \quad T(R,t) = T_R(t). \tag{11}$$

Joint solution of the equation (1), (7), (8) with the corresponding initial and boundary conditions determines the dynamics of the vapour bubble in the ultrasonic field. The problem formulated in such a manner can be solved only numerically with the use of contemporary electronic computers.

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DISCUSSION OF RESULTS

The present study, first of all, gives the results of numerical solutions of the above system of equations, derived for the vapour bubbles in liquid hydrogen, and, secondly, presents results of approximating solutions, obtained by making certain assumptions and hypotheses, shown below.

In the case of approximating calculations we shall limit ourselves with the study of such frequencies and amplitudes of the ultrasonic field, where one can dispense with the inertial terms in the equation (1). Disregarding also the term $\frac{2\sigma}{R}$ from the equations (1), (2), (4) we get

$$P_R\ (t) = P'\ (t) = P_o - P_m \sin\ (2\pi ft) \tag{12}$$

that is, we determine the law of change of the vapour pressure in the bubble in an ultrasonic field.

Using the correlation $P' = P_s\ (T')$, one can find the temperature $T_R(t)$, that is, one can fully determine the limiting conditions (11). In figure 1 is given a curve of phase equilibrium for hydrogen in the range of temperatures which are characteristic for hydrogen bubble chambers. The solid curve 1 shows the change of pressure with time P_R. When considering approximating calculations we shall assume that the periodic changes in pressure occur in the manner shown by the broken line 2, that is, the pressure changes with a step through the time internval τ, and acquires the value of $P_1 = P_o - P_m$, or $P_2 = P_o + P_m$. Moreover, the period of pressure change is equal to 2τ, so that $f = 1/2\tau$.

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Examining the vapour bubbles, for which the condition $R>(\pi D\tau)^{1/2}$ is fullfilled, one can evaluate the temperature gradients on the surface of the bubble, thus[4)]

$$\left(\frac{\partial T}{\partial r}\right)_R \cong \frac{T_o - T'_1}{(\pi D t)^{\frac{1}{2}}}, \qquad 0 < t < \tau, \tag{13}$$

$$\left(\frac{\partial T}{\partial r}\right)_R \cong - \frac{T'_2 - T_o}{(\pi D t)^{\frac{1}{2}}}, \qquad \tau < t < 2\tau, \tag{14}$$

with T_1, (T'_2) being temperature of vapour, which correspond respectively to pressure P'_1 (P'_2). Substituting (13) and (14) into equation (7), it turns out that the growth of the vapour bubble in the interval $(0,\tau)$ and its collapse in interval $(\tau,2\tau)$ is determined by the following correlation :

$$R = R_o\ (1 + F) + K_1\ (D_1 t)^{\frac{1}{2}}, \qquad 0<t<\tau, \tag{15}$$

$$R = R_m\ (1 - F) - K_2\ (D_2 t)^{\frac{1}{2}}, \qquad \tau<t<2\tau, \tag{16}$$

where

$$R_m = R_o\ (1 + F) + K_1\ (D_1 \tau)^{\frac{1}{2}}, \tag{17}$$

$$F = \frac{C_s}{3L}\ (T'_2 - T'_1) \quad \frac{P'_2 - P'_1}{3L\rho'} \left(\frac{\rho' L}{P_o} - 1\right), \tag{18}$$

Here

$$K_1 = \frac{2C_{P_1}\ \rho_1}{\pi^{\frac{1}{2}}\rho'_1\ L_1}\ (T_o - T'_1), \qquad K_2 = \frac{2C_{P_2}\rho_2}{\pi^{\frac{1}{2}}\rho'_2 L_2}\ (T'_2 - T'_o) \tag{19}$$

where indices 1 and 2 indicate that the values of parameters

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of liquid and gaseous hydrogen are taken at the temperature corresponding to T'_1 and T'_2. In formula (18) the values of these parameters are taken at the temperature $T_{cp} = T_\infty$. From equations (15), (16), (17) it follows, that the increase of the radius during the period of the ultrasonic field is equal to :

$$\Delta R = - \bar{R}F^2 + K_1 \,(1 - F - \kappa^{-1})\,(D_1\, \tau)^{\frac{1}{2}}; \tag{20}$$

where

$$\kappa = \frac{K_1}{K_2} \left(\frac{D_1}{D_2}\right)^{\frac{1}{2}} \quad , \tag{21}$$

with $\bar{R}$ being the average value of the bubble radius for one period. It follows from (20) that with the growth of R there comes a moment when the increase $\Delta R = 0$.

In this case the bubble reaches its asymptotic size $\bar{R}_\infty$,

$$\bar{R}_\infty = (D\tau)^{\frac{1}{2}} K_1 \left(\frac{1 - F - \kappa^{-1}}{F^2} \right) \tag{22}$$

Formula (22) allows one to evaluate the sizes, achieved by vapour bubbles during their growth in the ultrasonic field. It is essential to indicate, that at temperatures characteristic for bubble chambers, and at amplitudes of ultrasonic field which are sufficient for growth of vapour bubbles, the inequalities $F < 1$, $K_1 > 1$, $\kappa > 1$ are valid. Figure 2 shows the dependence of $\bar{R}$ (t) at two different liquid temperatures T_∞, obtained by means of a numerical solution of systems (1), (7), and (8). The initial value of radius R (0) was assumed to be equal to 10^{-4} - 10^{-5}. In the determination of initial conditions, results of [6] were used. It is apparent, that with the expiration of a certain time, the average radius of the bubble reaches its asymptotic value. In addition, the rectified and the static diffusion balance each other.

Figures 3 - 5 show the dependence of $\bar{R}_\infty$ on frequency and amplitude of the ultrasonic field and on the size of static pressure. Solid lines correspond to exact solutions, the broken lines to approximating solutions. In figure 5, the value of f, at the point where the curvature of the solid line $\bar{R}_\infty$ (f) changes, is the limit of applicability of the approximate method. With larger values for the frequencies of the ultrasonic field, the inertial terms in equation (1) become significant.

From figures 3 - 5 it follows, that in an ultrasonic field in liquid hydrogen, the asymptotic size of vapour bubbles is greater the lower the temperature T_∞, the lower the value of $P_o - P_s (T_\infty)$, and the greater the amplitude P_m. From the present calculations, it follows that at the amplitude of $P_m = 3{,}0$ atm, with the frequency f = 40,0 kHz and with the characteristic values for bubble chambers of $T_\infty = 26^\circ K$ and $P_o - P_s = 0.5$ atm, the asymptotic radius reaches the size $\bar{R} \cong 10^{-2}$ cm.

It is necessary to mention, that the derived results and the accepted assumptions can be checked by a comparison with experimental results. Unfortunately, at the present time there are no available quantitative experimental results dealing with the dynamics of vapour bubbles in liquid hydrogen in an ultrasonic field. The conclusions of this study, however, do not contradict the qualitative results of the publications[1), 2),3)].

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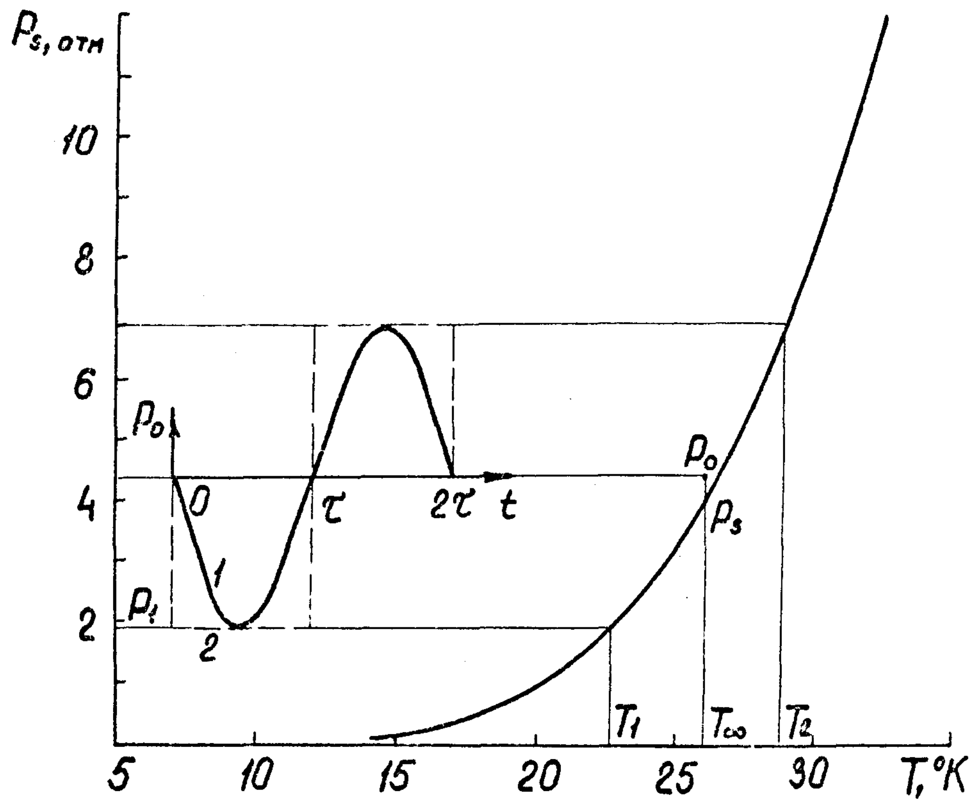

FIG. 1 The curve of phase equilibrium of hydrogen $P_S(T)$.

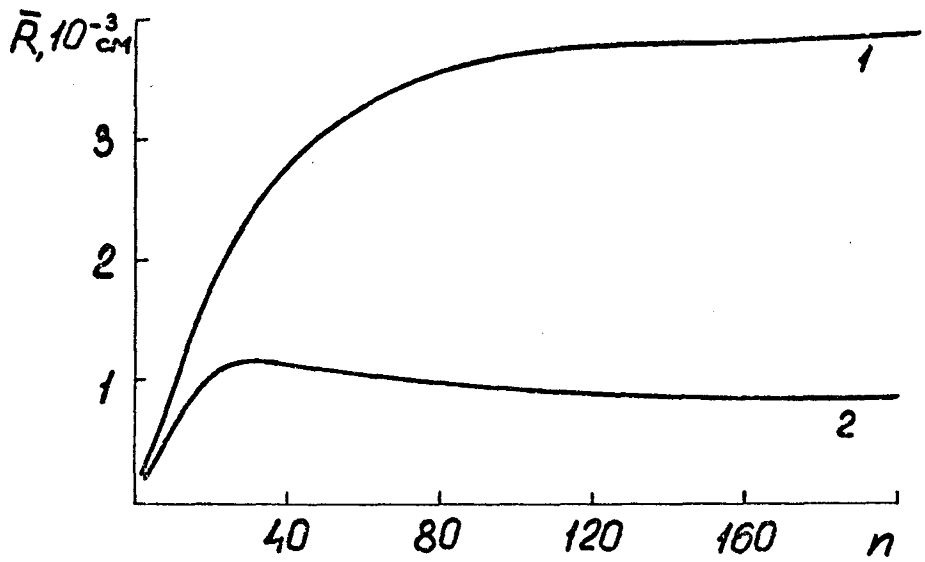

FIG.2 Dependence of the average radius of the bubble $\bar{R}$ on time (n - the number of periods of the ultrasonic field) with $R_o = 10^{-4}$ cm

$P_o - P_s = 0{,}5$ atm, $f = 40{,}0$ kHz

1 : $T = 26^oK$, $P_m = 3{,}0$ atm,

2 : $T = 28^oK$, $P_m = 4{,}0$ atm.

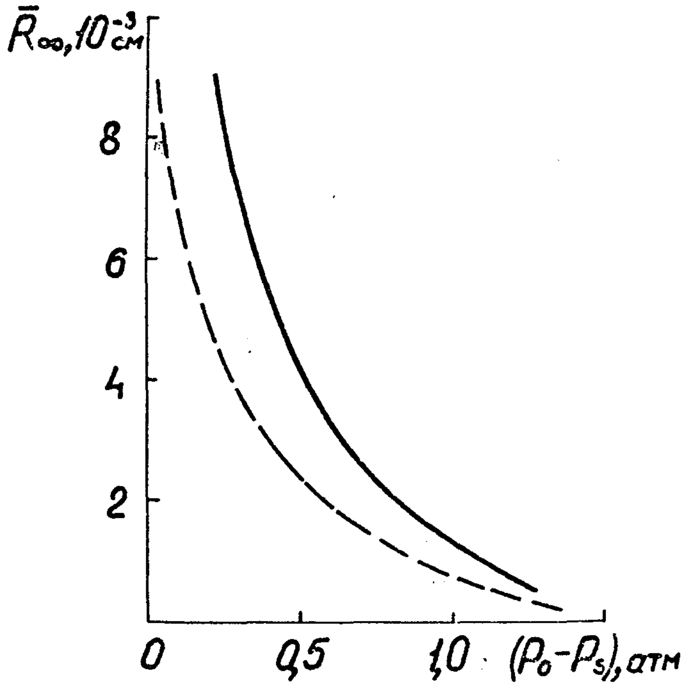

FIG.3 The size of the asymptotic radius $\bar{R}_{\infty}$ as a function of $(P_o - P_S)$. $T_{\infty} = 26^{o}K$, P_m = 3,0 atm, f = 40,0 kHz
The solid line - numerical calculations, the broken line - the approximating calculations.

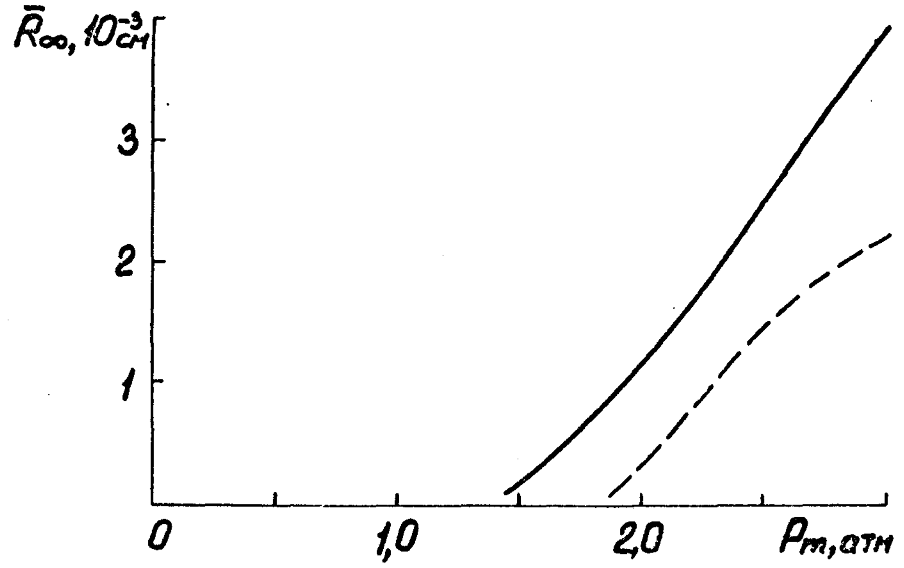

FIG. 4 The size of the asymptotic radius $\bar{R}_\infty$ as a function of P_m.
$T = 26°K$, $P_o - P_s = 0,5$ atm.
The solid line - numerical calculations, the broken line - the approximating calculations.

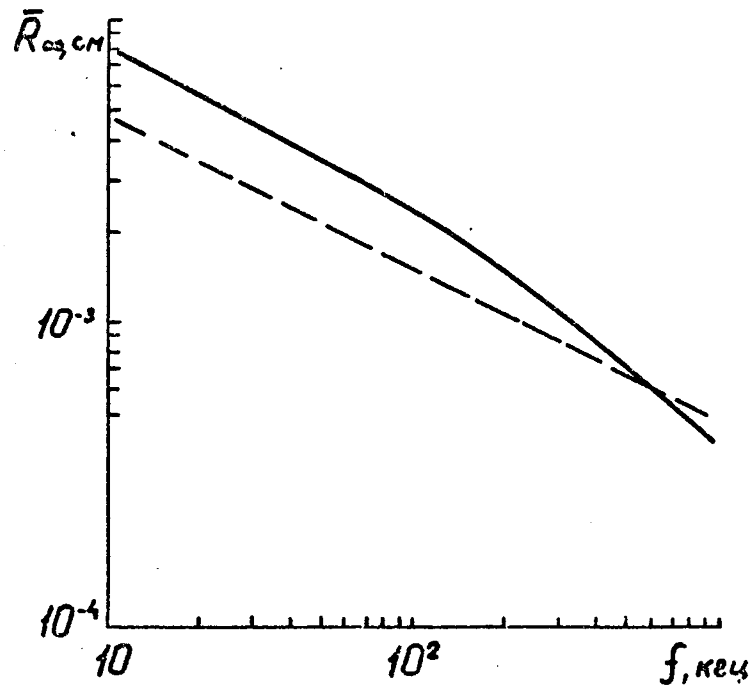

FIG.5 Dependence of the asymptotic radius $\bar{R}_\infty$ on the frequency of the ultrasonic field.
$T_\infty = 26^{\circ}K$, $P_o - P_s = 0{,}5$ atm, $P_m = 3{,}0$ atm.
The solid line - numerical calculations, the broken line - approximating calculations.