JOINT INSTITUTE FOR NUCLEAR RESEARCH, DUBNA

Report P2 - 5444

COMMENT ON ELASTIC NN- AND Nd-SCATTERING

O.V. Dumbrajs

Dubna 1970

Translated at CERN by B. Hodge
(Original: Russian)
Revised by N. Mouravieff

(CERN Trans. 71-22)

Geneva
March 1971

Elastic NN scattering is characterized by the fact that it is the only example of elastic scattering of elementary particles of one and the same isotopic multiplet that is accessible to experimental measurement. We can obtain several supplementary relations between the values which characterize elastic scattering.

Let us examine the processes

$$p + n \to p + n\,, \tag{1}$$

$$p + p \to p + p\,. \tag{2}$$

The amplitude of reaction (1) is*):

$$A_1(\theta) = \frac{1}{2} < 0,0 \mid S_0^0 \mid 0,0 > + \frac{1}{2} < 1,0 \mid S_0^0 \mid 1,0 > = \tag{3}$$
$$= \frac{1}{2} f_0 + \frac{1}{2}\sqrt{\frac{1}{3}}\, f_1\,,$$

where f_0, f_1 are amplitudes corresponding to the total isotopic spin $T_{pn} = 0$ and $T_{pn} = 1$. As p and n are terms of the same isomultiplet,

$$A_1(\pi - \theta) = -\frac{1}{2} f_0 + \frac{1}{2}\sqrt{\frac{1}{3}}\, f_1\,. \tag{4}$$

[see, for example, Dumbrajs and Podgoretskij[1] and Dumbrajs[2]].

For reaction (2) we have

$$A_2(\theta) = \sqrt{\frac{1}{3}}\, f_1\,. \tag{5}$$

From Eqs. (3) to (5) follow the relations

$$\sigma_1(\theta) + \sigma_1(\pi - \theta) = \frac{1}{2} \mid f_0 \mid^2 + \frac{1}{2}\sigma_2(\theta)\,, \tag{6}$$

$$\cos(\delta_0 - \delta_1) = \frac{\sigma_1(\theta) - \sigma_1(\pi - \theta)}{\sqrt{\sigma_2(\theta)}\ \sqrt{2\sigma_1(\theta) + 2\sigma_1(\pi - \theta) - \sigma_2(\theta)}}\,, \tag{7}$$

*) Here, and in the remainder of the text, the influence of nuclear spins has been disregarded.

where δ_0, δ_1 are the phases of the amplitudes f_0 and f_1, respectively.

Let $\theta = 0$; then the imaginary part of amplitude f_0 can, with the aid of the optical theorem, be expressed by the total cross-sections of reactions (1) and (2), and the real part by the real parts of the amplitudes of these reactions:

$$\mathrm{Im}\, f_0 = \frac{k}{2\pi}\,\sigma_{pn}^{tot} - \frac{k}{4\pi}\,\sigma_{pp}^{tot} \tag{8}$$

$$\mathrm{Re}\, f_0 = 2\,\mathrm{Re}\, f_{pn}(0) - \mathrm{Re}\, f_{pp}(0). \tag{9}$$

Finally, formulas (6) and (7) become, respectively*), the relations

$$a_{pn}\,a_{pp} = \frac{\sigma_{pp}(0) + \sigma_{pn}(0) - \sigma_{pn}(\pi)}{\frac{k^2}{8\pi^2}\,\sigma_{pn}^{tot}\,\sigma_{pp}^{tot}} - 1\,, \tag{10}$$

$$\cos\left(\delta_0(0) - \delta_1(0)\right) = \frac{\sigma_{pn}(0) - \sigma_{pn}(\pi)}{\sqrt{\sigma_{pp}(0)}\,\sqrt{2\sigma_{pn}(0) + 2\sigma_{pn}(\pi) - \sigma_{pp}(0)}} \tag{11}$$

where

$$a_{pn} \equiv \frac{\mathrm{Re}\, f_{pn}(0)}{\mathrm{Im}\, f_{pn}(0)}\,, \quad a_{pp} \equiv \frac{\mathrm{Re}\, f_{pp}(0)}{\mathrm{Im}\, f_{pp}(0)}\,.$$

Expressions (10) and (11) can be used, for example, to check general agreement between the experimental and theoretical data, as well as in several other cases, in particular for investigating pd scattering.

As is known[4], the total scattering cross-section of p on d can be expressed in the following manner:

$$\sigma_{pd}^{tot} = \sigma_{pn}^{tot} + \sigma_{pp}^{tot} - \frac{\langle r^{-2} \rangle}{4\pi}\,\sigma_{pn}^{tot}\,\sigma_{pp}^{tot}\,[1 - a_{pn}\,a_{pp}]\,, \tag{12}$$

*) A relation similar to (10) has already been obtained by Kanavets[3]. The author is grateful to V.A. Nikitin for drawing his attention to this paper.

where the last component is Glauber's correction, r^{-2} is the average value of the inverse square of the distance between the proton and the neutron in a deuteron. One of the applications of formula (12) is to calculate experimentally the value σ_{pn}^{tot} which is difficult to measure. The product of $a_{pn}a_{pp}$ is usually assumed to be zero [see, for example, Galbraith et al.[5]] since it is thought that, in the energy range for which Glauber's approximation is correct, the scattering amplitude is almost purely imaginary. Thus, yet one more approximation is introduced into Glauber's already rather inaccurate correction.

By means of the optical theorem, $a_{pn}a_{pp}$ can, of course, be expressed by the differential cross-sections in the forward direction and the total cross-sections, but only with an accuracy of up to the sign. However, when formula (10) is taken into account, this lack of accuracy is eliminated and formula (12) can be rewritten in the form:

$$\sigma_{pd}^{tot} = \sigma_{pn}^{tot} + \sigma_{pp}^{tot} + \frac{\langle r^{-2}\rangle}{4\pi} \{ \frac{8\pi^2}{k^2} [\sigma_{pp}(0) + \sigma_{pn}(0) - \sigma_{pn}(\pi)] - 2\sigma_{pn}^{tot}\sigma_{pp}^{tot} \} . \tag{13}$$

A similar modified expression for Glauber's correction can be obtained if, instead of formula (12), Glauber's "isotopically invariant" correction is used[6,7].

Let us point out once more that we obtained expressions (10), (11), and (13) on the assumption that there was no spin influence; consequently, any departure from them will point to the part played by spin effects in NN and Nd interactions.

I am grateful to M.I. Podgoretskij, N.M. Kuin and A.V. Tarasov for their useful comments.

REFERENCES

1) O.V. Dumbrajs and M.I. Podgoretskij, Yadernaya Fiz. 11, 232 (1970).

2) O.V. Dumbrajs, JINR communication R2-4621, Dubna 1969.

3) V.P. Kanavets, Yadernaya Fiz. 2, 931 (1965).

4) R.J. Glauber, Phys. Rev. 100, 242 (1955).

5) W. Galbraith et al., Phys. Rev. 138 B, 913 (1965).

6) C. Wilkin, Phys. Rev. Letters 17, 561 (1966).

7) R.J. Glauber and V. Franco, Phys. Rev. 156, 1685 (1967).