CERN - European Laboratory for Particle Physics

CLIC Note 407

PETS Output Power and Drive Beam Deceleration for Finite Q-Values and Tune Errors

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Abstract

PETS performance degradations caused by finite Q-values and small tune errors are estimated. A simple explanation of the recently discovered group velocity enhancement of the loss factor is given. .

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Beamloading fields along the structure

A **single** infinitely short **bunch** with charge q creates a **longitudinal field** amplitude in the main decelerating mode of

$$E = \frac{R'}{Q} q\omega \frac{1}{1-\beta_{gr}}$$

(R'/Q: circuit convention, travelling-wave, twice the standing-wave value)

for a test particle following tightly behind the bunch. Note that above factor $(1-\beta_{gr})^{-1}$ is a recent refinement with respect to the theory used in the past. An unexpected large beamloading was first observed in the Clic Test Facility (CTF2) by H. Braun and M. Valentini [1] and above dependance on group velocity was then found numerically by A. Millich [2]. - This factor is necessary for high group velocities, as used in PETS. - The annex gives a pedestrians explanation based on the group velocity notion.- **The steady-state** field is obtained via integration of above expression, as shown in the following section.

Case of no tune error and infinite Q-value

We consider the steady-state with constant bunch intensity and only after one fill time of the PETS.
At position z inside the structure and after the passage of many bunches the resulting steady-state beamloading field amplitude is given by (assuming no structure attenuation):

$$E = \frac{\frac{R'}{Q}}{v_{gr}} q\omega \frac{z}{\Delta t}.$$

The decelerating electric field amplitude at a travelling bin at z results from an average

$$\frac{z}{\Delta t}\left(\frac{1}{v_{gr}} - \frac{1}{c}\right)$$

elementary bunch waves (like above first expression) picked up by the bin as it travelled with v_{gr} from the structure entry to position z.

For simplicity we replace the bunch charge divided by the bunch spacing by the mean bunch current i

$$\frac{q}{\Delta t} \rightarrow i.$$

Finite Q-value

We introduce the classical attenuation factor

$$\alpha = \frac{\omega}{2Qv_{gr}}.$$

Integral to obtain the electrical field inside the lossy structure

The field at Z of a travelling-wave bin results from the integration of the many elementary contributions like (1) picked up by the bin during the passage from z=0 to Z. The contributions are attenuated between z and Z as e $^{-\alpha(Z-z)}$ with the result

$$E(Z) = \frac{\frac{R'}{Q}}{v_{gr}} i\omega \int_0^Z e^{-\alpha z}\, dz = \frac{\frac{R'}{Q}}{v_{gr}} i\omega \frac{1-e^{-\alpha Z}}{\alpha}.$$

Above expression as a power expansion in (αZ) yields, at the structure end (Z=ℓ),

$$E(Z=l) = \frac{\frac{R'}{Q}}{v_{gr}} i\omega l \left(1 - \frac{\alpha l}{2} + \frac{(\alpha l)^2}{6} - ..\right)$$

and an output power level at ℓ of

$$P = \frac{E^2(Z=l)v_{gr}}{2\frac{R'}{Q}\omega}.$$

Combining the two last expressions we obtain the following Mc Laurin expansion in kℓ for the output power

$$P = \frac{\frac{R'}{Q} i^2 \omega l^2}{2v_{gr}} \left(1 - \alpha l + \frac{(\alpha l)^3}{3} - \right),$$

which can be written in a new form in terms of fill time $t_f = \ell / v_{gr}$, current I and structure R/Q

$$P = \frac{1}{2}\frac{R}{Q} i^2 \omega t_f \left(1 - \alpha l + \frac{(\alpha l)^3}{3} - .. \right)$$

The power can also be given in terms of the drain time d = $\ell(1/v_{gr}-1/c)$ and drain time charge q_d = di as

$$P = \frac{\frac{R}{Q}\omega q_d^2}{2d(1-\beta_{gr})} \left(1 - \alpha l - \frac{(\alpha l)^3}{3} - ... \right)$$

Decelerating voltages

The peak total beam-induced voltage is obtained by integration of E over the structure length ℓ

$$U_{dec} = \frac{\frac{R'}{Q}}{v_{gr}} i\omega \left(\frac{(l + \frac{1}{\alpha}\left(e^{-\alpha l} - 1\right)}{\alpha} \right)$$

corresponding to the Mc Laurin expansion in $\alpha\ell$

$$U_{dec} = \frac{\frac{R'}{Q}}{v_{gr}} i\omega \frac{l^2}{2} (1 - \frac{\alpha l}{3} + \frac{(\alpha l)^2}{12} - ..)$$

or in terms of fill time and total R/Q

$$U_{dec} = \frac{1}{2}\frac{R}{Q} i\omega t_f (1 - \frac{\alpha l}{3} + \frac{(\alpha l)^3}{12} - ..).$$

It appears that the relative deceleration decreases as $\alpha\ell/3$ whereas the relative output amplitude decreases as $\alpha\ell/2$ with the consequence that the power decreases as αl; thus the beam to RF efficiency decreaseses only as $2\alpha\ell/3$.

One remedy to recover the desired power is to increase the R'/Q value as $\alpha\ell$.

Power loss and deceleration for mistuned PETS

For this case we introduce a fictitious wave number k for the propagation in the structure such that kz represents the phase slip of the **real** synchronous mode with the frequency error $\omega_{err\ synch}$ with respect to the **ideal** synchronous mode with no freq. error (the latter frequency being a harmonic of the bunch frequency)

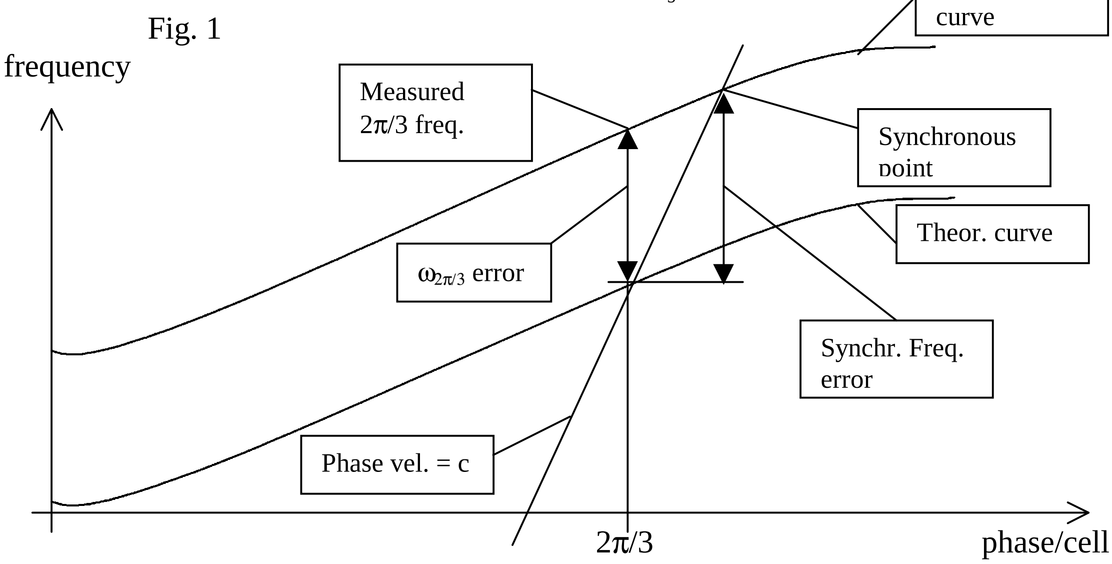
$$k = \omega_{errsnch} / v_{gr} .$$

Note that this synchronous frequency error is larger than the frequency error of the **standing wave** for the 2π/3 mode as measured in the lab. inside the structure with the correct periodicity but having some other imperfection.

Assuming that the real structure has the same group velocity as the theoretical one, we can approximately express, as can be seen in fig. 1 by simple geometrical considerations, the error in synchronous frequency as

$$\omega_{err} = \omega_{err\,2\pi/3} / (1 - \beta_{gr}).$$

Fig. 1



As a traveling bin moves along the structure with v_{gr} it will pick up elementary bunch waves having a slipping phase kz. At position Z its complex amplitude will be

$$E(Z) = \frac{R'}{Q}\frac{i\omega}{v_{gr}}\int_0^Z (\cos kz + j\sin kz)dz = \frac{R'}{Q}\frac{i\omega}{kv_{gr}}(\sin kZ + j(1-\cos kZ)).$$

The absolute amplitude is

$$|E(Z)| = 2\frac{R'}{Q}\frac{i\omega}{kv_{gr}}\sin\frac{kz}{2} \approx \frac{R'}{Q}\frac{i\omega}{v_{gr}}Z(1-\frac{(kZ)^2}{24}+\frac{(kZ)^4}{1920}-..).$$

We thus approximately lose output amplitude as $(k\ell)^2/24$ and output power as $(k\ell)^2/12$.

The drive beam deceleration must decrease like the output power, since no energy is lost in the structure, and there is no degradation of the efficiency due to tuning errors.

For a typical measured $2\pi/3$ frequency error of 20 MHz the CTF2 structure suffers a power loss of about 7%.

Reference:

[1] H. H. Braun & M. Valentini. Measurement of Drive Beam Deceleration in the CLIC Test Facility II during the 1998 Runs. Cern CTF-Note 99/05

[2] A. Millich & L. Thorndahl. Loss Factor Dependence on Group Velocity in Disc-Loaded Travelling-Wave Structures. Cern CLIC-Note 366

Acknowledgements:

Thanks to E. Jensen and W. Wuensch for discussions.

E. Jensen proposed the formal presentation of the new loss factor as the product of the usual (R/Q)/2 value times $(1-\beta_{gr})^{-1}$ times ω.

ANNEX

A simple explanation of the longitudinal loss factor as described in [1]

Consider an infinitely short bunch exciting a travelling wave in a structure of length ℓ with a loss factor k_δ and a normalised group velocity β_{gr} followed tightly by a test particle.

Consider the situation just when the exciting bunch exits the structure.

Fig. 2a (bunch excited travelling wave mode limited to the structure end at the exit of the bunch)

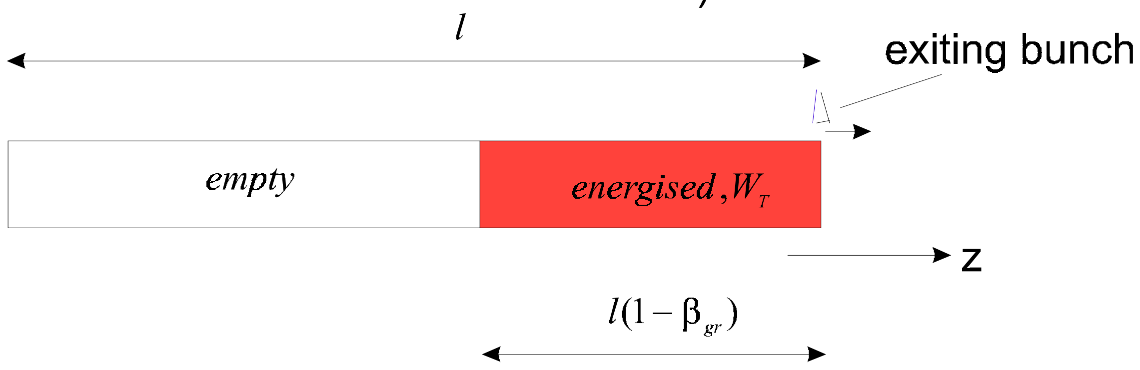
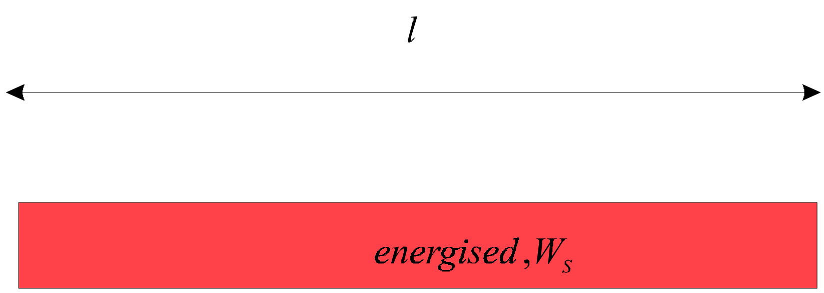


Fig. 2b (standing wave mode with same deceleration gradient but energised from an exterior generator, as calculated by urmel or mafia; with twice the longitudinal energy density, since there is one wave in each direction)



In the case of the bunch excitation of the travelling wave (Fig. 2a) the energy left in the structure amounts to

$$W_{tr} = k_{\delta tr} q_{tr}^{\ 2}.$$

The total voltage seen by the test particle following tightly behind the exciting bunch amounts to

$$V_{tr} = 2k_{\delta tr} q_{tr}.$$

Combining above 2 equations we obtain

$$k_{\partial tr} = \frac{V_{tr}^{\ 2}}{4W_{tr}}.$$

The loss factor is **inversely** proportional to the stored energy W_{tr}.

Comparing travelling- and standing wave-modes for **equal** structures with **equal** test particle deceleration ($V_{tr} = V_{st} = V$), as outlined in figs. 2a and 2b, we see that the relation between the stored energies W_{tr} and W_{st} can be written as

$$W_{tr} = (1-\beta_{gr})W_{st}/2.$$

Here the factor 2 in the denominator comes from the fact that there are 2 waves in the standing-wave case and the 1-β_{gr} factor from the wave compression due to the group velocity.

Inserting the fourth expression into the third we see the influence of the group velocity on the longitudinal loss factor of the travelling-wave.

$$k_{\partial st} = \frac{V^2}{2(1-\beta_{gr})W_{st}} = \frac{\frac{R_{st}}{Q}\omega}{(1-\beta_{gr})}, where \qquad W_{st} = \frac{V_{st}^{\ 2}}{2\frac{R_{st}}{Q}\omega}.$$

Note that the **travelling-wave** loss factor is here given in terms of the **standing-wave** R_{st}/Q. Furthermore we have the relation between the 2 loss factors

$$k_{\delta tr} = \frac{2k_{\delta st}}{1-\beta_{gr}}.$$