hep-th/9908163
CERN-TH/99-255

Superconformal Field Theories, Multiplet Shortening and the $\mathrm{AdS}_5/\mathrm{SCFT}_4$ Correspondence

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We review the unitarity bounds and the multiplet shortening of UIR's of 4 dimensional superconformal algebras $SU(2,2|N)$, $(N = 1,2,4)$ in view of their dual role in the AdS/SCFT correspondence. Some applications to KK spectra, non-perturbative states and stringy states are given.

To appear in the proceedings of the Moshé Flato *Conference, "Advances and Prospects in Physical Mathematics", Dijon (France), 5-8 September 1999.*
CERN-TH/99-255

August 99

1. Introduction

The classification of Super Poincaré and Super Anti-de-Sitter algebras [1,2,3] in diverse dimensions has played a major role since the early days of supersymmetry and supergravity.

Super Poincaré algebras have been widely used to classify supersymmetric field theories and in constructing generic supergravity theories in diverse dimensions and with different numbers of supersymmetries [4]. Their central extension (more precisely the central extension of the supertranslational algebras) had a major role with the inclusion of p-branes supergravity solutions; a particular case is related to the black hole classification based on the BPS properties of the solutions.

Supergravity solutions for BPS p-branes are considered to be limits of analogous solutions in quantum theories of gravity, such as string theory or M-theory. However, in certain cases, due to non renormalization theorems of N-extended supersymmetry, such solutions are exact or at least reliable approximations of stringy or M-theory solutions.

The recent interplay between Anti-de-Sitter supergravity and conformal field theories follows from the conjectured duality [5] between the string (or M-theory) background describing the AdS_{p+2} horizon geometry [6] of black p-branes and the conformal field theory living on their world-volume, which can be thought as the AdS-boundary. This correspondence claims that the *correlation functions* of composite operators in the CFT can be computed as *bulk* Green functions of AdS states of supergravity or string theory [7,8].

In the present discussion, we only consider the case of D3-branes in type IIB supergravity. The world-volume theories are $N = 1, 2, 4$ 4d superconformal field theories [8,9] and the corresponding bulk theory is type IIB on $\mathrm{AdS}_5 \times \mathrm{X}_5$, for internal manifolds X_5 giving $N = 2, 4, 8$ supergravities [10,11,12,13], respectively. Operators in the SCFT are associated with the KK excitations, coming from the harmonic expansion on X_5.

These theories and their UIR's are therefore related to the $SU(2, 2|N)$ algebras for $N = 1, 2, 4$ [14,15,16,17]. The case $N = 4$, as we will see, deserves a special treatment since the $U(1)$ factor of the $U(N) = U(1) \times SU(N)$ R-symmetry becomes an outer automorphism of the superalgebra [18,19,17,20] (this is a particular case of the $SU(N|M)$ superalgebra with $N = M$).

This contribution reviews the unitary bounds and multiplet shortening for highest weight UIR's of 4d superconformal algebras in view of the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence [9,21,22]. Special emphasis will be given to different properties of the $N = 1, 2, 4$ cases, due to the different supersingleton representations [23,14,18,19]. The latter are the basic degrees of freedom of the superconformal theories on the boundary. In this respect, the pioneering work of Moshe Flato, in collaboration with Chistian Frønsdal, has a crucial role in this analysis. The above authors extended, in particular, the unitary bounds of UIR's of $SU(2,2)$ to $SU(2,2|1)$, and classified the multiplet shortenings, which just correspond to the *thresholds* of the unitary bounds [14]. Extension of this analysis came later [18,19] and is the latter that we will use in the context of the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence.

In Sections II and III, the unitarity bounds for $N = 1, 2, 4$ will be reviewed. UIR's satisfying these bounds have an interpretation in terms of superconformal boundary fields [8,9,24]. Some examples will be discussed. More importantly, the case of multiplet shortening corresponds to CFT operators of protected dimensions. This happens either if a boundary operator is a *conserved current*, thus corresponding to a massless bulk field, or if it is a shortened multiplet of the *chiral* type [8,25]. Intermediate multiplets which are not chiral also exist that have protected dimensions [21,22]. They can be formally obtained by the product of a conserved supercurrent with a *chiral* operator.

For extended superconformal algebras with $N = 2, 4$ a new (exceptional) series of shortened multiplets also exists which has no analogous in $N = 1$. This has to do with the fact that $N = 2$ and $N = 4$ algebras admit *self-conjugate* supersingletons representations [16,18,19] (unlike the $N = 1$ case).

In Section IV, the application of different type of shortened multiplets to different boundary conformal field theories will be given. Shortened multiplets allow to make a detailed comparison between the AdS_5 bulk theory and the boundary superconformal field theory. Particular examples are the $N = 4$ KK towers [11,26,8,24], corresponding to an exceptional series of the $N = 4$ superalgebra as well as multitrace operators of the $N = 4$ Yang-Mills theory [27], some of them having protected dimensions [27,28,29]. A richer structure exists for $N = 1$ superconformal theories where shortened multiplets with anomalous dimensions can appear with precise associated supergravity states. This is the case for $\mathrm{X}_5 = T^{1,1} = SU(2) \times SU(2)/U(1)$, unique example [12] of a smooth manifold where both the bulk and the boundary theories have been worked out in full detail [30,31,32,22,33].

2. The unitary representations and shortening for $SU(2,2|N)$, $N = 1, 2$

In this Section we consider the unitarity bounds for the highest-weight representations of the $SU(2,2|N)$ superalgebra.

For the case of the $SU(2,2)$ algebra itself, a given UIR is denoted, following Flato and Frønsdal [23,14,34], as $D(E_0, J_1, J_2)$ where E_0, J_1, J_2 are the quantum number of the highest-weight state [16], given by a finite UIR of the maximal compact subgroup $SU(2) \times SU(2) \times U(1)$. The UIR's fall in three series [35,5,36],

$$
\begin{aligned}
&a)\ J_1 J_2 \neq 0 && E_0 \geq 2 + J_1 + J_2 \\
&b)\ J_2 J_1 = 0 && E_0 \geq 1 + J \\
&c)\ J_1 = J_2 = 0 && E_0 = 0
\end{aligned}
\tag{2.1}
$$

In the bulk interpretation, the inequality corresponds to massive representations in AdS$_5$, the bound in a) corresponds to massless bulk particles of *spin* $J_1 + J_2$ while the bound in b) corresponds to singletons of spin J.

Note that in the AdS/CFT correspondence the bulk-boundary quantum numbers (E_0, J_1, J_2) refer to the *compact* basis for the AdS states, while they refer to the non-compact basis $SL(2,C) \times O(1,1)$ for the boundary conformal operators [8,17]. The highest weight state in AdS corresponds to a conformal operator $O(x)$ at $x = 0$, so the AdS energy E_0 corresponds to the conformal dimension Δ_0 and the (J_1, J_2) quantum numbers correspond to the Lorentz spin of $O(x)$.

In the CFT, the bound a) corresponds to conformal conserved currents of spin $J = J_1 + J_2$,

$$
E_0 = 2 + J_1 + J_2 \qquad \partial^{\alpha_1 \dot\alpha_1} J_{\alpha_1 \ldots \alpha_{2J_1}, \dot\alpha_1 \ldots \dot\alpha_{2J_2}}(x) = 0, \qquad J_1 J_2 \neq 0 \tag{2.2}
$$

while the bound b) corresponds to massless spin J conformal fields on the boundary,

$$
\begin{aligned}
&E_0 = 1 + J \qquad && \partial^{\alpha_1 \dot\alpha_1} O_{\alpha_1 \ldots \alpha_{2J}} = 0 \qquad && J \neq 0 \\
& && \partial^2 O(x) = 0 && J = 0
\end{aligned}
\tag{2.3}
$$

The case c) corresponds to the identity representation.

Let us now consider the case of $SU(2,2|N)$ superalgebras [15,18]. In this case, the highest weight state is denoted by $D(E_0, J_1, J_2; r, a_1, ..., a_{N-1})$ where the quantum numbers in the bracket denote a UIR of $SU(2,2) \times U(1) \times SU(N)$, r being the quantum number of the $U(1)$ R-symmetry and $a_1, ..., a_{N-1}$ the Dynkin labels of a UIR of the non-abelian symmetry $SU(N)$. We will denote by R the $U(1)$ inside $U(N)$.

Note that for $N \neq 4$, the $SU(2,2|N)$ algebra is both a subalgebra and a quotient algebra of $U(2,2|N)$, since the supertrace generator (which is a central charge) can be eliminated by a redefinition of the $U(1)$ generator R of $U(N)$ [18,19]. This redefinition is not however possible for $N = 4$ since R drops from the supersymmetry anti-commutators and it becomes an outer automorphism of the algebra [19,37,20]. In this case we have therefore two inequivalent algebras (which do not include the $U(1)$ R generator), $PSU(2,2|4)$ and $PU(2,2|4)$, depending whether $r = 0$ or $r \neq 0$.

We will adopt the convention that r is always the quantum number of the $U(1)$ generator of $U(2,2|N)$, so it will be the $U(1)$ subgroup of $U(N)$ for $N = 1,2$ while it will be a central $U(1)$ for $N = 4$ [18].

In the boundary CFT language, UIR's can be realized as conformal superfields. The superhighest weight state corresponds to a superfield $\phi(x,\theta)$ at $x = \theta = 0$ [18,24,9,38].

The unitarity bounds for $SU(2,2|1)$ were found by Flato and Frønsdal [14]. They generalize the cases a),b) and c) of eq. (2.1). They are,

$$a) \qquad E_0 \geq 2 + 2J_2 + r \geq 2 + 2J_1 - r \qquad (\text{or } J_1 \rightarrow J_2,\, r \rightarrow -r) \qquad J_1, J_2 \geq 0 \qquad (2.4)$$

which implies

$$E_0 \geq 2 + J_1 + J_2, \qquad r \geq J_1 - J_2, \qquad 2 + 2J_1 - E_0 \leq r \leq E_0 - 2 - 2J_2 \qquad (2.5)$$

$$b) \qquad E_0 = r \geq 2 + 2J - r \qquad (J_2 = 0,\, J_1 = J, \text{ or } J_1 = 0,\, J_2 = J,\, r \rightarrow -r) \qquad (2.6)$$

which implies $E_0 \geq 1 + J$,

$$c) \qquad E_0 = J_1 = J_2 = r = 0 \qquad (2.7)$$

which corresponds to the identity representation.

Shortening in the case a) corresponds to

$$E_0 = 2 + 2J_2 + r, \qquad (r \geq J_1 - J_2) \qquad (\text{or } J_1 \rightarrow J_2,\, r \rightarrow -r) \qquad (2.8)$$

This is a semi-long AdS_5 multiplet [21] or, in conformal language, a *semiconserved* superfield [39,22],

$$\bar{D}^{\dot{\alpha}_1} L_{\alpha_1 \ldots \alpha_{2J_1}, \dot{\alpha}_1 \ldots \dot{\alpha}_{2J_2}}(x, \theta, \bar{\theta}) = 0, \qquad (\bar{D}^2 L_{\alpha_1 \ldots \alpha_{2J_1}} = 0 \text{ for } J_2 = 0) \qquad (2.9)$$

(in our conventions θ carries $\Delta = -1/2, r = 1, \bar{\theta}$ has $\Delta = -1/2, r = -1$).

Maximal shortening in case a) corresponds to $E_0 = 2 + J_1 + J_2$, $r = J_1 - J_2$. This is a conserved superfield which satisfies both left and right constraints:

$$\bar{D}^{\dot{\alpha}_1} J_{\alpha_1 \ldots \alpha_{2J_1}, \dot{\alpha}_1 \ldots \dot{\alpha}_{2J_2}} = D^{\alpha_1} J_{\alpha_1 \ldots \alpha_{2J_1}, \dot{\alpha}_1 \ldots \dot{\alpha}_{2J_2}} = 0 \tag{2.10}$$

The shortening in b) corresponds to *chiral superfields*. Maximal shortening in b) to *massless* chiral superfields, i.e. chiral singleton representations: $E_0 = r = 1 + J$. The corresponding superfield, for $E_0 = r$ satisfies,

$$\bar{D}^{\dot{\alpha}} S_{\alpha_1 \ldots \alpha_{2J}} = 0 \tag{2.11}$$

and, for $E_0 = 1 + J$, it also satisfies

$$D^{\alpha_1} S_{\alpha_1 \ldots \alpha_{2J}} = 0 \qquad (D^2 S = 0, \text{ for } J = 0) \tag{2.12}$$

These equations are the supersymmetric version of (2.2) and (2.3).

We may call, with an abuse of language, off-shell singletons chiral superfields in the sense that in an interacting conformal field theory singletons may acquire anomalous dimension and fall in (2.11).

It is also evident, from superfield multiplication, that by suitable multiplication of several free supersingletons one may get any other superfield of type (2.9), (2.10) or (2.11).

Note that superfields obeying (2.9),(2.11) may have anomalous dimensions since the shortening condition just implies a relation between E_0 and r without fixing their value.

The basic singleton multiplets for $N = 1$ gauge theories correspond to $J = 0, 1/2$ in (2.11), i.e. chiral scalar superfields S (Wess-Zumino multiplets) and Yang-Mills field strength multiplets W_α. Any other conformal operator is obtained by suitable multiplication of these two sets of basic superfields.

In type IIB on $T^{1,1}$ long, semi-long and chiral multiplets do indeed occur [30,31,22]. Chiral WZ singleton multiplets have in this case an anomalous dimension $\gamma = -1/4$ ($\Delta = 1 + \gamma$) and R-symmetry $R = 3/4$.

If we adopt the concept of *bulk* masslessness as corresponding to an UIR that is contained in the product of two supersingletons there are other massless bulk representations which are obtained by chiral multiplication of two singleton representations,

$$\begin{gathered} D(1 + J_1, J_1, 0; 1 + J_1) \otimes D(1 + J_2, J_2, 0; 1 + J_2) = D(2 + J_1 + J_2, J, 0; 2 + J_1 + J_2) \\ |J_1 - J_2| \leq J \leq J_1 + J_2 \end{gathered} \tag{2.13}$$

These representations do not occur in the AdS/SCFT correspondence unless $J_1 = 0, J_2 = 0, 1/2$ since they do not correspond to global symmetries of the SCFT. One can indeed show that the symmetry of $N = 1$ SYM theory, as in the dual of type IIB on $AdS_5 \times T^{1,1}$, do not allow such UIR's.

We now turn to the $SU(2,2|2)$ superalgebra. In this case the highest weight state is $D(E_0, J_1, J_2; r, l)$ where l is the spin label of the R-symmetry $SU(2)$. The series of UIR's are now,

$$\alpha) \qquad E_0 \geq 2 + 2J_2 + r + 2l \geq 2 + 2J_1 - r + 2l \text{ (or } J_1 \to J_2,\ r \to -r,\ l \to l) \qquad J_1, J_2 \geq 0 \tag{2.14}$$

which implies

$$E_0 \geq 2 + J_1 + J_2 + 2l,\ \ r \geq J_1 - J_2,\ \ 2 + 2J_1 + 2l - E_0 \leq r \leq E_0 - 2 - 2J_2 - 2l \tag{2.15}$$

$$\beta) \qquad E_0 = r + 2l \geq 2 + 2J - r + 2l \qquad (J_2 = 0, J_1 = J, \textit{ or } J_1 = 0, J_2 = J, r \to -r) \tag{2.16}$$

which implies $r \geq 1 + J$, $E_0 \geq 1 + J + 2l$,

$$\gamma) \qquad E_0 = 2l, \qquad J_1 = J_2 = r = 0 \tag{2.17}$$

The new phenomenon in $N = 2$ is that supersingletons appear in two different series, β) and γ). This is related to the fact that the γ) series does not reduce to the identity representation only, as for $N = 1$, but contains both massless bulk ($l = 1$) and massless boundary (supersingletons) ($l = 1/2$) representations. Let us discuss in detail the cases of massless bulk and massless boundary representations.

Supersingletons in β) correspond to the shortening condition

$$E_0 = r = 1 + J, \qquad l = 0 \tag{2.18}$$

This is the highest weight state $D(1+J, J, 0; 1+J, 0)$. The highest spin in these multiplets is $J + 1$. For $J = 0$ we get the Yang-Mills multiplet.

The $l = 1/2$ supersingleton in γ) is the hypermultiplet ($D(1, 0, 0; 0, 1/2)$).

Since supermassless bulk representations are defined as the product of two supersingletons we may obtain bulk massless representations by multiplying either two supersingletons (in a chiral and anti-chiral manner) in β) or by a supersingleton in β) and the $l = 1/2$ supersingleton in γ), or two $l = 1/2$ supersingletons in γ).

The spin 2 massless bulk states are obtained by chiral anti-chiral multiplication of the $J = 0$ chiral supersingleton in β), giving the shortened representations in α) with $J_1 = J_2 = r = l = 0$. This is the massless graviton multiplet: $D(2,0,0;0,0)$.

The massless spin 1 multiplet is obtained by multiplying two hypermultiplets and corresponds to the $l = 1$ UIR of case γ), giving $D(2,0,0;0,1)$.

Finally, the massless spin 3/2 multiplet is obtained by multiplying the $J = 0$ supersingleton in β) with the hypermultiplet, giving a bulk massless multiplet corresponding to the shortening $r = 1, J = 0, l = 1/2$ in β), i.e. $D(2,0,0;1,1/2)$. The superconformal realization of these superfields as current multiplets was given in [40,41,42].

Obviously these examples can be extended by replacing the chiral vector multiplet by an arbitrary spin J supersingleton. Also one can get, as for $N = 1$, *chiral* bulk massless multiplets by chiral multiplication of two spin J supersingletons as in (2.13).

In the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence the relevant supersingletons are the *chiral* vector multiplets (in β) for $J = 0$) and the hypermultiplets (in γ) for $l = 1/2$). Multiplet shortening in the AdS/CFT correspondence will be further discussed in Section IV. Here we note that the series γ) contains short multiplets with $E_0 = 2l$ for each value of l. These states can be explicitly constructed by multiplying $2l$ hypermultiplet singletons and their dimension is not renormalized and coincide with the canonical one. On the other hand, short multiplets in β), constructed using also chiral vector singletons, may have anomalous dimension since, due to (2.16), E_0 is related to the arbitrarily valued $U(1)$ charge r by $E_0 = r + 2l$.

3. UIR's of $PSU(2,2|4)$ and $PU(2,2|4)$ and shortening conditions.

The $N = 4$ superalgebra is of great interest because it corresponds to $N = 4$ superconformal Yang-Mills theory. In the dual description, it lives at the boundary of AdS_5 [5,7,8]. The supergravity theory emerges as the low energy limit of type IIB string theory compactified on $\mathrm{AdS}_5 \times \mathrm{X}_5$, where X_5 is a manifold preserving $N = 8$ such as S_5 or RP_5 [11,43].

The UIR's classes of the previous Section enter in this case as follows.

Let us consider a highest weight representation $D(E_0, J_1, J_2; r, p, k, q)$, where r is the central charge eigenvalue and (p, k, q) are the $SU(4)$ Dynkin labels. Then the three unitary series show up as follow [18]:

$$A) \qquad E_0 \geq 2 + 2J_2 + r + \frac{1}{2}(p + 2k + 3q) \geq 2 + 2J_1 - r + \frac{1}{2}(3p + 2k + q)$$
$$(\text{or } J_1 \to J_2,\ r \to -r,\ (p,k,q) \to (q,k,p)) \tag{3.1}$$

which implies

$$\begin{aligned} & E_0 \geq 2 + J_1 + J_2 + p + k + q, \qquad r \geq \frac{1}{2}(p-q) + J_1 - J_2 \\ & 2 + 2J_1 + \frac{1}{2}(3p + 2k + q) - E_0 \leq r \leq E_0 - 2 - 2J_2 - \frac{1}{2}(p + 2k + 3q) \end{aligned} \tag{3.2}$$

Shortening occurs when,

$$E_0 = 2 + 2J_2 + r + \frac{1}{2}(p + 2k + 3q) \tag{3.3}$$

and maximal shortening when

$$E_0 = 2 + J_1 + J_2 + p + k + q, \qquad r = \frac{1}{2}(p-q) + J_1 - J_2 \tag{3.4}$$

Generic massless bulk multiplets correspond to $p = k = q = 0$. They correspond to table 12 in [16].

When we have a strict inequality in (3.1), we obtain generic massive multiplets with highest spin $(J_1 + 2, J_2 + 2)$ in the (p, k, q) representation of $SU(4)$. Note that all these long multiplets (with $J_{MAX} \geq 4$) must necessarily be associated to stringy states, since in supergravity all the states have $J \leq 2$.

$$B) \qquad E_0 = r + \frac{1}{2}(p+2k + 3q) \geq 2 + 2J - r + \frac{1}{2}(3p + 2k + q)$$
$$(\text{for } J_2 = 0, J_1 = J \text{ or } J_1 \to J_2,\ r \to -r,\ (p,k,q) \to (q,k,p)) \tag{3.5}$$

which implies

$$E_0 \geq 1 + J + p + k + q \qquad r \geq 1 + J + \frac{1}{2}(p-q) \tag{3.6}$$

The maximal shortening occurs when

$$E_0 = 1 + J + p + k + q \tag{3.7}$$

and the chiral supersingletons occur for $p = k = q = 0$. They correspond to table 6 of [16]. When a strict inequality occurs in (3.6) and $p = k = q = 0$, we have UIR's described by

$N=4$ chiral (left-handed) superfields with highest spin $(J+2,0)$ in $SU(4)$ singlets with $\Delta = r-4$.

$$C) \qquad E_0 = p+k+q, \qquad r = \frac{1}{2}(p-q), \qquad J_1 = J_2 = 0 \quad (3.8)$$

which gives supersingleton representations for $p\,(\text{or } q) = 1, k = 0$ or $p = q = 0, k = 1$. The self-conjugate (Yang-Mills) supersingleton with spin 1 corresponds to the $D(1,0,0;0,0,1,0)$ highest weight and the spin 3/2 supersingleton to $D(1,0,0;1/2,1,0,0)$. They correspond to table 1 and 2 of [17].

The UIR's of the $PSU(2,2|4)$ superalgebra are obtained by setting $r=0$ in the previous shortening conditions. Correspondingly, we also abolish the entry corresponding to r in the symbol D denoting highest weight states. We get the three classes of UIR's,

$$A') \qquad E_0 \geq 2 + J_1 + J_2 + p + k + q, \qquad J_2 - J_1 \geq \frac{1}{2}(p-q) \qquad (3.9)$$

Maximal shortening occurs when,

$$E_0 = 2 + J_1 + J_2 + p + k + q, \qquad J_2 - J_1 = \frac{1}{2}(p-q) \qquad (3.10)$$

Massless bulk multiplets correspond to $p=k=q=0$ and $J_1 = J_2$.

$$B') \qquad \begin{gathered} E_0 = \frac{1}{2}(p+2k+3q) \geq 2 + 2J + \frac{1}{2}(3p+2k+q) \\ (J_2 = 0, J_1 = J \text{ or } J_1 \to J_2,\ (p,k,q) \to (q,k,p)) \end{gathered} \qquad (3.11)$$

with

$$E_0 \geq 1 + J + p + k + q \qquad 1 + J \leq \frac{1}{2}(q-p) \qquad (3.12)$$

Maximal shortening occurs when $1+J = \frac{1}{2}(q-p)$, with highest weight $D(3+3J+2p+k, J, 0; p, k, p+2+2J)$. No supersingletons appear in this series.

$$C') \qquad E_0 = 2p+k, \qquad p = q, \qquad J_1 = J_2 = 0 \quad (3.13)$$

The highest weight states are $D(2p+k,0,0;p,k,p)$. The $p=0, k\geq 2$ UIR's correspond to the KK states of type IIB on $\mathrm{AdS}_5 \times S_5$, the $k=2$ case being associated with the bulk graviton multiplet. The $p=0, k=1$ UIR corresponds to the only supersingleton of the $PSU(2,2|4)$ algebra [19,16]. The infinite sequence of UIR's with $p=0$, multiplets with $J_{MAX}=2$, have been obtained in [26] with the oscillator construction. They correspond to the harmonic [44] holomorphic superfields of [45]. The case $p \neq 0$ may be relevant for multiparticles supergravity states, as will be discussed in the next Section.

To summarize the structure of UIR's of $SU(2,2|N)$ algebras: generic massive supermultiplets, occurring when a strict inequality in (2.4)),(2.14)),(3.1)) holds, have multiplicity 2^{4N}, generic multiplets when the inequality in (2.6)),(2.16)),(3.5)) occurs have multiplicity 2^{2N} (the same is true for the series c), γ), C)). At the threshold of the unitarity bounds, these multiplicities shrink. For example, massless bulk multiplets have multiplicity 2^{2N}, while massless boundary multiplets (supersingletons) 2^N.

4. Applications to the AdS/CFT correspondence

The previous analysis of shortening conditions finds applications in the AdS/CFT correspondence, where many shortened multiplets are realized.

For $N = 1$ theories, the prototype is type IIB on $\mathrm{AdS}_5 \times T^{1,1}$, where all type of shortening a), b) and c) have been shown to occur for $J_1, J_2 \leq 1/2$ [31,22,32].

For $N = 2$ theories also all types of shortening occur. Shortening β) includes, for instance, $N = 2$ tensor multiplet KK recurrences, while shortening γ) corresponds to $N = 2$ vector multiplet KK recurrences [46,30].

The $N = 4$ case includes, for the $PSU(2,2|4)$ algebra, all the KK states of type IIB on $\mathrm{AdS}_5 \times S^5$ [11,26,8,25,24]. It also contain other states corresponding to multiparticle supergravity states, which can in principle be analyzed [27]; they correspond to multitrace conformal operators in the Yang-Mills theory [47,48,28,29].

Shortening in A$'$) typically occurs in free-field theory but not in the interacting $N = 4$ theory. For instance, the $D(E_0, 0, 0; 0, 0, 0)$ highest weight corresponds to the Konishi multiplet, which undergoes a shortening in a free theory ($E_0 = 2$) while is long in the interacting Yang-Mills theory since $E_0 > 2$ [49,41,50,9]. In the AdS/CFT correspondence, it is indeed the simplest example of operator corresponding to a stringy state. Another example of operator which is short only in free theory is the highest weight $D(3, 0, 0; 0, 0, 2)$ in B$'$), which contains the superpotential of the $N = 4$ theory when it is written in $N = 1$ notations. It is not a primary conformal superfield in the interacting Yang-Mills theory since it can be obtained by total antisymmetrization of the product of three self-conjugate supersingletons [9,51].

The sequence C$'$) has an interesting application to multitrace operators, since it predicts that primary operators with highest-weight $D(2p+k, 0, 0; p, k, p)$ are not renormalized.

In a single trace operator, representations with $p \neq 0$ are not primary since they involve at least a partial antisymmetrization of the Yang-Mills supersingletons, which makes it a descendent via equations of motion [8,25,24]. The same argument does not apply to multitrace operators. Consider the simplest case. Out of two single-trace primary operators in the 20_R $(p = 0, k = 2)$ we can construct by multiplication several multitrace operators. They are in the symmetric product $(20_R \times 20_R)_S = 105 + 84 + 20_R + 1$. The previous discussion implies that the 105 and 84 ($(0, 4, 0)$ and $(2, 0, 2)$) can fit into a shortening condition and are not renormalized, so that anomalous dimensions can only show up for the 20_R and singlet pieces. This result has been recently confirmed in [27,28,29] for the $(0, 4, 0)$ piece and in [28] for the 84 piece, by an explicit perturbative computation.

It is interesting to speculate at this point whether UIR's of the $PU(2, 2|4)$ algebra with $r \neq 0$ can occur. It is obvious that, since the Yang-Mills theory is built in terms of the self-conjugate multiplet with $r = 0$, any local operator constructed out of such multiplet will have $r = 0$ and will be in some representation of $PSU(2, 2|4)$. However, as speculated in [17], if some AdS_5 states are dual to dyonic non-perturbative states of the Yang-Mills theory, then necessarily a new sector with $r \neq 0$ will appear at the non-perturbative level. Natural candidates are the 1/4 BPS dyonic states of the Yang-Mills theory [52], which would correspond to the second exceptional supersingleton representation in C) (with $J_{MAX} = 3/2$). To strengthen this interpretation, it would be interesting to understand how the central $U(1)$ acts on spacetime type IIB fields. We could conjecture that non-perturbative states with $r \neq 0$ are obtained by considering type IIB (p, q) five-branes and strings. The mentioned 1/4 BPS states of the Yang-Mills theory certainly fall into this category, since they are obtained as junctions of (p, q) strings [52]. The central charge of the $U(2, 2|4)$ algebra is an $SU(4)$ singlet and may temptatively be interpreted as coming from a wrapped five-brane, in some generalization of the construction in [43]. The question whether such states are actually BPS saturated in AdS_5 is somewhat unclear.

Acknowledgements

S. F. is supported in part by the DOE under grant DE-FG03-91ER40662, Task C, the NSF grant PHY94-07194, and by the ECC Science Program SCI*-CI92-0789 (INFN-Frascati). We would like to thank M. Bianchi and D.Z. Freedman for helpful discussions.

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