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Logarithmic Unification From Symmetries Enhanced in the Sub-Millimeter Infrared *

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Abstract

In theories with TeV string scale and sub-millimeter extra dimensions the attractive picture of logarithmic gauge coupling unification at 10^{16} GeV is seemingly destroyed. In this paper we argue to the contrary that logarithmic unification *can* occur in such theories. The rationale for unification is no longer that a gauge symmetry is restored at short distances, but rather that a geometric symmetry is restored at large distances in the bulk away from our 3-brane. The apparent ‘running’ of the gauge couplings to energies far above the string scale actually arises from the logarithmic variation of classical fields in (sets of) two large transverse dimensions. We present a number of $N = 2$ and $N = 1$ supersymmetric D-brane constructions illustrating this picture for unification.

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1 Replacing the Desert with the Bulk

It has recently been realized that the fundamental scales of gravitational and string physics can be far beneath the conventional energies of $10^{17} - 10^{18}$ GeV. This can be accomplished if n spatial dimensions of size $\sim R$ are much larger than the fundamental Planck/string scales $\sim M_s$. At distances much larger than R , the weakness of gravity can be understood from Gauss' law, which relates the effective $4d$ Planck scale to the fundamental scale via

$$M_{pl}^2 \sim R^n M_s^{2+n} \quad (1)$$

The original motivation was to bring the fundamental scales close to the weak scale, in order to solve the hierarchy problem [1, 2, 3, 4]. Putting $M_s \sim \text{TeV}$, the radius R ranges from $\sim \text{mm}$ for $n = 2$ to $\sim (10\text{MeV})^{-1}$ for $n = 6$. These large dimensions are not in conflict with experiment if the SM interactions are confined to a 3-brane in the extra dimensions.

Perhaps the most surprising aspect of this framework is that, despite its profound modifications of physics at both sub-millimeter and TeV scales, it is experimentally viable, non-trivially surviving laboratory, astrophysical and cosmological constraints [3]. The last year has also seen the growing realization that the large space in the extra dimensions replaces the old ultraviolet desert as the new arena in which to address other outstanding mysteries of the Standard Model, such as the origin of flavor and absence of FCNC's [5], small neutrino masses [6], and proton stability [3, 5, 8]. The dynamics of the bulk also plays an important role in the early universe cosmology of this framework[9].

One important and general lesson has been that there are new, intrinsically higher-dimensional mechanisms for generating small parameters in the four dimensional theory, the smallness of which are not guaranteed by symmetries of the low energy theory. Instead, higher dimensional locality can guarantee that interactions between fields separated in the extra dimensions are suppressed. In this way, if symmetries are broken at $O(1)$ on distant branes, with the breaking transmitted to our brane by massive bulk fields, the breaking can be exponentially small on our wall [5]. In another context, this effect helps alleviate the SUSY flavor problem in models of wall to wall SUSY breaking [7]. As another example, proton decay can be suppressed if quarks and leptons are slightly 'split' in the extra dimensions [8].

In order to go further in making this framework as compelling as the standard picture with the string scale $\sim 10^{18}$ GeV and low-energy supersymmetry (SUSY) breaking, two important theoretical issues must be addressed:

- Why are the radii of the extra dimensions so large compared to the string scale?
- What about gauge coupling unification?

In this paper, we will focus on the issue of gauge coupling unification [10], and argue that the picture of logarithmic gauge coupling unification[†] may still emerge in string theory with a low string scale and large extra dimensions. As we will see, this provides another illustration of the way in which the bulk can mimic physics associated with the old Desert. Most of the formal results we discuss below are well known in the literature on D-brane constructions of gauge theories [12, 13, 14, 15, 16, 17, 18, 19], although we will be interested in going beyond the ‘decoupling’ limit. What we wish to point out is the possible application of these results to phenomenology.

There have recently been a number of papers addressing the possibility of ‘running’ far above the string scale [20, 21, 22]. Running the couplings above the string scale is however not enough, we need a reason for the couplings to *unify* far above the string scale. This is challenging because in string theory, the couplings usually unify at the string scale. We will argue that the new rationale for unification far above the string scale can be an enhanced geometrical symmetry at large distances away from our 3-brane in the bulk. We also further clarify the conditions under which the couplings can ‘run’ above the string scale in D-brane configurations with $N = 2$ and $N = 1$ supersymmetry.

2 Logarithmic ‘Running’ from the Infrared

There are two remarkable features of the usual picture of gauge coupling unification in supersymmetric extensions of the Standard Model: first that the couplings unify at all, and second that they unify so close to the String/Planck scale. Of course, given the precision with which the couplings have been measured, the near miss of the unification and string scales is usually considered a problem, but at zeroth order it is remarkable that the naive scales of gauge and gravitational unification are so close to each other. For the moment, we will ignore the difference between the GUT and Planck scales, and will return to this point later.

Of course, we have not actually *measured* the gauge couplings at energies approaching the GUT scale; all we know is that the measured strength of the gauge couplings α_i^{-1} at low energy satisfy, to high accuracy, the relation

$$\alpha_i^{-1}(\text{TeV}) = \alpha_0^{-1} - \frac{b_i}{2\pi} \log \left(\frac{M_{pl}}{\text{TeV}} \right). \quad (2)$$

Can this relationship possibly be reproduced in a theory with a low string scale $M_s \simeq \text{TeV}$ and large extra dimensions? In particular, we wish to reproduce the picture of *logarithmic* gauge coupling unification, as opposed

[†]Approaches to gauge coupling unification with power-law running have been discussed in [11].

to power-law unification [10]. Even if power-law running explains why the couplings unify, it can not explain why the unification appears to happen so close to the Planck scale. Put another way, suppose that instead of unifying at $\sim 10^{16}$ GeV, the couplings were found to unify at 10^8 GeV; power-law running could still accommodate this. The link between the unification scale and the gravitational scale in Eqn.(2) would then be wholly accidental. We do not wish to view this link as an accident, and will therefore try to reproduce Eqn.(2) in the context of theories with large extra dimensions.

Of course naively, once we hit the (low) string scale M_s , we can't continue to 'run' past the TeV scale using the RGE's of the 4-dimensional effective field theory, and there is no source for a UV logarithm of magnitude $\log(M_{pl}/M_s)$. In particular, scattering experiments performed at $\sqrt{s} \gg M_s$ no longer see just a gauge theory on our brane decoupled from the full set of string modes and from the bulk degrees of freedom, so there is no sense in speaking of usual QFT 'running' at these scales. However, the usual M_{pl} does exist as a physical scale; it is set by the size of the bulk. In units where $M_s = 1$, $M_{pl} \sim \sqrt{V_{bulk}}$. So, if the gauge coupling on our brane can have logarithmic sensitivity to the volume of the bulk, there is a hope that the correct magnitude logarithm is available. Notice that the source of the logarithm in this picture is an *infrared* effect.

Under what conditions can quantities on the brane depend on the size of the bulk [21]? Clearly, the brane must couple to light fields that can propagate in the bulk in order to have any sensitivity to the volume of the extra dimensions. Gravity is one model-independent field that must propagate in all n large extra dimensions. There may be other light fields as well, which can propagate in some of the large dimensions. Let us collectively refer to the light bulk fields as ϕ . Since the branes act as coherent sources for ϕ , they set up a ϕ profile in the bulk which is the same as that of a point source for ϕ in the t directions transverse to the source branes that ϕ propagates in. The precise nature of the dependence of brane quantities on the volume of the bulk then depends on the ϕ propagators in t dimensions: for $t > 3$, the Green's functions fall off as r^{2-t} so there is only power suppressed sensitivity to V_{bulk} . For $t = 2$ the propagator is a logarithm and this is precisely what we are looking for. Finally, for $t = 1$ there can be power IR divergences (which can have interesting physical consequences see e.g. [23]), but which are not of interest to us here.

We are thus lead to consider theories with light fields which can effectively propagate in two transverse dimensions. For simplicity, in the rest of this paper we will assume that there are only two large dimensions, although we emphasize that this is not a necessity. Our picture is summarized in Fig. 1.

The gauge coupling on our brane (evaluated at the cutoff $\sim M_s$) is the vacuum expectation value of some bulk field smeared out over a region of size $\sim l_s = M_s^{-1}$ around our brane (as a consequence of the non-trivial form-factor

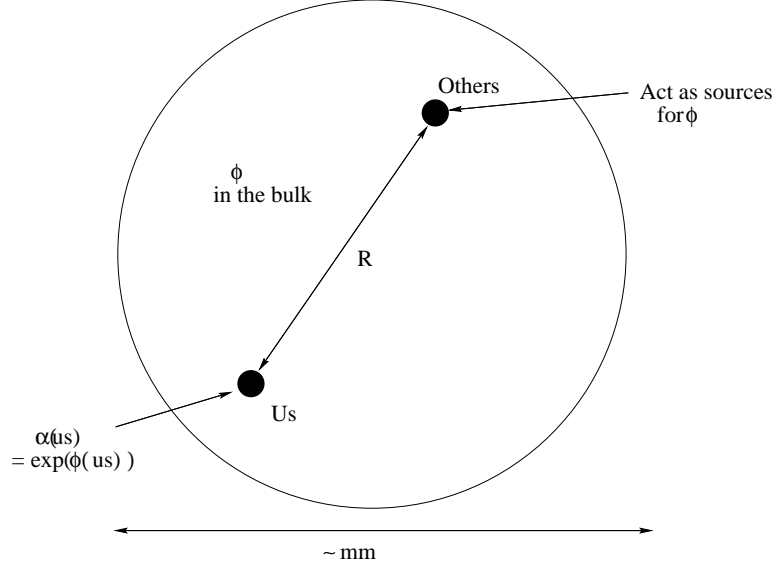


Figure 1: *The picture for ‘running’ from the infrared. The gauge coupling on our brane is determined by the value of a bulk field ϕ evaluated at the position of our brane. Other branes in the bulk a distance $R \gg M_s^{-1}$ away act as sources for ϕ , and if the transverse co-dimension relative to the source brane is 2, then the value of ϕ on our brane can be logarithmically sensitive to R . In principle, this logarithmic profile of ϕ can mimic field theoretic ‘running’ to an energy scale RM_s^2 far above the string scale.*

of the brane):

$$\alpha^{-1}(M_s) = e^{-\phi(us)}. \quad (3)$$

However, the presence of other branes far away in the bulk can modify the value of ϕ evaluated on our brane as compared to the value ϕ_0 asymptotically far away;

$$e^{-\phi(us)} = e^{-\phi_0} - \frac{c}{2\pi} \log(RM_s). \quad (4)$$

For two extra dimensions, $R^2 M_s \sim M_{pl}$, so

$$\log(RM_s) = \log\left(\frac{M_{pl}}{M_s}\right), \quad (5)$$

and therefore the correct magnitude logarithm between the weak and gravitational scales is indeed present in the theory. In fact, for two large extra dimensions, the phenomenological constraints are tight enough to force M_s at least up to ~ 50 TeV. Then, the logarithm $\log(M_{pl}/M_s)$ is closer to the desired $\log(M_{GUT}/\text{TeV})$, although accurate predictions must of course also then take into account the usual QFT running between M_Z and M_s . Another natural possibility is that the distant branes are not maximally removed from us in the extra dimensions, so that R is somewhat smaller.

Note that we are not restricted to a total of $n = 2$ large dimensions where gravity propagates. For instance, suppose we have $n = 4$ large dimensions, and we are located at the intersection of two orthogonal 5-branes. Fields

that live on the 5-brane propagate in effectively 2 large dimensions and can vary logarithmically. If the profiles on the two 5-branes are identical we can expect to obtain a logarithm $2 \times \log(RM_s) \sim \log(M_{Pl}/M_s)$, where we have used $M_{Pl}^2 \sim R^4 M_s^6$. Clearly this idea generalizes to $n = 6$ extra dimensions as well.

The fact of logarithmic variation is not enough however. In order to reproduce the correct ‘running’ of the gauge coupling, the coefficient c in Eqn.(5) must equal the β function coefficient of the gauge theory localized on our brane. At first sight, such an equality seems unlikely. After all, c is determined by the way in which the *distant* branes couple to the light bulk fields; why should this be related to the β function of the gauge group localized on *our* wall? This seems to require a miracle.

3 IR ‘running’ from $N = 2$ D -brane constructions

Precisely such a ‘miracle’ is, however, well known to occur in stringy brane constructions of gauge theories with 8 supercharges. A simple example is provided by the gauge theories living on D3 branes probing the geometry of parallel D7/O7 configurations [16]. (See Fig. 2) Let the D7/O7 planes fill out the 1,...,7 directions and the D3 brane fill out 1,...,3, and define $w = x^8 + ix^9$ labeling the space transverse to the 7 branes. Specifically, put an O7⁻ plane at $w = 0$, and place 4 D7 branes at w_i ($i = 1, \dots, 4$). With 4 D7’s, the total RR charge and tension of the D7/O7 branes cancel, so that for $|w| \rightarrow \infty$, all the bulk fields approach constant asymptotic values; in particular the complex coupling $\tau = a + ie^{-\phi}$ of the type IIB string approaches $ie^{-\phi(\infty)}$ at infinity. Now, put a single D3 brane at w . This configuration leaves 8 supercharges invariant, and the resulting gauge theory living on the D3 brane is $N = 2$ in four dimensions. When all the branes sit on top of each other, the resulting gauge theory living on the $D3$ brane is an $Sp(1) = SU(2)$, $N = 2$ gauge theory, with the vector multiplet coming from the 3-3 strings and 4 massless hypermultiplets from the 3-7 strings. This theory is conformal; the one loop beta function coefficient is $b = 0$. This statement has a counterpart from the long-distance gravity point of view: with $w_i = 0$ all the RR charges and tensions of the branes cancel against the orientifold, so that there is no variation of the bulk fields in transverse space. Now suppose we move f of the D7 branes very far away, i.e. $|w_i| = R \gg l_s$. The resulting gauge theory now has $(4 - f)$ hypermultiplets and beta function $b = f$. Now the tensions and RR charges no longer cancel locally, and there will be some variation of the light bulk fields, although of course asymptotically for $|w| \gg R$ the fields approach their fixed values at infinity. It is straightforward to calculate the profile for τ set up by this configuration of 7 branes. We are in fact only interested in

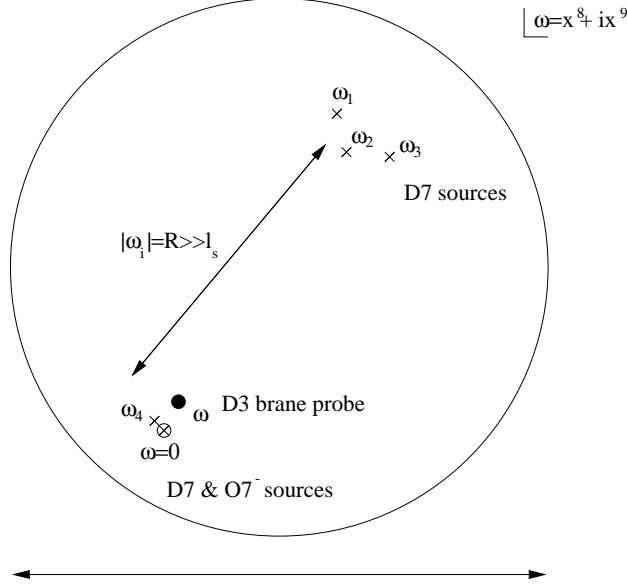


Figure 2: An $N = 2$ SUSY example where the bulk SUGRA equations reproduce the $(3+1)$ -d QFT holomorphic gauge coupling running on a probe D3 brane located at w . Source D7 branes are located at positions w_i in the $w = x^8 + ix^9$ plane, and an $O7^-$ orientifold plane is located at $w = 0$. At long distances $\gg R$ the total RR charge and tension of these D7/O7 branes cancel and there is no variation in the bulk fields. In this figure we have moved $f = 3$ of the D7 branes far away from the D3 and O7 brane, and so from the D3-brane gauge theory perspective 3 $N = 2$ hypermultiplets gain a large mass $m = RM_s^2$.

the long-distance behavior away from the branes (i.e. only the logarithms), so the full F theory computation [24] is not needed for our purposes. The result is find[‡]

$$\tau(w) = \tau(\infty) + \frac{i}{2\pi} \left(\sum_{i=1}^4 \log(w - w_i) - 4 \log(w) \right). \quad (6)$$

This in fact directly follows from the fact that the seven branes act as vortex solutions for the axion $a = \Re(\tau)$. This result precisely matches what we expect for the field theory running of the gauge theory with hypermultiplets of mass $w_i M_s^2$. The amazing thing of course is that this result of the long-distance gravity theory (which is valid for $|w_i| \gg l_s$) reproduces the naive extrapolation of the field theory running, despite the fact that the field theory description is valid only for $|w_i| \ll l_s$.

Why does this happen? How can a classical supergravity calculation reproduce the (quantum) running in a gauge theory? That classical gravity

[‡]From the full F -theory result the non-perturbative correction to this result can easily be extracted. They are negligibly small for a theory that has fundamental coupling of order the usual GUT coupling.

effects are equivalent to quantum gauge theory effects is a consequence of closed/open string duality in string theory: The world-sheet that represents a tree-level closed string exchange between the well separated branes can also be interpreted, via a re-labeling of ξ_1 and ξ_2 on the string worldsheet, as a 1-loop open string diagram of states on the branes that correspond to strings stretched between the branes. This is illustrated in Fig. 3. However, this alone is not enough to explain our miracle. At long distances, the tree-level closed string diagram is indeed well approximated by supergravity, but closed/open duality only tells us that this is equal to the *full* one-loop open string calculation, which sums both the lowest string excitations (which are the field-theory degrees of freedom) as well as all the massive open string excitations whose mass depends on M_s . In order for the field theory and gravity calculations to agree, it must be that sum over massive string excitations gives no corrections to τ . In the case of theories where the lightest stretched string states preserve 8 supercharges, this is in fact guaranteed by BPS considerations [16, 22]. The lightest states are BPS states with mass and (NS-NS) charge both linearly increasing with the string length ($M = Q$). The supersymmetry multiplets that these BPS states fill-out are $N = 2$ representations, while the excited strings with oscillator contributions to their mass are not BPS ($M > Q$) and group themselves into $N = 4$ multiplets, and therefore do not correct τ .

Notice that it is crucial here that the field theory is superconformal at the origin of moduli space $|w_i| = 0$, and that moving the branes away to non-zero $|w_i|$ is a soft breaking of the superconformal symmetry: The heavy hypermultiplets of mass $|w_i|M_s^2$ act as regulators for the low energy field theory, and the field theory is finite by itself. Ordinarily, low-energy field theory computations have logarithmic divergences which are cutoff by the softness of string theory at the *string* scale. If we are able to *ignore* string oscillators, it had better be that the field theory yields finite answers by itself.

The expression Eqn.(6) is valid when the two transverse dimensions, $x^{8,9}$ to the D7/O7 planes are non-compact. For realistic models we are interested in *compactifying* the two transverse dimensions with a large size R in order to reproduce the usual $1/r^2$ law for gravity at long distances, so let us consider what happens with this model when the two transverse dimensions to the 7 branes are compactified on a large torus with equal radii R . First note there is no obstruction to compactification since the total RR charge and tensions of the 7 branes cancel. For simplicity, we put the D3 brane at the origin, the O7 at $w = w_O$ and the D7's at $w = w_{D_i}$. We can enforce periodic boundary conditions by including (same sign) image charges for the O7 (D7_{*i*})'s located at $(m + in)R + w_{O(D_i)}$ with m, n integers. It is then important to check that the effect of these image charges is a small perturbation on top of what we have seen in the non-compact limit. This must be the case as long as the O7/D7's are sufficiently far from the “edges” of the torus, i.e. as long as

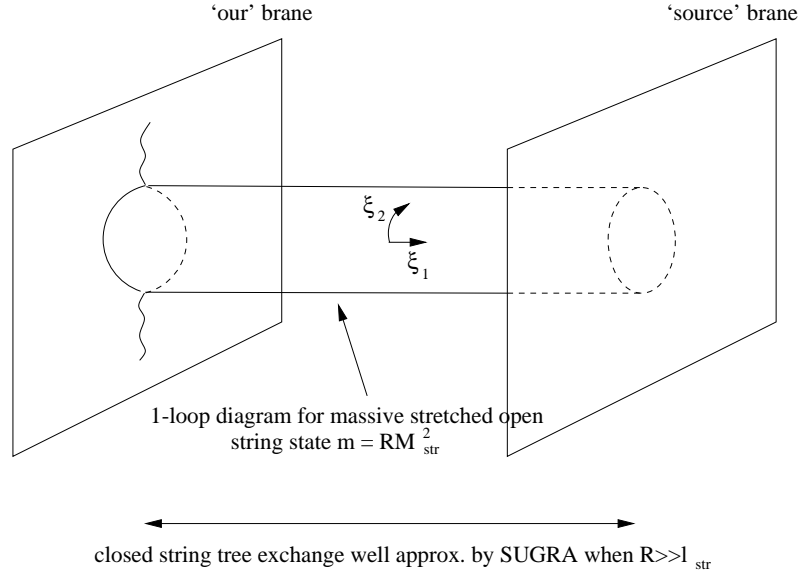


Figure 3: *The general picture describing the duality between 1-loop open string and tree-level closed string diagrams. The worldsheet coordinates for the string are ξ_1 and ξ_2 . If ξ_1 is taken to be the worldsheet ‘time’ coordinate then this diagram represents the exchange of a closed string state between our brane and a ‘source’ brane in the bulk. If the separation R of the branes is large $Rl_s \gg 1$, then this amplitude is well approximated by the (classical) bulk supergravity solution in the presence of the brane source. On the other hand, if ξ_2 is taken to be the worldsheet ‘time’ coordinate then this diagram represents the 1-loop contribution of a massive stretched open-string to the 2-point function of our brane-localized theory. In special cases (described in the text) only the zero-mode stretched string state contributes, the higher string oscillator states canceling, and the usual QFT beta-function is reproduced.*

$|w_O|/R, |w_{Di}|/R \ll 1$. Indeed, the exact expression for τ evaluated at the position of the $D3$ brane is

$$\tau(0) = \tau(\infty) + \frac{i}{2\pi} \sum_{m,n=-\infty}^{\infty} \sum_{i=1}^4 \log \left(\frac{m + in + \frac{w_{Di}}{R}}{m + in + \frac{w_O}{R}} \right). \quad (7)$$

The term with $m = n = 0$ is just the piece we already have in the non-compact limit, and it is trivial to Taylor expand the remaining contribution to find the shift

$$\delta\tau(0) = c \sum_i \left[\left(\frac{|w_{Di}|}{R} \right)^4 - \left(\frac{w_O}{R} \right)^4 \right] \quad (8)$$

where c is an $O(1)$ numerical factor. We expect this correction to hold in *any* realization of our logarithmic “running” scenario, since it takes into account the correct Green function on the compact space. Already, for $(|w|/R) \sim 1/3$ this correction is beneath the percent level and will be irrelevant in any potentially realistic model given the accuracy to which the SM couplings have been measured.

It is also instructive to consider what happens when only one of the transverse dimensions is compactified on a circle of radius r . At distances $|w| \gg r$, there is effectively only one transverse dimension, and τ varies linearly with $|w|$ rather than logarithmically. This is simply reflected in the field theory: The 3-7 strings can wind n times around the compact dimension, giving a tower of (BPS) winding states with masses n/r which are fundamental hypermultiplets under the $SU(2)$ gauge group. Including the effect of this entire tower of states changes logarithmic to power-law running in the field theory. Of course in field theory we must include in the running not only these states, but *all* states lighter than the ‘UV cutoff’ $|w|M_s^2$. However, as before the excited string oscillator states are non-BPS and form themselves in effective $N = 4$ SUSY multiplets. In the cases where the string excitations make no contribution to coupling renormalization, we can do a field theory calculation, keeping all the field theory states lighter than the field theory UV cutoff, despite the fact that this cutoff is far above the string scale.

This example illustrates that it is indeed possible that field theory “running” into the UV is exactly reproduced by logarithmic variation of bulk fields in the IR. However, there are two important issues to be addressed in moving towards more realistic theories:

- How crucial is $N = 2$ SUSY? Are there $N = 1$ models with the same property?
- It is not enough to reproduce the correct ‘running’ for the couplings; what is the rationale for the gauge couplings to appear to *unify* at M_{pl} ?

This latter question is quite serious and we will address it first. It is easy to see that the usual argument for unification in string theory must be lost in the picture we are discussing. The standard reason for unification is that there is a single field, the dilaton, whose vacuum expectation value sets the gauge couplings for all gauge group factors, enforcing equal gauge couplings at the string scale. In the present case with a low string scale, if the gauge couplings of the SM gauge groups living on our wall are set by a single bulk field ϕ , then no matter what other sources ϕ has, the gauge couplings are still guaranteed to unify at the string scale, which is now \sim TeV! Moreover, we can't have 'our' branes, on which the SM gauge group is realized, significantly separated from each other in the bulk since this would lead to e.g., very massive $(3, 2)$ states under $SU(3) \times SU(2)$. Thus we cannot realize different values of the low-energy SM gauge couplings from evaluating a single bulk field at different points in the bulk. An obvious way around this is if the different gauge couplings α_i are given by the vacuum expectation values of different bulk fields ϕ_i . But then why should we have unification? Clearly a new rationale is needed.

4 Unification from Symmetries in the Bulk

We now give an example of $N = 2$ theories where the gauge couplings unify and the rationale for unification is a geometric symmetry of the brane configuration in the far IR. These are based on the Hanany-Witten [13] construction of $N = 2$ theories which we briefly review here. In the simplest set up there are two NS5 branes, filling out 12345 and localized at $x^{7,8,9} = 0$, with one located at $x^6 = 0$ and the other at $x^6 = l$. Suspended between them are N_c D4 branes located at $x^{4,5,7,8,9} = 0$ filling out 123 and spanning $x^6 = 0 \rightarrow l$. In the absence of the NS5 branes, the gauge theory living on the D4 branes is a (4+1)-d gauge theory with 16 supercharges. The boundary conditions of the D4 branes ending on the NS5 branes reduces the SUSY down to 8 supercharges, and at long distances compared to l the theory is a (3+1)-d gauge theory with 8 supercharges, which is an $N = 2$ $U(N_c)$ gauge theory. The (3+1)-d gauge coupling is given by reduction from (4+1) to (3+1)-d

$$\alpha_{(3+1)}^{-1} = l\alpha_{(4+1)}^{-1} = lM_s\alpha_s^{-1}. \quad (9)$$

Moving the positions of the D4's corresponds to moving along the Coulomb branch (adjoint Higgsing) of the $U(N_c)$ gauge theory. Further suspending semi-infinite N_f D4 branes off the NS5 branes, located at $x^4 + ix^5 = v_i, i = 1, \dots, N_f$ adds N_f hypermultiplets of mass $M_s^2|v_i|$, coming from the 4-4 strings stretching between the semi-infinite and finite length D4 branes.

The D4 branes ending on the NS5 branes are under tension and therefore bend the NS5 brane in the x^6 direction. The end-points of the D4 branes are

three dimensional, and so are of co-dimension 2 inside the NS5's. Therefore we expect that this bending will be logarithmic. It is well known [14] that this bending encodes the β function of the $N = 2$ gauge theory. To see this physically, imagine first attaching N_c semi-infinite D4 branes on either side of the NS5 branes $x^{4,5} = 0$ (similar to the picture of Fig. 4). Clearly, the force on each of the NS5 branes cancel from the left and the right, and so there is no bending of the NS5 branes: Let us denote the distance between the NS5 branes as l_0 . Reflecting the non-bending NS5's, we have an $N = 2$ $U(N_c)$ gauge theory with $2N_c$ hypermultiplets, which is conformal. Now consider moving some of the semi-infinite D4's away from the origin; for simplicity move all N_c of the ones on the left and $N_c - N_f$ of the ones on the right a distance R away. The resulting gauge theory near $x^{4,5} = 0$ is $N = 2$ $U(N_c)$ with N_f hypermultiplets, which has one-loop beta function $b_0 = (2N_c - N_f)$. Now that the forces on the NS5's no longer cancel locally, they will bend. However, at distances $\gg R$, the net bending will cancel and they will still asymptote to being parallel and a distance l_0 apart. We can then sensibly ask for the distance l between the NS5 branes, at $x^{4,5} = 0$, as a function of the asymptotic distance l_0 and R ; the answer is

$$l = l_0 - \frac{(2N_c - N_f)\alpha_s}{2\pi} \log(R/l_s). \quad (10)$$

It is easy to understand the sign and dependence on N_c, N_f in the above: the D4's pull on the NS5's, so they are closer to each other at $x^{4,5} = 0$; the D4's between the NS5's pull each of them inward by an amount proportional to N_c while the D4's on the outside pull outward by N_f , so that the net pull is proportional to $(2N_c - N_f)$.

Using the relation Eqn.(9), we find that the value of the gauge coupling of the gauge theory at $x^{4,5} = 0$ varies with R as

$$\alpha_{(3+1)}^{-1} = \alpha_s^{-1} - \frac{(2N_c - N_f)}{2\pi} \log(R/l_s). \quad (11)$$

This is precisely the 'running' expected only keeping field theory states. In QFT language, at energies far above RM_s^2 the theory is conformal and the coupling doesn't run, its value being given by α_s . Beneath the mass of the $(2N_c - N_f)$ hypermultiplets, the theory runs exactly according to Eqn.(11). Once again, the $(2N_c - N_f)$ D4 branes that have been moved away act as 'regulators' of the theory left behind. From the long-distance, gravity point of view they serve as 'IR regulators', canceling the variation of bulk fields so they asymptote to well-defined values in the deep IR. From the gauge theory point of view, the low-energy theory has been embedded inside a softly broken superconformal theory; the massive hypermultiplets coming from the distant D4's regulate the low-energy theory, and the superconformality of the full theory ensure a well-defined value of the gauge coupling in the deep UV.

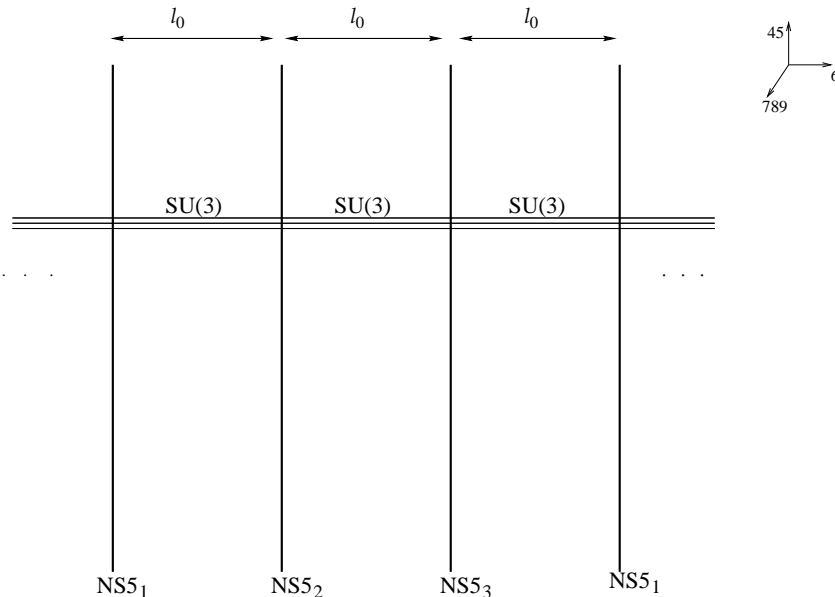


Figure 4: A toy $N = 2$ theory with unification far above the string scale based on the Hanany-Witten set-up. The thick lines are NS5 branes and the thin ones are D4 branes. The 6 direction is compactified on a circle, which we indicate by periodically repeating the configuration. The gauge group is $SU(3)^3$ with hypermultiplets transforming as $(3, \bar{3}, 1) + (1, 3, \bar{3}) + (\bar{3}, 1, 3)$. The NS5's are equally spaced so the three gauge couplings are identical. The forces on the NS5's due to the D4's cancel locally so there is no bending of the NS5's.

We can now present our toy example of unification. We will compactify the x^6 direction on a circle [§], and place 3 NS5's equally spaced on the circle as in Fig. 4. In the supersymmetric limit, there are no forces between the NS5's and we could place them with any relative spacing we please, but for the moment let us place them in the most symmetrical arrangement. Between each of the NS5's suspend 3 D4 branes. The theory is then $N = 2$ $SU(3)^3$ with hypermultiplets in the $(3, \bar{3}, 1) + (1, 3, \bar{3}) + (\bar{3}, 1, 3)$ representation. This particle content is an $N = 2$ version of 'trinification'. In the limit where all the D4's sit at $x^{4,5} = 0$, all the forces on the NS5 cancel and they do not bend. Again, reflecting this, the $N = 2$ theory is conformal. Let us now take one of the D4's between NS5_{2,3} and two between NS5_{3,1} a distance R away as in Fig. 5. The resulting gauge theory at the origin has gauge group $SU(3) \times SU(2) \times U(1)$, with hypermultiplets in the $(3, 2, 0) + (\bar{3}, 1, *) + (1, 2, *)$ representation. This is qualitatively similar to a one generation MSSM, except the hypercharges are wrong and the theory is $N=2$. Nevertheless, we can ask about gauge coupling unification in this toy world. The forces on the NS5's no longer cancel locally so they will bend locally as in Fig. 5. How-

[§]For simplicity, we will not compactify any of the other spatial dimensions. Even though we do not recover 4D gravity at long distances, doing this allows us to most clearly illustrate the picture for gauge coupling unification far above the string scale.

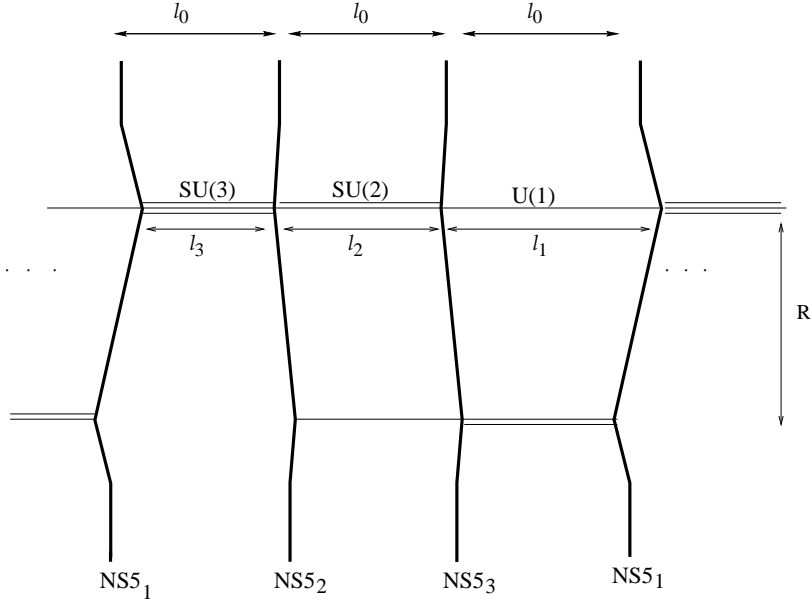


Figure 5: Moving some of the D4's a distance R away leaves an $SU(3) \times SU(2) \times U(1)$ gauge group living on the remaining branes. The forces on the NS5's no longer cancel locally and they bend. We see that $l_3 < l_2 < l_1$, so $g_3 > g_2 > g_1$.

ever, at distances much larger than R , the splitting between the D4's can not be resolved and the NS5's flatten out and continue to be equally spaced. We know that the bending precisely reproduces the field theory running in these models. The only question is what the deep UV value is for the gauge coupling. Because we have arranged for the NS5's to be equally spaced in the deep IR, in the field theory this corresponds to the boundary condition that the *couplings unify at the large mass scale* RM_s^2 . The rationale for unification at a scale far above the string scale is a geometrical Z_3 symmetry of the brane configuration at large distances. While the usual picture for unification invokes enhanced symmetries at *short* distances, here the rationale for unification is an enhanced geometrical symmetry of the brane construction viewed from *large* distances in the bulk (Fig. 6).

Notice that this model realizes the general possibility we mentioned earlier for evading the equality of gauge couplings at the string scale. At distances larger than the radius of the S^1 , we only see that we have the two large dimensions $x^{4,5}$. The three gauge couplings of $SU(3) \times SU(2) \times U(1)$ are then indeed given by the values of three bulk fields corresponding to the inter-NS5 brane distances, and this interpretation gives us the geometric rationale for unification.

But why should the NS5 branes be placed symmetrically around the circle? We can not fully answer this question until we address SUSY breaking; nevertheless, we can at least state some reasonable dynamical assumptions that generate the desired outcome. If SUSY is broken only on some branes, then the bulk will still be highly supersymmetric, and we can expect the

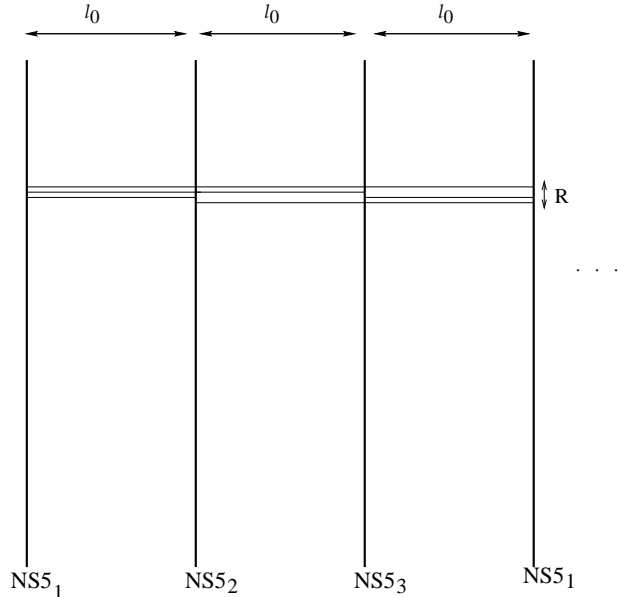


Figure 6: *The configuration of Fig. 5 viewed from afar. The NS5's flatten out and the Z_3 geometrical symmetry of the original brane configuration is regained at large distances in the bulk. This provides a rationale for the apparent unification of the gauge couplings at a scale much higher than the string scale*

brane bending to remain unchanged. On the other hand, there will now be forces between the NS5 branes, and the Z_3 symmetric configuration will at least be an extremum of the energy; we need this extremum to be a local minimum.

It is amusing to think about experimental signatures of this toy model. Since the gauge theory becomes effectively (4+1)-d at energies above l^{-1} , there will be KK excitations at this scale. But then why do we continue to get log (as opposed to power law) running? The reason is that the 16 supercharges of the theory on the D4's was broken to 8 supercharges by the boundary conditions imposed by their ending on the NS5's. In (3+1)-d language, we started with an $N = 4$ particle content which in $N = 2$ language consists of a vector multiplet V and a hypermultiplet Φ both in the adjoint representation. The boundary conditions are free for the V and fixed for Φ at $x^6 = 0, l$. This projects out the zero mode of Φ but leaves that of V , so the massless spectrum is $N = 2$. But the massive modes group themselves into $N = 4$ multiplets, and therefore make no contribution to the β function. There are three different KK towers for each of the $SU(3)$, $SU(2)$ and $U(1)$ factors, corresponding to the three strips between the NS5's. The KK masses then come in the ratio

$$M_3 : M_2 : M_1 = \alpha_3 : \alpha_2 : \alpha_1, \quad (12)$$

so this would be a “smoking-gun” signature of this framework for unification in this world.

5 $N = 1$ models

We now turn to the issue of whether $N = 1$ models can be constructed where the field-theory running is reproduced by supergravity. We do not have a general set of rules for when this is guaranteed to happen, but will present a class of models which work. First of all, a good place to look are brane constructions of $N = 1$ theories which are superconformal at the origin of moduli space. The superconformality guarantees that the field theory calculation of the running can be finite all by itself without requiring string theory to cut it off, so that there is at least a hope that string oscillators can be ignored in the tree-level close string/1-loop open string comparison. Perhaps the simplest possibility is to consider orbifolds of $N = 2$ models. Indeed, orbifolds of the Hanany-Witten construction were considered in [15]. Defining $u = x^4 + ix^5$ and $v = x^8 + ix^9$; the Z_M orbifold considered in [15] is $u \rightarrow \alpha u, v \rightarrow \alpha^{-1}v$ with $\alpha^M = 1$. Suspending NM D4's between the NS5's, the action of the orbifold group Z_M on the Chan-Paton indices of the $U(NM)$ gauge group is

$$\lambda^a \rightarrow \gamma^{-1} \lambda^a \gamma, \gamma = \text{diag} \left(1_N, \alpha 1_N, \dots, \alpha^{M-1} 1_N \right). \quad (13)$$

The resulting theory is an $N = 1$ $SU(N)^M$ theory with chiral content given by $(N, \bar{N}, 1, \dots, 1) + \text{cyclic permutations}$.

By lifting this configuration into M theory, Ref.[15] demonstrates explicitly that the bending of the NS5 branes reproduces the beta function of the $N = 1$ theory. We wish to observe that this can be seen on general grounds and suggests a perhaps wide class of $N = 1$ models with this feature. We find it convenient again to add ‘regulator’ branes to the system to ensure zero asymptotic bending in the IR and finiteness in the UV. In the present case, we add NM semi-infinite D4's on each of the NS5 branes and then orbifold. The resulting $N = 1$ theory is an orbifold of the finite $N = 2$ $SU(NM)$ theory with $2NM$ hypermultiplets, and is easily seen to be a superconformal $N = 1$ theory. Now, we move some of the regulator branes away from $u = 0$, which takes them away from the orbifold fixed point (of course we have to move D4's and their images together, so they move in groups of M). We can now ask what profile for the bulk fields is set up by these regulator branes. The crucial point is that since they are away from the fixed point, they can't act as sources for twisted sector fields, and therefore they set up exactly the same profile as the same configuration before the orbifold! Therefore, the ‘supergravity’ part of the calculation is unmodified. On the other hand, as is now well known [18], all correlation functions of an orbifolded field theory (the ‘daughter’ theory) agree with the theory before orbifolding (the ‘parent’ theory), at the level of planar diagrams, up to a re-scaling by a factor of $|\Gamma|^{-L}$ where L is the number of loops, and $|\Gamma| = M$ is the order of the

discrete orbifold group. This result, together with the fact that the ‘supergravity’ calculation is unaltered by the orbifold, allows the resulting $N = 1$ theory to inherit the equality between ‘supergravity’ bending and field-theory running from the $N = 2$ theory, as we now show. (We also note that in the case where not all branes can be moved away from the fixed point, the gauge theory running is still reproduced, at least in the $N = 2$ case [19], by the bulk supergravity equations.)

First of all, the gauge couplings g_p^2 and g_d^2 of the parent and daughter theories are related as

$$\frac{M}{g_d^2} = \frac{1}{g_p^2}. \quad (14)$$

Note that this implies that the ‘tHooft couplings of the theories are identical. This can most easily be seen by moving the N branes (together with all the M images) away from the origin. Since all branes are away from the fixed point, there is no local way to distinguish the daughter theory from the parent theory with the same brane configuration. From the daughter theory point of view, the $SU(N)^M$ group has been Higgsed to a single $SU(N)$, with gauge coupling $1/g_1^2 = M/g_d^2$. The same gauge configuration in the parent theory Higgses $SU(NM)$ to $SU(N)^M$ with gauge coupling $1/g_2^2 = 1/g_p^2$. But the gauge coupling of each factor of this $SU(N)^M$ must be exactly the same as that of the $SU(N)$ of the daughter theory, where the M factors of the group in the parent theory are interpreted as mirrors of the daughter gauge group. Therefore, we must have $1/g_1^2 = 1/g_2^2$ and Eqn.(14) follows.

Now, we know that the gauge coupling of the parent theory ‘runs’ from the IR according to the beta function b_p

$$\frac{1}{g_p^2} = \frac{1}{g_p^2(0)} - \frac{b_p}{8\pi^2} \log(R/l_s). \quad (15)$$

Since the bending is unchanged by the orbifold, we also know for the daughter theory that

$$\frac{M}{g_d^2} = \frac{M}{g_d^2(0)} - \frac{b_p}{8\pi^2} \log(R/l_s). \quad (16)$$

In order for this to reproduce the field theory ‘running’ for the daughter theory it must be that the beta function coefficient of the daughter theory b_d satisfies $b_p = Mb_d$. But this is an immediate consequence of the orbifold inheritance results quoted above. In particular, the one-loop beta function diagrams are all planar, so the beta function of each each factor of the daughter theory b_d is related to the parent theory b_p as

$$b_d = \frac{b_p}{M} \quad (17)$$

This is trivial to see in the present case: $b_p = 2NM$ and $b_d = 2N$.

This suggests that the correct ‘running’ can be obtained for a wide class of $N = 1$ theories. One starts with $N = 2$ models where supergravity is guaranteed to reproduce the field theory “running”, and orbifolds the theory down to $N = 1$. If the regulator branes are moved away from the orbifold fixed point, the $N = 1$ theory can inherit the equality between supergravity and field theory running from the $N = 2$ theory.

Another possibility is that the supergravity does *not* reproduce the $N = 1$ running, but that the mismatch comes in ‘complete $SU(5)$ multiplets’, that is, the mismatch is identical for $\alpha_{1,2,3}^{-1}$. This would preserve gauge coupling unification. It is easy to construct a toy example that works in this way. Going back to the Hanany-Witten set up, we can add N_f parallel D6 branes between the NS5 branes. When the D6’s fill out 123789, $N = 2$ SUSY is still preserved and the 4-6 strings give N_f hypermultiplets. The bending of the NS5 branes still reproduces the β -function of the field theory, the contribution of the N_f hypermultiplets in the supergravity description being understood as follows: The D6 branes are the largest objects in the system and set up a gravity and dilaton profile in the 456 space transverse to them, where the D6’s sit at the origin. In this transverse space, the NS5’s fill out a plane 45, and extremizing the NS5 brane action in the gravity/dilaton profile of the D6 brane causes logarithmic bending in the 6 direction, with the correct coefficient to equal the $-N_f$ contribution of N_f hypermultiplets.

Now consider rotating the D6’s so they are parallel to the NS5 branes, i.e. they fill out 123457. It is simple to check the conditions on the supercharges imposed by this brane configuration, and discover that only $N = 1$ SUSY is left unbroken. The particle content is still the same as in the $N = 2$ theory, but the superpotential interaction between the adjoint in the vector multiplet and the hypermultiplets $\bar{H}\Phi H$ is switched off, so the interactions only respect $N = 1$. In this configuration, however, the D6 branes *do not* bend the NS5’s in the 6 direction; they are parallel to them! The N_c D4’s ending on the NS5’s still bend them inward by $2N_c$, so the bending is proportional to $2N_c$ while the β function is still $(2N_c - N_f)$. Therefore the supergravity does not reproduce the field theory running in this case. However consider inserting N_f of these D6’s between each pair of NS5’s in our trinification model from the previous section.[¶] Now each of the $SU(3)$, $SU(2)$ and $U(1)$ factors have N_f extra hypermultiplets added, whose contribution is not reflected in the NS5 bending. But the extra contribution to the field theory beta function is N_f for all gauge group factors, so unification is preserved.

[¶]Note that in the $N = 2$ configuration the D6’s can not be placed between the NS5’s on a compact S^1 of a fixed radius, exactly because of the bending they induce. In our $N = 1$ set-up, however, they cause no bending and can be included.

6 Physical couplings and threshold corrections

So far we have focused on the analogue of 1-loop “running” of the gauge coupling coming from the IR. What about higher-order corrections?

What we have actually been computing is the logarithmically enhanced contribution to the holomorphic coupling of the low energy SUSY gauge theory. Of course, the precise way in which the brane “thickness” regulates the value of the gauge coupling on our brane gives a threshold correction in matching to the low-energy theory. However, all of these corrections are due to local physics close to the brane, where the string coupling is weak, and are not logarithmically enhanced by the size of the bulk. We do not expect them to be more important than e.g. the (unknown but small) MSSM threshold corrections. Therefore, our 1-loop expression is an excellent approximation to the holomorphic gauge couplings of the low-energy theory. Of course, these are not the physical gauge couplings; the physical couplings are determined by re-scaling all the fields of the low-energy theory to go back to canonical normalization [26], and are given by the Shifman-Vainshtein relation

$$\frac{1}{g_{ph}^2} + \frac{2t_2(A)}{8\pi^2} \log(g_{ph}) = \frac{1}{g_h^2} - \sum_i \frac{2t_2(i)}{8\pi^2} \log(Z_i), \quad (18)$$

where Z_i is the wavefunction renormalization of the i 'th matter multiplet in the low-energy theory. In field theory, all the higher loop running of the coupling is contained in the $\log(g_{ph})$ and $\log(Z_i)$ terms in the above, giving the NSVZ β -function. In our case, we expect that the Z_i are themselves determined by the vacuum expectation value of a logarithmically varying bulk field so that

$$Z_i = 1 - \frac{c_i}{8\pi^2} \log(R/l_s), \quad (19)$$

with c_i some constants coming from the supergravity solution. We do not know whether any miracles can guarantee that this expression reproduces the gauge theory result, although since Z is not holomorphic this seems doubtful. Nevertheless, because only $\log(Z_i)$ enters in the physical coupling, these corrections should be of the same order of magnitude as two-loop running in the field theory, and are also small.

7 Discussion

We have seen that the correct ‘field-theoretic running’ of the couplings may naturally be reproduced by the logarithmic variation of light fields in the deep IR. However, a new rationale for unification near the 4-d gravitational scale is required. We presented a toy model where unification is linked to a geometric symmetry of the brane configuration in the deep IR. This correspondence between UV effects in the gauge theory and IR effects in the gravitational

theory gives a fascinating re-interpretation of what we learn from RGE's. We normally think that we are limited to doing experiments at low energies, but the renormalization group allows us to extrapolate the couplings to much shorter distances, providing an indirect window to this remote realm. Our toy examples illustrate that sometimes this interpretation can be misleading. The fundamental short distance need not be remote at all, it could even be as low as a few TeV. On the other hand, we are then confined to a 3-brane, and can not probe *large* distances in the transverse space away from our brane. In our examples, the RGE's, reinterpreted as the result of logarithmic variation of bulk fields, provide an indirect window into this new remote realm a millimeter removed in the extra dimensions. In the old picture, the world look asymmetrical at large distances but symmetries emerge at short distances. In the new picture, the world close to our brane looks asymmetrical but symmetries emerge when we look at large distances in the bulk.

We comment in passing that the ideas presented here can also be used to provide controllable power-law unification with only one transverse dimension. The UV sensitivity of usual power-law unification becomes IR sensitivity in this picture, but since the IR physics is well-determined in any given model this can be controlled.

Finally, in asymptotically free gauge theories, we are used to generating energy scales much smaller than the UV cutoff by dimensional transmutation. The gauge coupling becomes strong at a scale Λ exponentially smaller than the cutoff and interesting physics happens. In the standard theories with large string scale, we expect that some SUSY gauge theory goes strong and triggers SUSY breaking far beneath the string scale, which stabilizes an exponentially large hierarchy between the electroweak and string scale. A similar phenomenon can in principle occur with low string scale and large extra dimensions. Suppose that there is a *non*-asymptotically free gauge group living on some collection of branes. Then, the bulk field setting the gauge coupling becomes strong exponentially far away in the bulk. This "dimensional transmutation in the bulk" naturally generates an IR scale exponentially larger than the string scale. It is tempting to speculate that this scale could determine an exponentially large effective compactification radius. More generally, logarithmic variation of fields in the bulk could force the theory into strong coupling exponentially far away in the extra dimensions, and interesting physics can happen. If an exponentially large radius can be generated in this way, we would have a true solution to the hierarchy problem, just as compelling as the standard picture in generating the various disparate scales observed in nature.

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