# CP VIOLATION IN THE HIGGS SECTOR OF THE MSSM

# **Apostolos Pilaftsis**

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

and

Department of Theoretical Physics, University of Thessaloniki, GR 54006 Thessaloniki, Greece

**Abstract.** Recently, it has been found that the tree-level CP invariance of the Higgs potential in the MSSM can be sizeably broken by loop effects due to soft-CP-violating trilinear interactions involving third generation scalar quarks. These soft-CP-violating couplings may be constrained by considering new two-loop contributions to the electron and neutron EDMs. The phenomenological consequences of such a minimal supersymmetric scenario of explicit CP violation at present and future colliders are briefly discussed.

# 1. Introduction

Many studies have been devoted to understand the origin of the observed CP asymmetry in the kaon system. In the existing literature, two generically different scenarios are known to describe CP violation in the Higgs

 $^1$  To appear in the proceedings of "Beyond the Desert," ed. H.V. Klapdor-Kleingrothaus, Castle Ringberg, Tegernsee, 6–12 June 1999, Germany.

<sup>1</sup> 

sector of a quantum field theory. In the first scenario, CP invariance is broken explicitly by complex bilinear terms or quartic couplings that involve Higgs doublets in an extended Higgs sector. Such a scenario predicts a CP-violating scalar-pseudoscalar mixing already at the tree level. Another interesting scenario is to have the CP symmetry of the Lagrangian be spontaneously broken by the ground state of the Higgs potential, while all parameters and couplings are real and respect CP invariance. To realize one of these two schemes, one needs to extend the Higgs sector of the Standard Model (SM) at least by one additional Higgs doublet. The most minimal supersymmetric extension of the SM, the so-called MSSM, with *R*-parity invariance, cannot realize any of the above two schemes at the tree level, despite the fact that the model contains two Higgs doublets. Beyond the Born approximation, the MSSM Higgs potential can break the CP symmetry either spontaneously [1] or explicitly [2]. The spontaneous CP-violating MSSM predicts a very light Higgs scalar below 10 GeV, which is ruled out experimentally [3].

Recently, it has been found [2, 4], however, that the tree-level CP invariance of the MSSM Higgs potential can be maximally broken at the one-loop level if soft-CP-violating Yukawa interactions involving stop and sbottom quarks are present in the theory. As an immediate consequence, the small tree-level mass difference between the heaviest Higgs boson and the CP-odd scalar may be lifted considerably through a large CP-violating scalar-pseudoscalar mass term [2, 4, 5]. This radiative scenario of explicit CP violation constitutes a very interesting possibility within the framework of the MSSM, and we shall briefly discuss its main phenomenological consequences at present and planned collider machines.

#### 2. The effective CP-violating Higgs potential

It is known that the MSSM introduces several new CP-odd phases in the theory that are absent in the Standard Model [6]. Specifically, the new CPodd phases may come from the following parameters: (i) the parameter  $\mu$ that describes the bilinear mixing of the two Higgs chiral superfields; (ii) the soft-supersymmetry-breaking gaugino masses  $m_{\lambda}$  for which we assume to have a common phase at the unification point; (iii) the soft bilinear Higgs-mixing mass  $m_{12}^2$ ; and (iv) the soft trilinear Yukawa couplings  $A_f$ of the Higgs particles to the scalar partners of matter fermions. Not all phases of the four complex parameters mentioned above are physical, i.e. two phases may be removed by suitable redefinitions of the fields. For example, one can rephase one of the Higgs doublets and the gaugino fields  $\lambda$ , in a way such that  $\arg(\mu)$  and  $\arg(A_f)$  are the only physical CP-violating phases in the MSSM.

An immediate consequence of Higgs-sector CP violation in the MSSM is the presence of mixing-mass terms between the CP-even and CP-odd Higgs fields [2]. Thus, one finds a  $(4 \times 4)$ -dimensional mass matrix for the neutral Higgs bosons. In the weak basis  $(G^0, a, \phi_1, \phi_2)$ , where  $G^0$  is the Goldstone field, the neutral Higgs-boson mass matrix  $\mathcal{M}_0^2$  takes on the form

$$\mathcal{M}_0^2 = \begin{pmatrix} \widehat{\mathcal{M}}_P^2 & \mathcal{M}_{PS}^2 \\ \mathcal{M}_{SP}^2 & \mathcal{M}_S^2 \end{pmatrix}, \qquad (1)$$

where  $\mathcal{M}_{S}^{2}$  and  $\widehat{\mathcal{M}}_{P}^{2}$  describe the CP-conserving transitions between scalar and pseudoscalar particles, respectively, whereas  $\mathcal{M}_{PS}^{2} = (\mathcal{M}_{SP}^{2})^{T}$  describes CP-violating scalar-pseudoscalar transitions. The characteristic size of these CP-violating off-diagonal terms in the Higgs-boson mass matrix was found to be [2, 4]

$$M_{SP}^{2} \simeq \mathcal{O}\left(\frac{m_{t}^{4}}{v^{2}} \frac{|\mu||A_{t}|}{32\pi^{2}M_{SUSY}^{2}}\right) \sin\phi_{CP} \\ \times \left(6, \frac{|A_{t}|^{2}}{M_{SUSY}^{2}}, \frac{|\mu|^{2}}{\tan\beta M_{SUSY}^{2}}, \frac{\sin 2\phi_{CP}}{\sin\phi_{CP}} \frac{|\mu||A_{t}|}{M_{SUSY}^{2}}\right), \quad (2)$$

where the last bracket summarizes the relative sizes of the different contributions, and  $\phi_{CP} = \arg(A_t\mu) + \xi$ . The parameter  $\xi$  is the relative phase between the two Higgs vacuum expectation values which is induced radiatively [2].

The CP-violating effects can become substantial if  $|\mu|$  and  $|A_t|$  are larger than the average of the stop masses, denoted as  $M_{\rm SUSY}$ . For example, the off-diagonal terms of the neutral Higgs-mass matrix may be of order  $(100 \text{ GeV})^2$ , for  $|\mu| \simeq |A_t| \lesssim 3M_{\rm SUSY}$ , and  $\phi_{\rm CP} \simeq 90^\circ$ . These potentially large mixing effects lead to drastic modifications of the predictions for the neutral Higgs-boson masses and for the couplings of the Higgs states to the gauge bosons [4]. As can be seen from Fig. 1, the effect of CP nonconservation on the lightest Higgs boson and on its related couplings to the gauge bosons is only important for relatively low values of  $M_{H^+}$ , e.g.  $M_{H^+} \lesssim 170$  GeV [4]. The upper limit on the lightest Higgs-boson mass does not change, as the relevant stop mixing parameter entering the definition of  $M_{H_1}$  is now given by

$$|\hat{A}_t| = |A_t - \mu^* / \tan \beta|.$$
 (3)

Notice that the scenario we have used in Fig. 1 is compatible with experimental upper limits on the electron and neutron electric dipole moments (EDMs). These EDM constraints will be discussed in the next section.

# 3. Two-loop EDM constraint

The MSSM generally gives large contributions to the electron and neutron EDMs, coming from squarks of the first two families [6, 7]. Even if the first



**Figure 1.** Numerical estimates of (a)  $g_{H_1ZZ}^2$  and (b)  $M_{H_1} \leq M_{H_2}$  as a function of  $\arg(A_t)$ .





**Figure 2.** Two-loop contribution to EDM and CEDM in supersymmetric theories (mirror graphs are not displayed.)

two families of squarks are arranged so as to give small FCNC and EDM effects [8], the two-loop graphs [7] shown in Fig. 2 may even dominate by several orders of magnitude over all other one-, two- and three-loop contributions, thereby significantly constraining Higgs-sector CP violation. In the SUSY scenario, with a large CP-violating phase only in the third family  $A_{\tau} = A_t = A_b = A$ , the CP-violating Lagrangian,

$$\mathcal{L}_{\rm CP} = -\xi_{\tilde{f}} v \, a \, (\tilde{f}_1^* \tilde{f}_1 - \tilde{f}_2^* \tilde{f}_2) + \frac{i g_w m_f}{2 M_W} R_f \, a \, \bar{f} \gamma_5 f \,, \tag{4}$$

gives rise to a new EDM contribution to the neutron and electron. In Eq. (4), a is the would-be CP-odd Higgs boson,  $M_W = \frac{1}{2}g_w v$  is the W-boson mass,  $\tilde{f}_{1,2}$  are the two mass-eigenstates of the third-family squarks,  $R_b = \tan \beta$ ,  $R_t = \cot \beta$ , and  $\xi_f$  is a model-dependent CP-violating parameter. In the MSSM, only  $\tilde{t}$  and  $\tilde{b}$  are expected to give the largest contributions, as the quantities  $\xi_f$  are Yukawa-coupling enhanced, viz.

$$\xi_{\tilde{f}} = -R_f \frac{\sin 2\theta_f m_f \operatorname{Im}(\mu e^{i\delta_f})}{\sin\beta \cos\beta v^2}, \qquad (5)$$

where  $\delta_f = \arg(A_f - R_f \mu^*)$ , and  $\theta_t$ ,  $\theta_b$  are the mixing angles between weak and mass eigenstates of  $\tilde{t}$  and  $\tilde{b}$ , respectively. Further details of the calculation may be found in [7].

Fig. 2 shows the dependence of the EDMs  $d_e$  (solid line),  $d_n^C$  (dashed line), and  $d_n$  (dotted line) on  $\tan\beta$  and  $\mu$ , for three different masses of the would-be CP-odd Higgs boson a,  $M_a = 100, 300, 500$  GeV. Since the coupling of the a boson to the down-family fermions such as the electron and d quark depends significantly on  $\tan\beta$ , we find a substantial increase of  $d_n$  and  $d_e$  in the large  $\tan\beta$  domain (see Fig. 2(a)). As can be seen



Figure 3. Numerical estimates of EDMs. Lines of the same type from the upper to the lower one correspond to  $M_a = 100, 300, 500$  GeV, respectively. 6

from Fig. 2(b), EDMs also depend on  $\mu$  through the  $a\tilde{f}^*\tilde{f}$  coupling in Eq. (4). Note that the numerical predictions for the size of EDMs do not depend on the sign of  $\mu$  for  $\arg(A) = 90^0$ . From our numerical analysis, we can exclude large  $\tan\beta$  scenarios, *i.e.*,  $40 < \tan\beta < 60$  with  $\mu$ , A > 0.5 TeV,  $M_a \leq 0.5$  TeV, and large CP phases. Nevertheless, the situation is different for low  $\tan\beta$  scenarios, *e.g.*  $\tan\beta \lesssim 20$ , where the two-loop Barr-Zee-type contribution to EDMs is not very restrictive for natural values of parameters in the MSSM. Finally, EDMs display a mild linear dependence on the mass of the *a* boson for the range of physical interest,  $0.1 < M_a \lesssim 1$  TeV.

#### 4. Higgs phenomenology and CP violation

The main effect of Higgs-sector CP violation is the modification of the couplings of the Higgs bosons to fermions and the W and Z bosons, i.e.  $ffH_i$ ,  $WWH_i$ ,  $ZZH_i$  and  $ZH_iH_j$ . The modified effective Lagrangians are given by

$$\mathcal{L}_{H\bar{f}f} = -\sum_{i=1}^{3} H_{(4-i)} \left[ \frac{g_w m_d}{2M_W c_\beta} \bar{d} (O_{2i} - is_\beta O_{1i} \gamma_5) d + \frac{g_w m_u}{2M_W s_\beta} \bar{u} (O_{3i} - ic_\beta O_{1i} \gamma_5) u \right],$$
(6)

$$\mathcal{L}_{HVV} = g_w M_W (c_\beta O_{2i} + s_\beta O_{3i}) \left( H_{(4-i)} W^+_\mu W^{-,\mu} + \frac{1}{2c_w^2} H_{(4-i)} Z_\mu Z^\mu \right),$$

$$\mathcal{L}_{HHZ} = \frac{g_w}{4c_w} \left[ O_{1i} (c_\beta O_{3j} - s_\beta O_{2j}) - O_{1j} (c_\beta O_{3i} - s_\beta O_{2i}) \right] \times Z^\mu (H_{(4-i)} \overleftrightarrow{\partial}_\mu H_{(4-j)}),$$
(8)

where  $c_w = M_W/M_Z$  and  $\overleftrightarrow{\partial}_{\mu} \equiv \overleftrightarrow{\partial}_{\mu} - \overleftrightarrow{\partial}_{\mu}$ . Note that the coupling of the Z boson to two real scalar fields is forbidden due to Bose symmetry.

We shall now discuss a generic example in order to better understand the effect of Higgs-sector CP violation on the mass of the lightest Higgs boson. We consider an intermediate-tan $\beta$  scenario, with tan $\beta = 4$ ,  $M_{\rm SUSY} = 0.5$  TeV,  $A_t = A_b = 1$  TeV and  $\mu = 2$  TeV. From Fig. 1 and for  $M_{H^+} = 140$  GeV, we observe that there exist regions for which the lightest Higgs-boson mass  $M_{H_1}$  is as small as 60–70 GeV and the  $H_1ZZ$  coupling is small enough for the  $H_1$  boson to escape detection at the latest LEP2 run with  $\sqrt{s} = 189$  GeV. In this scenario, the  $H_2$  boson is too heavy to be detected through the  $H_2ZZ$  channel. Furthermore, we find that either the coupling  $H_1H_2Z$  is too small or  $H_2$  is too heavy to allow Higgs detection in the  $H_1H_2Z$  channel [4]. An upgraded Tevatron machine has the potential capabilities to close most of such experimentally open windows.

It is worth stressing that the CP-violating MSSM Higgs potential retains its enhanced predictive power in the lightest Higgs  $(H_1)$  sector. As was mentioned in Section 2, CP violation decouples from the  $H_1$  sector for large values of  $M_{H^+} \approx M_{H_2} \sim M_{H_3}$ , but it is sizeable for  $M_{H^+} \lesssim 170$  GeV [4]. However, for much larger  $H^+$  masses, CP violation does not decouple in the  $H_2H_3$  system [2, 4, 5], giving rise to large CP-violating effects in the  $H_{2,3}uu$  and  $H_{2,3}dd$  couplings. In fact, the necessary condition for resonant CP violation through scalar-pseudoscalar (HA) mixing is [9]

$$2|\Pi_{HA}(s)| \gtrsim |M_{H}^{2} - M_{A}^{2} - \Pi_{HH}(s) + \Pi_{AA}(s)|, \qquad (9)$$

at  $s \approx M_H^2 \approx M_A^2$ , where  $\Pi_{HH,HA,AA}$  denote renormalized self-energy transitions of Higgs scalars of definite CP parity, namely *H* has CP parity +1 and *A* has CP parity -1. As was shown in [2], the condition (9) is comfortably satisfied within the framework of MSSM. Furthermore, CP-violating effects induced by radiative corrections to the Higgs-fermion vertices may also be important, particularly for large values of tan  $\beta$  [4].

Higgs-sector CP violation may also be tested at muon colliders by looking at observables of the kind [9]

$$\mathcal{A}_{\rm CP}^{\mu} = \frac{\sigma(\mu_L^- \mu_L^+ \to f\bar{f}) - \sigma(\mu_R^- \mu_R^+ \to f\bar{f})}{\sigma(\mu_L^- \mu_L^+ \to f\bar{f}) + \sigma(\mu_R^- \mu_R^+ \to f\bar{f})}, \qquad (10)$$

$$\mathcal{A}_{\rm CP}^f = \frac{\sigma(\mu^-\mu^+ \to f_L \bar{f}_L) - \sigma(\mu^-\mu^+ \to f_R \bar{f}_R)}{\sigma(\mu^-\mu^+ \to f_L \bar{f}_L) + \sigma(\mu^-\mu^+ \to f_R \bar{f}_R)}, \qquad (11)$$

where f may be top or bottom quarks. The former observable requires polarization of the initial muons. If the facility of polarization is not available at muon colliders, one may still observe CP violation through the second observable and reconstruct the polarization of the final fermions by looking at the angular momentum distribution of their decay products [10]. The magnitudes of these CP-violating observables strongly depend on the expressions  $2|(g_{H_iff}^S)(g_{H_iff}^P)|/[(g_{H_iff}^S)^2 + (g_{H_iff}^P)^2]$  (see also Fig. 4), where

$$\begin{aligned}
g_{H_{1}uu}^{S} &= O_{33}/s_{\beta}, \quad g_{H_{1}uu}^{P} = O_{13} \cot \beta, \quad g_{H_{2}uu}^{S} = O_{32}/s_{\beta}, \\
g_{H_{2}uu}^{P} &= O_{12} \cot \beta, \quad g_{H_{1}dd}^{S} = O_{23}/c_{\beta}, \quad g_{H_{1}dd}^{P} = O_{13} \tan \beta, \\
g_{H_{2}dd}^{S} &= O_{22}/c_{\beta}, \quad g_{H_{2}dd}^{P} = O_{12} \tan \beta.
\end{aligned} \tag{12}$$

To summarize, the MSSM with radiatively induced CP violation in the Higgs sector is a very predictive theoretical framework, with interesting consequences on collider experiments [11], CP asymmetries in B-meson decays [12], and dark-matter searches [13].

I wish to thank Darwin Chang, Wai-Yee Keung and Carlos Wagner for collaboration. I also thank the Theory Groups of SLAC and FERMILAB for their kind hospitality.



**Figure 4.** Numerical estimates of (a)  $2|(g_{H_2dd}^S)(g_{H_2dd}^P)|/[(g_{H_2dd}^S)^2 + (g_{H_2dd}^P)^2]$  and (b)  $2|(g_{H_2uu}^S)(g_{H_2uu}^P)|/[(g_{H_2uu}^S)^2 + (g_{H_2uu}^P)^2]$  as a function of  $\arg(A_t)$ .

#### References

- [1] N. Maekawa, Phys. Lett. B282 (1992) 387.
- [2] A. Pilaftsis, Phys. Rev. D58 (1998) 096010; Phys. Lett. B435 (1998) 88.
- [3] A. Pomarol, *Phys. Lett.* B287 (1992) 331; N. Haba, *Phys. Lett.* B398 (1997) 305; D. Chang and W.-Y. Keung, *Phys. Rev. Lett.* 74 (1995) 1928.
- [4] A. Pilaftsis and C.E.M. Wagner, hep-ph/9902371.
- [5] D.A. Demir, hep-ph/9901389; hep-ph/9905571.
- [6] J. Ellis, S. Ferrara and D.V. Nanopoulos, Phys. Lett. B114 (1982) 231; W. Buchmüller and D. Wyler, Phys. Lett. B121 (1983) 321; J. Polchinski and M. Wise, Phys. Lett. B125 (1983) 393; F. del Aguila, M. Gavela, J. Grifols and A. Mendez, Phys. Lett. B126 (1983) 71; D.V. Nanopoulos and M. Srednicki, Phys. Lett. B128 (1983) 61; M. Dugan, B. Grinstein and L. Hall, Nucl. Phys. B255 (1985) 413; J. Dai, H. Dykstra, R.G. Leigh, S. Paban and D. Dicus, *Phys. Lett.* **B237** (1990) 216; **B242** (1990) 547 (E); J.F. Gunion and D. Wyler, Phys. Lett. B248 (1990) 170; D. Chang, W.-Y. Keung and T.C. Yuan, Phys. Lett. B251 (1990) 608; P. Nath, Phys. Rev. Lett. 66 (1991) 2565; Y. Kizukuri and N. Oshimo, Phys. Rev. D46 (1992) 3025; T. Falk, K.A. Olive and M. Srednicki, Phys. Lett. B354 (1995) 99; T. Ibrahim and P. Nath, Phys. Lett. B418 (1998) 98; Phys. Rev. D57 (1998) 478; D58 (1998) 019901 (E); Phys. Rev. D58 (1998) 111301; T. Falk and K.A. Olive, hep-ph/9806236; M. Brhlik, G.J. Good and G.L. Kane, hep-ph/9810457; M. Brhlik, L. Everett, G.L. Kane and J. Lykken, hep-ph/9905215; hep-ph/9908326; S. Pokorski, J. Rosiek and C.A. Savoy, hep-ph/9906206; E. Accomando, R. Arnowitt and B. Dutta, hep-ph/9907446; T. Ibrahim and P. Nath, hep-ph/9907555.
- [7] D. Chang, W.-Y. Keung and A. Pilaftsis, Phys. Rev. Lett. 82 (1999) 900.
- [8] G.F. Giudice and S. Dimopoulos, *Phys. Lett.* B357 (1995) 573; G. Dvali and A. Pomarol, *Phys. Rev. Lett.* 77 (1996) 3728; G. Cohen, D.B. Kaplan and A.E. Nelson, *Phys. Lett.* B388 (1996) 588; P. Binétruy and E. Dudas, *Phys. Lett.* B389 (1996) 503.
- [9] A. Pilaftsis, *Phys. Rev. Lett.* **77** (1996) 4996; *Nucl. Phys.* **B504** (1997) 61;
   S.-Y. Choi and M. Drees, *Phys. Rev. Lett.* **81** (1998) 5509. For a recent analysis at *pp* coliders, see W. Bernreuther, A. Brandenburg and M. Flesch, hep-ph/9812387, and at *ee* colliders, T. Han, T. Huang, Z.H. Lin, J.X. Wang and X. Zhang, hep-ph/9908236.
- [10] B. Grzadkowski and J.F. Gunion, *Phys. Lett.* B350 (1995) 218.
- [11] For a recent analysis, see M. Carena, S. Mrenna and C.E.M. Wagner, hep-ph/9907422; see, also B. Grzadkowski, J.F. Gunion and J. Kalinowski, hep-ph/9902308; J. Kalinowski, hep-ph/9906336; S.Y. Choi and J.S. Lee, hep-ph/9907496.
- S. Baek and P. Ko, hep-ph/9904283; A. Ali and D. London, hep-ph/9907243; hep-ph/9903535; C.-S. Huang and L. Wei, hep-ph/9908246.
- T. Falk, A. Ferstl and K.A. Olive, hep-ph/9908311; K. Freese and P. Gondolo, hep-ph/9908xxx.
  - 10