

Principles of optics

*Electromagnetic theory of propagation,
interference and diffraction of light*

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