Cumulative Non-decoupling Effects of Kaluza-Klein Neutrinos in Electroweak Processes

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ABSTRACT

In Kaluza-Klein theories of low-scale quantum gravity, gravitons and iso-singlet neutrinos may propagate in a higher-dimensional space with large compact dimensions, whereas all particles of the Standard Model are confined on a $(1+3)$-dimensional subspace. After compactification of the extra dimensions, the resulting Yukawa couplings of the Kaluza-Klein neutrinos to the lepton doublets become naturally very suppressed by a higher-dimensional volume factor in agreement with phenomenological observations. We show that one-loop effects induced by Kaluza-Klein neutrinos, albeit tiny individually, act cumulatively in electroweak processes, giving rise to a non-decoupling behaviour for large values of the higher-dimensional Yukawa couplings. Owing to the non-decoupling effects of Kaluza-Klein neutrinos, we can derive strong constraints on the parameters of the theory that originate from the non-observation of flavour-violating and universality-breaking phenomena, which involve the W and Z bosons, and the e, μ and τ leptons.

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1   Introduction

Recently, Arkani-Hamed, Dimopoulos and Dvali [1] have considered the radical possibility that the fundamental scale of quantum gravity is no longer set by the Planck mass $M_{\mathrm{P}} = 1.2 \times 10^{19}$ GeV, but the true scale of quantum gravity, M_F, is many orders of magnitude smaller than M_{P} close to the TeV energies.* The observed weakness of gravity is then due to the presence of a number δ of large extra dimensions, within which only gravity can propagate and, most probably, fields that are singlets under the Standard Model (SM) gauge group, such as isosinglet neutrinos [7, 8, 9]. In this theoretical framework, the SM particles do not feel the presence of the extra dimensions, but are rather confined to a $(1 + 3)$-dimensional Minkowski subspace which constitutes our observable world. Most interestingly, the ordinary Planck mass M_{P} is related to the genuinely fundamental scale M_F through

$$M_{\mathrm{P}} \; \approx \; M_F \, (R \, M_F)^{\delta/2} \, , \tag{1.1}$$

where R denotes the compactification radii of the extra compact dimensions, which are all taken to be of equal size. The scenario, with $\delta = 1$ and M_F of order TeV, predicts a visible macroscopic compactification radius and is therefore ruled out. Many recent astrophysical [10, 11, 7, 8, 9, 12] and phenomenological [13] studies have been devoted to explore the viability of low-scale theories of quantum gravity, with $\delta \geq 2$.

In this paper, we shall study the phenomenological consequences of loop effects of higher-dimensional iso-singlet neutrinos on collider and lower energy experiments. Specifically, we find that both at the tree and quantum levels, higher-dimensional iso-singlet neutrinos can naturally induce observable phenomena of lepton-flavour violation and universality breaking in W- and Z-boson interactions. To quantitatively analyze the new-physics effects, we shall consider a minimal higher-dimensional scenario, in which the SM is extended by an iso-singlet neutrino N that propagates in $3 + \delta$ spatial dimensions. The iso-singlet neutrino N couples, with non-universal Yukawa couplings, to all three lepton SM doublets, L_e, L_μ and L_τ, which are localized on our 3-dimensional world. After com-

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*There have been earlier considerations in the literature that discussed the possibility of lowering the string but not the Planck scale in superstring theories. Most noticeably, Witten [2], and Hořava and Witten [3] considered a novel M-theory scenario compactified on an orbifold S^1/Z_2, in which the string scale was lowered by two orders of magnitude to energies of order 10^{16} GeV. Along these lines, Lykken [4] discussed an analogous scenario, in which the string scale was further lowered to the TeV range. In a much earlier work, Antoniadis [5] discussed the possibility of lowering the compactification radius of gauge interactions at the TeV scale in the context of string theories. In related supersymmetric scenarios, Dienes, Dudas and Gherghetta [6] have recently studied several aspects of gauge-coupling unification.

pactification, the resulting Yukawa couplings of the Kaluza-Klein (KK) neutrinos to the SM leptons come out to be highly suppressed by the volume factor of the extra dimensions $M_F/M_P \sim 10^{-14}$ [7, 8].

One might now think that the new-physics phenomena mediated by the KK neutrinos would also be extremely suppressed by the same volume factor of the extra dimensions. However, this is not true. After summing over the tower of the KK neutrinos, we obtain an effective theory whose Yukawa interactions are mediated by order-unity Yukawa couplings of the original Lagrangian before compactification. As a consequence, we expect a cumulative non-decoupling phenomenon of the KK neutrinos to occur in loops for large higher-dimensional Yukawa couplings, namely the KK neutrinos appear to violate the known decoupling theorem due to Appelquist and Carazzone [14].[†] The higher-dimensional non-decoupling phenomenon is rather analogous to the one studied in [15, 16, 17, 18, 19, 20], for singlet-neutrino scenarios [21, 22] with large $SU(2)_L$ Dirac masses and mixings. Because of the nondecoupling effects of heavy (KK) neutrinos, phenomena of new physics can be dramatically enhanced to an observable level, such as lepton-flavour-violating decays of the Z boson [15], universality-breaking effects in the diagonal leptonic decays of the Z boson [16], neutrinoless two- and three-body decays of the τ and μ leptons [17, 18], and universality-breaking effects in leptonic asymmetries measured on the Z pole [19]. In fact, the non-observation of the above new-physics effects places stringent bounds on the parameter space of the theory. Here, we shall perform an analogous study for the higher-dimensional singlet-neutrino scenario under consideration. The limits obtained by our analysis are rather generic and can easily carry over to related higher-dimensional models.

Another important feature of the singlet-neutrino models is the decoupling property of a very high iso-singlet mass [23]. In higher-dimensional models, the fundamental Planck mass, M_F, is the one, which is now playing the role of the iso-singlet mass scale. Thus, we expect that the KK neutrinos decouple from the loops as $M_F \to \infty$. In this limit, all new-physics phenomena mentioned above will be suppressed by inverse powers of M_F. However, for relatively small values of M_F, e.g. $M_F \lesssim 100$ TeV, the screening effect of the higher-dimensional Planck scale will be less dramatic, and experimental information is then needed to place a lower bound on M_F.

The organization of the paper is as follows: in Section 2, we describe the basic low-energy structure of a minimal model with one higher-dimensional iso-singlet neutrino. In

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[†]This theorem is not directly applicable to spontaneous-symmetry-breaking theories, as the one we are considering here. The reason is that not all operators of dimension 2 can be increased independently of those of dimension 3, since they are related via the Higgs mechanism.

Section 3, we derive constraints on the parameters of the KK theory, which arise from tree-level contributions to electroweak observables. In Section 4, we explicitly demonstrate the cumulative non-decoupling effect of KK neutrinos in a typical flavour-changing-neutral-current (FCNC) graph. In Section 5, we present analytic results of the loop contributions of the KK neutrinos to electroweak observables of new physics, and also set new limits on the parameters of the theory. Section 6 summarizes our conclusions.

2   Higher-dimensional model with one singlet neutrino

For our phenomenological study, we shall adopt a variant [9] of the model discussed in Ref. [7]. Nevertheless, the results of our analysis will equally well apply to other recently proposed scenarios [8]. For definiteness, we will be considering a model that minimally extends the SM-field content by one singlet Dirac neutrino, $N(x, y)$, which propagates in a $[1 + (3 + \delta)]$-dimensional Minkowski space. We denote by x^μ, with $\mu = 0, 1, 2, 3$, the one time and the three spatial coordinates of our observable world and by y^k, with $k = 1, \ldots, \delta$, the new large compact dimensions. The y-coordinates are compactified on a circle of radius R by applying the periodic identification: $y \equiv y + 2\pi R$. Furthermore, we consider that the higher-dimensional Dirac neutrino $N(x, y)$ generally has non-universal Yukawa couplings, $\bar{h}_l$, to the three ordinary lepton iso-doublets $L_l(x)$, with $l = e, \mu, \tau$.

For the purpose of illustration, we shall consider that the higher-dimensional Dirac neutrino $N(x, y)$ feels the presence of only one large compact dimension. Then, our results can easily be generalized to higher dimensions. The leptonic sector of the minimal model consists of the following fields:

$$L_l(x) \; = \; \left(\begin{array}{c} \nu_{lL}(x) \\ l_L(x) \end{array} \right), \qquad l_R(x), \qquad N(x, y) \; = \; \left(\begin{array}{c} \xi(x, y) \\ \bar{\eta}(x, y) \end{array} \right), \qquad (2.1)$$

where ν_{lL}, l_L and l_R describe 4-dimensional Weyl spinors of the charged leptons and their associate left-handed neutrinos, and ξ and η are two-component spinors in 5 dimensions. The 5-dimensional gamma matrices may be represented by

$$\gamma_\mu \; = \; \left(\begin{array}{cc} 0 & \bar{\sigma}_\mu \\ \sigma_\mu & 0 \end{array} \right), \quad \text{and} \quad \gamma_4 \; = \; \left(\begin{array}{cc} i\mathbf{1}_2 & 0 \\ 0 & -i\mathbf{1}_2 \end{array} \right), \qquad (2.2)$$

where $\sigma^\mu = (\mathbf{1}_2, \vec{\sigma})$ and $\bar{\sigma}^\mu = (\mathbf{1}_2, -\vec{\sigma})$, and σ_1, σ_2 and σ_3 are the usual Pauli matrices. The effective Lagrangian of our minimal model reads

$$\mathcal{L}_{\text{eff}} \; = \; \int_0^{2\pi R} dy \, \Big[\, \bar{N} \big(i\gamma^\mu \partial_\mu \, + \, i\gamma_4 \partial_y \big) N \; - \; m\bar{N}N \; + \; \delta(y - a) \Big(\sum_{l=e,\mu,\tau} \bar{h}_l L_l \tilde{\Phi} \xi \; + \; \text{H.c.} \Big)$$

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$$+ \delta(y - a)\, \mathcal{L}_{\text{SM}}(\Phi) \Big], \tag{2.3}$$

where $\tilde{\Phi} = i\sigma_2 \Phi^*$ and $\mathcal{L}_{\text{SM}}(\Phi)$ describes the SM Lagrangian. The dimensionful Yukawa couplings $\bar{h}_l$ may be related to the dimensionless ones, h_l, through

$$\bar{h}_l = \frac{h_l}{(M_F)^{\delta/2}}, \tag{2.4}$$

with $\delta = 1$. Without any further restriction on the parameters of the theory, the reduced couplings h_l are expected to be of order unity, as M_F is the only available energy scale to normalize the dimensionful couplings $\bar{h}_l$. In Eq. (2.3), we have included the bare Dirac bilinear $m\bar{N}N$. As we will see below, the effect of this term is to shift the mass of the lowest-lying KK state by an amount m. In principle, we could also have added another Lorentz- and gauge-invariant fermionic bilinear in Eq. (2.3), i.e. $MN^T C^{(5)-1} N$, where $C^{(5)} = -\gamma_1 \gamma_3$. The presence of the latter operator is not very essential for our phenomenological discussion. In fact, this last term drops out, if one imposes invariance of the Lagrangian (2.3) under a global transformation that respects lepton number: $N \to e^{i\theta} N$, $L_l \to e^{-i\theta} L_l$ and $l_R \to e^{i\theta} l_R$.

We can now express the 5-dimensional two-component spinors ξ and η of $N(x, y)$ in terms of a Fourier series expansion as follows:

$$\xi(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \xi_n(x)\, \exp\left(\frac{iny}{R}\right), \tag{2.5}$$

$$\eta(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \eta_n(x)\, \exp\left(\frac{iny}{R}\right). \tag{2.6}$$

Substituting Eqs. (2.5) and (2.6) into the effective Lagrangian (2.3) and then performing the y integration yields

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}}(\Phi) + \sum_{n=-\infty}^{\infty} \left\{ \bar{\xi}_n(i\bar{\sigma}^\mu \partial_\mu)\xi_n + \bar{\eta}_n(i\bar{\sigma}^\mu \partial_\mu)\eta_n - \left[\left(m + \frac{in}{R}\right)\xi_n \eta_{-n} + \text{H.c.} \right] \right.$$

$$\left. + \left(\sum_{l=e,\mu,\tau} \bar{h}_l^{(n)} L_l \tilde{\Phi} \xi_n + \text{H.c.} \right) \right\}, \tag{2.7}$$

where

$$\bar{h}_l^{(n)} = \frac{M_F}{M_P}\, h_l \exp\left(\frac{ina}{R}\right). \tag{2.8}$$

As was first noticed in [7, 8], the four-dimensional Yukawa couplings $\bar{h}_l^{(n)}$ are naturally suppressed by the volume factor M_F/M_P of the extra dimensions.

After spontaneous symmetry breaking (SSB), the effective Lagrangian of the KK neutrino-mass matrix reads

$$\mathcal{L}_{\text{mass}}^{\text{KK}} = \Psi_+^T \mathcal{M} \Psi_- + \text{H.c.}, \tag{2.9}$$

where $\Psi_+^T = (\nu_{lL}, \eta_0, \eta_1, \eta_{-1}, \ldots, \eta_n, \eta_{-n}, \ldots)$, $\Psi_-^T = (\xi_0, \xi_{-1}, \xi_1, \ldots, \xi_{-n}, \xi_n, \ldots)$ (with $n > 0$), and

$$\mathcal{M} = \begin{pmatrix} m_l^{(0)} & m_l^{(-1)} & m_l^{(1)} & \cdots & m_l^{(-n)} & m_l^{(n)} & \cdots \\ m & 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & m - \frac{i}{R} & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & m + \frac{i}{R} & \cdots & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ 0 & 0 & 0 & \cdots & m - \frac{in}{R} & 0 & \cdots \\ 0 & 0 & 0 & \cdots & 0 & m + \frac{in}{R} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \tag{2.10}$$

where $m_l^{(n)} = \bar{h}_l^{(n)} v/\sqrt{2}$, with $l = e, \mu, \tau$ and $\langle \Phi \rangle = v/\sqrt{2}$. Observe that the matrix $\mathcal{M}$ contains three rows more than the rectangular case. However, one can show that the additional rows correspond to three massless Weyl spinors, collectively denoted as ν_l, and can be treated independently of the rectangular part of the neutrino-mass matrix. The massless Weyl spinors ν_l are predominantly left-handed and hence describe the observable neutrinos.

To make this very last point explicit, we will first go from the weak basis Ψ_+ to another rotated basis, e.g. Ψ_+^R, which is defined by the unitary transformation: $\Psi_+ = U^\nu \Psi_+^R$, where $(\Psi_+^R)^T = (\nu_l, \eta_0^R, \eta_1^R, \eta_{-1}^R, \ldots, \eta_n^R, \eta_{-n}^R, \ldots)$ and

$$U^\nu = \begin{pmatrix} (\mathbf{1}_3 + \Xi^* \Xi^T)^{-1/2} & \Xi^* (\mathbf{1} + \Xi^T \Xi^*)^{-1/2} \\ -\Xi^T (\mathbf{1}_3 + \Xi^* \Xi^T)^{-1/2} & (\mathbf{1} + \Xi^T \Xi^*)^{-1/2} \end{pmatrix}, \tag{2.11}$$

with

$$\Xi = \left(\frac{m_l^{(0)}}{m}, \frac{m_l^{(-1)}}{m - \frac{i}{R}}, \frac{m_l^{(1)}}{m + \frac{i}{R}}, \cdots, \frac{m_l^{(-n)}}{m - \frac{in}{R}}, \frac{m_l^{(n)}}{m + \frac{in}{R}}, \cdots \right), \tag{2.12}$$

and $\Xi^* (\mathbf{1} + \Xi^T \Xi^*)^{-1/2} = (\mathbf{1}_3 + \Xi^* \Xi^T)^{-1/2} \Xi^*$. In Eq. (2.11), the root of a Hermitian matrix, e.g. $H = (\mathbf{1} + \Xi^T \Xi^*)$, is defined as $H^{1/2} = U_H \hat{H}^{1/2} U_H^\dagger$, where U_H is the unitary matrix that diagonalizes H, i.e. $\hat{H} = U_H^\dagger H U_H$. It is then easy to verify that $H^{1/2} H^{1/2} = H$, as it should be. In the newly introduced weak basis, the three upper rows of the rotated neutrino mass matrix $\mathcal{M}^R = (U^\nu)^T \mathcal{M}$ vanish identically, giving rise to three massless chiral fields ν_l, while the remaining matrix assumes the usual rectangular form that describes massive Dirac fields. In fact, for the case at hand, one has $m_l^{(n)} \ll m$ and the massless chiral fields ν_l are predominantly left-handed, i.e.

$$\nu_{lL} = (\mathbf{1}_3 + \Xi^* \Xi^T)^{-1/2} \left(\nu_l + \Xi^* \Psi_+'^R \right), \tag{2.13}$$

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with $\nu_{l(L)}^T = (\nu_{e(L)}, \nu_{\mu(L)}, \nu_{\tau(L)})$ and $(\Psi_+'^R)^T = (\eta_0^R, \eta_1^R, \eta_{-1}^R, \ldots, \eta_n^R, \eta_{-n}^R, \ldots)$. In the limit $m \to 0$ discussed in [7], there is a level-crossing effect and η_0 becomes massless, whereas one linear combination of the three ν_l fields acquires a small Dirac mass of order $m_l^{(0)}$; the other two linear combinations orthogonal to the last one remain massless. The rectangular part of the matrix $\mathcal{M}$, $\mathcal{M}_\chi$, spanned by the field vectors $\Psi_+'^R$ and Ψ_-, can be diagonalized independently by a bi-unitary transformation: $V_+^T \mathcal{M}_\chi V_- = \widehat{\mathcal{M}}_\chi$. We denote the resulting KK mass eigenfields by $\chi^{(n)}$. To leading order in $m_l^{(n)}/m$, the mass eigenvalues of $\mathcal{M}_N$ are given by

$$\widehat{\mathcal{M}}_\chi \approx \mathrm{diag}\left(m, \sqrt{m^2 + \frac{1}{R^2}}, \sqrt{m^2 + \frac{1}{R^2}}, \cdots, \sqrt{m^2 + \frac{n^2}{R^2}}, \sqrt{m^2 + \frac{n^2}{R^2}}, \cdots \right). \quad (2.14)$$

In this approximation, up to phase factors, the unitary matrices $V_\pm$ stay close to unity [12]. Note that unlike $\chi^{(0)}$, all other massive Dirac fields fall into degenerate pairs, i.e. $m_{(n)} = m_{(-n)}$, with $n > 0$. As we have mentioned above, since chiral neutrino ν_l is massless, the next-to-lightest state of the neutrino mass spectrum, $\chi^{(0)}$, exhibits a mass gap of order m, i.e. $m_{(0)} \approx m$.

In the following, we shall give the Lagrangians [24] that govern the interactions of the neutrinos, ν_l and $\chi^{(n)}$, and the charged leptons, l, with the gauge bosons, $W^\pm$ and Z, as well as with their respective would-be Goldstone bosons, $G^\pm$ and G^0. The interaction Lagrangians can be summarized as follows:

$$\mathcal{L}_{\mathrm{int}}^{W^\pm} = -\frac{g_w}{\sqrt{2}} W^{-\mu} \sum_{l=e,\mu,\tau} \left(B_{l\nu_l} \bar{l}\gamma_\mu P_L \nu_l + \sum_{n=-\infty}^{\infty} B_{l,n} \bar{l}\gamma_\mu P_L \chi^{(n)} + \mathrm{H.c.} \right), \quad (2.15)$$

$$\mathcal{L}_{\mathrm{int}}^Z = -\frac{g_w}{2c_w} Z^\mu \left[\sum_{l,l'=e,\mu,\tau} C_{\nu_l\nu_{l'}} \bar{\nu}_l\gamma_\mu P_L \nu_{l'} + \left(\sum_{l=e,\mu,\tau} \sum_{n=-\infty}^{\infty} C_{\nu_l,n} \bar{\nu}_l\gamma_\mu P_L \chi^{(n)} + \mathrm{H.c.} \right) \right.$$
$$\left. + \sum_{k=-\infty}^{\infty} C_{n,k} \bar{\chi}^{(n)}\gamma_\mu P_L \chi^{(k)} \right], \quad (2.16)$$

$$\mathcal{L}_{\mathrm{int}}^{G^\pm} = -\frac{g_w}{\sqrt{2}M_W} G^- \sum_{l=e,\mu,\tau} \left[B_{l\nu_l} m_l \bar{l} P_L \nu_l + \sum_{n=-\infty}^{\infty} B_{l,n} \bar{l}\left(m_l P_L - m_{(n)} P_R \right)\chi^{(n)} \right.$$
$$\left. + \mathrm{H.c.} \right], \quad (2.17)$$

$$\mathcal{L}_{\mathrm{int}}^{G^0} = \frac{ig_w}{2M_W} G^0 \left[\sum_{l=e,\mu,\tau} \left(\sum_{n=-\infty}^{\infty} C_{\nu_l,n} m_{(n)} \bar{\nu}_l P_R \chi^{(n)} + \mathrm{H.c.} \right) \right.$$
$$\left. - \sum_{k=-\infty}^{\infty} C_{n,k} \bar{\chi}^{(n)}\left(m_{(n)} P_L - m_{(k)} P_R \right)\chi^{(k)} \right], \quad (2.18)$$

where g_w is the $SU(2)_L$ coupling constant, $c_w^2 = 1 - s_w^2 = M_W^2/M_Z^2$, $P_{L(R)} = [1 - (+)\gamma_5]/2$ are the chirality projection operators, and m_l and $m_{(n)}$ indicate the masses of the charged

leptons and KK neutrinos, respectively. The matrices B and C appearing in Eqs. (2.15)–(2.18) are defined as

$$B_{l\nu_k} = \sum_{i=e,\mu,\tau} V_{li}^l U_{ik}^\nu \quad (\text{with } k = e,\mu,\tau), \qquad B_{l,n} = \sum_{i=e,\mu,\tau} V_{li}^l U_{i,n}^\nu, \tag{2.19}$$

$$C_{\nu_l\nu_{l'}} = \sum_{k=e,\mu,\tau} B_{k\nu_l} B_{k\nu_{l'}}^*, \quad C_{\nu_l,n} = \sum_{k=e,\mu,\tau} B_{k\nu_l} B_{k,n}^*, \quad C_{n,m} = \sum_{k=e,\mu,\tau} B_{k,n} B_{k,m}^*, \tag{2.20}$$

where V^l is a unitary matrix that occurs in the diagonalization of the charged lepton mass matrix. From Eqs. (2.19) and (2.20), it is interesting to observe that the gauge interactions between the $SU(2)_L$ doublets L_l and the KK states $\chi^{(n)}$ are suppressed by a factor $m_l^{(n)}/m_{(n)}$, while the corresponding couplings to two KK neutrino states $\chi^{(n)}$ and $\chi^{(m)}$ are further suppressed by the ratio $m_l^{(n)} m_l^{(m)}/(m_{(n)} m_{(m)})$.

3 Tree-level constraints

The interaction Lagrangians (2.15)–(2.18) give rise to important phenomena of new physics both at the tree level and beyond. In this section, we will determine the new-physics contributions of KK neutrinos to electroweak observables, which occur at the tree level, and so derive limits on the fundamental Planck scale M_F and the mixing parameters of the theory.

The most striking feature of the higher-dimensional scenario is the loss of lepton universality in electroweak processes involving W- and Z-boson interactions. However, KK neutrinos may also lead to observable modifications of the muon lifetime, the invisible width of the Z boson, the cross section of the νe deep inelastic scatterings, etc. For our analysis, it proves useful to define the mixing parameters, which were first introduced by Langacker and London [25],

$$\begin{aligned}
(s_L^{\nu_l})^2 &\equiv 1 - \sum_{l'=e,\mu,\tau} |B_{l\nu_{l'}}|^2 = \sum_{n=-\infty}^{\infty} |B_{l,n}|^2 \\
&= [V^l(\mathbf{1}_3 + \Xi^*\Xi^T)^{-1/2}(\Xi^*\Xi^T)(\mathbf{1}_3 + \Xi^*\Xi^T)^{-1/2}V^{l\dagger}]_{ll}.
\end{aligned} \tag{3.1}$$

In order to evaluate the last equality on the RHS of Eq. (3.1), we will approximate the sum over the KK states by a higher-dimensional energy integral which has an upper ultraviolet (UV) cutoff at M_F, above which string-threshold effects are expected to become more important. To be precise, we make the replacement

$$\sum_{n=-\infty}^{\infty} \rightarrow S_\delta R^\delta \int_0^{M_F} E^{\delta-1} dE, \tag{3.2}$$

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where $S_\delta = 2\pi^{\delta/2}/\Gamma(\frac{\delta}{2})$ is the surface area of a δ-dimensional sphere of unit radius. Furthermore, we consider that the charged lepton mass matrix is diagonal and non-negative, i.e. $V^l = \mathbf{1}_3$. Then, to leading order in $(h_l v)/M_F$, we find that $(s_L^{\nu_l})^2 \approx (\Xi^* \Xi^T)_{ll}$, with

$$(\Xi^* \Xi^T)_{ll} = |m_l^{(0)}|^2 S_\delta R^\delta \int_0^{M_F} \frac{dE\, E^{\delta-1}}{m^2 + E^2} = \begin{cases} \dfrac{\pi h_l^2 v^2}{M_F^2} \ln\left(\dfrac{M_F^2 + m^2}{m^2}\right), & \text{for } \delta = 2 \\[3mm] \dfrac{S_\delta}{\delta-2} \dfrac{h_l^2 v^2}{M_F^2} \left[1 + \mathcal{O}(m^2/M_F^2)\right], & \text{for } \delta > 2 \end{cases}$$

$$(3.3)$$

Equation (3.3) shows that deviations of the $Wl\nu$ and $Z\nu\nu$ couplings from their SM values are logarithmically enhanced in a theory with two compact dimensions [26]. Furthermore, we find that the parameters $(s_L^{\nu_l})^2$ are not suppressed by the volume factor M_F^2/M_P^2, as it would have been the case if we had not summed over all the KK states [26]. In fact, after summation over the KK states, we obtain an effective theory, in which the Yukawa interactions are mediated by couplings h_l of order unity. As we will see in Sections 4 and 5, the latter gives rise to observable non-decoupling effects at the one-loop electroweak order, and can lead to more severe limits on the parameters of the theory than those considered here.

The mixing parameters $(s_L^{\nu_l})^2$ may now be constrained by a number of experimental data, which are obtained from: (i) the precise measurement of the muon width $\Gamma(\mu \to e\nu\nu)$; (ii) the neutrino counting at the Z peak; (iii) charged-current universality in pion decays, i.e. $\Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$; (iv) charged-current universality in tau decays, i.e. $B(\tau \to e\nu\nu)/B(\tau \to \mu\nu\nu)$. In the following, we shall discuss in more detail the constraints obtained from limits on the new-physics phenomena mentioned above.

(i) *Precise measurement of the muon lifetime.* The presence of KK neutrinos leads to a modification of the muon width $\Gamma(\mu \to e\nu\nu)$, when compared to the SM result $\Gamma_{\text{SM}}(\mu \to e\nu\nu)$. In particular, we have

$$1 - \frac{\Gamma(\mu \to e\nu\nu)}{\Gamma_{\text{SM}}(\mu \to e\nu\nu)} = (s_L^{\nu_e})^2 + (s_L^{\nu_\mu})^2 \stackrel{<}{\sim} 0.01\,. \tag{3.4}$$

The upper limit derived above is very conservative, in the sense that we also estimated the impact of neglecting high-order terms. The major new-physics contributions come from one-loop corrections to the W-boson propagator. These one-loop corrections are quantified by the electroweak oblique parameters, such as S, T and U [27], and will be discussed in Section 5. Furthermore, one-loop vertex effects generically introduce further corrections to the mixing parameters $(s_L^{\nu_l})^2$, which could be of order 15% [28].

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(ii) *Neutrino counting at the Z peak.* In the higher-dimensional model under study, the coupling of the Z boson to the massless neutrinos ν_l is reduced by a factor $1 - (s_L^{\nu_l})^2$ relative to the SM case. The latter would result in an observable change of the Z-boson invisible width at LEP, which is translated into the upper limit [29]

$$1 - \frac{\Gamma(Z \to \nu\nu)}{\Gamma_{\mathrm{SM}}(Z \to \nu\nu)} = (s_L^{\nu_e})^2 + (s_L^{\nu_\mu})^2 + (s_L^{\nu_\tau})^2 < 0.034 \,, \qquad (3.5)$$

at the 2σ level. Equations (3.4) and (3.5) give rather reliable upper limits on the absolute size of $(s_L^{\nu_l})^2$. The remaining observables measure possible deviations from lepton universality.

(iii) *Charged-current universality in pion decays.* One may now define an analogous observable in order to probe e-μ universality, i.e.

$$1 - \frac{R_\pi}{R_\pi^{\mathrm{SM}}} = (s_L^{\nu_e})^2 - (s_L^{\nu_\mu})^2 \,, \qquad (3.6)$$

where

$$R_\pi = \frac{\Gamma(\pi \to e\nu)}{\Gamma(\pi \to \mu\nu)} \qquad (3.7)$$

and R_π^{SM} is the SM result. The experimental analysis of the observable in Eq. (3.6) yields the 2σ-upper limit [30]‡

$$(s_L^{\nu_e})^2 - (s_L^{\nu_\mu})^2 < 0.003 \pm 0.006 \,. \qquad (3.8)$$

(iv) *Charged-current universality in tau decays.* Yet, universality-breaking effects in the leptonic sector through charged-current interactions can be examined in the tau decays, $\tau \to e\nu\nu$ and $\tau \to \mu\nu\nu$. For this purpose, we first define the quantities:

$$R_{\tau e} = \frac{\Gamma(\tau \to e\nu\nu)}{\Gamma(\mu \to e\nu\nu)} \,, \qquad R_{\tau\mu} = \frac{\Gamma(\tau \to \mu\nu\nu)}{\Gamma(\mu \to e\nu\nu)} \,. \qquad (3.9)$$

By analogy, the non-SM contributions to the above two observables are constrained by [30]

$$1 - \frac{R_{\tau e}}{R_{\tau e}^{\mathrm{SM}}} = (s_L^{\nu_\tau})^2 - (s_L^{\nu_e})^2 < 0.032 \pm 0.048 \,, \qquad (3.10)$$

$$1 - \frac{R_{\tau\mu}}{R_{\tau\mu}^{\mathrm{SM}}} = (s_L^{\nu_\tau})^2 - (s_L^{\nu_\mu})^2 < 0.040 \pm 0.048 \,, \qquad (3.11)$$

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‡Most recently, it was noticed [31] that, if $m = 0$ in Eq. (2.3), the lower bounds on M_F that are deduced from charged-current universality in pion decays can be very tight, when phase-space effects of KK states lighter than the pion are taken into account. Here, we assume that $m \sim M_W \gg m_\pi$, so the next-to-lightest KK state is much heavier than π^+. Therefore, the strict limits found in [31] do not apply to our minimal model.

Observable	$h_e \;=\; h_\mu \;=\; h_\tau \;=\; h$ Lower limit on M_F/h [TeV]		$h_\mu = 0$ and $h_e = h_\tau$ Lower limit on M_F/h_τ [TeV]	
	$\delta = 2$	$\delta > 2$	$\delta = 2$	$\delta > 2$
$1 - \dfrac{\Gamma(\mu \to e\nu\nu)}{\Gamma_{\mathrm{SM}}(\mu \to e\nu\nu)}$	$8.86 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{3.54\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$	$6.27 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{2.50\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$
$1 - \dfrac{\Gamma(Z \to \nu\nu)}{\Gamma_{\mathrm{SM}}(Z \to \nu\nu)}$	$5.89 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{2.35\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$	$4.81 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{1.92\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$
$1 - \dfrac{R_\pi}{R_\pi^{\mathrm{SM}}}$	–	–	$18.68 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{7.45\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$
$1 - \dfrac{R_{\tau\mu}}{R_{\tau\mu}^{\mathrm{SM}}}$	–	–	$5.72 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{2.28\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$

Table 1: Limits on M_F and h_l at the 2σ level.

at the 2σ level. More recent experimental analyses of τ-e and τ-μ universality [32] lead to a significant improvement of the above 2σ-upper limits, i.e.

$$(s_L^{\nu_\tau})^2 - (s_L^{\nu_e})^2 \quad \text{or} \quad (s_L^{\nu_\tau})^2 - (s_L^{\nu_\mu})^2 \; < \; 0.012 \,. \tag{3.12}$$

These last limits are competitive with those obtained from considerations of charged-current universality in pion decays.

In Table 1 we exhibit the lower limits on the fundamental scale M_F, as well as the upper limits on the higher-dimensional Yukawa couplings h_l, for two representative scenarios. In the first scenario, we have considered complete universality of the Yukawa couplings, i.e. $h_e = h_\mu = h_\tau = h$. In the second scenario, we assume that the μ lepton does not mix with the singlet neutrino $N(x, y)$, and $h_e = h_\tau$. Of course, we might have considered the converse case, in which electron does not mix with $N(x, y)$ instead of muon, but the predictions that we would obtain then would not differ much. In both scenarios, we assume that the singlet Dirac mass m is of order M_W. From Table 1, we see that precision

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measurements on muon lifetime offer the most sensitive test of lepton-flavour universality in the first scenario. For instance, if $h = 1$ and $m = 100$ GeV, we derive the limits:

$$M_F \ \gtrsim \ 20 \, , \ 12.5 \, , \ 9.9 \ \text{TeV} \, , \tag{3.13}$$

for $\delta = 2$, 3 and 6 large compact dimensions, respectively. In the second scenario, the best bounds are obtained by looking at deviations from charged-current universality in pion decays. Thus, if we take $h_e = h_\tau = 1$ and $m = 100$ GeV, the following lower bounds on M_F may be derived:

$$M_F \ \gtrsim \ 45 \, , \ 26 \, , \ 21 \ \text{TeV} \, , \tag{3.14}$$

for $\delta = 2$, 3 and 6 large compact dimensions, respectively. The very same limits apply to a third possible scenario, with $h_e = h_\mu = 0$ and $h_\tau = 1$. For comparison, we note that the respective lower limits on M_F derived from upper bounds on the invisible Z-boson width (c.f. Eq. (3.5)) in the third scenario are weaker, i.e.

$$M_F \ \gtrsim \ 8.2 \, , \ 4.7 \, , \ 3.8 \ \text{TeV} \, , \tag{3.15}$$

for 2, 3 and 6 large compact dimensions. Notice that the lower limits on M_F displayed in Eqs. (3.13)–(3.15) for the three different scenarios increase linearly with the Yukawa couplings h_l. There may be additional tree-level processes that could constrain the e-μ-τ sectors, e.g. observable change of the νe deep-inelastic-scattering data. However, the additional constraints turn out to be comparable to those we listed above. In Section 5, we shall see that one-loop non-decoupling effects of KK neutrinos can lead to much stronger bounds than those given by Eqs. (3.13)–(3.15).

4　Cumulative non-decoupling effect of Kaluza-Klein neutrinos

It is now very instructive to explicitly demonstrate how higher-dimensional Yukawa couplings of order unity give rise to a nondecoupling effect mediated by KK neutrinos in electroweak processes. As an example, we will consider the lepton-flavour violating vertex Zll', shown in Fig. 1. Similar nondecoupling effects occur in box diagrams involving KK neutrinos. A more quantitative discussion of one-loop constraints on the parameters of the theory follows in the next section.

Adopting the Feynman gauge for simplicity, the dominant contribution to the lepton-flavour violating vertex Zll' comes from the Feynman diagram, displayed in Fig. 1. The
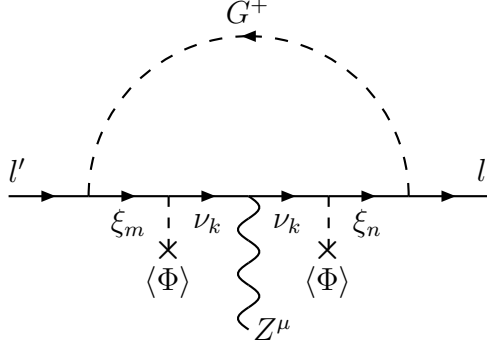
Figure 1: Feynman graph related to the dominant non-decoupling part of the effective Zll' coupling in the Feynman gauge.

non-decoupling part of the effective Zll' coupling may then be given by

$$\mathcal{T}(Zll') = \sum_{n,m=-\infty}^{\infty} \mathcal{T}_{(n,m)}(Zll'),\tag{4.1}$$

where

$$\mathcal{T}_{(n,m)}(Zll') = \frac{g_w}{4c_w}\,\bar{h}_l^{(n)}\,\bar{h}_{l'}^{(m)*}\left(\sum_{k=e,\mu,\tau}\bar{h}_k^{(n)*}\bar{h}_k^{(m)}\right)v^2$$

$$\times \int \frac{d^4k}{(2\pi)^4 i}\,P_R\frac{1}{\not{k}-m_{(n)}}\,P_L\,\frac{1}{\not{k}}\,\gamma_\mu P_L\,\frac{1}{\not{k}}\,P_R\,\frac{1}{\not{k}-m_{(m)}}\,P_L\,\frac{1}{k^2-M_W^2}\tag{4.2}$$

$$= (\gamma_\mu P_L)\,\frac{g_w}{64\pi^2 c_w}\,\frac{M_F^4}{M_P^4}\,\frac{h_l h_{l'}v^2}{M_W^2}\left(\sum_{k=e,\mu,\tau}h_k^2\right)\frac{\lambda_n\lambda_m\ln(\lambda_n/\lambda_m)+\lambda_m\ln\lambda_m-\lambda_n\ln\lambda_n}{(\lambda_m-\lambda_n)(1-\lambda_m)(1-\lambda_n)},$$

with $\lambda_n = m_{(n)}^2/M_W^2$. From the last equality in Eq. (4.2), one naively finds that the individual KK contributions $\mathcal{T}_{(n,m)}(Zll')$ are indeed tiny, since they are proportional to the volume-dependent suppression factor $M_F^4/M_P^4 \approx 10^{-64}$, for $M_F \approx 1$ TeV.

Let us now evaluate the sum over the KK states in Eq. (4.1). The loop function in the last equality of Eq. (4.2) receives its biggest support from KK neutrinos heavier than the W boson. Thus, converting the double sum over the KK states in Eq. (4.1) into a double integral (c.f. Eq. (3.2)) that has an infra-red (IR) cutoff at $M_W \sim m$, we find

$$\mathcal{T}(Zll') \approx -(\gamma_\mu P_L)\,\frac{g_w}{64\pi^2 c_w}\,\frac{M_F^4}{M_P^4}\,\frac{h_l h_{l'}v^2}{M_W^2}\left(\sum_{k=e,\mu,\tau}h_k^2\right)$$

$$\times \frac{M_W^2 S_\delta^2}{4M_F^2}\,(M_F R)^{2\delta}\int_{m^2/M_F^2}^{1} dz\,z^{\frac{\delta}{2}-1}\int_{m^2/M_F^2}^{1} dw\,w^{\frac{\delta}{2}-1}\,\frac{\ln(z/w)}{z-w}\,.\tag{4.3}$$

Employing the relation (1.1) among the parameters M_F, M_P and R, we observe that the huge suppression factor M_F^4/M_P^4 multiplying the individual KK contributions is cancelled against the total number of the active KK states, $(M_F R)^{2\delta}$! The double integral in Eq. (4.3) takes on values in the range 1–10^{-2}, for $2 \le \delta \le 6$. More precisely, to leading order in $\varepsilon \equiv m^2/M_F^2 \ll 1$, we have

$$I_\delta \equiv \frac{1}{4} \int_\varepsilon^1 dz\, z^{\frac{\delta}{2}-1} \int_\varepsilon^1 dw\, w^{\frac{\delta}{2}-1} \frac{\ln(z/w)}{z-w} = \begin{cases} \frac{1}{12}\pi^2\,, & \text{for } \delta = 2 \\ \frac{3}{8}\pi^2 - 1\,, & \text{for } \delta = 3 \\ \frac{1}{36}\pi^2 - \frac{1}{6}\,, & \text{for } \delta = 4 \\ \frac{3}{16}\pi^2 - \frac{5}{9}\,, & \text{for } \delta = 5 \\ \frac{1}{60}\pi^2 - \frac{1}{8}\,, & \text{for } \delta = 6 \end{cases} \qquad (4.4)$$

Notice that the above integral I_δ is geometrically enhanced for odd number of dimensions. Then, the effective Zll' coupling may be determined by

$$\begin{aligned} \mathcal{T}(Zll') &\approx -(\gamma_\mu P_L) \frac{g_w \alpha_w}{16\pi c_w} \frac{h_l h_{l'} v^2}{M_F^2} \left(\sum_{k=e,\mu,\tau} \frac{h_k^2 v^2}{M_W^2} \right) S_\delta^2\, I_\delta \\ &= -(\gamma_\mu P_L) \frac{g_w \alpha_w}{16\pi c_w} s_L^{\nu_l} s_L^{\nu_{l'}} \left[\sum_{k=e,\mu,\tau} (s_L^{\nu_k})^2 \right] \frac{M_F^2}{M_W^2}\, d_\delta\,, \end{aligned} \qquad (4.5)$$

where $\alpha_w = g_w^2/(4\pi)$ is the $\mathrm{SU}(2)_L$ fine structure constant, $(s_L^{\nu_l})^2$ are mixing parameters that have been estimated in Eq. (3.3), and d_δ are dimension-dependent correction factors that take the values:

$$d_2 = \frac{\pi^2}{12 \ln^2(M_F/m)} \approx \frac{0.822}{\ln^2(M_F/m)}\,, \qquad d_3 = \frac{3\pi^2}{8} - 1 \approx 2.701\,,$$

$$d_4 = \frac{\pi^2}{9} - \frac{2}{3} \approx 0.430\,, \qquad d_5 = \frac{27}{16}\pi^2 - 5 \approx 11.655\,,$$

$$d_6 = \frac{4\pi^2}{15} - 2 \approx 0.632\,. \qquad (4.6)$$

From Eq. (4.5), we observe that the strength of the effective Zll' coupling increases with the *fourth* power of the higher-dimensional Yukawa couplings, h_l, while it only decreases as $1/M_F^2$. Thus, the lower bounds on M_F, which are derived from limits on new-physics signals mediated by Z-boson interactions, increase *quadratically* with h_l.

An analogous cumulative non-decoupling phenomenon, with a kinematic dependence related to the effective Zll' coupling, takes place in box diagrams, which involve KK modes and two oppositely charged would-be Goldstone bosons, $G^\pm$, in the loop. Finally, we should remark that the effective $\gamma ll'$ vertex considered in [26] only scales quadratically with the Yukawa coupling h_l. As we will see in the next section, the bounds on M_F derived from the non-observation of photonic muon decays, $\mu \to e\gamma$, are found to be less restrictive than those obtained from $\mu \not\to eee$ and the absence of μ-to-e-conversion events in nuclei.

5 One-loop constraints

After having gained some insight of the cumulative non-decoupling mechanism of KK neutrinos, we shall now present analytic results for the most precisely tested lepton-flavour-violating and unversality-breaking processes that involve the $W^\pm$ and Z bosons, and the e, μ and τ leptons. These electroweak processes, which are strictly forbidden in the SM, are induced by KK states at the one-loop electroweak order. Based on these results, we are then able to derive very stringent limits on the parameters of the theory. Specifically, we confront the predictions of the higher-dimensional model with experimental data for the following set of observables of new physics: (i) photonic decays of the muon and tau leptons: $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$; (ii) decays of the μ and τ leptons into three lighter charged leptons, e.g. $\mu \to eee$, $\tau \to eee$, etc.; (iii) coherent μ-to-e conversion in nuclei; (iv) lepton-flavour-violating decays of the Z boson, $Z \to ll'$, universality-breaking effects among different diagonal leptonic decays of the Z boson, $Z \to ll$, and among its associate leptonic asymmetries; (v) the electroweak oblique parameters S, T and U [27], and especially $T \propto \delta\rho$ [33].

5.1 Photonic decays of the μ and τ leptons

As shown in Fig. 2, KK neutrinos can give rise to the photonic FCNC decays $\mu \to e\gamma$ and $\tau \to e\gamma$ or $\mu\gamma$. The transition element for the generic decay $l(p_l) \to l'(p_{l'})\gamma(q)$ may conveniently be given by

$$\mathcal{T}(l \to l'\gamma) \;=\; \frac{ie\alpha_w s_w^2}{16\pi M_W^2}\, G_\gamma^{ll'}\, \varepsilon_\gamma^\mu\, \bar{u}_{l'}\, i\sigma_{\mu\nu}q^\nu \Big[m_{l'}(1+\gamma_5) + m_l(1-\gamma_5) \Big] u_l \,, \qquad (5.1)$$

where $G_\gamma^{ll'}$ is the composite form factor

$$G_\gamma^{ll'} \;=\; \sum_{n=-\infty}^{\infty} B_{l,n}^* B_{l',n} G_\gamma(\lambda_n) \,, \qquad (5.2)$$

with

$$G_\gamma(x) \;=\; -\frac{2x^3 + 5x^2 - x}{4(1-x)^3} \;-\; \frac{3x^3 \ln x}{2(1-x)^4} \,. \qquad (5.3)$$

From this very last equation, it is obvious that the dominant contribution to $G_\gamma^{ll'}$ comes from values of $\lambda_{(n)} \gg 1$. Neglecting then terms subleading in $\lambda_{(n)}$ and using the fact that $G_\gamma(x) = 1/2$ in the infinite limit of x, the composite form factor $G_\gamma^{ll'}$ may easily be evaluated to be

$$G_\gamma^{ll'} \;=\; \frac{1}{2}\, s_L^{\nu_l} s_L^{\nu_{l'}} \,, \qquad (5.4)$$
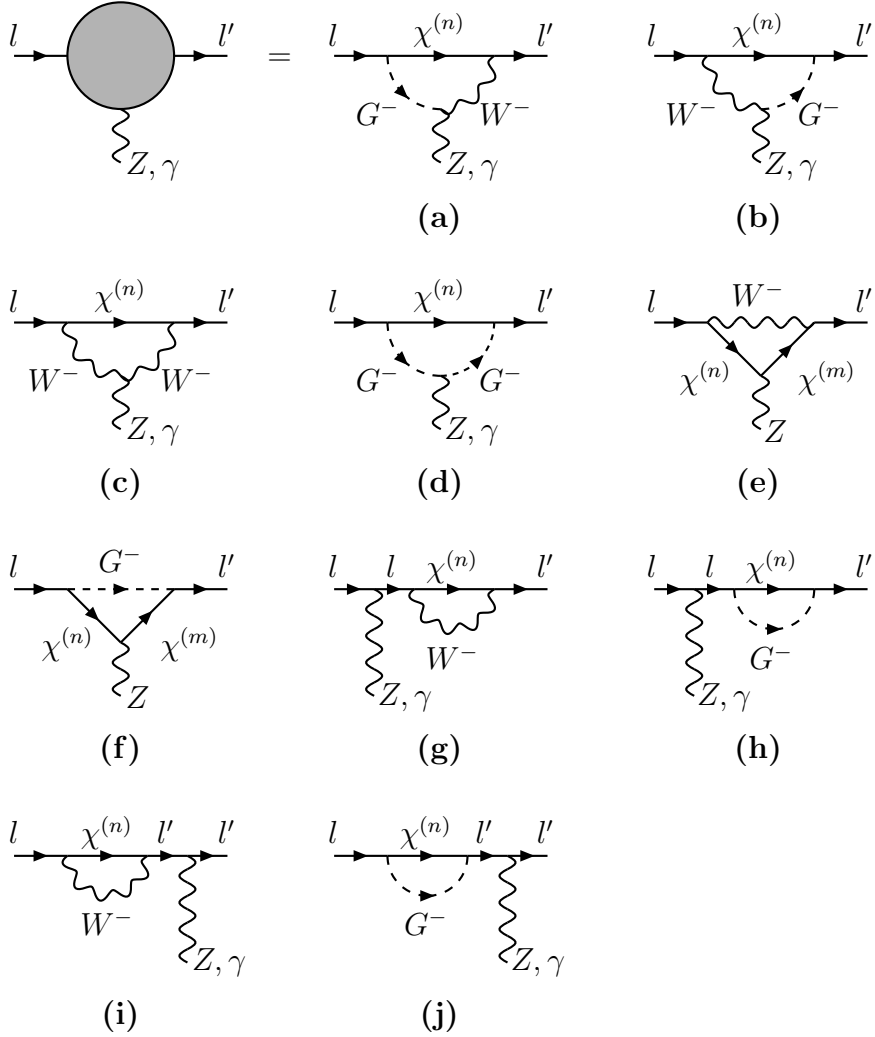
Figure 2: Feynman graphs pertaining to the effective $\gamma ll'$ and Zll' couplings.

where the mixing parameters, $s_L^{\nu_l}$, are defined in Eq. (3.1) and further estimated in Eq. (3.3).

Employing Eq. (5.4), the branching ratio of $l \to l'\gamma$ may be determined by

$$B(l \to l'\gamma) \;=\; \frac{\alpha_w^3 s_w^2}{256\pi^2} \frac{m_l^4}{M_W^4} \frac{m_l}{\Gamma_l} |G_\gamma^{ll'}|^2 \;\approx\; \frac{\alpha_w^3 s_w^2}{1024\pi^2} \frac{m_l^4}{M_W^4} \frac{m_l}{\Gamma_l} (s_L^{\nu_l})^2 (s_L^{\nu_{l'}})^2 \,. \tag{5.5}$$

On the experimental side, we have [29]

$$B_{\exp}(\mu \to e\gamma) \;<\; 4.9 \,(6.0) \times 10^{-11} \,, \quad B_{\exp}(\tau \to e\gamma) \;<\; 2.7 \,(3.3) \times 10^{-6} \,,$$

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$$B_{\exp}(\tau \to \mu\gamma) \ < \ 3.0\,(3.7) \times 10^{-6}\,, \tag{5.6}$$

at the 90% confidence level (CL). The numbers in the parentheses refer to upper limits at the 2σ level. Only the experimental bound on $B(\mu \to e\gamma)$ can lead to stronger constraints than those presented in Section 3. Using the experimentally measured value for the muon width, $\Gamma_\mu = 2.997 \times 10^{-19}$ GeV, we obtain the 2σ-upper limit on the product $s_L^{\nu_e} s_L^{\nu_\mu}$:

$$s_L^{\nu_e} s_L^{\nu_\mu} \ < \ 4.5 \times 10^{-4}\,. \tag{5.7}$$

If we take a scenario with $h_e = h_\mu = 1$ and $m = 100$ GeV, this very last bound translates into the lower limits on M_F:

$$M_F \ \gtrsim \ 75\,,\ 43\,,\ 33\ \text{TeV}\,, \tag{5.8}$$

for 2, 3 and 6 large compact dimensions, respectively. This result is in qualitative agreement with Ref. [26], even though their scenario differs from our minimal higher-dimensional model.

In the following, we shall see that owing to the cumulative non-decoupling effect of KK neutrinos, the non-observation of the decay $\mu \to eee$ and the absence of μ-to-e-conversion events in nuclei can lead to a much tighter bound than that given by Eq. (5.8).

5.2   Neutrinoless three-body decays of the μ and τ leptons

We shall calculate the non-decoupling loop effects of KK neutrinos in three-body decays of the μ and τ leptons: $\mu \to eee$, $\tau \to eee$, $\tau \to e\mu\mu$, $\tau \to \mu\mu\mu$, and $\tau \to ee\mu$.

For convenience, we first consider the generic decay process $\tau(p_\tau) \to l(p_l)l_1(p_{l_1})\bar{l}_2(p_{l_2})$, which is induced by the loop graphs shown in Figs. 2 and 3; analytic expressions for $\mu \to eee$ may then be obtained by an obvious interchange of the τ-μ kinematic parameters. As can be seen from Fig. 3, there are three amplitudes that contribute to the decay $\tau \to ll_1\bar{l}_2$: the photon-exchange amplitude, $\mathcal{T}_\gamma$; the Z-boson mediated one, $\mathcal{T}_Z$, and an amplitude describing the box contribution, $\mathcal{T}_{\text{box}}$. The three contributing amplitudes are given by

$$\mathcal{T}_\gamma(\tau \to ll_1\bar{l}_2) \ = \ -\frac{i\alpha_w^2 s_w^2}{4M_W^2}\delta_{l_1 l_2}\bar{u}_{l_1}\gamma^\mu v_{l_2}\ \bar{u}_l\Big\{ F_\gamma^{\tau l}\Big(\gamma_\mu - \frac{q_\mu \slashed{q}}{q^2}\Big)(1 - \gamma_5)$$
$$-iG_\gamma^{\tau l}\sigma_{\mu\nu}\frac{q^\nu}{q^2}\Big[m_\tau(1 + \gamma_5) + m_l(1 - \gamma_5)\Big] \Big\}u_\tau\,, \tag{5.9}$$

$$\mathcal{T}_Z(\tau \to ll_1\bar{l}_2) \ = \ -\frac{i\alpha_w^2}{16M_W^2}\ \delta_{l_1 l_2}F_Z^{\tau l}\bar{u}_l\gamma_\mu(1 - \gamma_5)u_\tau\ \bar{u}_{l_1}\gamma^\mu(1 - 4s_w^2 - \gamma_5)v_{l_2}\,, \tag{5.10}$$

$$\mathcal{T}_{\text{box}}(\tau \to ll_1\bar{l}_2) \ = \ -\frac{i\alpha_w^2}{16M_W^2}\ F_{\text{box}}^{\tau ll_1 l_2}\ \bar{u}_l\gamma_\mu(1 - \gamma_5)u_\tau\ \bar{u}_{l_1}\gamma^\mu(1 - \gamma_5)v_{l_2}\,, \tag{5.11}$$
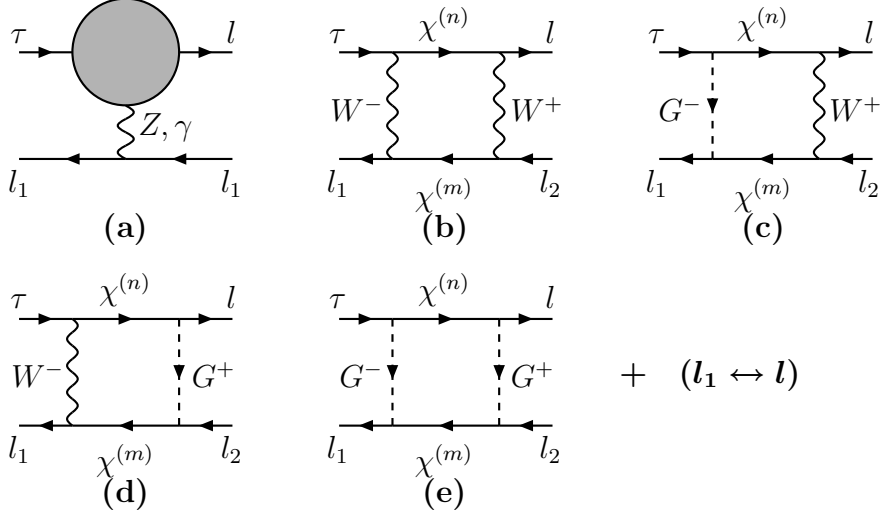
Figure 3: Feynman graphs pertaining to the decay $\tau \to ll_1\bar{l}_2$.

where $q = p_\tau - p_l$ and [18]

$$F_\gamma^{\tau l} = \sum_{n=-\infty}^{\infty} B_{\tau,n}^* B_{l,n} F_\gamma(\lambda_n) , \qquad (5.12)$$

$$\begin{aligned} F_Z^{\tau l} = \sum_{n,m=-\infty}^{\infty} B_{\tau,n}^* B_{l,m} &\Big\{ \delta_{nm} \Big[F_Z(\lambda_n) + 2G_Z(0, \lambda_n) \Big] \\ &+ C_{n,m}^* \Big[G_Z(\lambda_n, \lambda_m) - G_Z(0, \lambda_n) - G_Z(0, \lambda_m) \Big] \Big\}, \end{aligned} \qquad (5.13)$$

$$\begin{aligned} F_{\text{box}}^{\tau l l_1 l_2} = \sum_{n,m=-\infty}^{\infty} &\Big\{ \delta_{nm} \Big(B_{\tau,n}^* B_{l,m} \delta_{l_1 l_2} + B_{\tau,n}^* B_{l_1,m} \delta_{l l_2} \Big) \Big[F_{\text{box}}(0, \lambda_n) - F_{\text{box}}(0,0) \Big] \\ &+ B_{\tau,n}^* B_{l_2,m}^* \Big(B_{l,n} B_{l_1,m} + B_{l_1,n} B_{l,m} \Big) \Big[F_{\text{box}}(\lambda_n, \lambda_m) \\ &- F_{\text{box}}(0, \lambda_n) - F_{\text{box}}(0, \lambda_m) + F_{\text{box}}(0,0) \Big] \Big\}. \end{aligned} \qquad (5.14)$$

Note that the composite form factor $G_\gamma^{\tau l}$ is defined in Eq. (5.2). In deriving the expressions in Eqs. (5.13) and (5.14), we made use of the unitarity relations, which the B and C matrices defined in Eqs. (2.19) and (2.20) obey. Furthermore, the analytic forms of the loop functions F_γ, F_Z, G_Z, and F_{box}, which may also be found in [18, 34], are given by

$$F_\gamma(x) = \frac{7x^3 - x^2 - 12x}{12(1-x)^3} - \frac{(x^4 - 10x^3 + 12x^2)\ln x}{6(1-x)^4} , \qquad (5.15)$$

$$F_Z(x) = -\frac{5x}{2(1-x)} - \frac{5x^2 \ln x}{2(1-x^2)} , \qquad (5.16)$$

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$$G_Z(x, y) = -\frac{1}{2(x-y)}\left[\frac{x^2(1-y)\ln x}{1-x} - \frac{y^2(1-x)\ln y}{1-y}\right], \tag{5.17}$$

$$F_{\text{box}}(x, y) = \frac{1}{x-y}\left[\left(1 + \frac{xy}{4}\right)\left(\frac{1}{1-x} + \frac{x^2\ln x}{(1-x)^2} - \frac{1}{1-y} - \frac{y^2\ln y}{(1-y)^2}\right)\right.$$
$$\left. - 2xy\left(\frac{1}{1-x} + \frac{x\ln x}{(1-x)^2} - \frac{1}{1-y} - \frac{y\ln y}{(1-y)^2}\right)\right]. \tag{5.18}$$

As we discussed in Section 5, to leading order in M_F/M_W, the dominant contributions to the composite form factors in Eqs. (5.12)–(5.14) come from KK states heavier than the W boson. More explicitly, we have

$$F_\gamma^{\tau l} \approx -\frac{7}{12}s_L^{\nu_\tau}s_L^{\nu_l} - \frac{c_\delta}{6}s_L^{\nu_\tau}s_L^{\nu_l}\ln\lambda_F, \tag{5.19}$$

$$F_Z^{\tau l} \approx \left(\frac{5}{2} - \frac{3c_\delta}{2}\ln\lambda_F\right)s_L^{\nu_\tau}s_L^{\nu_l} - \frac{d_\delta}{2}s_L^{\nu_\tau}s_L^{\nu_l}\sum_{k=e,\mu,\tau}(s_L^{\nu_k})^2\lambda_F, \tag{5.20}$$

$$F_{\text{box}}^{\tau l_1 l_2} \approx -s_L^{\nu_\tau}s_L^{\nu_l}\delta_{l_1 l_2} - s_L^{\nu_\tau}s_L^{\nu_{l_1}}\delta_{l l_2} + \frac{d_\delta}{2}s_L^{\nu_\tau}s_L^{\nu_l}s_L^{\nu_{l_1}}s_L^{\nu_{l_2}}\lambda_F, \tag{5.21}$$

where $\lambda_F = M_F^2/M_W^2$, $c_\delta = 1/2$, for $\delta = 2$, and $c_\delta = 1$, for $\delta > 2$. In Eqs. (5.20) and (5.21), d_δ are dimension-dependent correction factors, which are given in Eq. (4.6).

The branching ratios for the decays $\tau \to e\mu\mu$ and $\tau \to eee$ were calculated in [18, 17]. Their analytic expressions may be cast into the form:

$$B(\tau \to e\mu\mu) = \frac{\alpha_w^4}{24576\pi^3}\frac{m_\tau^4}{M_W^4}\frac{m_\tau}{\Gamma_\tau}\left\{|F_{\text{box}}^{\tau e\mu\mu} + F_Z^{\tau e} - 2s_w^2(F_Z^{\tau e} - F_\gamma^{\tau e})|^2\right.$$
$$+ 4s_w^4|F_Z^{\tau e} - F_\gamma^{\tau e}|^2 + 8s_w^2\,\text{Re}\left[(F_Z^{\tau e} + F_{\text{box}}^{\tau e\mu\mu})G_\gamma^{\tau e\,*}\right]$$
$$\left. - 32s_w^4\,\text{Re}\left[(F_Z^{\tau e} - F_\gamma^{\tau e})G_\gamma^{\tau e\,*}\right] + 32s_w^4|G_\gamma^{\tau e}|^2\left(\ln\frac{m_\tau^2}{m_\mu^2} - 3\right)\right\}, \tag{5.22}$$

$$B(\tau \to eee) = \frac{\alpha_w^4}{24576\pi^3}\frac{m_\tau^4}{M_W^4}\frac{m_\tau}{\Gamma_\tau}\left\{2|\tfrac{1}{2}F_{\text{box}}^{\tau eee} + F_Z^{\tau e} - 2s_w^2(F_Z^{\tau e} - F_\gamma^{\tau e})|^2\right.$$
$$+ 4s_w^4|F_Z^{\tau e} - F_\gamma^{\tau e}|^2 + 16s_w^2\,\text{Re}\left[(F_Z^{\tau e} + \tfrac{1}{2}F_{\text{box}}^{\tau eee})G_\gamma^{\tau e\,*}\right]$$
$$\left. - 48s_w^4\,\text{Re}\left[(F_Z^{\tau e} - F_\gamma^{\tau e})G_\gamma^{\tau e\,*}\right] + 32s_w^4|G_\gamma^{\tau e}|^2\left(\ln\frac{m_\tau^2}{m_e^2} - \frac{11}{4}\right)\right\}. \tag{5.23}$$

The branching ratio for the decay $\mu \to eee$ may be obtained by making in Eq. (5.23) the obvious replacements: $m_\tau \to m_\mu$ and $m_\mu \to m_e$. Taking the dominant nondecoupling parts of the composite form factors into account, we obtain for the branching ratios:

$$B(\tau \to e\mu\mu) \simeq \frac{\alpha_w^4}{98304\pi^3}\frac{m_\tau^4}{M_W^4}\frac{m_\tau}{\Gamma_\tau}\frac{M_F^4}{M_W^4}\,d_\delta^2\,(s_L^{\nu_\tau})^2(s_L^{\nu_e})^2\left\{(s_L^{\nu_\mu})^4\right.$$

$$+ 2(1 - 2s_w^2)(s_L^{\nu_\mu})^2 \Big[\sum_{l=e,\mu,\tau} (s_L^{\nu_l})^2 \Big] + 8s_w^4 \Big[\sum_{l=e,\mu,\tau} (s_L^{\nu_l})^2 \Big]^2 \Big\}, \quad (5.24)$$

$$B(\tau \to eee) \simeq \frac{\alpha_w^4}{98304\pi^3} \frac{m_\tau^4}{M_W^4} \frac{m_\tau}{\Gamma_\tau} \frac{M_F^4}{M_W^4} d_\delta^2 (s_L^{\nu_\tau})^2 (s_L^{\nu_e})^2 \Big\{ \frac{1}{2}(s_L^{\nu_e})^4$$

$$+ 2(1 - 2s_w^2)(s_L^{\nu_e})^2 \Big[\sum_{l=e,\mu,\tau} (s_L^{\nu_l})^2 \Big] + 12s_w^4 \Big[\sum_{l=e,\mu,\tau} (s_L^{\nu_l})^2 \Big]^2 \Big\}. \quad (5.25)$$

The present experimental upper limits on the branching ratios for the lepton-flavour-violating three-body decays of μ and τ leptons are [29]

$$B_{\exp}(\mu \to eee) < 1.0\,(1.2) \times 10^{-12}, \qquad B_{\exp}(\tau \to e\mu\mu) < 1.8\,(2.2) \times 10^{-6},$$

$$B_{\exp}(\tau \to eee) < 2.9\,(3.5) \times 10^{-6}, \qquad B_{\exp}(\tau \to \mu ee) < 1.5\,(1.8) \times 10^{-6},$$

$$B_{\exp}(\tau \to \mu\mu\mu) < 1.9\,(2.3) \times 10^{-6}, \qquad (5.26)$$

at the 90% confidence (2σ) level. In the scenario, with complete Yukawa-coupling universality, i.e. $h_e = h_\mu = h_\tau = h$, the most severe bound on M_F comes from $B_{\exp}(\mu \to eee)$. For $h = 1$ and $m = 100$ GeV, we obtain the lower limits:

$$M_F \gtrsim \begin{cases} 250\text{ TeV}, & \text{for } \delta = 2 \\ 600\text{ TeV}, & \text{for } \delta = 3 \\ 220\text{ TeV}, & \text{for } \delta = 4 \\ 830\text{ TeV}, & \text{for } \delta = 5 \\ 205\text{ TeV}, & \text{for } \delta = 6 \end{cases} \qquad (5.27)$$

The above limits are stronger for odd number of dimensions, as the δ-dependent correction factors d_δ in Eq. (4.6) come out to be geometrically enhanced in this case. If the limits in Eq. (5.27) are implemented in this scenario, the predictions obtained for $B(\tau \to eee)$ and $B(\tau \to e\mu\mu)$ are hopelessly small of order 10^{-12} to be tested in any future experiment.

The high upper bounds on M_F given in Eqs. (5.8) and (5.27) may be completely avoided in a scenario, in which $h_\mu = 0$ and $h_e = h_\tau = h$. In this case, M_F is only constrained by limits presented in Table 1. Implementing these constraints for $h = 1$ and $m = 100$ GeV (see also Eq. (3.14)), we find

$$B(\tau \to eee) \lesssim \left(7.2 \times 10^{-11}, 7.9 \times 10^{-8}, 6.5 \times 10^{-10}, 1.8 \times 10^{-7}, 6.8 \times 10^{-10} \right),$$

$$B(\tau \to e\mu\mu) \lesssim \left(5.9 \times 10^{-11}, 2.3 \times 10^{-8}, 6.6 \times 10^{-11}, 2.3 \times 10^{-7}, 7.2 \times 10^{-10} \right). (5.28)$$

The numbers in the parentheses correspond to numerical estimates in a theory with $\delta = 2, 3, 4, 5, 6$ large extra dimensions, respectively. From Eq. (5.28), we see that the predictions for $\tau \to e\mu\mu$ and $\tau \to eee$, with $\delta = 5$, might have some chance to be probed in future experiments.

5.3 Coherent μ-to-e conversion in nuclei

Coherent $\mu \to e$ conversion in nuclei, e.g. $\mu^- \ {}^{48}_{22}\text{Ti} \to e^- \ {}^{48}_{22}\text{Ti}$, constitutes one of the most sensitive experiments that can set severe bounds on lepton-flavour-violating physics [35, 36]. The existence of KK neutrinos may give rise to a sizeable $\mu \to e$ conversion in nuclei, which comes from the following γ-, Z- and box-mediated transition amplitudes:

$$
\begin{aligned}
\mathcal{T}_\gamma(\mu \to e) &= \frac{i\alpha_w^2 s_w^2}{4M_W^2} \ \bar{u}_e \Big[F_\gamma^{\mu e} \gamma_\nu (1 - \gamma_5) - iG_\gamma^{\mu e} \sigma_{\nu\lambda} \frac{q^\lambda}{q^2} m_\mu (1 + \gamma_5) \Big] u_\mu \\
&\quad \times \Big(\frac{2}{3} \bar{u}_u \gamma^\nu u_u - \frac{1}{3} \bar{u}_d \gamma^\nu u_d \Big),
\end{aligned} \tag{5.29}
$$

$$
\begin{aligned}
\mathcal{T}_Z(\mu \to e) &= \frac{i\alpha_w^2}{16M_W^2} \ F_Z^{\mu e} \ \bar{u}_e \gamma_\nu (1 - \gamma_5) u_\mu \Big[\Big(1 - \frac{8}{3} s_w^2 \Big) \bar{u}_u \gamma^\nu u_u \\
&\quad - \Big(1 - \frac{4}{3} s_w^2 \Big) \bar{u}_d \gamma^\nu u_d \Big],
\end{aligned} \tag{5.30}
$$

$$
\mathcal{T}_{\text{box}}(\mu \to e) = -\frac{i\alpha_w^2}{16M_W^2} \ \bar{u}_e \gamma_\nu (1 - \gamma_5) u_\mu \Big[F_{\text{box}}^{\mu euu} \ \bar{u}_u \gamma^\nu u_u + F_{\text{box}}^{\mu edd} \ \bar{u}_d \gamma^\nu u_d \Big], \tag{5.31}
$$

where

$$
F_{\text{box}}^{\mu euu} = \sum_{n=-\infty}^{\infty} B_{\mu,n}^* B_{e,n} \Big[F_{\text{box}}(0, \lambda_n) - F_{\text{box}}(0, 0) \Big],
$$

$$
\begin{aligned}
F_{\text{box}}^{\mu edd} &= \sum_{n=-\infty}^{\infty} \Big\{ B_{\mu,n}^* B_{e,n} \Big[F_{\text{box}}(0, \lambda_n) - F_{\text{box}}(0, 0) \Big] + B_{\mu,n}^* B_{e,n} |V_{td}|^2 \Big[F_{\text{box}}(\lambda_t, \lambda_n) \\
&\quad - F_{\text{box}}(0, \lambda_n) - F_{\text{box}}(0, \lambda_t) + F_{\text{box}}(0, 0) \Big] \Big\},
\end{aligned} \tag{5.32}
$$

with $\lambda_t = m_t^2/M_W^2$ and $|V_{td}| \approx 0.01$ [29]. In Eqs. (5.29)–(5.31), we have only considered the vectorial coupling of the u and d quarks, which is coherently enhanced in nuclei, whereas the axial part of the quark couplings describes spin-dependent interactions, and therefore, their total contribution is almost vanishing [35, 36].

Following [35, 36], we consider the kinematic approximations: $q^2 \approx -m_\mu^2$ and $p_e^0 \approx |\vec{p}_e| \approx m_\mu$, which are applicable for $\mu \to e$ conversion in nuclei. Then, for nuclei with nucleon numbers (N, Z), we obtain

$$
B_{\mu e}(N, Z) \equiv \frac{\Gamma[\mu(N, Z) \to e(N, Z)]}{\Gamma[\mu(N, Z) \to \text{capture}]} \approx \frac{\alpha_{\text{em}}^3 \alpha_w^4 m_\mu^5}{32\pi^2 M_W^4 \Gamma_{\text{capt.}}} \frac{Z_{\text{eff}}^4}{Z} |F(-m_\mu^2)|^2 |Q_W|^2, \tag{5.33}
$$

where $\alpha_{\text{em}} = 1/137$ is the electromagnetic fine structure constant, Z_{eff} is the effective atomic number of coherence (e.g. $Z_{\text{eff}} \approx 17.6$ for ${}^{48}_{22}\text{Ti}$) [37], $|F(-m_\mu^2)| \approx 0.54$ is the nuclear form factor [38], and $Q_W = V_u(2Z + N) + V_d(Z + 2N)$, with

$$
V_u = \frac{2}{3} s_w^2 \Big(F_\gamma^{\mu e} - G_\gamma^{\mu e} - F_Z^{\mu e} \Big) + \frac{1}{4} \Big(F_Z^{\mu e} - F_{\text{box}}^{\mu euu} \Big), \tag{5.34}
$$

$$V_d = -\frac{1}{3} s_w^2 \left(F_\gamma^{\mu e} - G_\gamma^{\mu e} - F_Z^{\mu e} \right) - \frac{1}{4} \left(F_Z^{\mu e} + F_{\text{box}}^{\mu e dd} \right). \tag{5.35}$$

The most strict upper limit on $B_{\mu e}(N, Z)$ is obtained from experimental data of $\mu \to e$ conversion in $^{48}_{22}\text{Ti}$ [39]:

$$B_{\mu e}^{\text{exp}}(26, 22) < 4.3 \, (5.24) \times 10^{-12}, \tag{5.36}$$

at the 90% confidence (2σ) level. For numerical predictions, we use the experimental value of the muon nuclear capture rate, $\Gamma[\mu \, ^{48}_{22}\text{Ti} \to \text{capture}] \approx 1.705 \times 10^{-18}$ GeV [40].

For definiteness, we shall now consider the scenario with all Yukawa couplings equal to unity and $m = 100$ GeV. Then, the experimental upper bound on $B_{\mu e}(26, 22)$ gives the following lower limits on M_F:

$$M_F \gtrsim \begin{cases} 361 \text{ TeV}, & \text{for } \delta = 2 \\ 826 \text{ TeV}, & \text{for } \delta = 3 \\ 301 \text{ TeV}, & \text{for } \delta = 4 \\ 1182 \text{ TeV}, & \text{for } \delta = 5 \\ 282 \text{ TeV}, & \text{for } \delta = 6 \end{cases} \tag{5.37}$$

These limits represent the most severe bounds that one is able to derive on the fundamental quantum gravity scale M_F from laboratory experiments.

5.4 Lepton-flavour violation and breaking of universality at the Z peak

Loop effects of KK neutrinos may induce a number of lepton-flavour and universality-breaking phenomena on the Z-boson pole. The most striking experimental signals of new physics would be FCNC Z-boson decays into different leptons, e.g. $Z \to ll'$. Further new-physics signals would be the detection of breaking of universality in the leptonic partial widths $Z \to l\bar{l}$ and in the Z-boson leptonic asymmetries, which may be probed by looking at the observables [16, 19]:

$$U_{\text{br}}^{(ll')} = \frac{\Gamma(Z \to l\bar{l}) - \Gamma(Z \to l'\bar{l}')}{\Gamma(Z \to l\bar{l}) + \Gamma(Z \to l'\bar{l}')} - U_{\text{br}}^{\text{PS}} = \frac{|\mathcal{T}(Z \to l\bar{l})|^2 - |\mathcal{T}(Z \to l'\bar{l}')|^2}{|\mathcal{T}(Z \to l\bar{l})|^2 + |\mathcal{T}(Z \to l'\bar{l}')|^2}, \tag{5.38}$$

$$\Delta\mathcal{A}_{ll'} = \frac{\mathcal{A}_l - \mathcal{A}_{l'}}{\mathcal{A}_l + \mathcal{A}_{l'}} = \left(\frac{1}{\mathcal{A}_l^{\text{SM}}} - 1 \right) U_{\text{br}}^{(ll')}, \quad \text{with} \quad l \neq l', \tag{5.39}$$

where $U_{\text{br}}^{\text{PS}}$ indicates known phase-space corrections due to the finiteness of the charged-lepton masses, and $\mathcal{A}_l^{\text{SM}} = 0.14$ is the SM prediction [29]. In deriving the last equality of Eq. (5.39), we used the theoretical fact that the induced coupling of the Z boson to

charged leptons is predominantly left-handed, as is the case in our higher-dimensional singlet-neutrino model (c.f. Eq. (5.10)).

The branching ratio of the FCNC decay $Z \to ll'$ is given by

$$
\begin{aligned}
B(Z \to l\bar{l}' \text{ or } \bar{l}l') &= \frac{\alpha_w^3}{48\pi^2 c_w^2} \frac{M_Z}{\Gamma_Z} |\mathcal{F}_Z^{ll'}(M_Z^2)|^2 \\
&\approx \frac{\alpha_w^3}{768\pi^2 c_w^2} \frac{M_Z}{\Gamma_Z} \frac{M_F^4}{M_W^4} d_\delta^2 (s_L^{\nu_l})^2 (s_L^{\nu_{l'}})^2 \Big[\sum_{k=e,\mu,\tau} (s_L^{\nu_k})^2 \Big]^2,
\end{aligned}
\tag{5.40}
$$

where the loop function $\mathcal{F}_Z^{ll'}(M_Z^2)$ was calculated in [15, 18]. To obtain the last equality in Eq. (5.40), we used the approximation: $\mathcal{F}_Z^{ll'}(M_Z^2) \approx \mathcal{F}_Z^{ll'}(0) = F_Z^{ll'}/2$, where $F_Z^{ll'}$ is given by Eq. (5.13). Furthermore, the universality-breaking observable $U_{\mathrm{br}}^{(ll')}$ is found to be

$$
\begin{aligned}
U_{\mathrm{br}}^{(ll')} &= \frac{\alpha_w}{2\pi} \frac{1 - 2s_w^2}{(1 - 2s_w^2)^2 + 4s_w^4} \left[\mathcal{F}_Z^{ll}(M_Z^2) - \mathcal{F}_Z^{l'l'}(M_Z^2) \right] \\
&\approx \frac{\alpha_w}{8\pi} \frac{1 - 2s_w^2}{(1 - 2s_w^2)^2 + 4s_w^4} \frac{M_F^2}{M_W^2} d_\delta \left[(s_L^{\nu_l})^2 - (s_L^{\nu_{l'}})^2 \right] \Big[\sum_{k=e,\mu,\tau} (s_L^{\nu_k})^2 \Big].
\end{aligned}
\tag{5.41}
$$

The current experimental situation on $B(Z \to ll')$ and $U_{\mathrm{br}}^{(ll')}$ is as follows [29]:

$$
B_{\exp}(Z \to e\mu) < 1.7 \times 10^{-6}, \quad B_{\exp}(Z \to e\tau) < 9.8 \times 10^{-6}, \quad B_{\exp}(Z \to \mu\tau) < 1.2 \times 10^{-5},
\tag{5.42}
$$

and, almost independently of the lepton flavours l and l',

$$
U_{\mathrm{br}}^{(ll')\exp} = \frac{1}{2}\left(1 - \frac{\Gamma(Z \to l'l')}{\Gamma(Z \to ll)}\right) = 5.0 \times 10^{-3},
\tag{5.43}
$$

at the 2σ level. The experimental limits on the universality-breaking parameter $\Delta \mathcal{A}_{ll'}$ are slightly weaker than those derived by $U_{\mathrm{br}}^{(ll')}$, if one uses the relation given by Eq. (5.39), i.e. $\Delta \mathcal{A}_{ll'} < 3.0 \times 10^{-2}$.

In view of the constraints derived in Sections 5.1–5.3, new-physics phenomena in Z-boson decays with electrons and muons only in the final state are far beyond the realm of detection. Therefore, we shall discuss the numerical predictions obtained for the FCNC Z-boson decay $Z \to e\tau$ and $U_{\mathrm{br}}^{(\mu\tau)}$ in a more favourable phenomenologically scenario with $h_e = h_\tau = h$ and $h_\mu = 0$. Considering $h = 1$ and $m = 100$ GeV and lower limits on M_F stated in Eq. (3.14), we obtain the following upper limits for $\delta = 2$ through 6:

$$
\begin{aligned}
B(Z \to e\tau) &\lesssim \left(2.9 \times 10^{-10},\ 1.3 \times 10^{-7},\ 3.6 \times 10^{-9},\ 1.1 \times 10^{-6},\ 3.9 \times 10^{-9}\right), \\
U_{\mathrm{br}}^{(\mu\tau)} &\lesssim \left(5.0 \times 10^{-5},\ 1.1 \times 10^{-3},\ 1.8 \times 10^{-4},\ 3.1 \times 10^{-3},\ 1.9 \times 10^{-4}\right).
\end{aligned}
\tag{5.44}
$$

Obviously, singlet-neutrino theories with 5 large compact dimensions give predictions close to the current experimental sensitivity.

Finally, it is illuminating to discuss the upper bounds that can be obtained for M_F in a more weakly constrained scenario, i.e. $h_e = h_\mu = 0$ and $h_\tau \neq 0$. In addition, we assume that this scenario is only constrained by experimental limits on the invisible width of the Z boson (c.f. Eq. (3.15)). If $h_\tau = 1$ and $m = 100$ GeV, the experimental bounds on $U_{\rm br}^{(e\tau)}$ and $U_{\rm br}^{(\mu\tau)}$ yield

$$
M_F \; \gtrsim \; \begin{cases}
3.0 \text{ TeV}, & \text{for } \delta = 2 \\
9.5 \text{ TeV}, & \text{for } \delta = 3 \\
3.2 \text{ TeV}, & \text{for } \delta = 4 \\
12.3 \text{ TeV}, & \text{for } \delta = 5 \\
2.8 \text{ TeV}, & \text{for } \delta = 6
\end{cases}
\tag{5.45}
$$

These limits are comparable to those deduced by experimental data on the invisible Z-boson width in Eq. (3.15) for the even number of large compact dimensions, but are tighter by a factor 2 to 3 for the odd ones.

5.5   Contribution to the electroweak oblique parameters

Finally, KK neutrinos may manifest their presence by inducing sizeable contributions to the electroweak oblique parameters S, T and U [27]. We find that a cumulative non-decoupling effect of KK states also occurs here, very analogously to the previous flavour-dependent observables. For the kind of the new physics we are considering, T turns out to be the most sensitive electroweak oblique parameter, which is related to Veltman's ρ parameter [33] through

$$
\rho - 1 \; = \; \alpha_{\rm em}(M_Z)\, T \; = \; \frac{\Pi_{WW}^{\rm KK}(0)}{M_W^2} - \frac{\Pi_{ZZ}^{\rm KK}(0)}{M_Z^2} \; .
\tag{5.46}
$$

In Eq. (5.46), $\Pi_{WW}^{\rm KK}(0)$ and $\Pi_{ZZ}^{\rm KK}(0)$ indicate the KK-neutrino contributions to the W- and Z-boson self-energies, respectively.

The most recent experimental constraint on T is $T = -0.21 \pm 0.16\,(+0.10)$, for $M_H = M_Z$ (300 GeV) [29]. This corresponds to the 2σ-upper bound on $(\rho - 1)$:

$$
(\rho - 1)_{\rm exp} \; \leq \; 2.42 \times 10^{-3}\,,
\tag{5.47}
$$

for a Higgs-boson mass $M_H = 300$ GeV. On the other hand, to leading order in M_F/M_W, the electroweak oblique parameter $(\rho - 1)$ is found to be

$$
\rho - 1 \; \approx \; \frac{\alpha_w}{16\pi}\, \frac{M_F^2}{M_W^2}\, d_\delta \Big[\sum_{l=e,\mu,\tau} (s_L^{\nu_l})^2 \Big]^2 \,.
\tag{5.48}
$$

Combining this last theoretical prediction with the limit in Eq. (5.47), we find

$$
\left(\frac{M_F}{1 \text{ TeV}} \right) \;\gtrsim\; \begin{cases} 2.21 \left(\sum\limits_{l=e,\mu,\tau} h_l^2 \right), & \text{for } \delta = 2 \\[4ex] 0.387 \, d_\delta^{1/2} \left(\sum\limits_{l=e,\mu,\tau} h_l^2 \right) \dfrac{S_\delta}{\delta - 2} \,, & \text{for } \delta > 2 \end{cases}
\tag{5.49}
$$

The above constraints are much weaker than those derived from limits on non-oblique new-physics observables.

Before closing Section 5, we wish to comment on the fact that theoretical uncertainties related to the one-loop results may exist. In the loop calculations, one usually considers that the KK theory is truncated at some scale M_F' close to M_F, where M_F' represents the energy scale of the active KK states, beyond which string effects are still negligible [6]. In the present analysis, we assumed that $M_F' = M_F$. Depending on the dynamics of a given string theory, however, one generally has $M_F' \leq M_F$, i.e. $x_F \equiv M_F'/M_F \leq 1$. Thus, for $x_F \neq 1$, the non-decoupling contributions discussed in Section 4 (e.g. the Zll' coupling given by Eq. (4.5)) must be multiplied by an additional factor $(x_F)^{2\delta}$. For example, if $x_F = 0.9$ and $\delta = 6$, this additional multiplicative factor is ~ 0.3. On the other hand, there may exist compensating factors which can be obtained by modestly rescaling the normalization of the higher-dimensional Yukawa couplings h_l in Eq. (2.4). For instance, if h_l is rescaled by a factor 1.35 for $\delta = 6$, then the factor $(x_F)^{2\delta}$, with $x_F = 0.9$, drops out completely. Because of the above theoretical uncertainties which are generally inherent to all truncated KK theories, the numerical predictions for the new-physics phenomena and the derived limits should rather be viewed as order-of-magnitude estimates.

6 Conclusions

We studied the phenomenological implications of Kaluza-Klein theories of low-scale quantum gravity which may naturally predict, in addition to gravitons, singlet neutrinos. In these theories, the singlet neutrinos can propagate in a higher-dimensional space, which is endowed with a number δ of large compact spatial dimensions. Such low-scale KK theories that include higher-dimensional singlet neutrinos are very appealing, since they can naturally provide suppression mechanisms for understanding the smallness in mass of the observed light neutrinos [7, 8], explain the observed anomalies in solar and atmospheric neutrino data through neutrino oscillations [8, 12], and finally account for the baryon asymmetry in the universe through leptogenesis [9].

Another very interesting feature of KK theories of low-scale quantum gravity is that the presence of KK-neutrino states can give rise to a number of testable new-physics signals at collider and lower energies, such as lepton-flavour violation in muon, tau and Z-boson decays, coherent $\mu \to e$ conversion in nuclei, and effects of universality breaking in Z- and W-boson interactions. We confronted the predictions of a minimal higher-dimensional singlet-neutrino model for the afore-mentioned new-physics phenomena with current experimental data. We found that KK neutrinos heavier than the W boson act cumulatively in the loops, thereby leading to a non-decoupling phenomenon, for large values of the original higher-dimensional Yukawa couplings of the theory. Because of this cumulative non-decoupling phenomenon of the KK states, we were able to derive very stringent constraints on lepton-flavour violation in the μ-e sector, which originate from upper limits on $\mu \to e$ conversion in nuclei and $\mu \not\to eee$. In fact, the so-derived limits can be much stronger than those obtained from $\mu \not\to e\gamma$ by an earlier consideration [26], for higher-dimensional Yukawa couplings of order unity. Specifically, for $h_e = h_\mu = h_\tau = 1$, we derived the rather tight lower limits: $M_F \gtrsim$ 361, 826, 301, 1182, 282 TeV, for 2–6 large extra dimensions, respectively. Notice that the constraints related to the odd number of dimensions are stronger due to a geometric enhancement of the double integral I_δ in Eq. (4.4).

New-physics effects in e-τ and μ-τ sectors are less constrained than those found for the e-μ one. In general, one may avoid most of the latter constraints by setting the higher-dimensional Yukawa couplings h_e or h_μ to zero. However, even if h_e and h_μ are vanishingly small, the non-observation of universality breaking in leptonic Z-boson decays, which involve τ leptons in the final state, together with experimental data on the invisible Z-boson width are sufficient to place a lower bound on M_F. For $h_\tau = 1$ and $h_\mu = h_e = 0$, the lower limits on M_F were found to be: $M_F \gtrsim$ 8.2, 9.5, 4.7, 12.3, 3.8 TeV, for 2–6 large compact dimensions, respectively. Even though the analysis was performed in a minimal higher-dimensional model, these last bounds, as well as those pertaining to the e-μ sector, are generically rather model-independent, as long as order-unity Yukawa couplings, h_l, are present in the original theory before compactification. In this context, it is important to emphasize again that all of the above 'one-loop' lower limits on M_F scale quadratically with h_l, for $h_l \gtrsim 1$. This is an important consequence of the cumulative non-decoupling effect that results from summing up KK states heavier than the electroweak scale. For instance, if $h_l \sim 3$, the lower bounds on M_F will then increase by one order of magnitude. On the other hand, if the lower bounds on M_F are implemented in the analysis for $h_l = 1$, the branching ratios for $\tau \to e\mu\mu$ and $Z \to e\tau$ were predicted to be as large as 2.3×10^{-7} and 1.1×10^{-6}, respectively. Next-generation colliders have the potential capabilities to probe these predictions of KK theories of low-scale quantum gravity and to impose new

constraints on their parameters.

Acknowledgements

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