Precision determination of the πN scattering lengths and the charged πNN coupling constant

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We critically evaluate the isovector GMO sumrule for the charged πNN coupling constant using recent precision data from π^-p and π^-d atoms and with careful attention to systematic errors. From the π^-d scattering length we deduce the pion-proton scattering lengths $\frac{1}{2}(a_{\pi^-p} + a_{\pi^-n}) = (-20 \pm 6(\text{statistic}) \pm 10\ (\text{systematic})) \cdot 10^{-4} m_{\pi_c}^{-1}$ and $\frac{1}{2}(a_{\pi^-p} - a_{\pi^-n}) = (903 \pm 14) \cdot 10^{-4} m_{\pi_c}^{-1}$. From this a direct evaluation gives $g_c^2(GMO) = 14.20 \pm 0.07(\text{statistic}) \pm 0.13(\text{systematic})$ or $f_c^2 = 0.0786 \pm 0.0008$.

1. INTRODUCTION.

The most direct and transparent way of determining the πNN coupling constant directly from data is to use the basic forward dispersion relation called the Goldberger-Miyazawa-Oehme (GMO) sum-rule:

$$g_c^2 = -4.50 \cdot J^- + 103.3 \cdot (\frac{a_{\pi^-p} - a_{\pi^+p}}{2}), \tag{1}$$

where J^- is a weighted integral over the difference of $\pi^\pm$p total cross sections. The assumption of isospin symmetry is not necessary. The scattering lengths $a_{\pi^\pm p}$ have become experimentally accessible recently due to high precision experiments on π^-p and π^-d atoms as will be discussed below. The main obstacle in the evaluation of this sum rule in the past has been the necessity to rely, at least in part, on scattering lengths constructed from higher energy πN data via dispersion relations, a difficult procedure in view of the high precision needed and the heavy cancellations in the symmetric combination of the scattering lengths. We report here on a critical precision evaluation of the scattering lengths using the deuteron information and on the first direct evaluation of the GMO sum rule.

2. DETERMINATION OF THE πN SCATTERING LENGTHS.

It is essential to know the πN scattering lengths a^+ and a^- and the πNN coupling coupling constant to high precision and with well controlled uncertainties in order to make

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accurate tests of chiral symmetry predictions. The potentially most accurate experimental source for a determination of the isoscalar combination a^+ is the π^-d scattering length[1,2]. To leading order this scattering length is given by the coherent sum

$$a_{\pi^- d} \;=\; a_{\pi^- p} \;+\; a_{\pi^- n}. \tag{2}$$

Since the two components cancel to nearly 1% precision, even a rather summary description of the deuteron gives a high absolute precision. The main correction is the double scattering term, which in the static pointlike approximation is

$$D \;=\; 2\,\frac{(1+m_\pi/M)^2}{(1+m_\pi/M_d)}\,[(\frac{a_{\pi^- p}+a_{\pi^- n}}{2})^2 - 2(\frac{a_{\pi^- p}-a_{\pi^- n}}{2})^2] < 1/r >, \tag{3}$$

where M_d is the deuteron mass. This term dominates heavily and its approximate value is D = -0.0257 m_π^{-1} to be compared to the experimental $a_{\pi^- d} \;=\; -0.0261\; m_\pi^{-1}$. The most recent theoretical investigation of the various correction terms inside multiple scattering theory is due to Baru and Kudryatsev (B-K)[3]; we take their work as the departure for a critical and quantitative assessment of the theoretical uncertainties. Typical results are given in Table 1.

Table 1
Typical contributions to $a_{\pi d}$ scattering length in units $10^{-4} m_\pi^{-1}$.

Contributions	Present work	B-K [3]
$a_{\pi^- d}$(double scattering; static)	-257(7)	-252
Fermi motion	61(7)	50
dispersion correction	-56(14)	not included
isospin violation	3.5	3.5
form factor	23(6)	32(8)
higher orders	4(1)	6
sp interference	small	-44
non-static effects	11(6)	10
p wave double scattering	-3	-3
virtual pion scattering	-7(2)	not considered
$a_{\pi d}$(experimental)	-261(5)	

The principal corrections are the following:

a) the Fermi motion of the nucleons, which produces a well defined correction due to induced p wave scattering.

b) the dispersive correction due to the $\pi^- d \to nn$ absorption. This term has been evaluated in 3-body Fadeev calculations[4]. It is the single largest source of uncertainty. This correction was not included in B-K.

c) B-K advocate a rather large correction from 'sp' interference, in which one scattering is s wave and the other one p wave due to 'Galilean invariant' off mass shell contributions. The procedure is model dependent. We find that the dominant contribution is generated

by the spin averaged isovector p wave πN Born term, which has no ambiguity in its of-mass-shell structure. The contribution vanishes for this term and no correction should be made for this effect. This is our most important change from the results of B-K.

d) corrections for non-pointlike interactions and form factors. No major uncertainty is introduced by these effects with liberal variation of parameters.

e) non-static effects. These produce only rather small effects.

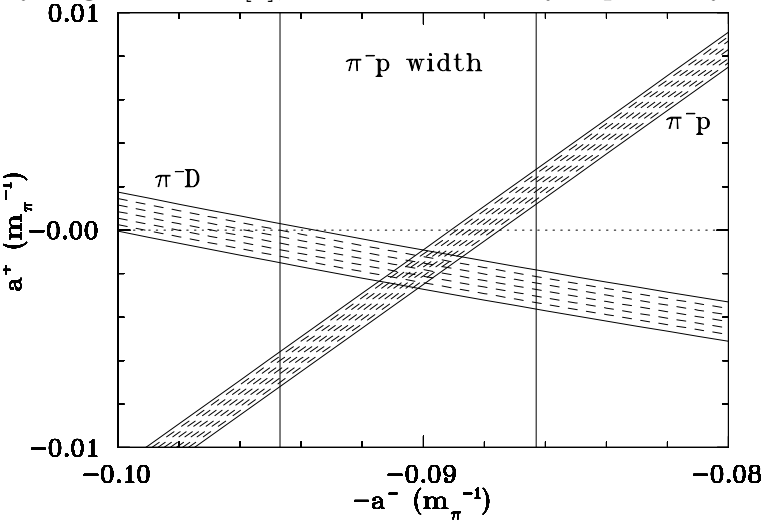
f) corrections for virtual pion scattering, isospin violation, p wave double scattering and higher order terms are all small and controllable corrections.

In particular, the assumption of isospin symmetry is only needed in correction terms and, on the level of expected violations, has a negligible influence on the conclusions.

Based on this, we obtain well controlled values for the symmetric and antisymmetric combinations of scattering lengths $\frac{1}{2}(a_{\pi^-p} \pm a_{\pi^-n}) \simeq a^{\pm}$ deduced from the data. The preliminary values are

$$\frac{a_{\pi^-p} + a_{\pi^-n}}{2} = (-0.0020 \pm 10 \pm 6)m_\pi^{-1}; \quad \frac{a_{\pi^-p} - a_{\pi^-n}}{2} = (0.0903 \pm 10 \pm 9)m_\pi^{-1}. \tag{4}$$

Here the first error is systematic and the second one statistical. As seen from the Figure these values are in excellent agreement with the value based on the width of the pionic hydrogen 1s state[2], but the uncertainty is presently much smaller.



3. EVALUATION OF THE πNN COUPLING CONSTANT.

We report as well the first evaluation of the GMO sum rule based directly on observables. This gives a controlled and model-independent value for the the charged coupling constant g_c^2. We rewrite the sum rule in the following convenient and robust form, which emphasizes its directly determined empirical ingredients:

$$g_c^2 = -4.50 \cdot J^- + 103.3 \cdot a_{\pi^-p} - 103.3 \cdot (\frac{a_{\pi^-p} + a_{\pi^+p}}{2}). \tag{5}$$

Here the total cross section integral J^- is in mb and the scattering lengths in units of $m_{\pi_c}^{-1}$. The systematic error in J^- entering this relation is presently a major limitation on its accuracy of eq. (5) and we have critically examined its contributions. We find J^-

=-1.083(25)mb, which agrees well with values used in previous evaluations of the sum rule, but with errors under control. Assuming charge symmetry, i.e., $a_{\pi^+p} = a_{\pi^-n}$, we find from eq.(4) above the following preliminary value for the charged coupling constant

$$g_c^2 = 4.87(11) + 9.12(8) + 0.21(12) = 14.20(18). \quad (6)$$

Since this value is directly based on data it supersedes previous evaluations of the sum-rule. Our value for g_c^2 with systematic errors under control is only with difficulty consistent with the low values in the range 13.5-13.6 advocated in ref.[5], since it differs by over 3 standard deviations. There is a possible discrepancy by 2 standard deviations with the value 13.75(9) from πN dispersion analysis[6]. The origin is not clear. This group evaluates the scattering lengths within the framework and find the same value 13.75 from their GMO relation. The heavy cancellation of the components in the term $1/2(a_{\pi^-p} + a_{\pi^-n})$ is however quite subtle to reproduce to high precision in this dispersive approach and the value they find would represent a 50% contribution to the deuteron scattering length. This contradicts the more direct deuteron result of eq. (4). This might indicate a too strong reliance on isospin symmetry in the dispersive analysis, since this extremely small amplitude is less than $1{:}10^3$ of the amplitude in the resonant region. On the other hand, our result is consistent to 1 standard deviation with the value from np charge exchange 14.52(26)[7]. It is also consistent to one standard deviation with the Goldberger-Treiman relation which gives a coupling constant 13.99(17) assuming that its discrepancy is due entirely to a πNN monopole form factor with a cut-off of 800 MeV/c as typically found from the Cloudy Bag Model[8], meson-theoretical ones[9] etc..

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