

IFT-UAM/CSIC-99-27

hep-th/9907158

The Confining String from the Soft Dilaton Theorem

Enrique Álvarez^{◇,♣}¹ and César Gómez^{◇,♠}²

[◇] *Theory Division, CERN, 1211 Geneva 23, Switzerland*

and

Instituto de Física Teórica, C-XVI, Universidad Autónoma de Madrid

E-28049-Madrid, Spain³

[♣] *Departamento de Física Teórica, C-XI, Universidad Autónoma de Madrid*

E-28049-Madrid, Spain

[♠] *I.M.A.F.F., C.S.I.C., Calle de Serrano 113*

E-28006-Madrid, Spain

Abstract

A candidate for the confining string of gauge theories is constructed via a representation of the ultraviolet divergences of quantum field theory by a closed string dilaton insertion, computed through the soft dilaton theorem. The resulting (critical) confining string is conformally invariant, singles out naturally $d = 4$ dimensions, and can not be used to represent theories with Landau poles.

¹E-mail: enrique.alvarez@uam.es, @cern.ch

²E-mail: iffgomez@roca.csic.es

³Unidad de Investigación Asociada al Centro de Física Miguel Catalán (C.S.I.C.)

The simplest way to address the problem of a string representation of a non abelian gauge theory [16] consist in looking for a confining string background, determined by vanishing σ -model beta functions, such that the condition of dilaton beta function equal to zero could be reinterpreted as the standard renormalization group equation:

$$\mu \frac{dg}{d\mu} = \beta(g) \quad (1)$$

for the Yang Mills running coupling constant. This program implies a deep geometrization of quantum field theory renormalization program, pointing out to gravity as the underlying dynamics controlling the running of coupling constants ².The next and more difficult step, in the confining string program, ends up stablishing the precise relation between gauge singlets and the confining closed string spectrum.

Back to the early days of string theory [12],[18], [1]the problem of infinities of one loop bosonic string amplitudes, was treated in perfect analogy with the quantum field theory renormalization program. An important subproduct of that analysis, controlling the structure of string *renormalizability*, was the so called soft dilaton theorem.

Let us consider a one loop open bosonic string amplitude for N external gluons $A^1(p_1, p_2, \dots, p_N)$. On the moduli space of the cylinder this amplitude becomes singular when the length of the cylinder $t^{-1} \rightarrow \infty$ corresponding to an annulus with the size of the internal hole going to zero. In string theory the singularity of this amplitude is exactly given by the tree level amplitude of emission of a soft dilaton of momentum $k = 0$ that is subsequently absorbed by the vacuum:

$$Sing (A^1(p_1, p_2, \dots, p_N)) = \lim_{k \rightarrow 0} J \Delta(k) A^0(p_1, p_2, \dots, p_N; k) \quad (2)$$

with J the dilaton-vacuum amplitude, $\Delta(k)$ the dilaton propagator and $A^0(p_1, p_2, \dots, p_N; k)$ the tree level amplitude for N gluons and one dilaton of momentum k .

The soft dilaton theorem [1] precisely states that:

$$\lim_{k \rightarrow 0} A^0(p_1, p_2, \dots, p_N; k) = \pi g l_s^{\frac{d-2}{2}} \left[l_s \frac{\partial}{\partial l_s} - \frac{d-2}{2} g \frac{\partial}{\partial g} \right] A(p_1 \dots p_N) \quad (3)$$

²Previous work on the relationship between the renormalization group and the holographic principle include [19],[2],[3] ,[4] and [6].

with g the open string coupling constant, and $\alpha' \equiv l_s^2$, thus implying that the singularity in equation (2) can be absorbed in a double *renormalization* of α' and g .

As it is clear from equation (2), the singularity of $A^1(p_1, p_2, \dots, p_N)$ comes from the dilaton propagator $\Delta(k)$ at $k = 0$. We have schematically represented in the enclosed figure the pertinent string diagrams.

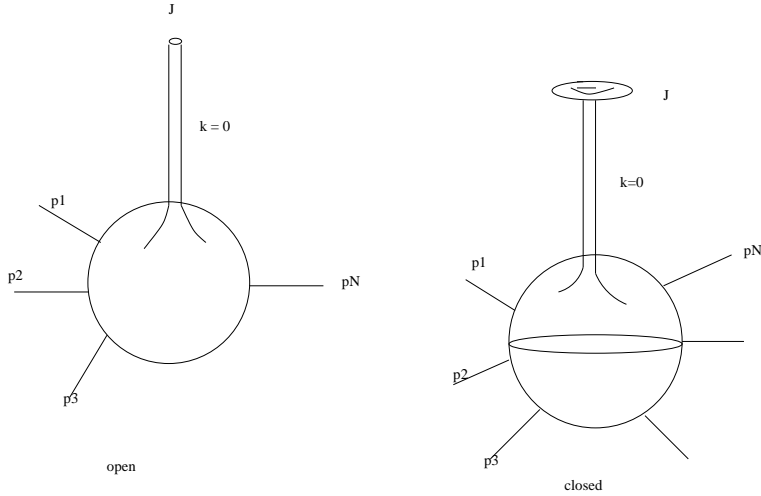


Figure 1: Topology of the singularities in the open and closed string diagrams

Using the integral representation of $\Delta(k)$ and introducing a cutoff μ as:

$$\Delta_\mu(k)|_{k=0} = \int_{1/\mu}^1 \frac{dz}{z} \quad (4)$$

(where μ should be interpreted as a cutoff on the *size* of the long tube in Figure 1), the renormalized string tension is given by:

$$\frac{1}{\alpha'_R(\mu)} \sim \frac{Jg^2}{\alpha'_0} \int_{1/\mu}^1 \frac{dz}{z} + \frac{1}{\alpha'_0} \quad (5)$$

From the soft dilaton theorem (3) and equation (5) we get a *running* string coupling

constant ³ $g(\mu)$ with beta function ⁴ given by:

$$\beta(g) = -g^3 \frac{(d-2)\pi}{4} \quad (6)$$

It is interesting to observe that equation (6) for $d > 2$ is of asymptotically free type.

The modern approach, after Fischler-Susskind ([10]), to the string tension renormalization (5) stems from identifying $\log \mu$ with the world sheet Weyl factor (the Liouville field ϕ) and to look for a background redefinition with a new Ricci tensor determined by:

$$\frac{\delta(\frac{1}{\alpha'_R(\mu)})\eta_{\mu\nu}}{\delta \log \mu} \quad (7)$$

in such a way that the σ -model beta function equations for the string are satisfied.

After the discovery of D -p-branes it is perfectly natural to interpret the one loop open string amplitude $A^1(p_1, p_2, \dots, p_N)$ as a world volume amplitude with p_1, p_2, \dots, p_N momenta in $p+1$ dimensions. In this case the singularity in the one loop open string amplitude can be related to a soft dilaton insertion with momentum k in the *bulk* transversal directions. This D-brane picture naturally yields the interpretation of μ as a transversal coordinate. In this set up the renormalization of α can be interpreted as defining a background metric of the type:

$$ds^2 = \frac{1}{\alpha'(\mu)} d\vec{x}_{p+1}^2 + d\phi^2 + d\vec{y}_{26-p-2}^2 \quad (8)$$

(where $d\vec{x}_d^2$ is the flat metric in \mathbb{R}^d) for $\phi = \log \mu$, the world sheet Weyl factor or Liouville field (cf.[16]). (The general idea of using strings with variable tension is due to Polyakov (cf. [17])). The condition of world sheet conformal invariance for the background (8) ought to be consistent with the dilaton dependence $\Phi(\mu)$ dictated by soft dilaton theorem (3) i.e

³In references [12], [18], the renormalization of the string coupling constant, g , was interpreted as a consequence of the tachyon singularity in the one-loop open string amplitude, in the $t \rightarrow 0$ limit. In [1], however, the appearance of g in the soft dilaton theorem was related to the transversality of the external dilaton. We believe that the potential closed string tachyon interpretation of the renormalization of g in parallel to the transversality condition for the dilaton, could benefit from better understanding (cf. [17],[11]).

⁴The first renormalization of the string coupling constant was made in [12] by analytically regularizing the logarithmically divergent integral. What we have done here is just to regularize it by means of a cutoff in the lower end.

by the renormalized string coupling. As we will see in a moment this is in fact the case, but only if $p = 3$!

Before going into more details let us highlight the main point of this note. From the Dirac-Born-Infeld action for a $D - 3$ brane we get the standard relation between Yang Mills coupling constant and the closed string coupling constant g^2 :

$$g_{YM}^2 = g^2 \tag{9}$$

where in addition g^2 is given by the vacuum expectation value of the dilaton field. The confining string interpretation of four dimensional pure Yang Mills theory will be based on interpreting the Yang Mills beta function as a *stringy* beta function governing the string one loop renormalization of g as due to soft dilaton insertions. Once we establish this identification we use the soft dilaton theorem to find the associated running string tension $\alpha'(\mu)$. The confining string background metric will be finally defined by a metric as (8). As a consistency check of the whole procedure we will show that the σ -model beta functions vanish for the so defined confining string background.

It is important to stress that in this approach we read the confining string geometry directly from the perturbative ultraviolet behavior of pure Yang Mills gauge theory. The identification of the string beta function, governing the *renormalization* of the string coupling constant, with the pure gauge beta function should be understood as a generalization to quantum field theory of expression (2). In fact what we are doing is reinterpreting the standard ultraviolet singularities of a quantum field theory in terms of a soft dilaton insertion in the corresponding confining string. More precisely what we understand as a confining string is the string interpretation, in terms of a dilaton propagator (in bulk direction) and a dilaton tadpole, of the standard ultraviolet singularities of non abelian gauge theories. In this sense we can think of strings as a physical model of quantum field theory renormalizability. The D -brane description ([15]) of gauge theories will be specially suited for our purposes.

The DBI action for a D -p-brane:

$$S_{DBI} = T_p \int d^{p+1}\xi e^{-\Phi} \sqrt{\det[g_{ab} + b_{ab} + 2\pi\alpha'F_{ab}]} \tag{10}$$

through an expansion in α' , conveys the relation between Yang Mills bare coupling and the dilaton vacuum expectation value:

$$\frac{1}{g_0^2} = T_p e^{-\Phi_0} (2\pi\alpha')^2 \quad (11)$$

The one loop corrections to the Yang Mills coupling are given in the background field method by:

$$\left(\frac{1}{g_0^2} + \frac{1}{2}C_{G,1} - C_{G,0}\right)(F^2 + \text{higher derivatives}) \quad (12)$$

where the coefficients $C_{G,i}$ represent the contribution of the determinants of the gluon ($i = 1$) and ghosts ($i = 0$) quadratic terms:

$$C_{G,i} = c_i \log(M^2/k^2) \quad (13)$$

where the scale M can be appropriately chosen to cancel the bare coupling dependence in (12).

The simplest possible matching between equations (11) and (12) suggests a renormalization group dependence of the dilaton field such as:

$$T_p e^{-\Phi(k^2)} (2\pi\alpha')^2 F^2 \quad (14)$$

This dependence will be interpreted as providing a running string coupling constant $g(\mu^2 = k^2)$ that, according to our previous discussion will be identified with the running string coupling constant one would obtain in the string renormalization of string one loop infinities satisfying equation (2). In order to put this identification on more solid grounds we need to match the renormalization group scale in quantum field theory with the cutoff used in equation (4) for the dilaton propagator. In order to do that we will again profit from de D -brane description.

To be concrete we will consider the two gluon amplitude corresponding to the process depicted in figure:

$$A \sim \alpha'^{1-d/2} \epsilon_1 \cdot \epsilon_2 \frac{g^2}{(4\pi)^{d/2}} \int dt \int_0^t d\nu t^{-d/2} \prod_{n=1}^{\infty} (1 - e^{-2nt})^{2-D} e^{2\alpha' p_1 \cdot p_2 G(\nu)} \partial^2 G(\nu) e^{-y^2 t/\alpha'} \quad (15)$$

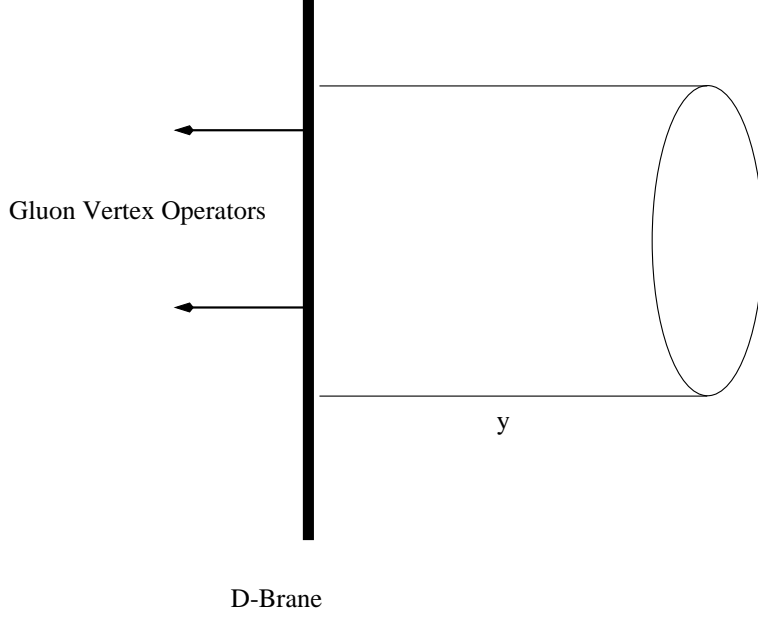


Figure 2: The planar two-gluon annulus amplitude

where $G(\nu)$ is the cylinder Green function, ν is the relative position of the vertex operators, characterized by polarizations and momenta ϵ_1, p_1 and ϵ_2, p_2 . The D-brane effect is conveyed by the exponential factor depending on y , the distance between branes.

Let us now consider the limit:

$$\alpha' = 0; t^{-1} = 0; y = 0 \quad (16)$$

with

$$\alpha' t = \bar{t}; \frac{y}{\alpha'} = u \quad (17)$$

In equation (15) $d = p + 1$ for p the dimension of D -brane world volume space and D represent the dimension of the target space-time. In the limit (16), (17) amplitude (15) for p_1 and p_2 on shell becomes ([9]):

$$A \sim \frac{g^2}{(4\pi)^{d/2}} \epsilon_1 \cdot \epsilon_2 \int \frac{d\bar{t}}{\bar{t}} \int_0^1 d\hat{\nu} \bar{t}^{1-d/2} e^{-u^2 \bar{t}} [(D-2)(1-2\bar{\nu})^2 - 8] \quad (18)$$

where we have used the relationship for the Neumann Green's functions:

$$[e^{2\bar{t}} + (D-2)] \partial^2 G \sim (D-2)(1-2\hat{\nu})^2 - 8 \quad (19)$$

From equation (18) we easily get:

$$A \sim -\frac{g^2}{(4\pi)^{d/2}} \epsilon_1 \cdot \epsilon_2 \Gamma(1-d/2) (u^2)^{d/2-1} [(D-2) - 4] D - 2) B(2, 2) - 24] \quad (20)$$

(where $B(u, v)$ is Euler's Beta function), which by standard dimensional regularization leads for $d = 4$ to:

$$A \sim -\frac{g^2}{(4\pi)^{d/2}} \epsilon_1 \cdot \epsilon_2 u^2 \log(u^2/\Lambda^2) [(D-2) - 4(D-2)B(2, 2) - 24] \quad (21)$$

From equation (21) it is clear that variable u as defined in (17) is playing the rôle of the renormalization group variable. On the other hand u as it is plain from definition (17) is nothing else but the near horizon transversal variable introduced in reference [13] possessing the natural meaning of energy in transversal bulk direction which is precisely the meaning of the cutoff variable μ^2 introduced in equation (4). Using equation (14) the corresponding dilaton field defined by (21) will be given by:

$$e^{-\Phi(u^2)} = -[(D-2) - 4(D-2)B(2, 2) - 24] \log(u^2/\Lambda^2) \quad (22)$$

Following our strategy what we will do now is to identify the running of the dilaton (22)⁵ with the string renormalization of the string coupling constant due to soft dilaton tadpole. Next and using the soft dilaton theorem we induce from that a running dependence of world volume metric on u . Finally we check the consistency of the so defined confining string by solving the σ -model beta functions. Going through these steps the confining string background metric⁶ we get is:

$$ds^2 = \log(u) d\vec{x}_4^2 + \frac{du^2}{u^2} + d\vec{y}_{21}^2 \quad (23)$$

with a dilaton field of the type:

$$\Phi(u) = -\log \log(u) \quad (24)$$

The σ -model beta equations are:

$$R_{\mu,\nu} - \nabla_\mu \nabla_\nu \Phi = 0 \quad (25)$$

and

$$\nabla^2 \Phi + (\nabla \Phi)^2 = c_{extra} \quad (26)$$

⁵It is worth remarking that for $D = 4$ the equation (21) gives directly the well known precise factor for Yang Mills (11/3). In $D = 26$ instead we get zero (the correct value for $n_s = 22$ adjoint scalars). In what follows we will stick to $D = 4$.

⁶With euclidean signature.

It is known in general (cf, for example, [7],[8]) that consistency of the equations imply that c_{extra} must be constant (equal to $\frac{2(D-10)}{3}$ in the (world-sheet) supersymmetric case and to $\frac{26-D}{3}$ in the ordinary bosonic string). Curiously enough, the above metric (23) with the dilaton field *exactly* as in (24) is a solution of the first equation (25) for *any* value of p .

The second equation, (26) however can be easily shown to be given by:

$$\frac{1 - (p + 1)/2}{(\log u)^2} + \frac{1}{(\log u)^2} \quad (27)$$

conveying the fact that it is a constant for $p + 1 = 4$ only, in which case it is zero. This means that the string must necessarily be critical, and we have to add the extra flat (*spectator*) dimensions, 21 of them in the bosonic case ⁷. This is one remarkable fact of this background. Another is, of course, that the *sign* of the dilaton in equation (24) is fixed, so that it is not possible to construct *non-confining* strings to describe ultraviolet slave quantum field theories using this set of ideas.

The confining string itself should allow the computation of many gauge invariant observables in gauge theories, such as the Wilson loop. Assuming a static configuration bounded by $\sigma = \pm l/2$, and with a temporal span equal to T , it is useful to work in the gauge in which $\tau = x^0$ and $\sigma = x^1$. If at the same time we define a new coordinate by $z \equiv \log u$ such that the metric reads

$$ds^2 = z d\bar{x}_4^2 + dz^2, \quad (28)$$

and assuming a symmetric imbedding of the world sheet into the target spacetime of the confining string, $z = z(\sigma)$, then the induced metric on the worldsheet is given by:

$$ds^2 = z d\tau^2 + (z + z'^2) d\sigma^2 \quad (29)$$

It can be argued ([13]) that the semiclassical approximation to the Wilson loop action is given by the area of the world sheet computed precisely with the induced metric, that is:

$$S = T \int_{-l/2}^{l/2} d\sigma \sqrt{z(z + z'^2)} \quad (30)$$

⁷It should be clear that the spectator dimensions change neither the confining string target space five-dimensional metric nor the Yang-Mills beta function. It is important to realize that the metric (23) does *not* correspond to a $D-3$ brane in a 26-dimensional ambient space. Actually, the only source for a nontrivial dilaton is the five-dimensional metric given by the running string tension.

(We are neglecting here the contribution of the dilaton to the action, possibly important).

The variational principle (30) enjoys a first integral, namely:

$$p = \frac{-z^2}{\sqrt{z^2 + zz'^2}} \quad (31)$$

The formal solution to this differential equation is:

$$\int^{z(\sigma)} df \frac{1}{\sqrt{f^3 - p^2 f}} = \int_{-l/2}^{\sigma} \frac{d\sigma}{p} \quad (32)$$

The lower limit of the first integral should be chosen in one of the regions in which z'^2 is allowed to be positive by the first integral, namely $(-p, 0)$ or (p, ∞) . Choosing by concreteness it to be 0, the solution can be easily shown to be given by an elliptic integral of the first kind:

$$\sigma + l/2 = -(2p)^{1/2} F(\cos^{-1}(|z(\sigma)/p|), \frac{\sqrt{2}}{2}) \quad (33)$$

which implies that:

$$\left| \frac{z(\sigma)}{p} \right| = \operatorname{sn} \frac{\sigma + l/2}{(2p)^{1/2}} \quad (34)$$

If we neglect the square of the modulus, $m^2 = 1/2$, the trigonometric approximation can be applied:

$$z(\sigma) = p \sin \frac{\sigma + l/2}{\sqrt{2p}} \quad (35)$$

The relationship between p and l is found by demanding that $z(l/2) = 0$, yielding $p = \frac{l^2}{2\pi^2}$.

The action then reduces to:

$$S = T \int_{-l/2}^{l/2} d\sigma \frac{z^2}{p} = \frac{Tl^3}{4} \quad (36)$$

which indeed corresponds to a confining potential.

It would be interesting to check how world sheet supersymmetry (as well as GSO projections) modify our analysis. In particular, it should be stressed that multiplicative factors in the beta function of the gauge theory yield additive factors in the formula (24), which do not contribute to the string beta function.

A quite surprising aspect of our analysis is the way pure Yang Mills perturbative information is promoted to string dynamics. This implies a connection between perturbative aspects of gauge dynamics (such as the beta function) with non perturbative, (*stringy*) dynamics, such as infrared strong forces. The main ingredient for doing that has been

the soft dilaton theorem that allows to read coupling constant renormalization as variable string tension and in that sense as confining string background metric. It is perhaps worth remarking that vanishing σ -model beta functions - essential to the quantum consistency of the string - have been used in our approach only at the end as a consistency check for the metric and dilaton field directly derived from the quantum field theory data and not as an extra governing principle. Moreover four dimensional space time appears in this scheme as the only consistent solution. All this seems to indicate that some string dynamics is already encoded in the renormalization of asymptotically free four dimensional gauge theories.

Acknowledgments

This work has been partially supported by the European Union TMR program FMRX-CT96-0012 *Integrability, Non-perturbative Effects, and Symmetry in Quantum Field Theory* and by the Spanish grant AEN96-1655. The work of E.A. has also been supported by the European Union TMR program ERBFMRX-CT96-0090 *Beyond the Standard model* and the Spanish grant AEN96-1664. C.G would like to thank the organizers of the Abdus Salam ICTP Extended Workshop in String Theory for hospitality while this work was being written up.

References

- [1] M. Ademollo, A. D'Adda, R. D'Auria, F. Gliozzi, E. Napolitano, S. Sciuto and P. Di Vecchia, *Soft Dilatons and Scale Renormalization in Dual Theories*, Nucl. Phys. B94 (1975)221.
- [2] E.T.Akhmedov, *A Remark on the AdS/CFT correspondence and the Renormalization Group Flow*, hep-th/9906217, Phys.Lett. B442 (1998) 152.
- [3] E. Alvarez and C. Gómez, *Non-critical Confining Strings and the Renormalization Group*, hep-th/9902012; Nucl. Phys. B550 (1999),169.

- [4] E. Alvarez and C. Gómez, *Geometric Holography, the Renormalization Group and the c-Theorem*, hep-th 9807226; Nucl. Phys. B541 (1999),441.
- [5] E. Alvarez and C. Gómez, (*work in progress*)
- [6] V. Balasubramanian and P. Kraus, *Spacetime and the Holographic Renormalization Group*, hep-th/9903190.
- [7] C.G.Callan, E.J. Martinec, M.J. Perry , D. Friedan , *Strings in background fields*, Nucl.Phys.B262:593,1985
- [8] G. Curci and G. Paffuti, *Consistency between the string background field equation of motion and the vanishing of the conformal anomaly*, Nucl.Phys.B286:399,1987
- [9] P. Di Vecchia, L. Magnea, A. Lerda, R. Russo and R. Marotta, *String Techniques for the Calculation of Renormalization Constants in Field Theory*,hep-th/9601143;
- [10] W.Fischler , L. Susskind, *Dilaton Tadpoles, String Condensates and Scale Invariance,1*, Phys.Lett.B171:383,1986;
Dilaton Tadpoles, String Condensates and Scale Invariance,2, Phys.Lett.B173:262,1986.
- [11] I. Klebanov and A. Tseytlin, *D-Branes and Dual Gauge Theories in Type 0 Strings*, Nucl.Phys.B546:155-181,1999 hep-th/9811035
- [12] A. Neveu and J. Scherk, *Gauge Invariance and Uniqueness of the Renormalization of Dual Models with Unit Intercept*, Nucl. Phys. B36 (1972) 317.
- [13] J. Maldacena, *The large N limit of superconformal field theories and supergravity*, Adv.Theor.Math.Phys.2:231-252,1998, hep-th/9711200.
- [14] J. Maldacena, *Wilson loops in large N field theories*, Phys.Rev.Lett.80:4859-4862,1998, hep-th/9803002;
S. J. Rey, Jungtay Yee , *Macroscopic strings as heavy quarks in large N gauge theory and anti de-sitter supergravity*, hep-th/9803001;

- E. Alvarez, C. Gómez and T. Ortín, *String representation of Wilson loops*, Nucl. Phys.B545:217-232,1999 ;[hep-th/9806075](#)
- [15] J. Polchinski, *String Theory* (Cambridge University Press).
- [16] A.M. Polyakov, *Gauge Fields and Strings* (Harwood).
- [17] A.M. Polyakov, *The Wall of the Cave*, [hep-th/9808057](#)
- [18] J. Shapiro, *Renormalization of Dual Models*, Phys. Rev. D11 (1975) 2937.
- [19] L. Susskind and E. Witten, *The holographic bound in Anti-de Sitter space*, [hep-th/9805114](#)