Light–Cone Broadening and TeV Scale Extra Dimensions

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Abstract

We examine the effect of light–cone broadening induced by quantum–gravity foam in the context of theories with "large" extra dimensions stretching between two parallel brane worlds. We consider the propagation of photon probes on one of the branes, including the response to graviton fluctuations, from both field– and string–theoretical viewpoints. In the latter approach, the dominant source of light–cone broadening may be the recoil of the D–brane, which scales linearly with the string coupling. Astrophysical constraints then place strong restrictions on consistent string models of macroscopic extra dimensions. The broadening we find in the field–theoretical picture seems to be close to the current sensitivity of gravity–wave interferometers, and therefore could perhaps be tested experimentally in the foreseeable future.

I. INTRODUCTION

The considerable theoretical interest in the recent suggestion [1] of possible extra dimensions of macroscopic (sub–millimetre) size stems largely from the apparent extreme difficulty of ruling out such a suggestion experimentally. It is the point of this note to examine this suggestion from the point of view that quantum gravity might be treated as a stochastic medium [2–5]. As three of us have argued in the past [6], when one considers interactions of closed–string particle states with D–branes, the recoil of the latter may induce distortions of the space–time around the brane, which manifest themselves as new quantum degrees of freedom, carrying information and giving a stochastic nature to the process. One of the

most important features of this approach is the induced light–cone fluctuations [7], which may be detectable as stochastic fluctuations in the velocity of light propagating through this "medium".

Such phenomena may also characterize conventional point–like approaches to quantum gravity. Indeed, as previously argued in [8], one encounters light–cone fluctuations when one expands about a squeezed graviton coherent state, which arguably characterizes physically interesting models of quantum–gravity foam. The application of such ideas to flat Minkowskian space–time with compactified extra dimensions was first considered in [9, 10], with the conclusion that the stochastic fluctuations in the light–cone produce stochastic fluctuations in the arrival times of photons which are considerably larger than the conventional Planck scale of four–dimensional gravity. In [10], a similar calculation has been performed in the context of the models of [1], but the calculation has been done only in the case of just one extra dimension, which may be ruled out by macroscopic astrophysical observations [11].

In the present work, we consider the stochastic fluctuations of the light–cone in a rather different framework, that of two parallel 3–branes separated by a distance l. The branes live in a world with $n \leqslant 6$ extra transverse dimensions, in the standard string theory picture. As discussed in [1], only closed–string states (gravitons) can propagate in the bulk. The geometry of interest is depicted in Fig. 1. We initially ignore the recoil of the branes, and only study the effect of graviton fluctuations about flat $(3+1)$–dimensional space–time on the brane, which is assumed to have Minkowskian signature $(+---)$. The issue of recoil effects is taken up later.
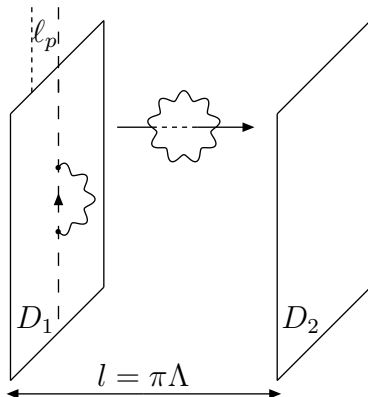

FIG. 1. Schematic representation of the geometry of interest for our computation, where D_1 and D_2 are brane worlds separated by the size of the extra dimensions l, in which only closed–string states (gravitons) can propagate. A photon is depicted travelling parallel to D_1 and separated from it by a small distance ℓ_p, reflecting the quantum uncertainty in the location of the brane.

In the picture of [1], the Planck scale on the branes is constrained to be the standard Planck mass scale $M_p^{(4)} \sim 10^{19}$ GeV, which is related to the underlying $(4+n)$–dimensional scale $M_p^{(4+n)}$ via the size $l = \pi \Lambda$ of the postulated extra dimensions:

$$\left(M_p^{(4)} \right)^2 \sim (\pi \Lambda)^n \left(M_p^{(4+n)} \right)^{2+n}. \tag{1.1}$$

We are interested in the superstring–motivated cases of $n \leqslant 6$ extra dimensions, and we work in units such that $\hbar = c = M_p^{(4+n)} = 1$.

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We first review briefly the analysis in [8, 9], which considered gravitons in a squeezed coherent state, relevant for discussions of quantum–gravitational space–time foam. Such gravitons induce quantum fluctuations in the space–time metric, in particular fluctuations in the light–cone, which may have observable effects Δt on the arrival times of photons. Consider a flat background space–time with a linearized perturbation corresponding to the invariant metric element

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \left(\eta_{\mu\nu} + h_{\mu\nu}\right)dx^\mu dx^\nu = dt^2 - d\vec{x}^2 + h_{\mu\nu}dx^\mu dx^\nu.$$

Let $2\sigma(x, x')$ be the squared geodesic separation for any pair of space–time points x and x', and let $2\sigma_0(x, x')$ denote the corresponding quantity in an unperturbed flat space–time background. In the case of small gravitational perturbations about the flat background, one may expand $\sigma = \sigma_0 + \sigma_1 + \sigma_2 + \ldots$, where σ_n denotes the nth–order term in an expansion in the gravitational perturbation $h_{\mu\nu}$. Then, as shown in [8], the root–mean–square deviation from the classical propagation time Δt is related to $\langle\sigma^2\rangle$ by:

$$\Delta t = \frac{\sqrt{\langle\sigma^2\rangle - \langle\sigma_0^2\rangle}}{L} \simeq \frac{\sqrt{\langle\sigma_1^2\rangle}}{L} + \ldots \tag{1.2}$$

where $L = |x' - x|$ is the distance between the source and the detector. The expression (1.2) is gauge invariant [9]. For convenience, the transverse trace–free gauge is used, for which $h_{0\nu} = h_i^i = \partial^\mu h_{\nu\mu} = 0$, where Greek indices refer to $(3+n+1)$–dimensional space–time and Latin indices refer to the $3+n$ spatial components only.

The light–cone broadening effect is computed in [9] both for a cylindrical topology (or one periodic compactified dimension) and in the presence of a plane boundary. The situation of most interest for us is that in which the photon travels in a direction orthogonal to the compactified dimension, i.e., parallel to the plane boundary, since only closed–string states (gravitons) propagate in the extra dimensions [1]). The following results were derived in [9]:

$$\frac{\Delta t}{t_p} \simeq \frac{1}{4\sqrt{2}}\sqrt{\frac{L}{l}}, \tag{1.3}$$

for the cylindrical topology, where l is the size of the compact dimension, and $L \gg l$ is the distance travelled by the photon, and in the vicinity of a plane boundary (at which the gravitons were forced to obey Neumann boundary conditions):

$$\frac{\Delta t}{t_p} \simeq \sqrt{\frac{\ln[L/z]}{6\pi^2}}, \tag{1.4}$$

where z is the distance of the photon trajectory from the plane, for which it is assumed that $z \ll L$. The technique used in the above computations was that of the image method, in which the Kaluza–Klein modes resulting from the compact dimension are taken into account in the graviton two–point function

$$G_{ijkl}(x, t; x', t') = \langle h_{ij}(x, t)\, h_{kl}(x', t')\rangle$$

as follows [9]:

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$$G_{xxxx}(t, \vec{x}, z_i; t', \vec{x}', z_i') = \sum_{m_i=-\infty}^{\infty} {}' G_{xxxx}(t, \vec{x}, z_i; t', \vec{x}', z_i' + m_i l), \qquad (1.5)$$

where z_i label the compact dimensions and the prime on the sums indicates that the $m_i = 0$ terms are omitted. This technique simplifies considerably the calculation of the contribution from the Kaluza–Klein modes for the case where the compactified dimension is periodic.

A similar computation has been performed in [10], this time concentrating on a cylindrical topology in more than four dimensions, i.e., with some extra compact dimensions, as motivated by the proposal of [1]. It was found that, for a five–dimensional world with one dimension compact, the light–cone broadening is

$$\Delta t \sim t_p \frac{L}{l}, \qquad (1.6)$$

where t_p is the Planck time. Note that this differs from the cylindrical case (1.3) above [9], for which the light–cone broadening grows as the square root of L. The conclusion of [10] is that this scaling will in general vary according to the dimensionality of the brane.

II. FIELD–THEORETICAL CONTRIBUTION

Here we deal with a different topology (see Fig. 1), in which the extra dimensions do not have periodic boundary conditions. Instead, we adopt Neumann boundary conditions for the graviton field at each brane hyperplane. Therefore we do not use the image method, but instead obtain an estimate of the induced light–cone broadening by identifying the dominant contributions in the various discrete mode sums involved, which will be sufficient for our purposes. The light–cone broadening effect is given by (1.2), with L denoting the distance travelled by the photon and where σ_1 is related to the graviton two–point function by [9]

$$\langle \sigma_1^2 \rangle = \frac{1}{8} L^2 \int_{r_i}^{r_f} dr \int_{r_i}^{r_f} dr' \, n^a n^b n^c n^d \, \langle h_{ab}(x) h_{cd}(x') + h_{ab}(x') h_{cd}(x) \rangle$$

$$\doteq \frac{1}{8} L^2 \int_{r_i}^{r_f} dr \int_{r_i}^{r_f} dr' \, n^a n^b n^c n^d \, G_{abcd}(x, x'), \qquad (2.1)$$

where the integrations are taken along the geodesic, the spatial direction of which is defined by the spatial unit vectors n.

As already mentioned, the topology we wish to consider in evaluating σ_1 is that of n extra dimensions of size $l = \pi \Lambda$ [1], as shown in Fig. 1. We compute the effect of this topology on the arrival time of a photon travelling parallel to one of the boundaries but separated from it by a distance $z \sim \ell_p$. This last feature is supposed to model the effect of quantum fluctuations in the position of the brane world [12]. This uncertainty is motivated by the framework of general relativity, in which there cannot be rigid planes.

Our conventions are as follows:

$$x^\mu \doteq (t, \vec{x}, z_1, \ldots, z_n)$$
$$k^\mu \doteq (\omega, \vec{k}, m_1/\Lambda, \ldots, m_n/\Lambda) : \qquad m_i \in \mathbb{Z}, \qquad (2.2)$$

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and, on account of the n extra dimensions, the graviton modes are normalized with an extra factor of $l^{-n/2} = (\pi\Lambda)^{-1/2}$. Following [9], the required two–point function can be expressed as follows:

$$G_{xxxx}(x, x') = 2\left(D(x, x') - 2F_{xx}(x, x') + H_{xxxx}\right), \tag{2.3}$$

where

$$D(x, x') = -\frac{1}{4\pi^2}\left[(t - t')^2 - (\vec{x} - \vec{x}')^2\right]^{-1} \tag{2.4}$$

is the Hadamard function for a free scalar field, and the functions F and H are defined by

$$F_{ij} \doteq \text{Re} \prod_{a=1}^{n}\left[\frac{1}{l}\sum_{m_a=-\infty}^{\infty}\right]\int d^3k \, \frac{k_i k_j}{2\omega^3}e^{ik\cdot(x-x')}e^{-i\omega(t-t')}, \tag{2.5}$$

$$H_{ijkl} \doteq \text{Re} \prod_{a=1}^{n}\left[\frac{1}{l}\sum_{m_a=-\infty}^{\infty}\right]\int d^3k \, \frac{k_i k_j k_k k_l}{2\omega^5}e^{ik\cdot(x-x')}e^{-i\omega(t-t')}. \tag{2.6}$$

We have restricted ourselves above to consideration of the $xxxx$ component of the two–point function, since we are interested in photons propagating parallel to the brane, and we conveniently set this direction to be parallel to the x–axis, assuming rotational invariance on the brane. Adopting Neumann boundary conditions for the graviton modes at the boundaries of each compact dimension, the required function F_{xx} is given by

$$F_{xx} = \text{Re} \frac{\partial_x \partial'_x}{\pi^n \Lambda^n}\left[\prod_{a=1}^{n}\sum_{m_a=-\infty}^{\infty}\cos\left[m_i(z_i - z'_i)/\Lambda\right]\right]\int d^3k \, \frac{1}{2\omega^3}e^{i\vec{k}\cdot(\vec{x}-\vec{x}')-i\omega(t-t')}, \tag{2.7}$$

where as a result of the transverse trace–free gauge choice [9]

$$\omega = \sqrt{k^2 + \alpha_n^2}; \qquad \alpha_b^2 \doteq \frac{1}{\Lambda^2}\sum_{a=1}^{b}m_a^2 \doteq \frac{1}{\Lambda^2}\overline{\alpha}_b^2.$$

Performing the angular part of the integration, we obtain

$$F_{xx} = \sum_{b=0}^{n}\frac{1}{4\pi^2}\frac{1}{\pi^n\Lambda^n}\partial_x\partial'_x\prod_{a=1}^{n-b}\left[2\sum_{m_a=1}^{\infty}\cos\left[m_a Z_a/\Lambda\right]\right] \times$$

$$\times \int dk \, \frac{k}{\left(k^2 + \alpha_{n-b}^2\right)^{3/2}}\frac{\sin[kR]}{R}\cos[(k^2 + \alpha_{n-b}^2)^{1/2}T], \tag{2.8}$$

where we have written $T \doteq (t - t')$, $R \doteq |x - x'|$, and $Z_i \doteq z_i - z'_i$. In the sum over b, the bth term has b of the mode numbers m_a vanishing.

The term with $b = n$ has no sums to evaluate, and when combined with the equivalent term from H and the term D as in (2.3), it reduces to the expression computed in [9] for a single plane boundary, and ultimately gives [9]

$$\Delta t = \frac{1}{(\pi\Lambda)^n} \sqrt{\frac{\ln\left[L/\ell_p\right]}{6\pi^2}}, \tag{2.9}$$

where L is the distance travelled by the photon.

We wish now to determine the effects of the non–zero modes in the sum above, i.e., those which are sensitive to the presence of the other plane boundaries. Note first from (2.8) that the effects of high modes $m_a \gg 1$ are (as expected) small compared to the lowest few modes with $m_a = 1, 2, \ldots$. Note also from (2.8) that, after differentiation with respect to x and x', there will be some terms after the integrations which will feel the presence of the ultraviolet cutoff. In this way, we will be able to make a direct connection with the relation between the four–dimensional Planck scale and the underlying TeV scale of gravity [1], as shown in (1.1). In order to obtain an estimate of the effects of the first non–zero modes in each of the extra dimensions, we compute only the $m_a = 1$ terms in the sums above. The pertinent expression is as follows:

$$\frac{d^2}{dR^2} \int dk \, \frac{k}{(k^2 + \Lambda^{-2})^{3/2}} \frac{\sin[kR]}{R} \, \cos\left[\left(k^2 + \Lambda^{-2}\right)^{1/2} T\right]$$
$$= \frac{1}{\Lambda^2} \frac{d^2}{d\rho^2} \int dy \, \frac{y}{(y^2 + \rho^2)^{3/2}} \sin[y] \cos\left[\left(y^2 + \rho^2\right)^{1/2} \frac{T}{R}\right], \tag{2.10}$$

where $\rho \doteq R/\Lambda$ and $y \doteq kR$. In order to estimate the magnitude of this contribution, we split the integral at ρ and approximate the kernel appropriately in each domain:

$$\frac{1}{\Lambda^2} \frac{d^2}{d\rho^2} \left\{ \int_0^\rho dy \, \frac{y \sin[y] \cos[t/\Lambda]}{\rho^3} + \int_\rho^C dy \, \frac{\sin[y] \cos[yT/\Lambda\rho]}{y^2} \right\}, \tag{2.11}$$

where $C \sim M_p^{(4)} R$ is the ultraviolet cutoff, which can be related via (1.1) to the size of the compact dimensions. The differentiations can be performed with ease, and neglecting the trigonometric dependence one obtains an order–of–magnitude estimate of the leading contribution for $cT \gg \Lambda$:

$$\frac{1}{\Lambda^2} \int_\rho^C dy \, \frac{T^2}{\Lambda^2 \rho^4} \sim \frac{M_p^{(4)}}{R} \sim \frac{(\pi\Lambda)^{n/2}}{R}, \tag{2.12}$$

where we have adopted the approximation $R = T$ in the kernel, consistent with the classical (unbroadened) light–cone. Now we can reassemble the function F and integrate over x and x' as in (2.1), which yields a light–cone broadening of the form

$$\frac{\Delta t}{t_p} \sim \frac{2^{n/2}}{2\pi} \left(\frac{L}{\ell_p}\right)^{1/2} \sqrt{\ln[L/\ell_p]} \left(\frac{\ell_p}{\pi\Lambda}\right)^{n/4}, \tag{2.13}$$

which for the case $n = 6$ reduces to:

$$\frac{\Delta t}{t_p} \sim \frac{4}{\pi^{5/2}} \left(\frac{L}{\Lambda}\right)^{1/2} \left(\frac{\ell_p}{\Lambda}\right) \sqrt{\ln[L/\ell_p]}. \tag{2.14}$$

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We have neglected contributions from the function H, since they die away much more quickly with increasing m_i and will not affect our rough estimate. Note that in the expression above we have not removed the factor of $(\pi\Lambda)^{-3}$ from the graviton normalization, which is common to the zero–mode (2.9), this contribution, and also the vacuum contribution. In a string–theoretical picture, this scale Λ could pick up a dynamical significance, and could vary with time. As was noted in [10], time variation of Λ could also result from cosmological expansion. Note finally that the L dependence is very similar to that obtained in [9] for a compactified topology, as seen in (1.3) above.

The ratio $\ell_p/\pi\Lambda$ can be calculated in terms of the fundamental Planck scale $M_p^{(4+n)}$, as follows. Following [1], we have

$$\ell_p = 2 \times 10^{-17} \left(\frac{1\,\text{TeV}}{M_p^{(4+n)}}\right) \text{cm}$$

$$\pi\Lambda = 10^{30/n-17} \left(\frac{1\,\text{TeV}}{M_p^{(4+n)}}\right)^{1+2/n} \text{cm} \tag{2.15}$$

whence, if $M_p^{(4+n)} = \mu\,\text{TeV}$,

$$\left(\frac{\ell_p}{\pi\Lambda}\right) = 2 \times 10^{-30/n}\mu^{2/n}. \tag{2.16}$$

For $n = 6$, this ratio is bigger than 2×10^{-5} and grows with μ as $\mu^{1/3}$, and for $\mu \sim 100$ it can be as large as 10^{-4}.

To gain an understanding whether the light–cone broadening effect might in principle be measurable, we consider some specific cases. For gamma–ray bursters with redshifts $z \sim 1$, $L \sim 10^{28}$cm, and, using the above formulae, we estimate $\Lambda \sim 10^{-12}$cm, $\ell_p \sim 10^{-17}$cm for $n = 6$, and hence

$$\Delta t \sim 10^{15} t_p, \tag{2.17}$$

In the extra dimension picture the (fundamental) Planck time is very much larger than normal, being of order 10^{-27} seconds, so that the light–cone broadening is 10^{-12} seconds. This is far below the sensitivity of experiments measuring gamma–ray bursts [13], which is in the millisecond region. It is easy to see that the effect is even smaller for $n < 6$. The above estimates have been made for $\mu = 1$; if the underlying scale is significantly higher than this, both estimates would get larger. In the case of gravity–wave interferometers, the sensitivity of the experiments is much better [14], namely of the order of 10^{-18} metres. For this case we have $L \sim 10^3$cm, and

$$\Delta t \sim 10^2 t_p \sim 10^{-25}\,\text{s}, \tag{2.18}$$

which is in principle testable at current or future gravity–wave interferometers, provided there is a controlled way to distinguish this effect from conventional noise sources.

Before closing this section, we would like to remark briefly on the computation of [10] concerning extra compact dimensions. Due to the periodic boundary conditions imposed in

that case, there are Kaluza–Klein modes which are resummed using the image method. The case of more than one extra dimension complicates the analysis and was not considered in detail in [10]. For one extra dimension, the light–cone broadening effect was found to scale linearly with the distance travelled by the photon, in contrast to our estimate (2.13). We stress therefore that this scaling depends crucially on the boundary conditions as well as the number of non–compact dimensions. For example, when the image method is used for the case of a periodic fifth dimension, there are four k integrals in the non–compact space, and the effect of the discretization is incorporated with the image sum (1.5). In the case considered above, with only Neumann boundary conditions, there are three k integrations and the sums have to be performed explicitly.

III. NON–CRITICAL STRING AND THE D–BRANE RECOIL CONTRIBUTION

So far we have ignored recoil of the 3–brane, which is present in all realistic cases due to the scattering with the closed–string state that propagates in the bulk. As discussed in [6] in the case of D–brane string solitons, their recoil after interaction with a closed–string graviton state [15] is characterized in a world–sheet context by a σ–model deformed by pairs of logarithmic operators [16]:

$$C_\epsilon^I \sim \epsilon \Theta_\epsilon(X^I), \qquad D_\epsilon^I \sim X^I \Theta_\epsilon(X^I), \qquad I = 0, \dots, 3 \qquad (3.1)$$

defined on the boundary $\partial \Sigma$ of the string world–sheet. Here X^I obey Neumann boundary conditions on the string world–sheet, and denote the brane coordinates. The remaining $y^i, i = 4, \dots, 9$ denote the transverse directions.

In the case of D–particles, examined in [6], I takes the value 0 only. In such a case, the operators (3.1) act as deformations of the conformal field theory on the world–sheet: $U_i \int_{\partial \Sigma} \partial_n X^i D_\epsilon$ describes the shift of the D–brane induced by the scattering, where U_i is its recoil velocity, and $Y_i \int_{\partial \Sigma} \partial_n X^i C_\epsilon$ describes quantum fluctuations in the initial position Y_i of the D–particle. It has been shown [12] that energy–momentum is conserved during the recoil process: $U_i = k_1 - k_2$, where $k_1(k_2)$ is the momentum of the propagating closed–string state before (after) the recoil, as a result of the summation over world–sheet genera. We also note that $U_i = g_s P_i$, where P_i is the momentum and g_s is the string coupling, which is assumed here to be weak enough to ensure that D–branes are very massive, with mass $M_D = 1/(\ell_s g_s)$, where ℓ_s is the string length.

In the case of D–p–branes, the pertinent deformations are slightly more complicated. As discussed in [15], the deformations are given by $\sum_I g_{iI}^1 \int_{\partial \Sigma} \partial_n X^i D_\epsilon^I$ and $\sum_I g_{Ii}^2 \int_{\partial \Sigma} \partial_n X^i C_\epsilon^I$. The $0i$ component of the 'tensor' couplings g_{Ii}^α, $\alpha = 1, 2$ include the collective momenta and coordinates of the D–brane, as before, but now there are more couplings g_{Ii}^α, $I \neq 0$, describing the 'bending' of the D–brane under the emission of a closed–string state propagating in the transverse direction, as seen in Fig. 2.

The correct specification of the logarithmic pair (3.1) entails a regulating parameter $\epsilon \to 0^+$, which appears inside the $\Theta_\epsilon(t)$ operator: $\Theta_\epsilon(X^I) = \int \frac{d\omega}{2\pi} \frac{1}{\omega - i\epsilon} e^{i\omega X^I}$. In order to realize the logarithmic algebra between the operators C and D, one takes [15]: $\epsilon^{-2} \sim \ln[\Lambda/a] \doteq \beta$, where Λ (a) are infrared (ultraviolet) world–sheet cutoffs. The recoil operators (3.1) are

slightly relevant, in the sense of the renormalization group for the world–sheet field theory, with small conformal dimensions $\Delta_\epsilon = -\frac{\epsilon^2}{2}$.
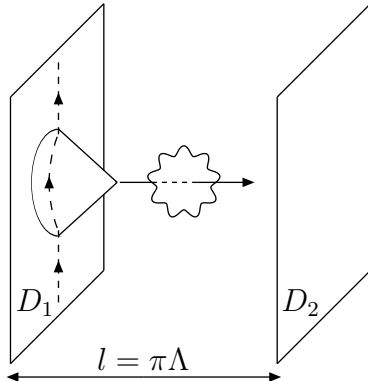

FIG. 2. Schematic representation of the recoil effect: the photon's trajectory (dashed line) is distorted by the conical singularity in the brane that results from closed–string emission into the bulk.

The relevant two–point functions have the following form:

$$\langle C_\epsilon(z) C_\epsilon(0) \rangle \overset{\epsilon \to 0}{\sim} 0 + \mathcal{O}(\epsilon^2)$$

$$\langle C_\epsilon(z) D_\epsilon(0) \rangle \overset{\epsilon \to 0}{\sim} \frac{\pi}{2} \sqrt{\frac{\pi}{\epsilon^2 \beta}} \left(1 + 2\epsilon^2 \log |z/a|^2 \right)$$

$$\langle D_\epsilon(z) D_\epsilon(0) \rangle = \frac{1}{\epsilon^2} \langle C_\epsilon(z) D_\epsilon(0) \rangle \overset{\epsilon \to 0}{\sim} \frac{\pi}{2} \sqrt{\frac{\pi}{\epsilon^2 \beta}} \left(\frac{1}{\epsilon^2} - 2\eta \log |z/a|^2 \right) \tag{3.2}$$

which is the logarithmic algebra [16] in the limit $\epsilon \to 0^+$, modulo the leading divergence in the $\langle D_\epsilon D_\epsilon \rangle$ recoil correlator. In fact, it is this leading divergent term that will be of importance for our purposes below.

Since the recoil operators are relevant in a world–sheet renormalization–group sense, they require dressing with a Liouville field [17] in order to restore conformal invariance, which has been lost in the recoil process. One then makes the crucial step of identifying the world–sheet zero mode of the Liouville field with the target time t, which is justified in [6, 12] using the logarithmic algebra (3.2) for the case at hand. This identification leads to the appearance of a curved space–time background, with metric elements that generalize straightforwardly those for D–particles, that were given in [18]:

$$G_{ij} = \delta_{ij}, \quad G_{00} = -1, \quad G_{0i} = \sum_I \epsilon(\epsilon g_{Ii}^2 + g_{Ii}^1 X^I) \Theta_\epsilon(X^I) \tag{3.3}$$

where the suffix 0 denotes temporal components. In the limit $\epsilon \to 0$, the leading–order terms are the ones proportional to the g_{Ii}^1 bending couplings. From now on we restrict ourselves to these.

For simplicity, we again consider photons moving along the x direction, as in the previous section, in which case the relevant metric perturbations are

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$$h_{0x} = \epsilon \sum_I g_{Ii}^1 X^I \Theta_\epsilon(X^I) \tag{3.4}$$

To evaluate σ_1^2 in terms of the two–point function of h_{0x}, we consider the null geodesic in the presence of the small metric perturbations (3.4). To leading order in the bending/recoil couplings g_{Ii}^1, one has:

$$\langle \sigma_1^2 \rangle \sim L^2 \int_x^{x'} dy \int_x^{x'} dy' \, \langle h_{0x}(y,t) h_{0x}(y',t') \rangle \tag{3.5}$$

In the case of D–brane recoil/bending, the computation of the quantum average $\langle \ldots \rangle$ may be made in the Liouville–string approach described in [6,7]. In this case, the quantum average $\langle \ldots \rangle$ is replaced by a world–sheet correlator calculated with a world–sheet action deformed by (3.1). It is clear from (3.4) that the two–point metric correlator appearing in (3.5) is just the $\langle D_\epsilon D_\epsilon \rangle$ world–sheet recoil two–point function described in (3.2). The result is therefore

$$\Delta t_{\text{recoil/bending}} \sim \frac{L}{c} \left(\sum_I |g_{Ii}^1|^2 \right)^{1/2} \tag{3.6}$$

To obtain an order of magnitude estimate of the effect, we take into account the fact that, for $I = 0$, the coupling $g_{0i}^1 \sim U_i$ is the recoil velocity of the world 3–brane. Viewed as a very massive non–relativistic string soliton, in a dual string theory with coupling g_s, the three–dimensional brane world would have [6,12] a recoil velocity $U_i \sim g_s E/M_s$, where M_s is the fundamental string scale, and E is the typical low–energy scale of the photon propagating on the brane.

Consistent embeddings of the picture of [1] into a string–theoretical framework have been made in [19,20]. There are various string theories which seem theoretically consistent with the picture of [1]. We now argue that the recoil expected in any realistic model compatible with general relativity in the $(3+1+n)$–dimensional space–time places strong restrictions on such models. We examine two explicit cases, namely type I$'$ and type II strings. In the first case [19], D–3–brane configurations appear to be consistent solutions of the model, but with the restriction that only gravitational closed–string states can propagate in the bulk, exactly as advocated in [1]. In this case, the string coupling g_s is given by the four dimensional Yang–Mills gauge fine structure constant at the string scale $g_s = 4\alpha_G$, so that (3.6) yields $\Delta t \sim \alpha_G(LE)/M_s$. Since the astrophysical data on GRBs and other sources are sensitive to $\Delta t \sim (LE)/M_{QG}$ with $M_{QG} \sim 10^{15}$ GeV [13], it seems that, in such a type I$'$ scenario, M_s cannot lie in the TeV range as originally proposed [1].

In the case of type II closed strings, the picture of large extra dimensions can be accommodated [20] provided one uses an extremely weak string coupling:

$$g_s \sim \alpha_G^{-1/2} \frac{M_s}{M_p^{(4)}} \tag{3.7}$$

which is of order 10^{-14} for $M_s \sim$ TeV. In such a scenario there are D–brane solutions, in particular D–5 Neveu–Schwarz branes in which two of the longitudinal dimensions (as well as the extra transverse dimensions) are assumed to be of the string size. This implies

that the brane looks effectively three–dimensional for low–energy physics, as in the type I′ string case. Within our recoil framework, couplings of the form (3.7) will lead to light–cone broadening (3.6) which depends solely on the four–dimensional Planck scale,

$$\Delta t_{\text{recoil/bending}} \sim \frac{L}{c} \frac{E}{M_p^{(4)}}$$

Comparing with the bounds set in [13], we see that this possibility is not excluded, but is also not far beyond the present experimental sensitivity. We also note that, in this latter framework, higher–order quantum phenomena, due to higher world–sheet topologies [6,7], are suppressed by extra powers of the (weak) string coupling $g_s \sim 10^{-14}$ and hence are negligible. It is possible to consider models [20] with some of the transverse dimensions larger which would imply higher values for the string coupling g_s. For $M_s = 10$ TeV, the astrophysical data [13] and our recoil formalism place the phenomenological restriction $g_s \lesssim 10^{-11}$ on the maximum string coupling through the resulting light–cone broadening effect (3.6).

IV. CONCLUSIONS

We have examined in this letter light–cone broadening effects in the context of non–periodic extra dimensions between two parallel brane worlds, one of which represents the observable universe. We have considered the phenomenon from both field– and string–theoretical viewpoints, by analysing the rôle of coherent graviton fluctuations on the propagation of photons on one brane.

In the field–theoretical case, we have estimated that the light–cone broadening scales with the distance L traversed by the photons as

$$\Delta t \sim \sqrt{L \ln L}.$$

For astrophysical sources such as gamma–ray bursters, the order of the effect is about 10^{-12} seconds, which falls well below the sensitivity of observations. However, the sensitivity of gravity–wave interferometer experiments is much better, and for these experiments we estimate a light–cone broadening of the order of 10^{-25} seconds, which may well lie within their sensitivity.

In the string case, we have found that the dominant contributions to the phenomenon come from the recoil of the D–brane due to the scattering of closed–string states (gravitons) propagating in the bulk. The recoil distorts the space–time around the D–brane, resulting in a mean–field effect which implies stochastic fluctuations in the arrival time of photons of energy E on the brane of order

$$\Delta t \sim g_s L \frac{E}{M_s}.$$

Such phenomena place strong restrictions on string–theoretical models of extra dimensions [19,20], particularly type I′ models.

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