B Physics at e^+e^- Colliders

Roger Forty

CERN, Geneva

ABSTRACT

B physics at e^+e^- colliders is reviewed. The experiments at such machines, past and present, are discussed. Then an overview is given of the physics that they are doing, firstly concerning the production of beauty flavoured particles, then the decay. Finally, to compensate for the general nature of the review, more detail is given on a topic that I find exciting, B^0–$\overline{B}^0$ oscillation.

1. Introduction

The title of this review represents a daunting task, as B physics—the study of particles containing the beauty quark—is a large and increasingly popular field. As an example, I counted the number of talks in the parallel sessions at the Glasgow conference last year[1], and out of a total of 241 there were 74 which involved heavy quarks; by this measure B physics makes up almost a third of all high energy physics! Furthermore, almost all of it has been done, or is being done, at e^+e^- colliders[a]. To cover the whole subject in a short review is challenging, and I must necessarily be either selective or superficial, whilst hopefully giving an overview of the field.

I have selected results for illustration without attempting to be exhaustive, and have concentrated on recent developments, particularly from LEP. Since I am a member of the ALEPH collaboration some bias might be noticeable in the selection of examples: you should bear in mind that the other LEP collaborations generally have similar results, that are (almost) as good.

2. e^+e^- experiments

The b quark was discovered in 1977 (at Fermilab, a counter-example to the e^+e^- domination of B physics) through the observation of the Υ, the $b\overline{b}$ bound state. Its ground-state is too light to decay to hadrons containing the b quark ("naked" beauty), but there are excited states of the resonance, as illustrated in Fig. 1; the first that is heavy enough to decay to $B\overline{B}$ is the $\Upsilon(4S)$. This state decays to B^+B^-

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[a] This is an exaggeration: although very little B physics has come from fixed-target experiments there is an increasing contribution from $p\overline{p}$ colliders, although mostly still relying on J/ψ's—the results shown at this meeting by CDF[2] were impressive.

Figure 1: The cross section for $e^+e^- \to$ hadrons as a function of the centre-of-mass energy, showing the Υ and its excited states.

close to 50% of the time, the rest to $\mathrm{B^0_d\overline{B}{}^0_d}$.[b] The large rate for B production has led to machines being operated with centre-of-mass energy sitting on the peak of the $\Upsilon(4S)$: in particular DORIS (DESY) and CESR (Cornell). It should be noted, however, that beneath the $\Upsilon(4S)$ there is a significant background ($\sim$ 3–4 times the signal) from continuum production. Furthermore, as their production is close to threshold the B and $\overline{\mathrm{B}}$ are almost at rest.

Experiments that were installed at these machines include respectively ARGUS (which ran for about ten years, finishing data-taking in 1992) and CLEO (which has been running over a similar period, but is still going strong after a number of upgrades). The $\mathrm{b\overline{b}}$ production cross section is about 1 nb, and the peak luminosity achieved at CESR is an impressive $3 \times 10^{32}\,\mathrm{cm^{-2}s^{-1}}$. This has led to a current dataset for CLEO of about $2\,\mathrm{fb^{-1}}$ on the $\Upsilon(4S)$, corresponding to about 2.2 million $\mathrm{B\overline{B}}$ pairs produced. They have also taken $\sim 1\,\mathrm{fb^{-1}}$ on the continuum (at $E_{\mathrm{cm}} = 10.54\,\mathrm{GeV}$, i.e. below the $\mathrm{B\overline{B}}$ threshold) for background subtraction. A corner of the latest incarnation of CLEO is shown in Fig. 2. As can be seen, it has the traditional layout for an e^+e^- experiment, with a cylindrical geometry and axial magnetic field, and central tracking surrounded by calorimeters. It is, by the modern scale of things, a modestly sized device, with the tracking chambers occupying a region of radius about 1 m. The electromagnetic calorimeter is noteworthy, being composed of 7800 caesium iodide crystals with impressive energy resolution ($\sigma_E/E = 1.5\%$ at 5 GeV). Its high granularity allows complex final states involving many neutrals to be reconstructed, as illustrated in Fig. 3.

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[b] A subscript d is added to the $\mathrm{B^0}$ to distinguish it from the $\mathrm{B^0_s}$—the symbol $\mathrm{B^0}$ is kept to refer to neutral B mesons indiscriminantly.

Figure 2: A section through one quarter of the CLEO detector: the beam pipe is at the bottom of the figure, and the interaction point at the left.

Figure 3: An event display from CLEO, of a fully reconstructed B^+B^- event containing numerous photons: the calorimeter crystals are displayed in a perspective view, as if one were looking down the barrel.

Figure 4: The total cross section for $\mathrm{e^+e^-}$ interaction as a function of the centre-of-mass energy, showing the enormous enhancement at the Z resonance (note the logarithmic scale).

At higher energy, above the Υ but below the Z, one relies on continuum production, with falling cross section as shown in Fig. 4. Here the production rate is proportional to the quark charge squared, so $\mathrm{b\overline{b}}$ contributes only 1/11 of the rate, whilst $\mathrm{c\overline{c}}$ gives about 30% due to the charm quark charge of 2/3, and provides a serious background. B physics is therefore difficult in this energy regime, although some was attempted at PEP and PETRA in the eighties—in particular, the b hadrons are no longer at rest, so their lifetime can be measured.

Eventually one reaches the Z pole, and the cross section takes off as seen in Fig. 4; this is the realm of LEP (CERN) and the SLC (SLAC). As we shall see, the decay $\mathrm{Z \rightarrow b\overline{b}}$ contributes about 22% to the total hadronic width, whilst charm makes up only about 17%, so a "Z-factory" such as LEP is also a good B-factory. SLC, a single-pass collider, has suffered from relatively low statistics compared to LEP, but has the possibility of interesting B physics due to its small beam spot and polarized $\mathrm{e^-}$ beam. At LEP there are four experiments, ALEPH, DELPHI, L3 and OPAL, which have been running since 1989. In principle this year is the last of data-taking at the Z peak, and at the end of the year the centre-of-mass energy will start its upward climb towards the $\mathrm{W^+W^-}$ threshold. This year also marks the change of operation

Figure 5: A cutaway perspective view of the ALEPH detector.

of the machine from "pretzel" mode (with 8 equally spaced bunches of e^+ and e^-, kept apart at parasitic collision points by transverse oscillations of the beams in the arcs) to "bunch trains", where the single bunches are instead replaced with trains of closely spaced bunches (or "waggons"), the current aim being for four trains, each with four waggons. The $b\overline{b}$ production cross section is about 7 nb at the Z, and the peak luminosity achieved so far at LEP is about $2 \times 10^{31}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$, leading to a current dataset per experiment of $\sim 110\,\mathrm{pb}^{-1}$, or about 0.8 million $b\overline{b}$ pairs produced. This year we hope for a further doubling of the dataset, although with the new machine operation the startup has been slow.

A cutaway view of ALEPH is given in Fig. 5, and as can be seen the scale of the experiment is large. Buried in the middle is a detector of great importance for B physics, the silicon microvertex detector. This is composed of planes of silicon wafers arranged into a cylindrical geometry, each implanted with microstrips on both front and back faces, with orthogonal strip orientation on the two faces. A spatial resolution of about $12\,\mu$m is achieved in both projections, which allows the characteristic lifetime of b hadrons to be straightforwardly identified.

3. B production

3.1. Z → b$\overline{\rm b}$ *partial width*

One of the simplest questions one might think to ask about Z decays is what fraction give beauty quarks—to answer this precisely turns out to be challenging experimentally; it is important, however, because Z → b$\overline{\rm b}$ has a contribution from vertex corrections involving the top quark, of the type shown in Fig. 6. This contribution is suppressed for other quark final states, and thus

$$R_{\rm b} \equiv \frac{\Gamma({\rm Z} \to {\rm b}\overline{\rm b})}{\Gamma({\rm Z} \to q\overline{q})} \tag{1}$$

has a strong top-quark mass dependence. Furthermore, contributions from the Higgs cancel in the ratio, so $R_{\rm b}$ is insensitive to the Higgs mass.

The experimental measurement is simple in principle: one selects hadronic Z decays (which is straightforward at LEP) and then one tags the b decays in the selected sample. The b-tagging can be achieved using various techniques:

1. Leptons from the semileptonic decay b→ $X\ell\nu$. This is the traditional approach; due to the hard b fragmentation (discussed below) and large b mass the leptons have high momentum p and high transverse momentum $p_{\rm T}$ relative to their associated jet, allowing clean separation from background contributions. The efficiency of a lepton tag is, however, limited by the semileptonic branching ratio.

2. Event shapes. This approach is more difficult as the difference between b decays and lighter quark decays are rather subtle: due to its mass the b quark tends to produce events that are more "spherical" in nature, for example, and thus a variable such as boosted sphericity has some discriminating power; these days neural networks are often used, with many such variables as input.

3. Lifetime tagging. Here the microvertex detector information is exploited: at LEP the b lifetime corresponds to a decay length of a few millimetres, which can

Figure 6: Vertex correction to the Z → b$\overline{\rm b}$ process.

Figure 7: Efficiency versus purity for a lifetime-based b-tagging algorithm; the dashed curve is the result when one hemisphere only of the event is used, the solid curve when both hemispheres are used.

easily be seen using the precise tracking that such detectors provide. Typically a variable is constructed that represents the probability that all tracks in the event, or in the "hemisphere" of the b decay (i.e. the half event, defined by a plane perpendicular to, for example, the thrust axis) come from a single vertex. In the case of light quark events such a variable would be close to unity, whilst for $\mathrm{b\overline{b}}$ events it will be small on average.

The performance of such a lifetime tag is illustrated in Fig. 7, where the efficiency is plotted against the purity for selecting b decays, when the information from either one or both hemispheres of an event is used[3]. As can be seen, a purity of 90% can be achieved whilst maintaining an efficiency close to 50%. For a lepton tag at such purity the efficiency would only be about 10%; the lifetime tagging technique is significantly more powerful than other approaches to b-tagging.

For the extraction of R_{b} the efficiency of the tag must be known accurately. This has led to the development of the "double-tagging" technique, where the two hemispheres of an event are used separately. One counts the fraction of single hemispheres that are tagged, r_{s}, and the fraction of events for which *both* hemispheres are tagged, r_{d}. Then if there is no background, and no correlations between the two hemispheres of an event, the efficiency ε_{b} and R_{b} can be measured directly from the data:

$$\varepsilon_{\mathrm{b}} = \frac{r_{\mathrm{d}}}{r_{\mathrm{s}}} \quad , \quad R_{\mathrm{b}} = \frac{r_{\mathrm{s}}^{2}}{r_{\mathrm{d}}} \ . \tag{2}$$

Of course in reality there is background, particularly from charm, and the hemisphere correlations are not negligible—for example, if a mistake is made in estimating the production vertex of the event, it effects both hemispheres in opposite directions.

Figure 8: The top-quark mass dependence of $R_{\rm b}$ in the standard model, with the experimental result superimposed.

Nevertheless, this approach results in a measurement of $R_{\rm b}$ that is less dependent on the modelling of b decays than would be the case if the efficiency was estimated from Monte Carlo simulation.

The current LEP average is $R_{\rm b} = 0.2192 \pm 0.0018$ [4]. The standard model prediction is displayed in Fig. 8, as a function of the top-quark mass. Since we now have a measurement of $m_{\rm t} = (180 \pm 12)\,\mathrm{GeV}$ (the average of CDF [5] and D0 [6] results), the experimental result can be superimposed on the figure as shown. As can be seen, there is an approximately 2σ discrepancy between the data and the standard model prediction; since this is more-or-less the only such discrepancy at LEP it has led to some hysteria amongst theorists, extending the standard model in all directions. It is clear that an improved experimental measurement would be welcome, to verify that there really is a discrepancy; as the measurement is systematically limited, however, such improvement will be hard to achieve.

3.2. Forward-backward asymmetry

Whilst on the subject of electroweak measurements with b quarks, another observable is the forward-backward asymmetry. This is defined for a process $\mathrm{e^+e^-} \to f\overline{f}$, where f is a fermion, by counting the number $N_{\rm F}$ of final-state fermions that are produced in the forward direction (defined relative to the incoming electron) and

Figure 9: The polar angle distribution for $\mathrm{b\overline{b}}$ events.

comparing the number produced backward, N_B. Then

$$A^f_\mathrm{FB} = \frac{N_\mathrm{F} - N_\mathrm{B}}{N_\mathrm{F} + N_\mathrm{B}} \ . \tag{3}$$

On the Z peak, a non-zero asymmetry results from the interference of the vector and axial couplings:

$$A^f_\mathrm{FB} \approx \frac{3}{4} A_\mathrm{e} A_f \ , \quad A_f = \frac{2 a_f v_f}{a_f^2 + v_f^2} \ , \tag{4}$$

where the vector and axial couplings are related by

$$\frac{v_f}{a_f} = 1 - 4|Q_f| \sin^2\theta_\mathrm{W} \ . \tag{5}$$

The b-quark charge $Q_\mathrm{b} = -1/3$ leads to a strong sensitivity of A^b_FB to $\sin^2\theta_\mathrm{W}$—about three times that of the muon asymmetry, for example.

To measure the asymmetry the b decays must again be tagged. In addition, the particle/antiparticle state of the b hadron must be determined—this is straightforward with lepton tagging, since the sign of the lepton from the semileptonic decay of a b hadron reflects the initial b-quark charge (apart from the influence of $\mathrm{B^0}$–$\mathrm{\overline{B}^0}$ mixing, discussed below, which must be corrected for). The direction of the b quark is usually estimated from the thrust axis of the event, signed using the lepton charge. A typical distribution as a function of the polar angle θ is shown in Fig. 9 (after acceptance correction), which displays a clear asymmetry. Fitting with the predicted angular dependence $\sim \frac{3}{8}(1 + \cos^2\theta) + A^\mathrm{b}_\mathrm{FB} \cos\theta$, the asymmetry can be extracted.

Figure 10: Schematic illustration of the production of hadrons in e^+e^- collisions.

The result from that analysis is $A_{\mathrm{FB}}^{\mathrm{b}} = (8.43 \pm 0.68 \pm 0.14)\%$ [7][c]; illustrating that this measurement is still statistically limited. The current LEP average is $(9.67 \pm 0.38)\%$, which can be used to extract $\sin^2\theta_{\mathrm{W}} = 0.2327 \pm 0.0007$ [4], one of the most precise measurements of this fundamental parameter.

3.3. Fragmentation

Up to this point, the discussion has concerned only b quarks. Of course, what are seen in the experimental apparatus are b hadrons, since the quarks are confined. The produced $\mathrm{b\overline{b}}$ quark pair must therefore hadronize, as sketched in Fig. 10. The shaded region in the figure is the realm of non-perturbative QCD, which unfortunately cannot be calculated exactly; models are therefore used, such as JETSET string-fragmentation (popular with the LEP collaborations).

One feature that can be studied is the "fragmentation function", i.e. the fraction of the initial b quark energy that ends up being carried by the b hadron. This has been studied at LEP using the decay $\mathrm{B} \to \mathrm{D}^*\ell\nu$, where the sum of energies of the reconstructed D^*, lepton, and neutrino (measured from the missing energy in the hemisphere) allows the b-hadron energy to be determined. The extracted fragmentation function (after an iterative acceptance correction) is shown in Fig. 11 [8]. As can be seen the measured distribution is in reasonable agreement with the Peterson form, which is widely assumed in Monte Carlo simulations. On average one finds that the b hadron takes $(70.2 \pm 0.8)\%$ of the b-quark momentum—this is "hard" fragmentation by comparison with for example the charm quark, where the equivalent fraction is close to 50%.

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[c] When two errors are given for a measurement in this report, the first is statistical and the second systematic.

Figure 11: B energy fraction for reconstructed $\mathrm{B} \to \mathrm{D}^{*}\ell\nu$ decays, with superimposed fit (solid line) and Peterson function (dotted histogram).

3.4. Spectroscopy

At the $\Upsilon(4S)$ the B^{+} and $\mathrm{B}^{0}_{\mathrm{d}}$ are produced copiously. They have been fully reconstructed at CLEO, summing a large number of different decay channels: $\mathrm{B} \to \mathrm{D}^{(*)}\pi$, $\mathrm{D}^{(*)}\rho$, $\mathrm{D}^{(*)}\mathrm{a}_1$, $\mathrm{J}/\psi\mathrm{K}$, and so on. The reconstructed invariant mass plot is shown in Fig. 12 for the B^{+}, showing a signal of 834 ± 42 fully-reconstructed B^{+} mesons[9]. From such analyses CLEO measure the masses:

$$\begin{aligned} m(\mathrm{B}^{+}) &= (5278.7 \pm 0.2 \pm 2.0)\,\mathrm{MeV} \\ m(\mathrm{B}^{0}_{\mathrm{d}}) &= (5279.2 \pm 0.2 \pm 2.0)\,\mathrm{MeV}\ , \end{aligned} \tag{6}$$

where the systematic error is dominated by the uncertainty on the beam energy, which cancels in the difference: $m(\mathrm{B}^{+}) - \mathrm{m}(\mathrm{B}^{0}_{\mathrm{d}}) = (0.41 \pm 0.31)\,\mathrm{MeV}$. Clean signals for fully-reconstructed B^{+} and $\mathrm{B}^{0}_{\mathrm{d}}$ are now also being observed at LEP, as illustrated in Fig. 13.

At the Z, however, there are also heavier states produced; it is expected that the B^{+}, $\mathrm{B}^{0}_{\mathrm{d}}$, $\mathrm{B}^{0}_{\mathrm{s}}$ and Λ^{0}_{b} are produced in the proportions $(39:39:12:10)\,\%$. The first direct evidence for the $\mathrm{B}^{0}_{\mathrm{s}}$ was supplied by LEP, from the study of $\mathrm{D}^{+}_{\mathrm{s}}$–$\ell^{-}$ correlations. A recent example is shown in Fig. 14, where the $\mathrm{D}^{+}_{\mathrm{s}}$ invariant mass plot is shown in events where a lepton is found in the same jet, with the "right" or "wrong" sign for coming from the decay $\mathrm{B}^{0}_{\mathrm{s}} \to \mathrm{D}^{-}_{\mathrm{s}}\ell^{+}X$, i.e. with the opposite- or same-sign as the

Figure 12: Invariant mass of reconstructed B^+ candidates from CLEO.

D_s^-. The clear enhancement seen for the right-sign decays provides evidence for the production of B_s^0.

More recently, B_s^0 candidates have been fully reconstructed, as shown for example in Fig. 15 from OPAL[10]. There is a particularly beautiful event from ALEPH[11] with

Figure 13: Invariant mass of (a) reconstructed B^+ and (b) B_d^0 candidates from ALEPH.

Figure 14: Invariant mass of reconstructed $\mathrm{D_s^+}$ candidates in events with a reconstructed lepton, (a) where the $\mathrm{D_s^+}$ and lepton have opposite charge and (b) where they have the same charge.

a decay in the channel $\mathrm{B_s^0} \rightarrow \psi'\phi$. Through a lucky kinematical configuration this one event gives a very precise measurement of the mass:

$$m(\mathrm{B_s^0}) = (5368.4 \pm 5.6 \pm 1.5)\,\mathrm{MeV}\ , \tag{7}$$

which dominates the current LEP average $m(\mathrm{B_s^0}) = (5368.5 \pm 5.3)\,\mathrm{MeV}$.

Figure 15: Invariant mass of reconstructed $\mathrm{B_s^0}$ candidates from OPAL.

Figure 16: (a) Invariant mass of reconstructed Λ candidates in events with a reconstructed lepton, of right-sign (above) or wrong-sign (below); (b) invariant mass of reconstructed Λ_b^0 candidates (for two different cuts on their momentum).

Before leaving the mesons, what about the B_c^+? Since we know from LEP that there are only three generations of quarks, and the top-quark mass is so heavy that it will decay before hadronizing, the b hadrons are the last system of hadrons, and the B_c^+ is the last meson that remains to be discovered! Its predicted mass is $\sim 6.3\,\mathrm{GeV}$, and production rate of a few hundred per million hadronic Z decays, so the search is now underway at LEP. The decay of the B_c^+ is interesting as it has two heavy quarks, and they compete in determining its lifetime: either the b or c quark can decay first, or they can annihilate one another; all of which leads to a predicted lifetime that is shorter than other b mesons, but with large uncertainty. ALEPH has searched for the decay $B_c^+ \to J/\psi \ell \nu X$, and has found two candidates with clear three-muon final states. However, with an expected background of about 0.5 events no signal can yet be claimed and a limit is set on the branching ratio product:

$$B(\mathrm{Z} \to \mathrm{B}_c^+ X)\, B(\mathrm{B}_c^+ \to \mathrm{J}/\psi \ell \nu X) < 7 \times 10^{-5} \ (90\% \ \mathrm{CL}) \ . \qquad (8)$$

Amongst the b baryons the production of Λ_b^0 is expected to dominate. First evidence at LEP came from Λ–ℓ correlations, as shown in Fig. 16 (a). Here the right-sign combinations are $\Lambda\ell^-$ and $\overline{\Lambda}\ell^+$; the wrong-sign plot also shows an enhancement at the Λ mass, due to the production of Λ's in the fragmentation process (combined accidentally with a lepton); however, one expects that there should be roughly an equal probability of right- or wrong-sign correlation for these events, so the *excess* of the right-sign events provides evidence for Λ_b^0 production.

Concerning full reconstruction of the Λ_b^0, there has been some controversy in the past, with UA1 claiming a signal for $\Lambda_b^0 \to J/\psi\Lambda$ with a branching ratio of $(1.8 \pm 1.0)\%$ [12], which was not confirmed by CDF: they set a limit for the same

Figure 17: Difference in invariant mass between B^* and B candidates.

branching ratio of $< 0.5\%$ at 90% CL[13]. Now LEP has some candidates, as illustrated in Fig. 16 (b) from ALEPH for the channel $\Lambda_\mathrm{b}^0 \to \Lambda_\mathrm{c}^+\pi^-$. Although the significance of this signal is only 2–3σ there are also some candidates from DELPHI and OPAL, giving a combined mass measurement of[14]:

$$m(\Lambda_\mathrm{b}^0) = (5626 \pm 19)\,\mathrm{MeV}\ . \tag{9}$$

Baryon spectroscopy is rich, and is just starting at LEP. As well as the Λ_b^0 there are first indications of the Ξ_b, signals for which have been seen by ALEPH and DELPHI in Ξ^-–ℓ^+ correlations.

3.5. Excited states

The vector partner of the pseudoscalar b mesons is denoted B^*, and was established by CUSB and CLEO with a mass difference: $m(\mathrm{B}^*) - m(\mathrm{B}) \approx 46\,\mathrm{MeV}$. It therefore decays by emission of a photon, with $B(\mathrm{B}^* \to \mathrm{B}\gamma) = 100\%$. At LEP energies the photon is boosted but still has energy less than about 800 MeV, and is therefore difficult to reconstruct in the calorimeters. New results from LEP rely on inclusive B reconstruction: jets are tagged as containing b decays using the lifetime information, and then tracks are selected from within the jet as coming from the b decay using, for example, their rapidity: tracks from the b hadron have generally higher rapidity than those from fragmentation. To the inclusively reconstructed B candidate, a reconstructed photon is added, either from the electromagnetic calorimeter (difficult, although L3 has managed) or from reconstructing a conversion $\gamma \to \mathrm{e}^+\mathrm{e}^-$. The reconstructed mass-difference plot from such an analysis is shown in Fig. 17[15],

Figure 18: Difference in invariant mass between B^{**} and B candidates.

and shows a clear enhancement from the B^{*}. The mass difference is measured to be:

$$m(\mathrm{B}^{*}) - m(\mathrm{B}) = (45.2 \pm 0.4 \pm 0.9)\,\mathrm{MeV} \tag{10}$$

and the production rate:

$$\frac{N(\mathrm{B}^{*})}{N(\mathrm{B}^{*}) + N(\mathrm{B})} = (74 \pm 7)\,\% \ , \tag{11}$$

in agreement with the value of 3/4 expected from simple spin-counting.

Finally concerning B production we come to a hot topic: B^{**} states. This is the generic name given to higher excited states of the B with orbital excitation, which had not been seen before last year. They have been studied at LEP by extending the B^{*}-style analyses to search for $\mathrm{B}^{**} \to \mathrm{B}^{(*)}\pi$, using the same inclusive B reconstruction but instead of adding a photon, adding a charged track from the production vertex (assumed to be a pion). A typical mass-difference plot from such an analysis[15] is shown in Fig. 18, and again displays a clear enhancement above the expected background (modelled using Monte Carlo simulation). The interest in these measurements is that Heavy Quark Effective Theory, the recent advance in the theoretical understanding of B decays[16], makes definite predictions about the B^{**} spectrum: there should be two doublets of states distinguished by the light-quark angular momentum j. The doublet with $j = \frac{1}{2}$ ($J^P = 0^+$, 1^+) are expected to be broad states with width $\sim 150\,\mathrm{MeV}$, whilst those with $j = \frac{3}{2}$ ($J^P = 1^+$, 2^+) should be narrow, with width $\sim 20\,\mathrm{MeV}$. The observed resonance structure in Fig. 18 is too broad to be a single narrow state; the resolution is insufficient to draw stronger conclusions, so higher resolution (more exclusive) analyses are underway.

Requiring that the additional track be a kaon rather than a pion has led to the observation of first indications for the $\mathrm{B_s^{**}}$, and this topic will remain of great interest in the coming months. Overall the rate of $\mathrm{B^{**}}$'s is such that about 30% of B's at LEP come from $\mathrm{B^{**}}$ decays.

4. B decay

The weak decays of quarks occur via the charged current:

$$J^\mu = \overline{U}\,\frac{1}{2}\gamma^\mu(1-\gamma^5)\,V_{\mathrm{CKM}}D\ , \tag{12}$$

where V_{CKM} is the Cabbibo-Kobayashi-Maskawa matrix that describes the strength of the coupling between up-type (U) and down-type (D) quarks; its elements are fundamental parameters of the standard model. Experimentally the following hierarchy is observed:

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}\ , \tag{13}$$

where $\lambda = \sin\theta_{\mathrm{C}} = 0.2205 \pm 0.0018$ is the sine of the Cabbibo angle [17]. Unitarity implies that the matrix can be described using four independent parameters; the popular Wolfenstein parametrization is motivated by the observed hierarchy of the elements, and represents an expansion in powers of λ, with the other three parameters denoted A, ρ and η:

$$V_{\mathrm{CKM}} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{14}$$

to $\mathcal{O}(\lambda^3)$. Parameter η represents the imaginary part of the matrix, the non-trivial phase which is only present if there are at least three generations of quarks. Non-zero η corresponds to the existence of CP violation, seen so far only in the kaon system. The standard model predicts observable CP violation in b decays, and its study is the goal of future B-factories. Since CP violation is required to explain the dominance of matter over antimatter in the universe, and thus our existence, it provides another reason for considering B physics important!

4.1. Spectator model

Moving now to the decay of b *hadrons*, in the spectator model the b quark in the hadron is treated as if it decays freely: the other quark in the meson (or diquark in a baryon) simply acts as a "spectator" to the decay, playing no part. Then the semileptonic decay rate of the b hadron can be simply related to that of muon decay,

Figure 19: Diagrams for the decays (a) $\mu \to \mathrm{e}\nu_{\mathrm{e}}\nu_{\mu}$ and (b) $\mathrm{b} \to q\ell\nu$.

illustrated in Fig. 19:

$$\Gamma(\mu \to \mathrm{e}\nu_{\mathrm{e}}\overline{\nu}_{\mu}) = \frac{\mathrm{G_F^2\, m_\mu^5}}{192\,\pi^3}\,\eta_\mu \ , \qquad (15)$$

where η_μ is a phase-space correction, close to unity. For the b hadron decay this becomes:

$$\begin{aligned} \Gamma(\mathrm{b} \to q\ell\nu) &= \frac{G_{\mathrm{F}}^2\, m_{\mathrm{b}}^5}{192\,\pi^3}\,\eta_q |V_{q\mathrm{b}}|^2 \qquad (16) \\ &= \frac{B(\mathrm{b} \to q\ell\nu)}{\tau_{\mathrm{b}}} \ , \end{aligned}$$

where the muon mass has been replaced by the b quark mass, and the relevant V_{CKM} element has been introduced; τ_{b} is the average b hadron lifetime.

4.2. Semileptonic branching ratio

To estimate the inclusive semileptonic branching ratio of b hadrons, one can consider the final states accessible to the virtual W: $\mathrm{e}\nu$, $\mu\nu$, $\tau\nu$, $\mathrm{u\overline{d}}$ and $\mathrm{c\overline{s}}$. Folding in the respective phase-space factors, and a factor three for colour for the quark final states, one would naively predict $B(\mathrm{b} \to X\ell\nu) \approx 15\%$. There are QCD corrections, and non-spectator decays (discussed below) that increase the hadronic rate, and reduce the predicted branching ratio to about 12%.

For the experimental measurement, the traditional approach is to fit the (p, p_{T}) distribution of reconstructed leptons. The main background comes from the "cascade" decay via charm, $\mathrm{b} \to \mathrm{c} \to \ell$, which tends to give a lower p_{T} lepton than the direct $\mathrm{b} \to \ell$ decay, and so can be separated in the global fit. However, the (p, p_{T}) spectra of the two contributions need to be modelled using Monte Carlo, and this leads to a dependence on the b decay model assumed.

A new approach has been pioneered by the $\Upsilon(4S)$ experiments, using events with two leptons: for the first lepton a momentum cut of $> 1.4\,\mathrm{GeV}$ is applied, so that it is almost certainly from direct b decay; then the charge correlation of the two leptons is studied. If the second lepton has opposite charge to the first, it is most likely to be

Figure 20: Momentum spectra reconstructed for b $\rightarrow \ell$ decays (solid points) and b $\rightarrow$ c $\rightarrow \ell$ decays (open points) from CLEO.

from direct b decay, whilst if it has the same charge is is probably from the cascade decay (angular cuts are applied to ensure that the two leptons are from different B decays). A correction must be applied for B^0–$\overline{\mathrm{B}}^0$ mixing, but as a result of this technique the momentum spectra of the direct and cascade decays can be unfolded down to low momentum, as shown in Fig. 20. This leads to reduced model-dependence for the extracted semileptonic branching ratio.

The current averages from the $\Upsilon(4S)$ and LEP experiments are[18]:

$$\begin{aligned} B(\mathrm{b} \rightarrow \mathrm{c}\ell\nu) &= (10.31 \pm 0.10 \pm 0.25)\,\%\ (\Upsilon) \\ &= (11.33 \pm 0.22 \pm 0.41)\,\%\ (\mathrm{LEP}) \end{aligned} \tag{17}$$

As can be seen, there is marginal consistency between these two values[d], and between them and the theoretical expectation. Further work is in progress to clarify this discrepancy, applying similar techniques with reduced model-dependence at LEP.

4.3. Average b lifetime

Charmed hadron lifetimes were well measured before the b hadrons were studied, and have lifetimes of order 10^{-12} s. Since, as seen in Eq. 16, $\tau_Q \propto 1/m_Q^5$, a short b lifetime was expected; it was found to be surprisingly long due to the small coupling, $V_{\mathrm{cb}} \sim \lambda^2$. The traditional technique for measuring the inclusive b lifetime is to study the impact parameter, or distance of closest approach to the production vertex, of high p_{T} leptons. The production vertex position is typically determined using the

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[d] This discrepancy is potentially greater than is seems at first sight, since it is expected that the semileptonic partial widths for the different b hadron species should be equal, and so the exclusive semileptonic branching ratios should scale with the lifetimes; at LEP there are Λ_{b}^0's produced, with (as we shall see) a shorter lifetime than the average; so one would expect if anything the average for $B(\mathrm{b} \rightarrow \mathrm{c}\ell\nu)$ at LEP to be slightly lower than that at the $\Upsilon(4S)$.

Figure 21: Three-dimensional impact parameter distribution for high p_{T} lepton candidates from ALEPH.

beam spot, the area over which the e^+ and e^- beams collide ($\sim 150 \times 10\,\mu$m at LEP). The average impact parameter expected is $\sim c\tau \approx 450\,\mu$m, reduced to $\sim 300\,\mu$m if the projection into the plane transverse to the beam is made (as has usually been the case due to the general preference for tracking in that plane). With the typical resolution of a few hundred microns that was achieved by experiments at PEP and PETRA that first measured the b lifetime, the resulting distributions were rather Gaussian in character, with a small offset due to the lifetime.

The advent of silicon microvertex detectors has led to a dramatic improvement in this type of analysis, as illustrated in Fig. 21[19]. Now with double-sided silicon detectors the 3-dimensional impact parameter can be measured, and with the high spatial resolution the exponential character of the lifetime is clearly visible. The result of the illustrated analysis is:

$$\tau_{\mathrm{b}} = (1.533 \pm 0.013 \pm 0.022)\,\mathrm{ps}\ , \tag{18}$$

in good agreement with (and comparable precision to) the recent world average value of $(1.537 \pm 0.021)\,\mathrm{ps}$[17]. The measurement of this quantity has had a rather chequered history, with earlier values for the world average being as low as $\sim 1\,\mathrm{ps}$; it now finally appears to have stabilized. I will return in Section 4.7 to the extraction of V_{cb} from the measurements of $B(\mathrm{b} \to \mathrm{c}\ell\nu)$ and τ_{b}.

4.4. Non-spectator decays

The spectator model is only an approximation: there are some decays in which the "spectator" quark participates. An obvious example is the fully leptonic decay of a B meson, where the quarks in the meson must annihilate. This leads to a suppression of the rate by a factor proportional to the square of the decay constant f_{B} for the meson. The rate is further reduced due to helicity suppression: a spin 0 particle cannot decay to a massless pair of fermions; of course the charged lepton is not massless, but the rate is suppressed by a factor proportional to its mass squared. Thus

$$\Gamma(\mathrm{B} \to \ell\nu) \propto f_{\mathrm{B}}^2\, m_\ell^2\, |V_{\mathrm{ub}}|^2 \ . \tag{19}$$

This is largest for $\ell = \tau$, for which the predicted branching ratio is $\sim 10^{-5}$ (but could be larger in extensions to the standard model, for example involving the charged Higgs). The experimental signature is large missing energy, due to the neutrinos from both the initial leptonic decay and the subsequent tau decay. The reconstructed missing energy spectrum from an ALEPH analysis is shown in Fig. 22. In the region marked (1) there is a significant excess of events over the predicted background, allowing a measurement to be made of the inclusive branching ratio:

$$B(\mathrm{b} \to \tau^- \overline{\nu} X) = (3.12 \pm 0.36 \pm 0.38)\,\% \ . \tag{20}$$

The region (2), of higher missing energy, is where the fully leptonic decays should be seen; for the present there is no significant excess, and limit is set:

$$B(\mathrm{B}^+ \to \tau^+\nu) < 1.5 \times 10^{-3}\ (90\%\ \mathrm{CL}) \ . \tag{21}$$

Figure 22: Missing energy distribution from ALEPH, in the search for $\mathrm{b} \to \tau^- \overline{\nu} X$ decays.

Figure 23: Diagrams for B decay: (a) colour-allowed spectator decay (b) colour-suppressed (c) W-exchange.

In the simple spectator model, all b hadron lifetimes would be equal. This was found not to be true in the charm system, where $\tau(\mathrm{D}^+) \approx 2.5\,\tau(\mathrm{D}^0)$. This is understood to be a result of the influence of non-spectator decays. The effect of the strong interaction is to introduce other decay diagrams for the hadronic b decays, as illustrated in Fig. 23 (a) and (b). In the case of the B^+ the "colour-suppressed" diagram gives the same final state as the original (colour-allowed) spectator diagram; they can therefore interfere, and the interference is believed to be destructive, leading to a longer lifetime for the B^+. There are also non-spectator diagrams of the type shown in Fig. 23 (c); these are helicity suppressed for the meson decays, but the suppression is lifted by the extra quark in a baryon, leading to a shorter predicted lifetime for the Λ_b^0. These considerations have been studied in the framework of HQET[20], with the conclusion that the lifetime differences should scale with f_Q^2/m_Q^2, and should thus be much smaller for the b system than for charm. The predicted hierarchy is:

$$\tau(\mathrm{B}^+) > \tau(\mathrm{B}_\mathrm{s}^0) \approx \tau(\mathrm{B}_\mathrm{d}^0) > \tau(\Lambda_\mathrm{b}^0)\ , \tag{22}$$

with the first inequality expected to be about 5%, and the last about 10%.

4.5. Exclusive b lifetimes

A typical exclusive lifetime measurement is illustrated in Fig. 24 (a) for the B_s^0. In this case the D_s^+ decay is reconstructed first, enriching the sample in B_s^0. The D_s^+ is then vertexed with a lepton of the correct charge to form the candidate B_s^0 decay vertex; the decay length d can then be determined if the primary vertex position is known (usually relying on knowledge of the beam-spot position). The reconstructed proper time for the decay is then given by:

$$t = \frac{d\,m_\mathrm{B}}{p_\mathrm{B}\,c}\ , \tag{23}$$

where m_B and p_B are the mass and reconstructed momentum of the B_s^0, typically determined with the help of missing energy for the neutrino. The resolution on

Figure 24: (a) Schematic illustration of a $\mathrm{B_s^0} \rightarrow \mathrm{D_s^+}\ell^-\nu$ decay; (b) proper time distribution for $\mathrm{B_s^0}$ candidates from ALEPH, with the different contributions to the fit displayed.

proper time is given by:

$$\frac{\sigma_t}{\tau} = \frac{\sigma_d}{\langle d \rangle} \oplus \frac{\sigma_p}{p}\frac{t}{\tau} \ . \tag{24}$$

The two terms combined in quadrature are respectively the decay-length resolution and the momentum resolution—the latter component scales with proper time and thus dominates at long proper times; for a typical LEP detector the two terms are roughly equal at a proper time of one lifetime τ (for semileptonic decays) and give an overall resolution on t of 10–20%. The result of a representative analysis, for the $\mathrm{B_s^0}$ lifetime, is shown in Fig 24 (b), and gives [21]:

$$\tau(\mathrm{B_s^0}) = (1.60 \ {}^{+0.17}_{-0.15} \ {}^{+0.03}_{-0.02})\,\mathrm{ps} \ . \tag{25}$$

Over recent years the measurement of exclusive b lifetimes has developed into an industry, and rather than show all of the different measurements I just summarize the results in Fig. 25 [22]. As can be seen, the predicted lifetime hierarchy is observed, although the measured Λ_b^0 lifetime is rather lower than expected.

4.6. Charmless b decays

The CKM matrix element $|V_\mathrm{ub}|$ is expected in the framework of the standard model to be non-zero: if it (or any other matrix element) were exactly zero then it can be shown that CP violation arising from the non-trivial phase of the CKM matrix would vanish. Thus charmless b decays (to u quarks) are expected. To measure $|V_\mathrm{ub}|$ the traditional approach is to study the lepton momentum spectrum at the $\Upsilon(4S)$: $\mathrm{b} \rightarrow \mathrm{u}\ell\nu$ can give $p_\ell > 2.4$ GeV, beyond the $\mathrm{b} \rightarrow c\ell\nu$ endpoint, since $m_\mathrm{u} < m_\mathrm{c}$. The

Figure 25: Summary of the world averages for b lifetimes; the value marked “average” is calculated under a standard assumption for the production fractions of the different species, and agrees well with the inclusive measurement.

endpoint region from CLEO is shown in Fig 26, and shows a clear excess of events in the region above the b→ $c\ell\nu$ endpoint. From this analysis a value of:

$$\left|\frac{V_{\mathrm{ub}}}{V_{\mathrm{cb}}}\right| = 0.08 \pm 0.02 \tag{26}$$

is extracted.

CLEO have also made a direct search for charmless hadronic B decays, suppressing background using the dE/dx measurement from their tracking detector and with event shape cuts. The resulting mass plot is shown in Fig. 27, and shows an enhancement at the $\mathrm{B_d^0}$ mass that is fitted to give:

$$B(\mathrm{B_d^0} \to \mathrm{K}^+\pi^-, \pi^+\pi^-) = (1.8 \ ^{+0.6}_{-0.5} \pm 0.2) \times 10^{-5} \ . \tag{27}$$

At present there are insufficient statistics to quote separate branching ratios for the individual $\mathrm{K}^+\pi^-$ and $\pi^+\pi^-$ decay modes. At LEP, oppositely charged tracks with a detached vertex have been searched for by ALEPH. A few candidates are seen with $m_{\pi\pi} > 5\,\mathrm{GeV}$, above the endpoint for b → c, consistent with the branching ratio quoted in Eq. 27.

Figure 26: End-region of the lepton momentum spectrum from CLEO (a) with "tight" and (b) with "loose" cuts; the continuum background is shown as the open points with fit, the $\Upsilon(4S)$ data is shown as the solid points, and the Monte Carlo prediction for b$\rightarrow$ c$\ell\nu$ decay is shown as a histogram.

Figure 27: Invariant mass for $\mathrm{B}^0_\mathrm{d} \rightarrow \mathrm{K}^+\pi^-, \pi^+\pi^-$ candidates from CLEO (the $\mathrm{K}^+\pi^-$ contribution is shaded).

4.7. *Extracting* $V_{\rm cb}$

From before (Eq. 16) we have:

$$\eta_{\rm c}|V_{\rm cb}|^2 + \eta_{\rm u}|V_{\rm ub}|^2 = \frac{192\,\pi^3}{G_{\rm F}^2 m_{\rm b}^5}\,\frac{B({\rm b}\to \ell\nu X)}{\tau_{\rm b}}\ , \tag{28}$$

where the phase-space and QCD correction coefficients $\eta_q(m_q/m_{\rm b}, \alpha_{\rm s})$ are anticorrelated with $m_{\rm b}$. Using values of $m_{\rm b} = (5.0 \pm 0.3)\,{\rm GeV}$ and $m_{\rm b} - m_{\rm c} = (3.3 \pm 0.1)\,{\rm GeV}$ from ARGUS (interpreted within the ACCMM model)[23], this equation corresponds to the circular constraint shown in Fig. 28. Using the result for $V_{\rm ub}/V_{\rm cb}$ from the previous section, one finds:

$$|V_{\rm cb}| = 0.041 \pm 0.002\ ^{+0.004}_{-0.003}\ . \tag{29}$$

Note that although the error on $B({\rm b} \to \ell\nu X)/\tau_{\rm b}$ is only $\sim 4\%$ the error on $V_{\rm cb}$ is about 10% due to the $m_{\rm b}^5$ dependence.

An alternative approach has been developed using ${\rm B} \to {\rm D}^*\ell\nu$ decays: in the heavy quark limit ($m_Q \to \infty$) the q^2-dependence should be described by a universal function $\xi(q^2)$ (the "Isgur-Wise" function). A variable y is introduced, the product of the four-velocities of the B and D*:

$$y \equiv v_{\rm B}\, v_{{\rm D}^*} = \frac{m_{\rm B}^2 + m_{{\rm D}^*}^2 - q^2}{2\,m_{\rm B} m_{{\rm D}^*}}\ . \tag{30}$$

Figure 28: Constraints on the $(V_{\rm ub}, V_{\rm cb})$ plane.

Figure 29: $|V_{\rm cb}|\,\hat{\xi}(y)\,\eta$ as a function of y from CLEO.

This has the property that $y = 1$ at $q^2 = q^2_{\rm max}$, and $\xi(1) = 1$. Then

$$\frac{d\Gamma}{dy}(\mathrm{B} \to \mathrm{D}^*\ell\nu) \propto \eta^2 |V_{\rm cb}|^2 \xi^2(y) \ . \tag{31}$$

The resulting distribution is shown in Fig. 29 for data from CLEO[18]. Extrapolation to the intercept at $y = 1$ allows a value for $V_{\rm cb}$ to be read off. This must be adjusted for finite masses, $\hat{\xi}(1) = 0.93 \pm 0.04$[24], and for the QCD correction, $\eta = 0.985 \pm 0.015$, leading to:

$$|V_{\rm cb}| = 0.0400 \pm 0.0025 \pm 0.0020 \ , \tag{32}$$

in good agreement with the inclusive measurement quoted above. Within the Wolfenstein parametrization of the CKM matrix discussed above, $V_{\rm cb} = A\lambda^2$, and thus $A = 0.8 \pm 0.1$.

4.8. The unitarity triangle

To complete the description of the CKM matrix in the Wolfenstein parameterization, we need to determine the remaining two parameters ρ and η. The matrix elements must satisfy unitarity conditions, one of which is:

$$V_{\rm ud}V^*_{\rm ub} + V_{\rm cd}V^*_{\rm cb} + V_{\rm td}V^*_{\rm tb} = 0 \tag{33}$$

This corresponds to a triangle in the (ρ, η) plane, as shown in Fig. 30, where the current knowledge of the apex of the triange is indicated by the dashed 90% CL contour.

CP violation should be seen in the B system, for example a difference in rate should be observed for the decay of $\mathrm{B}^0_{\rm d}$ and $\overline{\mathrm{B}}^0_{\rm d}$ to $\mathrm{J}/\psi\mathrm{K}^0_{\rm s}$ (a CP eigenstate). The

Figure 30: The unitarity triangle.

CP asymmetry in this case is directly proportional to $\sin 2\beta$, one of the angles of the unitarity triangle, almost independent of hadronization uncertainty. This, and other CP violation measurements which can determine the other angles of the triangle, require greater statistics than are currently available, and are the domain of future B experiments[25]. But how can we hope to measure (ρ, η) now? Since $|V_{\mathrm{ub}}|$, which determines the length of one side of the triangle, can already be measured using charmless b decays, the critical step is to determine $|V_{\mathrm{td}}|$ which would give the length of the remaining side. Since the supply of top quarks is limited at present, we need to rely on loop diagrams to access V_{td}.

One such loop diagram is illustrated in Fig. 31 (a), a so-called penguin diagram, which can give $\mathrm{b} \to \mathrm{s}\gamma$, $\mathrm{d}\gamma$. CLEO have searched for such decays, in modes $\mathrm{B} \to \mathrm{K}^*\gamma$, $\rho\gamma$, $\omega\gamma$. They suppress the background from initial-state radiation using event-shape cuts, and find the mass plots shown in Fig. 32[18]. A clear signal is seen for $\mathrm{B} \to \mathrm{K}^*\gamma$, with branching ratio

$$\mathcal{B}(\mathrm{B} \to \mathrm{K}^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}\ , \tag{34}$$

Figure 31: Loop diagrams: (a) "penguin" for $\mathrm{b} \to \mathrm{d}\gamma$, and (b) "box" for B^0–$\overline{\mathrm{B}}^0$ oscillation.

Figure 32: Invariant mass for (a) $\mathrm{B} \to \mathrm{K}^*\gamma$ and (b) $\mathrm{B} \to \rho\gamma, \omega\gamma$ from CLEO; the different shading denotes the different decay channels.

whilst for the $\mathrm{b} \to \mathrm{d}\gamma$ modes no signal is seen, leading to the limit

$$\frac{B(\mathrm{B} \to \rho\gamma, \omega\gamma)}{B(\mathrm{B} \to \mathrm{K}^*\gamma)} < 0.34 \text{ (90\% CL)} , \tag{35}$$

and hence

$$\left| \frac{V_{\mathrm{ts}}}{V_{\mathrm{td}}} \right| > 1.5 . \tag{36}$$

An alternative loop process which can be used to constrain this ratio is B^0–$\overline{\mathrm{B}}^0$ oscillation, the subject of the next section.

5. B^0–$\overline{\mathrm{B}}^0$ oscillation

The B^0 can transform into its antiparticle, via box diagrams like the one shown in Fig. 31 (b). Thus the B^0 and $\overline{\mathrm{B}}^0$ mix, and the states with well-defined mass (neglecting CP violation) are:

$$\begin{aligned} \mathrm{B}_1 &= \frac{1}{\sqrt{2}}(\mathrm{B}^0 + \overline{\mathrm{B}}^0) \\ \mathrm{B}_2 &= \frac{1}{\sqrt{2}}(\mathrm{B}^0 - \overline{\mathrm{B}}^0) \end{aligned} \tag{37}$$

Figure 33: Illustration of B^0–$\overline{\mathrm{B}}^0$ oscillation for two different values of x, relevant to the B^0_d and B^0_s respectively.

with mass difference $\Delta m = m_1 - m_2$, and decay widths which are expected to be similar: $\Gamma_1 \approx \Gamma_2 = \Gamma$. Their time-development is governed by the Schrödinger equation:

$$i\frac{d}{dt}\begin{pmatrix}\mathrm{B}^0\\ \overline{\mathrm{B}}^0\end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix}\mathrm{B}^0\\ \overline{\mathrm{B}}^0\end{pmatrix}, \tag{38}$$

with the result that an initially pure B^0 state decays as a B^0 or $\overline{\mathrm{B}}^0$ with probability:

$$\begin{aligned}\mathcal{P}(\mathrm{B}^0 \to \mathrm{B}^0) &= \mathrm{e}^{-t/\tau}\frac{(1+\cos\Delta m\, t)}{2} \\ \mathcal{P}(\mathrm{B}^0 \to \overline{\mathrm{B}}^0) &= \mathrm{e}^{-t/\tau}\frac{(1-\cos\Delta m\, t)}{2}\end{aligned} \tag{39}$$

Thus one finds oscillatory behaviour, with frequency Δm. The oscillation frequency is often expressed in terms of the lifetime using the dimensionless parameter $x \equiv \Delta m/\Gamma$. From experiment $x_\mathrm{d} \approx 0.7$, whilst the predicted value for the B^0_s frequency from fits to the unitarity triangle is much higher, $5 < x_\mathrm{s} < 50$ [26]. This oscillatory behaviour is illustrated in Fig. 33.

The calculation of the box diagram for the $\mathrm{B^0_d}$ gives:

$$\Delta m_\mathrm{d} \propto m_\mathrm{t}^2 \, |V_\mathrm{td}|^2 \; . \tag{40}$$

The first measurement of (time-integrated) mixing for the $\mathrm{B^0_d}$ by ARGUS in 1987 led to lower limit on the top-quark mass of $m_\mathrm{t} > 50\,\mathrm{GeV}$ (which was a surprise!).

5.1. $\mathrm{B^0_d}$–$\mathrm{\overline{B}^0_d}$ *mixing*

Until recently, all measurements of $\mathrm{B^0}$ mixing were time-integrated:

$$\int \mathcal{P}(\mathrm{B^0} \rightarrow \mathrm{\overline{B}^0})\, dt \equiv \chi = \frac{1}{2}\frac{x^2}{1+x^2} \quad (0 < \chi < 0.5) \; . \tag{41}$$

Its measurement requires the particle/antiparticle state of the $\mathrm{B^0}$ to be determined ("tagged") at both its production and decay. The classical tag is the charge of the lepton from semileptonic decay. A like-sign pair of leptons is then a signature of mixing, and one studies the "like-sign fraction":

$$R \equiv \frac{N_{\pm\pm}}{N_{\pm\pm} + N_{\pm\mp}} \; . \tag{42}$$

At the $\Upsilon(4S)$ $\mathrm{B^0_d}$ and $\mathrm{B^+}$ are produced, with coherent $\mathrm{B\overline{B}}$ production, and thus $\chi_\mathrm{d} = (1+\lambda)R$, where $\lambda = 1.13 \pm 0.19$ is a correction for $\mathrm{B^+B^-}$ production[27]. At LEP both the $\mathrm{B^0_d}$ and $\mathrm{B^0_s}$ are produced, and one measures $\overline{\chi} = f_\mathrm{d}\chi_\mathrm{d} + f_\mathrm{s}\chi_\mathrm{s}$, where f_d and f_s are the production fractions of the $\mathrm{B^0_d}$ and $\mathrm{B^0_s}$. Here the $\mathrm{b\overline{b}}$ production is incoherent, so $R \sim 2\,\overline{\chi}(1-\overline{\chi})$.

For a given assumption on f_d and f_s (for which the standard prejudice is to assume values of 39% and 12% respectively) a measurement of $\overline{\chi}$ corresponds to a line on the $(\chi_\mathrm{d}, \chi_\mathrm{s})$ plane, as illustrated for the current world average in Fig. 34. Also shown in the figure is the result obtained using an alternative to lepton tagging, jet-charge: this is the sum of track charges q in a hemisphere, weighted according to their momenta raised to a power κ,

$$Q_J \equiv \frac{\sum q\, p_{\|}^{\kappa}}{\sum p_{\|}^{\kappa}} \; , \tag{43}$$

where typically κ is taken as 0.5–1.0. As can be seen in the figure, both results are consistent with the value of χ_d, if $\chi_\mathrm{s} \approx 0.5$ as predicted in the standard model. However, the result of a fit to these measurements gives $x_\mathrm{s} > 0.9$, a rather weak limit, since $x_\mathrm{s} \rightarrow \infty$ as $\chi_\mathrm{s} \rightarrow 0.5$. To measure $\mathrm{B^0_s}$ mixing one needs to study the time-dependence directly.

An illustrative time-dependent analysis uses dilepton events, from ALEPH[27]. The $\mathrm{B^0}$ decay length is measured using a topological vertexing technique, and the boost using a combination of the charged and neutral energy, plus the missing energy in

Figure 34: Constraints on the $(\chi_{\rm d}, \chi_{\rm s})$ plane: the different shaded bands give the $\pm 1\,\sigma$ experimental measurements, and the hyperbolic region is that favoured by the standard model; a fit to the data gives the elliptical 95% CL contour shown, in good agreement with the prediction.

each hemisphere for the neutrino. The like-sign fraction is plotted as a function of the reconstructed proper time in Fig. 35. The slow oscillation of the $\mathrm{B^0_d}$ is clearly visible. There is, of course, a contribution from both $\mathrm{B^0_d}$ and $\mathrm{B^0_s}$ decays; for the measurement of $\Delta m_{\rm d}$ maximal $\mathrm{B^0_s}$ mixing is assumed (i.e. $x_{\rm s} \rightarrow \infty$). Then the fit gives:

$$\Delta m_{\rm d} = (0.44 \pm 0.05\ {}^{+0.09}_{-0.08})\,\mathrm{ps}^{-1}\ . \tag{44}$$

A factor of $\hbar$ converts this frequency to a mass difference, giving $\Delta m_{\rm d} \sim 3 \times 10^{-4}\,\mathrm{eV}$!

Time-dependent mixing is another field of research that has mushroomed at LEP: in the last two years there have been many (of order 20) determinations of $\Delta m_{\rm d}$, leading to the current world average[22] of $\Delta m_{\rm d} = (0.476 \pm 0.029)\,\mathrm{ps}^{-1}$. The full prediction of the standard model, from the calculation of box diagrams such as that illustrated in Fig. 31 (b), is:

$$\Delta m_{\rm d} = \frac{G_{\rm F}}{6\pi^2} m_{\rm B}\, m_{\rm t}^2\, F\left(\frac{m_{\rm t}^2}{m_{\rm W}^2}\right) \eta\, \mathcal{B} f_{\rm B}^2\, |V_{\rm tb}^* V_{\rm td}|^2\ , \tag{45}$$

Figure 35: The like-sign fraction versus proper time for dileptons from ALEPH. The superimposed fit assumes maximal $\mathrm{B_s^0}$ mixing; in the insert a zoom on the short proper time region is given, showing the result of a fit with Δm_s free.

where $m_\mathrm{t} = (180 \pm 12)\,\mathrm{GeV}$ (from an average of the CDF and D0 results), the QCD correction $\eta = 0.55$ [28], $|V_\mathrm{tb}| \approx 1$ from unitarity, and $\mathcal{B} f_\mathrm{B}^2$ (the product of "bag-factor" and decay constant for the B) represents the hadronic uncertainty, which is estimated from lattice QCD and sum rules [29] to be $(1.0 \pm 0.2)(180 \pm 50\,\mathrm{MeV})^2$. With these values one finds:

$$|V_\mathrm{td}| = (0.97 \pm 0.03 \pm 0.07 \pm 0.30) \times 10^{-2} \ , \tag{46}$$

where the contributions to the error are given separately for Δm_d, m_t and $\mathcal{B} f_\mathrm{B}^2$; clearly the latter dominates, and until there is progress on the understanding of that factor, the precision on $|V_\mathrm{td}|$ from this measurement will not improve. However, if $\mathrm{B_s^0}$ mixing could be measured, then

$$\frac{\Delta m_\mathrm{s}}{\Delta m_\mathrm{d}} = \frac{(\eta\, m_\mathrm{B}\, \mathcal{B} f_\mathrm{B}^2)_{\mathrm{B_s}}}{(\eta\, m_\mathrm{B}\, \mathcal{B} f_\mathrm{B}^2)_{\mathrm{B_d}}} \left| \frac{V_\mathrm{ts}}{V_\mathrm{td}} \right|^2 \ , \tag{47}$$

where the factor in front of the CKM matrix-element ratio should be close to unity (up to SU(3)-breaking effects); it is estimated to be 1.3 ± 0.2 [26]. Thus a strong constraint could be made on the ratio $V_\mathrm{ts}/V_\mathrm{td}$.

Figure 36: A fully reconstructed B_s^0 decay from ALEPH, with a zoom on the vertex region.

5.2. $\mathrm{B_s^0}$–$\mathrm{\overline{B}{}_s^0}$ *mixing*

One of the handful of fully reconstructed $\mathrm{B_s^0}$ decays is shown in Fig. 36. For this event the production state is tagged by an identified fragmentation $\mathrm{K^+}$, and also be an $\mathrm{e^-}$ in the opposite hemisphere, and is such that it demonstrates a $\mathrm{B_s^0} \rightarrow \mathrm{\overline{B}{}_s^0}$ transition. However, there is only one such event reported to date, so one needs a more inclusive analysis to extract the oscillation frequency. An example is the time-dependent dilepton analysis discussed above: here there is a contribution from both $\mathrm{B_d^0}$ and $\mathrm{B_s^0}$; allowing a second frequency component in the fit allows one to probe for $\mathrm{B_s^0}$ oscillation.

The dependence of the negative log-likelihood of the fit on Δm_s is shown for that analysis in Fig. 37. No significant minimum is seen, but high values of Δm_s are preferred. After studying the expected behaviour of the log-likelihood using fast simulation, a lower limit of $\Delta m_\mathrm{s} > 5.6\,\mathrm{ps^{-1}}$ is set at 95% CL. Currently the best limit on Δm_s comes from an extension of this analysis, using jet-charge tagging, which

Figure 37: The negative log-likelihood as a function of Δm_s from the ALEPH dilepton analysis; the data curve shows a minimum at $\Delta m_\mathrm{s} \approx 8\,\mathrm{ps^{-1}}$, but higher values cannot be excluded, so a lower limit is set: the points show the 95% CL contour determined using Monte Carlo simulation, and the intersection of data curve and this contour gives the lower limit.

Figure 38: Constraints on the apex of the unitarity triangle that would result from measurements of both x_s and $\sin 2\beta$.

gives[27] $\Delta m_\mathrm{s} > 6\,\mathrm{ps}^{-1}$. This corresponds to $x_\mathrm{s} > 9$ for $\tau(\mathrm{B_s^0}) = 1.5\,\mathrm{ps}$. Using the value measured for Δm_d given before leads to the limit $\Delta m_\mathrm{s}/\Delta m_\mathrm{d} > 11$, and hence:

$$\left|\frac{V_\mathrm{ts}}{V_\mathrm{td}}\right| > 2.8 \;, \tag{48}$$

which is the best existing limit on this ratio. Covering the remaining range of Δm_s will not be easy, and if its value is towards the top of the predicted range it will only be accessible at the LHC-B experiment.

6. Conclusions

B physics is an important, diverse field, and $\mathrm{e^+e^-}$ colliders have played a critical role in its development. They will continue to do so, with the continuing work at LEP, SLC and CESR, and at future B-factory experiments BaBar and Belle. Many crucial measurements remain to be made, which I have touched on in this review: resolving the R_b discrepancy, the discovery of the $\mathrm{B_c^+}$, $\mathrm{B_s^0}$–$\overline{\mathrm{B}}{}_\mathrm{s}^0$ oscillations, and of course CP violation in the B system. Eventually, with a measurement of both x_s and $\sin 2\beta$, the unitarity triangle might look like Fig. 38, and then from other measurements, of charmless b decays, or other CP-violating asymmetries, one might find that the triangle is not triangular after all, and get the first sign of new physics.

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8. References

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