EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN – PS DIVISION

DM/lmg **CERN/PS 99-044 (DI)**

STABILITY OF COOLED BEAMS

J. Bosser, C. Carli, M. Chanel, N. Madsen, S. Maury, D. Möhl, G. Tranquille

Invited paper presented at the ECOOL'99 Workshop, Uppsala, Sweden,
19-22 May 1999

Geneva, Switzerland
17 June 1999

STABILITY OF COOLED BEAMS

J. Bosser, C. Carli, M. Chanel, N. Madsen, S. Maury, D. Möhl*, G. Tranquille
CERN, CH 1211 Geneva 23, Switzerland.

Abstract

Because of their high density together with extremely small spreads in betatron frequency and momentum, cooled beams are very vulnerable to incoherent and coherent space-charge effects and instabilities. Moreover, the cooling system itself, i.e. the electron beam in the case of e-cooling, presents large linear and non-linear 'impedances' to the circulating ion beam, in addition to the usual beam-environment coupling impedances of the storage ring. Beam blow-up and losses, attributed to such effects, have been observed in virtually all the existing electron cooling rings. The adverse effects seem to be more pronounced in those rings, like CELSIUS, that are equipped with a cooler capable of reaching the presently highest energy (100 to 300 keV electrons corresponding to 180 to 560 MeV protons). The stability conditions will be revisited with emphasis on the experience gained at LEAR. It will be argued that for all present coolers, three conditions are necessary (although probably not sufficient) for the stability of intense cold beams: (i) operation below transition energy, (ii) active damping to counteract coherent instability, and (iii) careful control of the e-beam neutralisation. An extrapolation to the future 'medium energy coolers', planned to work for (anti)protons of several GeV, will also be attempted.

PACS: 29.27.Bd, 29.20.Dh, 52.35.-g

Keywords: Beam instability; Electron cooling; Coupling impedance; Space charge; Neutralisation instability

---

* Corresponding author. Tel. +41 22 767 3035; fax: +41 22 767 9145; email: dieter.mohl@cern.ch

1. Introduction

Beam instabilities due to space-charge and wake fields, induced and experienced by the circulating particles, have become the subject of accelerator school articles [1] and text books [2]. It is by now customary to analyse coherent beam stability in terms of longitudinal and transverse *coupling impedances* (or equivalently in terms of wake potentials), which are estimated from the beam and its environment and compared to *maximum tolerable impedances* calculated for desired beam conditions. The application to cooled beams is reviewed in workshop contributions, which put the emphasis on questions of high density [3] so that crystalline beams could emerge, on methods to measure coupling impedances and stability diagrams with cold beams [4] and on the additional impedances presented by the cooling system [5]. The present note tries to update this information, taking account of the experience gained at LEAR [6], and to extrapolate it to the planned Medium Energy Electron Cooling (MEEC) rings [7].

2. Overview of the situation reported for existing cooling rings

The situation in existing cooling rings, as we extract it from private or published communications [8-15], is summarised in Table 1. Different machines have widely different parameters (energy, intensity, particle species...) so that a comparison is difficult and may be even misleading. Yet from a glance at the table, one might conclude that the stability problem is more critical for the coolers designed for high electron energy.

3. Impedances and threshold relations, calculated for LEAR and a MEEC-ring

The basic relations to estimate the beam coupling impedances for some components are recalled in Table 2. One notes that the space-charge impedance is strongly energy dependent ($\beta^{-1}\gamma^{-2}$ or $\beta^{-2}\gamma^{-2}$), whereas the other contributions depend only on β . Typical values for LEIR (i.e. LEAR working with Pb54+ ions at 4.2 MeV/nucleon) and for a "generic MEEC-ring " for 9 GeV antiprotons are compiled in Table 3. Whereas in LEIR the space-charge contributes a very large reactive component, the situation in the MEEC-ring is dominated by the resistive impedance of the vacuum chamber and of other equipment.

The "Keil-Schnell" threshold relation for the longitudinal impedance and the "Schnell-Zotter" threshold for the transverse one are recalled in Table 3. For constant ion beam-current, the tolerable impedances increase proportionally to $\beta^{2}\gamma$ and γ respectively, which constitutes another bonus at higher energy. The calculated impedances (Table 4) exceed the threshold values in LEIR by a factor 2 to 10, thus indicating that a challenge exists. The MEEC parameters are safely below the thresholds, provided that the extrapolated low "equipment impedances" can be obtained. We note here, that factors like γ^{-2} are the result of a subtle cancellation between the electric and magnetic space-charge field and may be upset, e. g. if beam neutralisation is present.

4. Summary of the stability situation observed experimentally at LEAR

As design goal, a budget of $|Z_n/n| = 60\ \Omega$ was specified, including all contributions, except space-charge, which unavoidably leads to a large reactive contribution at low energy. Care was taken in the choice of equipment. For instance, all ceramic chamber sections are coated on the inside with a thin metallic layer (<10 Ω/square surface resistance). Critical components, e.g. the injection kicker and its tank were investigated using the wire method which simulates the beam by an RF-current on a wire [16]. Damping resistors were then installed to reduce the impedance seen by the beam. For coasting beam operation the RF cavities are short-circuited by relays. In bunched beam operation, strong voltage feedback keeps the effective beam impedance small.

Subsequently the compound beam coupling impedance has been repeatedly checked "in situ" using Beam Transfer Function (BTF) measurements [17]. The measured imaginary part was of the order of the space-charge impedance expected from theory and the resistive part R_n/n was about 30 Ω , when the machine was well set-up. On one occasion a larger resistance (R_n = 1400 Ω at 7 MHz) was observed and traced down to an imperfect short-circuit of the RF cavity. The Z_t-impedance was also "surveyed" by transverse BTF-measurements. For a well-behaved machine, a reactive impedance as expected from space-charge, was identified together with a resistive part consistent with the basic relation $R_t = \{2\, c / (\, \omega_o\, b^2)\}\, R_n/n$ between transverse and longitudinal impedances [1] [2].

In this situation, no *longitudinal* instability was observed, although the "Keil-Schnell" criterion was exceeded by factors up to 10. This is explained by the "thermometer shape" of stable area in the complex plain Im(Z_n/n) vs. Re(Z_n/n), where a "shaft" near the negative part of the imaginary axis exceeds the stable "Keil-Schnell"-circle ("thermometer ball") by a large amount. This picture holds for operation below transition energy, above transition the "thermometer shaft" points to positive Im(Z_n/n) and stability for the negative space-charge impedance beyond the "Keil-Schnell"-circle is absent. The longitudinal ("negative mass") instability for $\gamma > \gamma_t$ was clearly observed in the CERN Initial Cooling Experiment (1977-79) where electron cooling could only work, when the working point was changed to $\gamma < \gamma_t$.

With the impedances obtained in LEAR, strong *transverse* instabilities occurred once the intensity exceeded a few 10^8 protons. A large number of modes was observed at all energies accessible with electron cooling (5.3 - 50 MeV). Therefore, a feedback system acting from 0.1 - 70 MHz was implemented, to stabilise the first 100 or so dipole modes. It was then possible, to store up to about 3 10^9 protons with the small emittances given by the equilibrium between intra-beam scattering and cooling in the energy range accessible. Higher intensities, up to 8 10^{10} protons, could be cooled to the intra-beam scattering limit, when the stochastic cooling system with a band up to 500 MHz was used (with reduced gain!) as additional dipole damper.

5. Influence of neutralisation of the electron beam

The space-charge potential of the electron beam influences the longitudinal electron velocity profile $v_{||}(r)$ and in addition causes an $E\times B$-drift with a transverse velocity $v_t \alpha\ r/(\gamma^2 B)$, due to the radial electric space-charge field and the external magnetic field of the cooler. Both effects complicate the velocity matching between the electrons and the circulating beam particles and it would be desirable, to eliminate them by neutralising the electron beam. This can be achieved by ions from the residual gas created and trapped by the electron beam. For this purpose, sets of electrodes polarised to reflect the ions and to clear the slow ionisation electrons, are installed at the gun and collector end outside the interaction region. If the cooling electrons do not "see" the same boundary radius inside and outside the interaction region, similar trapping potentials, induced by the electron beam, can lead to "natural ionisation".

As pointed out e.g. by Burov [18], multi-stream instabilities with a large variety of possible modes (linked to the various eigen frequencies of the system consisting of: the electron-beam, the different species of neutralising ions, secondary electrons, and the circulating beam particles) render the stability very delicate. Neutralisation experiments at LEAR [19] have shown, that such instabilities lead to sudden changes of the neutralisation level. The jumps occur at regular intervals, typically once every 1 to 10 seconds, related to the ionisation time of the e-beam (about 3 s at the LEAR pressure around 5×10^{-12} torr). Associated with these bursts are energy jumps of the cooled circulating beam due to the change of the effective acceleration potential for the electrons.

The repetitive energy jumps lead to heating and sometimes even to losses from both the cooling and the cooled beam. To avoid this harmful effect a very *strict control of the neutralisation level* is necessary. In LEAR this could be achieved by using "shaker" electrodes. These are electrode pairs, similar in shape to position pick-ups, acting as a transverse kicker. They are excited with a sinusoidal RF-voltage of a few volts and a frequency of some 100 kHz, in the range of the bounce frequencies of the ions in the electron beam. The action is two-fold: they heat up the neutralising ions and thus Landau damp their motion and they expel continuously the surplus ions. In this way neutralisation levels up to 0.4 (respectively 1, i.e. full neutralisation) could be stabilised in LEAR at 2.5 keV (resp. 25 keV) electron energy and 0.4 A (resp. 1.5 A) electron current.

Even when the neutralisation electrodes are not used, the cooling can be hampered by an instability from the natural neutralisation. This was observed at LEAR where originally an electron beam chamber with different radii inside and outside the interaction region was used. In addition to inducing energy jumps, neutralisation, both forced and natural, can be undesired for cooling of a heavy ion beam due to charge exchange between the beam and the neutralising ions. A neutralisation level of (nominally) zero was therefore chosen for lead ion cooling in LEIR [20]. It could be achieved by applying a field of 12 - 15 V to the LEAR "shaker". However, to obtain cleaner conditions, the electron beam chamber was modified [21] to present a smooth conducting surface to the electrons. In this way, the natural neutralisation and its jumps were eliminated and the shaker was only required to stabilise high neutralisation levels if desired. We recommend, that modern coolers should be designed to avoid natural neutralisation.

6. Tune shifts

With the high beam density obtained by cooling, the incoherent tune shift (Laslett's formula, Table 3) and tune spread is large and resonances limit the process of cooling for intense beams. This was the case in LEAR with 5 10^{10} protons at 50 MeV when the coherent tune was set just above the second order resonance 2Qv = 5 [22]. It was then observed that the vertical emittance could not be cooled at all and the horizontal emittance decreased only slightly. Roughly speaking cooling stopped at a beam size such that the tune depression corresponds to the distance from the resonance. By proper choice of the working point, it was possible in LEAR to accommodate a Laslett shift of about 0.1 even without compensation of resonances. This is much more than the "storage ring limit" $|\Delta Q|$=0.01 observed in the ISR. But it is smaller than the "synchrotron limit", $|\Delta Q|$=0.5, found for machines like the PS Booster where one compensates resonances and where, moreover, the dwelling time in the high space-charge regime, near injection, is only a few tens of ms.

Also included in Tables 3 and 4 is the tune shift $|\Delta Q|_{\text{e-beam}}$ of the circulating beam due to the presence of the electron beam, acting as a 'plasma lens'. In analogy with the beam-beam effect in colliders, where $|\Delta Q|_{\text{bb}}$=0.005 to 0.01 is regarded as the limit for hadron machines, one might expect, that this tune has to be kept very small. However the experience with protons at LEAR indicates that a $|\Delta Q|_{\text{e-beam}}$ of the order of 0.03 is well tolerable, provided that one re-tunes the working point. This difference is, at least partially, explained by the fact, that the space-charge field of the electron beam is much more linear than the field of the beams in a collider. Thus with a careful control of the working point, relatively large tune shifts are tolerable in the cooling rings. To stack high intensity, it can be necessary to prevent 'over-cooling'. This can be achieved by selective heating of the stack or by cooling with a "hollow electron beam" (unpublished proposal mentioned to us by V. Parkhomchuk), so that the equilibrium size does not get too small.

Conclusion

Experience with LEAR suggests a number of measures, necessary to avoid unpleasant limitations in the cooling capacity: Strict impedance 'hygiene' (even though the space charge impedance is unavoidably very high); operation below transition energy; an active damping system of rather large bandwidth for (at least) the transverse planes; efficient control of the electron beam and its neutralisation. Moreover, stacking to high intensity, requires careful choice of the working point and probably selective heating or similar measures to avoid 'over-cooling' of the stack. A Laslett tune shift of 0.15 seems well reachable. With a stack, which fills a sizeable fraction (say half) of the acceptance, this allows for accumulation of respectable intensities. For a medium energy ring with parameters like the FNAL recycler [7], the stability of the cooled beam is less critical, provided that the extrapolated low equipment impedances can be obtained.

References

[1] A. Hofmann, "Beam Instabilities" and "Landau Damping", in: Proc. CERN Acc. School, fifth advanced course, CERN 95-06 pp. 275-331 and references given therein.
K.Y. Ng, "Fields, Impedances, and Structurers" in: Phys. of Particle Acc., Summer Schools 1987 and 1988. Amer. Inst, Phys. Conf. Proc. 184, N.Y.1989, pp. 473-524.

[2] A.W. Chao, "Physics of Collective Beam Instabilities in High Energy Accelerators", J. Wiley & Sons, New York, 1993.

[3] D. Möhl, "Limitations and Instabilities in Cooled Beams", in: Proc. Workshop on Crystalline Ion Beams, Wertheim, Germany, Oct. 4-7, 1988 (GSI Report 1989).

[4] M. Chanel, "Beam Instabilities and their Cures", in: Proc. Workshop on Crystalline Beams and Related Topics, Erice, Italy, Nov. 12-21 1995, World Scientific, 1996, pp. 71-92.

[5] A.V. Burov, "Electron Cooler Impedances", in: Proc. Workshop on Crystalline Beams and Related Topics, Erice, Italy, Nov. 12-21 1995, World Scientific, 1996, pp. 205-216.

V. Parkhomchuk, D. Pestrikov, ”Coherent Instabilities at Electron Cooling”, in: Proc. Workshop on Beam Cooling and Related Topics, Montreux, Switzerland Oct. 4-8, 1993, CERN 94-3 pp. 327-329.

[6] For a summary see [4] above and:
U. Oeftiger, "Measurement of Beam Properties and Beam Environment in LEAR and COSY...." doctoral thesis, Uni. Bonn, 1994, published as FZ-Juelich Report, Oct. 1994.

[7] I. Meshkov (ed.) Proc. Workshop on Medium Energy Electron Cooling (MEEC'98), Dubna, Russia, Sep. 14-15, 1998, to be published.

J.A. MacLachlan, "Fermilab R&D Program in Medium Energy Electron Cooling". Nucl. Instr. Meth. A 391 (1997), pp. 131-137.

S. Nagaitsev, "Electron Cooling for the Fermilab Recycler", Nucl. Instr. Meth. A 391 (1997), pp. 142-146.

[8] H. Danared, Private Communication.

[9] S. P. Møller, Private Communication.

[10] M. Grieser, Private Communication.

[11] See ref. [4] and [6] above and
M. Chanel, U. Oeftiger, "Results from Beam Transfer Function Measurements at the Low Energy Antiproton Ring (LEAR)", in: Proc. Workshop on Beam Cooling and Related Topics, Montreux, Switzerland, Oct. 4-8, 1993, CERN 94-3 pp. 287-292.

[12] J. Stein, R. Maier, S. Martin, D. Prasuhn, J.D. Witt, "Electron Cooling in Cosy", in: Proc. Workshop on Medium Energy Electron Cooling (MEEC'98), Dubna, Russia, Sep. 14-15, 1998, to be published.

[13] S. Nagaitsev, T. Ellison, M. Ball, V. Derenchuk, G. East, M. Ellison, B. Hamilton and P. Schwandt, ”Space-charge Effects and Intensity Limits of Electron-Cooled Bunched Beams”, in: Proc. 1997 Part. Acc. Conf., PAC'97, Dallas, pp. 2937-2975.

[14] D. Reistad, L. Hermansson, T. Bergmark, O. Johansson, A. Simonsson, A.V. Burov, ”Measurements of Electron Cooling and 'Electron Heating' at CELSIUS”, in: Proc. Workshop on Beam Cooling and Related Topics and related topics”, Switzerland, Oct. 4-8, 1993, CERN 94-3, pp. 183-187.

M. Bengtsson, T. Bergmark, H. Calén, C. Ekström, C.-J. Fridén, K. Gajewski, L. Hermansson, P. Jahnke, T. Lofnes, G. Norman, D. Reistad, R. Wedberg, L. Westerberg, V. Ziemann, A. Johansson, K. Rathsman, J. Zlomanczuk, ”New Developments at Celsius”, in: Proc. 1998 Europ. Part. Acc. Conf., EPAC'98, Stockholm, pp. 514-516.

[15] U. Schaaf, K. Beckert, D. Budicin, H. Eickhoff, F. Franzke, I. Hofmann, G. Kalisch, F. Nolden, P. Spädtke, M. Stech, "Impedance and Instability Studies at the ESR" in: Proc. 1992 Europ. Part. Acc. Conf. EPAC'92, Berlin, pp.123-125.

[16] A. Faltens, E.C. Hartwig, D. Möhl, A.M. Sessler, "An Analog Method for Measuring the Coupling Impedance ...", in: Proc. 8th Internat. Conf. on High-Energy Acc., CERN, 1971, pp. 338-344.

F. Caspers "Bench Methods for Beam Coupling Impedance Measurements" in: Proc. Joint US-CERN Part. Acc. School, Hilton Head Island, SC, USA, Nov. 7 - 14, 1990; Lecture Notes in Phys. Springer Verl., Berlin, 1992 p. 80-109.

[17] D. Möhl, A.M. Sessler, "The Use of RF-Knock-Out for Determination of the Characteristics of Transverse Coherent Instabilities..." in: Proc. 8th Internat. Conf. on High-Energy Acc., CERN, 1971, pp. 338-344.
D. Boussard, "Schottky Noise and Beam Transfer Function Measurement" in: Proc. CERN Acc. School, Advanced Acc. Phys. CERN 87-03 pp. 416-445.

[18] A.V. Burov, "Secondary Particle Instabilities in Storage Rings with Electron Cooling", in: Proc. Workshop on Beam Cooling and Related Topics, Montreux, Switzerland Oct. 4-8, 1993, CERN 94-3 pp. 230-234.

[19] J. Bosser, I. Meshkov, D. Möhl, V.V. Parkhomchuk, E. Syresin, G. Tranquille, "Neutralization of the LEAR-ECOOL Electron Beam Space Charge", internal report CERN/PS/AR/Note 93-08.

J. Bosser, Y. Korotaev, R. Ley, R. Maccaferri, I. Meshkov G. Molinari, E. Syresin, G. Tranquille, F. Varenne, "Experimental Study of the Neutralised Electron Beam for Electron Cooling ", in: Proc. 1996 Europ. Part. Acc. Conf., EPAC'96, Sitges, Spain, pp.1193-1195.

J. Bosser, S. Maury, I. Meshkov, D. Möhl, E. Mustafin, E. Syresin, F. Varenne, P. Zenkevich, "Stability Conditions for a Neutralised Electron Beam", in: Proc. 1995 Part. Acc. Conf., PAC,95, Dallas , pp. 2940-2943.

F. Varenne, "Neutralisation de la Charge d' Éspace du Faisceau d'Electrons du Refroidisseur electronique de LEAR", doctoral thesis, University of Clermont-Ferrand, France, Nov. 1995, reprint: internal report CERN/PS/AR/Note 96-22.

I. Meshkov, D. Möhl, G. Tranquille, E. Syresin (eds.), "Neutralization of the Electron-Beam Space Charge, Report of the CERN/PS/JINR/ITEP 1994-95 Collaboration", internal report CERN/PS/DI/Note 98-16.

[20] R. Maccaferri, "Ecool Momentum Feedback, Control and Operation", internal report CERN/PS/AR/Note 95-07.

[21] G. Tranquille, "Suppression of Natural Neutralisation of the Electron Beam Space Charge via Polarised Tubes", internal report CERN/PS/OP/Note 95-47.

[22] M. Chanel, "Study of Beam Envelope Oscillations by Measuring the Beam Function with Quadrupolar Pick-up and Kicker" Proc. 1996 Europ. Part. Acc. Conf., EPAC'96, Sitges, Spain, pp. 1193-1195.

Table 1: General stability situation in different storage rings with electron-cooling

Ring	maximum electron energy [keV]	stability problem reported	remarks, antidotes reported	Reference
CRYRING	5	weak	loss when stacking high intensity	[8]
ASTRID	5	weak	intensity limitation due to space charge	[9]
TSR	15	weak	transv. instability in some situations	[10]
LEAR	30	controlled	damper, impedance reduction	[11]
COSY	100	strong	intensity limitation	[12]
IUCF	200	strong, but controlled	damper	[13]
CELSIUS	300	strong, partly controlled	selective heating by RF	[14]
ESR	300	strong, but controlled	damper, impedance "hygiene"	[15]

Table 2: Some formulae for contributions to the beam-environment coupling impedances Z_n/n and Z_t . Here Z_n/n is the usual longitudinal coupling impedance, [1]-[4], at n-times the revolution frequency divided by the harmonic n. The longitudinal impedance $Z_n = -\frac{1}{I_n}\oint\langle E_n\rangle ds$ is defined by the longitudinal electric field induced by a beam current $I_n\, exp(\, in\omega_0 t\,)$ averaged transversely over the beam and integrated around the ring. In a similar way $Z_{t,n-Q} = \frac{-1}{i\beta\, I_0\, x_{n-Q}}\oint(\, E_{n-Q} + v\times B_{n-Q}\,)_\perp ds$ is given by the transverse Lorentz force, induced by an oscillation $x_{n-Q}\; exp\{\; i\,(\, n-Q\,)\omega_0 t\,)\}$ of the centre of the coasting beam with circulating current I_0. Note that the time dependence $exp(\,+i\omega\; t\,)$ is used here so that an inductive impedance has a positive imaginary part. Some authors use $exp(\,-i\omega\; t\,)$, then *i* --> *-i* has to be substituted in the impedance relations.

Impedance contribution	Longitudinal coupling impedance Z_n/n [Ω]	Transverse coupling impedance Z_t [Ω/m]	Remark
space charge	$-i\,\frac{Z_o}{2\beta\gamma^2}\left(\; 1 + 2\,ln(\, b/a\,)\right)$	$-i\,\frac{Z_o\; R}{\beta^2\gamma^2}\left(\frac{1}{a^2} - \frac{1}{b^2}\right)$	perfectly conducting chamber, circular cross-section
resistive wall	$(\; 1+i\;)\,\frac{1}{2}\,\frac{Z_o\beta}{b}\,\delta(\omega_n\,)$	$(\; 1+i\;)\;\frac{Z_o R}{b^3}\,\delta(\omega_{n-Q}\,)$	wall thickness larger than skin depth
cavities	$\frac{1}{n}\;\frac{R_s}{1 - iQ_{cav}(\omega_r/\omega - \omega/\omega_r\,)}$	$\frac{2R}{nb^2\beta}\;\frac{R_s}{1 - iQ_{cav}(\omega_r/\omega - \omega/\omega_r\,)}$	Q = 1 $\omega_r = \beta cR/b^2$ for broad band model
inj./eject. kicker	$\approx\;\frac{\ell_k}{2\pi R}\;Z_o\,\beta$	$\approx\;\frac{\ell_k}{2\pi R}\;\frac{R}{h_k^2}\,Z_o$	max. of real part, see K.Y. Ng [1}. Height=width of kicker gap
electron beam	$\approx 2.2\,\frac{Z_o}{\beta^2}\,a_e\eta_c\;\sqrt{n_e r_e}$	$\approx 5.5\,\frac{Z_o}{\beta^2}\,\frac{R}{a_e}\,\eta_c\;\sqrt{n_e r_e}\;\; S$	maximum of real part S: factor of order 1 [5]

Notes to Tables 2-4: Notations and parameters (LEAR | MEEC) used

A: mass number of ion (208 | 1),
a: radius of cooled ion beam, FWHM (4 | 20 mm)
a_e : radius of e–beam (25 | 10 mm),
b : radius of vacuum chamber (50 | 100 mm),
B_F : bunching factor (average -/peak current) (1 | 1)
f_o : revolution frequency (360 | 90 kHz),
$F_L \approx 1$, $F_r \approx 1$: form factors
h_k: half height and width of kicker gap (40 | 40 mm),
F_{sc}, $F_{e\text{-}sc}$: image force correction factors, $F_{e\text{-}sc} \approx$(1 | 1.3), $F_{sc} \approx$(1 | 3.7)
$I_o = Nef_o$: circulating particle current (3.4 10^{-5} | 5.8 10^{-3} Amp)
ℓ_c: interaction length of cooler (1.5 | 66 m)
ℓ_k : kicker length (1 | 5 m),
$m_o c^2$ = 938 MeV : proton rest energy
n : longitudinal mode number (n =1 for strongest resistive wall mode)
$n\text{-}Q$: transverse mode number, $(n - Q)$ = (0.3 | 0.6) for strongest resistive wall mode
n_e : electron beam density (3.3 10^{13} | 1.3 10^{13} m^{-3}),
N: Number of particles circulating (6 10^{8} | 4 10^{11});
q: charge state number of ion (54 | 1)
Q: betatron tune of storage ring (2.7 | 25.4)
Q_{cav} : quality factor of resonator ($Q_{cav} = 5$ for parasitic resonance of LEAR cavity at 7 MHz)
$Q' = Q/(\Delta p/p)$: chromaticity (–7 | –2)
R : storage ring mean radius (12.5 | 528 m),
R_s : shunt resistance of resonator (1.4 KΩ for parasitic resonance at 7 MHz)
$r_e = 2.82\ 10^{-15}$ m: classical electron radius
$r_p = 1.54\ 10^{-18}$ m: classical proton radius
S : Burov's sum factor [5] (S = 1 taken),
$Z_0 = 377\ \Omega$: impedance of free space
Z_n : longitudinal coupling impedance at $\omega = n\ \omega_o$
Z_t : transverse coupling impedance at $\omega = (n - Q)\ \omega_o$ (with $n > Q$)
$\beta = v/c$: relativistic parameter (0.094 | 0.994), $\gamma = (1 - \beta^2)^{-1/2}$
β_c : storage ring focusing function at cooler (5 | 20 m)
$\delta(\omega) = \sqrt{2/(\omega\ \mu_o \sigma\ )}$ skin depth at freq. ω {$\mu_0 = 1.26\ 10^{-6}$ As/Vm, σ = (1.4 10^6 | 1.4 10^6 $(\Omega\mathrm{m})^{-1}$)}
$\Delta p/p$: momentum spread FWHM (0.5 10^{-3} | 1 10^{-3})
$\Delta Q_a = (\partial Q/\partial a^2)a^2$: amplitude dependent Q – spread (neglected for LEIR and MEEC)
ΔQ_n : momentum dependent spread of mode frequency $(n - Q)f_o$, (≥ 3.6 10^{-3} | ≥ 2 10^{-3})
$\eta = -(\Delta f_o / f_o)/(\Delta p/p) = 1/\gamma_{tr}^2 - 1/\gamma^2$: off-momentum factor of storage ring (–1 | –8.7 10^{-3})
$\eta_c = \ell_c/2\pi R$: circumference factor of cooler (0.02 | 0.02)
ω : angular frequency, $\omega = n\ \omega_o$ for longitudinal- $\omega = (n - Q)\omega_o$ for transverse (dipole) modes
ω_r : resonance frequency of resonator (7 MHz for parasitic resonance of LEAR cavity).
ω_o : angular revolution frequency

Table 3: Impedance and tune shift limits

Limit on	Criterion	Remark
Longitudinal impedance	$\left\lvert \frac{Z_n}{n} \right\rvert \le F_L \; \frac{A}{q^2} \frac{\beta^2\gamma \; (m_o c^2/e) \lvert\eta\rvert}{q^2 I_0} \left(\frac{\Delta p}{p}\right)^2_{FWHM}$ $F_L \approx 1, \qquad I_0 = Nef_0 \;!$	Coasting beam. "Keil-Schnell" circle. For bunched beam, local (peak) current and Δp/p can be inserted for rough estimate
Transverse impedance	$\lvert Z_t \rvert \le 4F_t \frac{A}{q^2} \frac{\gamma \; m_o c^2/e}{I_0} \frac{Q}{R} (\Delta Q_n)_{FWHM}$ $\Delta Q_n = \left(\left[\lvert \underset{\substack{\uparrow \\ \Delta(\text{frev})}}{(n-Q)\eta} + \underset{\substack{\uparrow \\ \text{chromaticity}}}{Q'} \rvert (\Delta p/p) \right]^2 + \underset{\substack{\uparrow \\ \text{octupoles}}}{\left[(\partial Q/\partial a^2)\, a^2 \right]^2} \right)^{1/2}$ $F_t \approx 1$	Coasting beam! "Schnell-Zotter" criterion
Laslett tune shift	$\Delta Q_{Laslett} \approx \frac{r_p}{2\pi} \frac{q^2}{A} \frac{N \cdot (R/Q)\, F_{sc}}{\beta^2\gamma^3 \; a^2 \; B_F} \le 0.15$	Round beam, uniform density, radius a
Tune shift due to e-beam	$\Delta Q_{e-beam} \approx \frac{r_p}{2} \frac{q}{A} \frac{n_e \ell_c \; \beta_c \; F_{e-sc}}{\beta^2\gamma^3 \; B_{F-e}} \le 0.03$	Negative for antiprotons!

Table 4: Calculated impedances, thresholds and tune shifts for LEIR and "MEEC"

Impedance contributions and limits	Longitudinal coupling impedance Z_n/n [Ω]	Transverse coupling impedance Z_t [Ω/m]	remark
space charge *LEIR* *"MEEC'*	 $-i\ 1.2\times10^{4}$ $-i\ 9.0$	 $-i\ 3.2\times10^{10}$ $-i\ 5.3\times10^{6}$	perfectly conducting chamber, circular cross-section
resistive wall *LEIR* *"MEEC'*	 $(1+i)\ 0.25$ $(1+i)\ 2.5$	 $(1+i)\ 4.9\times10^{4}$ $(1+i)\ 3.2\times10^{5}$	maximum (occuring for lowest mode frequency)
cavities *LEIR* *"MEEC'*	 75 (18)	 - -	parasitic resonance with imperfect short-circuit of LEAR-cavity
inj/ej kicker *LEIR* *"MEEC'*	 0.5 0.6	 3.7×10^{4} 1.9×10^{5}	max. of real part, estimate for simplified. kicker geometry
electron beam *LEIR* *"MEEC"*	 14 0.03	 6.8×10^{5} 4.2×10^{5}	maximum of real part ! S=1 taken
tolerable *LEIR* *"MEEC'*	$\lvert Z_n/n\rvert$ 4.3×10^{3} 1.3×10^{4}	$\lvert Z_t\rvert_{\text{min}}$ 6.1×10^{9} 6.0×10^{8}	"Keil-Schnell" and "Schnell-Zotter" criterion for most critical mode
tune shifts *LEIR* *"MEEC'*	Laslett 7×10^{-2} 5.9×10^{-6}	due to e-beam 6×10^{-3} 1.7×10^{-5}	