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PROCEEDINGS

Top Mixing

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ABSTRACT: We discuss the existing limits on top flavour-changing neutral couplings in models with new vector-like quarks. Large hadron and e^+e^- colliders can improve these bounds by more than one order of magnitude.

1. Introduction

The top quark is the heaviest fermion discovered up to now. Its mass $m_t = 173.8 \pm 5.2$ GeV [1] is 3.5×10^5 times the electron mass and makes more demanding the question of why the fermion mass hierarchy is so large. Not to say if we compare m_t with the light neutrino masses. At present there is no compelling theoretical reason for such spreading. The Standard Model (SM) simply accommodates fermion masses and mixings. One is tempted, however, to relate the large value of the top mass, which is not very different from the electroweak symmetry breaking scale, with the mass generation mechanism. This and any other new physics are then expected to show up at the top first.

In contrast, top couplings are constrained rather poorly by present experimental data, especially the flavour-changing neutral (FCN) ones. In the SM there are no tree level FCN couplings and the effective ones induced at one loop are negligible. However, the top quark can have large tree level FCN couplings in models with extra vector-like quarks [2, 3, 4]. (These are fermions whose left-handed (LH) and right-handed (RH) parts have the same transformation properties under SU(2)$_L$. They are present in many extensions of the SM and are the only fermions whose mass can be banished to high energy.) Hence, it is theoretically important to constrain them experimentally. In Sections 2 and 3 we discuss the existing limits on these couplings in these simple and well-defined SM extensions [5]. In Section

4 we comment on the bounds which can be obtained at large hadron and e^+e^- colliders. Existing limits will be eventually improved by at least one order of magnitude [6, 7, 8, 9, 10].

2. Model-independent limits

Direct limits on top FCN couplings can be derived from top decays. At present the most stringent bounds result from the non-observation of the decays $t \to Zq$, $t \to \gamma q$, $t \to gq$ at the Fermilab Tevatron. The neutral trilinear interactions between the top and a light quark q are given by the Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\text{FCNC}} = \bar{t} &\left[\frac{g}{2c_W} \gamma_\mu (X_{tq}^L P_L + X_{tq}^R P_R) Z^\mu \right. \\
&+ \frac{g}{2c_W} (\kappa_{tq}^Z - i\tilde{\kappa}_{tq}^Z) \frac{i\sigma_{\mu\nu} q^\nu}{m_t} Z^\mu \\
&+ e(\kappa_{tq}^\gamma - i\tilde{\kappa}_{tq}^\gamma) \frac{i\sigma_{\mu\nu} q^\nu}{m_t} A^\mu \\
&+ \left. g_s (\kappa_{tq}^g - i\tilde{\kappa}_{tq}^g) \frac{i\sigma_{\mu\nu} q^\nu}{m_t} T^a G^{a\mu} \right] q \\
&+ \text{h.c.}\,, \quad\quad\quad\quad\quad (2.1)
\end{aligned}
$$

with $P_{R,L} = (1 \pm \gamma_5)/2$ and T^a the Gell-Mann matrices satisfying $\text{Tr}\,(T^a T^b) = \delta^{ab}/2$. Tevatron bounds [11, 12]

$$
\begin{aligned}
\text{Br}(t \to Zq) &\leq 33\% \\
\text{Br}(t \to \gamma q) &\leq 3.2\% \\
\text{Br}(t \to gq) &\leq 15\% \quad\quad\quad (2.2)
\end{aligned}
$$

translate into $|X_{tq}^L|, |X_{tq}^R| \leq 0.84$, $|\kappa_{tq}^Z|, |\tilde{\kappa}_{tq}^Z| \leq 0.78$, $|\kappa_{tq}^\gamma|, |\tilde{\kappa}_{tq}^\gamma| \leq 0.26$, $|\kappa_{tq}^g|, |\tilde{\kappa}_{tq}^g| \leq 0.15$.

The effective couplings proportional to $\sigma_{\mu\nu}$ in Eq. 2.1 are absent at tree level in renormalizable theories. Hence they are suppressed by one-loop factors $\sim \alpha/\pi$, and also by the GIM mechanism if there are no tree-level (γ_μ) FCN couplings. Thus we will neglect them in the following. However, SM extensions with vector-like quarks allow for large (tree-level) γ_μ FCN couplings. This is so because the addition of vector-like quarks has little effect on radiative corrections. In particular, the new quarks decouple when their masses are taken to infinity. Although FCN couplings are constrained by precise electroweak data, indirect limits are at the mercy of possible cancellations between the top and other new particle contributions (see Ref. [6] for an effective Lagrangian approach). It may be then convenient to derive these constraints in specific models. In the next Section we discuss these limits for SM extensions with vector-like fermions.

3. Limits in extended models with vector-like quarks

Let us consider a general SM extension with N standard families, n vector-like doublets and n_u up and n_d down vector-like singlets. (In practice, our limits will be saturated in the minimal extensions with $N = 3$ and one extra doublet or singlet.) The quark content of this model is summarized in Table 1. The weak eigenstates are then $q^0_{L,R} = (q^{(d)}_{L,R}, q^{(s)}_{L,R})$, with $q^{(d)}_{L,R}$ doublets and $q^{(s)}_{L,R}$ singlets, and $q = u, d$.

LH doublets	$N + n$
RH doublets	n
up LH singlets	n_u
up RH singlets	$N + n_u$
down LH singlets	n_d
down RH singlets	$N + n_d$

Table 1: Quark content of the model

Note that the total number of up-type quarks $\mathcal{N}_u = N + n + n_u$ and down-type quarks $\mathcal{N}_d = N + n + n_d$ do not need to be equal in general. As in the SM, only isodoublets have $SU(2)_L$ couplings, then the gauge neutral current Lagrangian in the weak eigenstate basis can be written as

$$\mathcal{L}_Z = -\frac{g}{2c_W} \left(\bar{u}_L^{(d)} \gamma^\mu u_L^{(d)} + \bar{u}_R^{(d)} \gamma^\mu u_R^{(d)} \right.$$
$$- \bar{d}_L^{(d)} \gamma^\mu d_L^{(d)} - \bar{d}_R^{(d)} \gamma^\mu d_R^{(d)}$$
$$\left. - 2 s_W^2 J_{\text{EM}}^\mu \right) Z_\mu . \quad (3.1)$$

With this notation the only apparent difference with the SM Lagrangian is in the new RH currents. However, when we express this Lagrangian in the mass eigenstate basis the particular features of this model manifest. In this basis,

$$\mathcal{L}_Z = -\frac{g}{2c_W} \left(\bar{u}_L X^{uL} \gamma^\mu u_L + \bar{u}_R X^{uR} \gamma^\mu u_R \right.$$
$$- \bar{d}_L X^{dL} \gamma^\mu d_L - \bar{d}_R X^{dR} \gamma^\mu d_R$$
$$\left. - 2 s_W^2 J_{\text{EM}}^\mu \right) Z_\mu , \quad (3.2)$$

where $u = (u, c, t, T, \dots)$ and $d = (d, s, b, B, \dots)$ are $\mathcal{N}_u$ and $\mathcal{N}_d$ dimensional vectors, respectively. The X's are matrices in flavour space. Apart from the new RH terms, the most important difference with the SM Lagrangian is that now the LH currents are not flavour diagonal, *i. e.* the X's are nondiagonal. To show this, let us write the unitary transformation between the mass and weak interaction eigenstates, $q^0_L = \mathcal{U}^{qL} q_L$, $q^0_R = \mathcal{U}^{qR} q_R$, with $\mathcal{U}^{qL}$ and $\mathcal{U}^{qR}$ $\mathcal{N}_q \times \mathcal{N}_q$ unitary matrices. Then,

$$X_{\alpha\beta}^{uL} = (\mathcal{U}_{i\alpha}^{uL})^* \mathcal{U}_{i\beta}^{uL} ,$$
$$X_{\alpha\beta}^{uR} = (\mathcal{U}_{j\alpha}^{uR})^* \mathcal{U}_{j\beta}^{uR} ,$$
$$X_{\sigma\tau}^{dL} = (\mathcal{U}_{k\sigma}^{dL})^* \mathcal{U}_{k\tau}^{dL} ,$$
$$X_{\sigma\tau}^{dR} = (\mathcal{U}_{l\sigma}^{dR})^* \mathcal{U}_{l\tau}^{dR} , \quad (3.3)$$

where (i, k) and (j, l) sum over the left- and right-handed doublets, respectively, $\alpha, \beta = u, c, t, T, \dots$ and $\sigma, \tau = d, s, b, B, \dots$ In the SM, $\mathcal{U}^{qL}, \mathcal{U}^{qR}$ are 3×3 matrices and i, k run from 1 to 3, hence $X_{qq'}^{uL,dL} = \delta_{qq'}$ by unitarity for any mass eigenstates q, q'. This is the well-known GIM mechanism. Besides, $X_{qq'}^{uR,dR} = 0$ for there are no RH doublets. In general, with $n > 0$ and $N + n < \mathcal{N}_u, \mathcal{N}_d$ the X's are products of submatrices with less rows than columns and thus they are nondiagonal in general.

At this point one would have to wonder about the constraints on the X's from precise electroweak data, and in particular from rare processes. However, from the apparently trivial Eqs. 3.3 we will

find a set of inequalities which imply limits on the top couplings strong enough to fulfil all experimental data. To be definite, we work with $\mathcal{U}^{uL}$, and to simplify the notation, we write

$$\mathcal{U}^{uL} = \left(\begin{array}{cccc} a_u & a_c & a_t & \cdots & a_{\mathcal{N}_u} \\ b_u & b_c & b_t & \cdots & b_{\mathcal{N}_u} \end{array} \right) \begin{array}{l} \}N+n \\ \}n_u \end{array} , \quad (3.4)$$

with a_α, b_α column vectors (remember that $N + n$ is the number of up LH doublets and n_u the number of up LH singlets). With this notation, orthogonality between columns of $\mathcal{U}^{uL}$ is written as $a_\alpha \cdot a_\beta + b_\alpha \cdot b_\beta = \delta_{\alpha\beta}$, and X^{uL} can be written as $X^{uL}_{\alpha\beta} = a_\alpha \cdot a_\beta$ with the complex scalar product '·' (in particular this shows that $|X^{uL}_{\alpha\beta}| \leq 1$ and $X^{uL}_{\alpha\alpha} \geq 0$). It is then easy to apply the Schwarz inequality to obtain

$$\begin{aligned} |X^{uL}_{\alpha\beta}|^2 = |a_\alpha \cdot a_\beta| &\leq |a_\alpha|^2 |a_\beta|^2 \\ &= X^{uL}_{\alpha\alpha} X^{uL}_{\beta\beta} , \end{aligned} \quad (3.5)$$

and for $\alpha \neq \beta$,

$$\begin{aligned} |X^{uL}_{\alpha\beta}|^2 = |a_\alpha \cdot a_\beta| = |b_\alpha \cdot b_\beta| &\leq |b_\alpha|^2 |b_\beta|^2 \\ &= (1 - |a_\alpha|^2)(1 - |a_\beta|^2) \\ &= (1 - X^{uL}_{\alpha\alpha})(1 - X^{uL}_{\beta\beta}) . \end{aligned} \quad (3.6)$$

The same can be done for down quarks and RH couplings, so we drop superscripts and write in complete generality

$$\begin{aligned} |X_{qq'}|^2 &\leq (1 - X_{qq})(1 - X_{q'q'}) , \\ |X_{qq'}|^2 &\leq X_{qq} X_{q'q'} . \end{aligned} \quad (3.7)$$

It is important to remark that these inequalities are valid for any number of generations N, doublets n and singlets n_u, n_d. In particular, if $X_{qq} = 0, 1$, Eqs. 3.7 imply $X_{qq'} = 0$ independently of $X_{q'q'}$. In this way, Eqs. 3.7 can be seen as the generalization of the GIM mechanism for models with vector-like quarks. In the case of the top quark, these equations allow to derive bounds on X_{tu}, X_{tc}, despite our ignorance on X_{tt}.

To use the inequalities in Eqs. 3.7 we have to extract the values of the diagonal elements X_{qq} from experimental data. From atomic parity violation [1, 13] and the SLAC polarized electron experiment [14] the diagonal X elements of the u and d quarks in Table 2 can be extracted, whereas the measurement of R_b, R_c, $A^{0,b}_{\rm FB}$, $A^{0,c}_{\rm FB}$

at the CERN e^+e^- collider LEP and SLC [15] provides the diagonal X elements of the b and c quarks (see Ref. [5] for details). We observe that the values of X^R_{uu}, X^L_{dd}, X^R_{cc} and X^R_{bb} in Table 2 are unphysical. This is worst for X^R_{bb}, which is 2σ away from the physical region $[0, 1]$, a direct consequence of the 2σ discrepancy between the measured and the SM values of $A^{0,b}_{FB}$. It is then necessary a careful application of the inequalities. We define the 90% C. L. upper limit on $X_{qq'}$ as the value x such that the probability of finding $X_{qq'} \leq x$ *within* the physical region is 0.9. With this definition and a Monte Carlo generator for the Gaussian distributions of R_b, R_c, $A^{0,b}_{FB}$, $A^{0,c}_{FB}$ (correlated) and $X^{L,R}_{uu}$, $X^{L,R}_{dd}$ (assuming no correlation) we obtain the bounds in Table 3, where we also quote existing limits in the literature [4, 11, 16].

$$\begin{aligned} X^L_{uu} &= 0.965 \pm 0.032 \\ X^R_{uu} &= -0.049 \pm 0.026 \\ X^L_{dd} &= 1.035 \pm 0.022 \\ X^R_{dd} &= 0.209^{+0.096}_{-0.154} \\ X^L_{cc} &= 0.998 \pm 0.013 \\ X^R_{cc} &= -0.013 \pm 0.019 \\ X^L_{bb} &= 0.996 \pm 0.005 \\ X^R_{bb} &= -0.039 \pm 0.018 \end{aligned}$$

Table 2: Diagonal elements in Eq. 3.2

It is worth to emphasize that with Eqs. 3.7 and precise electroweak data we can set limits on top couplings that (*i*) are much stronger than present direct limits, and (*ii*) can be saturated in minimal SM extensions with one doublet or singlet and still fulfil all present experimental constraints [5].

4. Future limits

Improvements on the determination of the top FCN couplings should come from large hadron and e^+e^- colliders. At these top factories the top FCN vertices manifest in top decays and in single top production. For $\sigma_{\mu\nu}$ vertices and large center of mass energies single top production is a more efficient way to look for FCN couplings because the extra q^ν factor in the vertex increases the large transverse momentum distributions im-

Coupling	X^L	X^R	Source
uc	1.2×10^{-3}	1.2×10^{-3}	δm_D
	0.033	0.019	Inequalities
ut	0.84	0.84	$t \to uZ$
	0.28	0.14	Inequalities
ct	0.84	0.84	$t \to cZ$
	0.14	0.16	Inequalities
ds	4.1×10^{-5}	4.1×10^{-5}	$K^+ \to \pi^+ \nu \bar{\nu}$
	0.14	0.62	Inequalities
db	1.1×10^{-3}	1.1×10^{-3}	δm_B
	0.0081	0.062	Inequalities
sb	1.9×10^{-3}	1.9×10^{-3}	$B^0 \to \mu^+ \mu^- X$
	0.076	0.12	Inequalities

Table 3: Present bounds on nondiagonal elements in Eq. 3.2

proving the signal to background ratios, whereas in top decays the energy available is typically smaller and the characteristic q^ν factor makes no appreciable difference. For definiteness, we restrict ourselves to renormalizable Ztq couplings, which are the relevant ones in extended models with vector-like quarks. (The other couplings can be discussed in an analogous way.) $e^+ e^-$ machines are a cleaner environment but less efficient top factories. LEP2 can improve present Tevatron limits at most by a factor of 3 [17] and Tevatron Run II with a luminosity of 2 fb^{-1} will improve LEP2 bounds by a factor of 2 [10]. Finally, LHC with a luminosity of 100 fb^{-1} will improve the former bounds by a factor of 20 [18]. Linear colliders will have to be consistent with the LHC limits but are not expected to improve them. However, if a sizeable coupling is found, linear colliders will be able to disentangle better the P and CP structure of the vertex.

Acknowledgments

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