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# Supergravity and Supersymmetry Breaking in Four and Five Dimensions

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## ABSTRACT

We discuss supersymmetry breaking in the field-theoretical limit of the strongly-coupled heterotic string compactified on a Calabi-Yau manifold, from the different perspectives of four and five dimensions. The former applies to light degrees of freedom below the threshold for five-dimensional Kaluza-Klein excitations, whereas the five-dimensional perspective is also valid up to the Calabi-Yau scale. We show how, in the latter case, two gauge sectors separated in the fifth dimension are combined to form a consistent four-dimensional supergravity. In the lowest order of the  $\kappa^{2/3}$  expansion, we show how a four-dimensional supergravity with gauge kinetic function  $f_{1,2} = S$  is reproduced, and we show how higher-order terms give rise to four-dimensional operators that differ in the two gauge sectors. In the four-dimensional approach, supersymmetry is seen to be broken when condensates form on one or both walls, and the goldstino may have a non-zero dilatino component. As in the five-dimensional approach, the Lagrangian is not a perfect square, and we have not identified a vacuum with broken supersymmetry and zero vacuum energy. We derive soft supersymmetry-breaking terms for non-standard perturbative embeddings, that are relevant in more general situations such as type I/type IIB orientifold models.

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# Introduction

In their impressive analysis of the effective field theory limit of the strongly-coupled heterotic  $E_8 \times E_8$  string theory, Horava and Witten [1, 2] constructed a consistent eleven-dimensional supergravity on a manifold  $M_4 \times X \times S^1/Z_2$ , coupled to ten-dimensional Yang-Mills models on the fixed hyperplanes of the  $S^1/Z_2$  orbifold. Witten [1] also solved the equations of motion along the eleventh dimension on the orbifold  $S^1/Z_2$ , and found the correct six-dimensional compactification that preserves four unbroken supercharges in the presence of non-trivial background components of the antisymmetric tensor field  $G_{ABCD}$ . He also calculated the gravitational constant in four dimensions and the gauge couplings on both the visible and hidden walls.

On the basis of general arguments, this should reduce to some four-dimensional  $N = 1$  supergravity theory in the infrared limit, but the above results were obtained without constructing explicitly the effective Lagrangian in four dimensions. Knowing that the final effective theory is four-dimensional  $N = 1$  supergravity, one way to obtain the complete Lagrangian is simply to read the Kähler potential, superpotential and gauge kinetic functions off from the direct  $d = 11 \rightarrow 4$  reduction of the relevant eleven-dimensional terms. With the full four-dimensional Lagrangian at hand, one can study the properties of its vacuum, such as supersymmetry breaking, and thereby understand the infrared limit of the strongly-coupled heterotic superstring.

We follow this route in the first part of this paper, and contrast the generic features of supersymmetry breaking due to gaugino condensation [3] in the strongly-coupled case [4] with the better-known weakly-coupled case [5]. An important difference is that supersymmetry is generically broken by non-zero expectation values of both  $F^S$  and  $F^T$  in the strongly-coupled string. The effective potential of the four-dimensional supergravity in the strongly-coupled case cannot be written simply as a perfect square, complicating the minimization problem, which is not solved simply by minimizing the  $|F^S|^2$  term alone. This observation may be welcome, given the phenomenological interest [6] in the dilaton-dominated scenario for supersymmetry breaking. However, we also demonstrate that the potential does not vanish generically.

The four-dimensional approach is appropriate if the condensation scale  $\Lambda$  is smaller than the threshold  $m_5 = 1/R_5$  for five-dimensional Kaluza-Klein excitations. However, studying the compactification chain  $d = 11 \rightarrow 5 \rightarrow 4$  [7, 8, 9, 10, 11, 12] is justified whatever the scale  $\Lambda$  of the nonperturbative physics. This approach allows us to study how the low-energy four-dimensional model arises out of the two  $d = 10 \rightarrow 4$  sectors which are spatially disconnected in the original  $d = 11 \rightarrow 5$  theory, in other words, how the two gauge sectors are glued together in course of the compactification process.

It was noted in the earliest days of M phenomenology that the size of the eleventh (fifth) dimension must be larger than the Calabi-Yau radius, if one enforces unification of the gauge and gravitational couplings [1]. Therefore, it is physically more interesting to compactify first the six Calabi-Yau dimensions:  $d = 11 \rightarrow 5$ , and only later the fifth dimension of the resulting five-dimensional supergravity:  $d = 5 \rightarrow 4$ . It has also been stressed [12] that the  $d = 11 \rightarrow 5$  compactification is, in the presence of a non-trivial background for  $G$ , mathematically more consistent than the path  $d = 11 \rightarrow 10 \rightarrow 4$ . It can be thought of as a Calabi-Yau compactification with non-vanishing values of the four-form field  $G_{ABCD}$  in the internal Calabi-Yau directions [12, 13, 14, 15], i.e., on a manifold with torsion. Such a construction gives gauged five-dimensional supergravity, as was first emphasized in [9], see also [11], whose equations of

motions have domain-wall solutions. These may be viewed as creating spontaneously the  $S^1/Z_2$  orbifold and reproduce the Witten solution in the approximation linear in  $x^{11}$ . The domain wall which is the vacuum solution of the gauged supergravity is a BPS state, i.e., it breaks four out of the eight supercharges possessed by the simple ungauged supergravity/Abelian Yang-Mills theory in five dimensions.

The complete compactification of that five-dimensional model down to four dimensions has not yet been performed. An important aspect of the attempt to establish the detailed relation between the Horava-Witten model [2] and the four-dimensional supergravity is that the Horava-Witten Lagrangian, although anomaly-free and supersymmetric, is only a limited approximation to the strongly-coupled string. It contains terms which are of the order  $\kappa^{2/3}$  with respect to the gravitational action, plus those terms of order  $\kappa^{4/3}$  which are necessary to maintain supersymmetry in eleven dimensions.

However, when one integrates out the fifth dimension to obtain various terms in the four-dimensional Lagrangian, such as two- and four-fermion terms, one finds that those portions of these terms that differ from one wall to the other arise at order  $\kappa^{4/3}$  and higher. Thus, in order to reproduce them reliably, one would need to go beyond the linearized solution given by Witten [1], even beyond the original eleven-dimensional field-theoretical Lagrangian. The problem is underlined by the observation that supersymmetry in various dimensions interrelates terms that are of different orders in the expansion parameter  $\kappa^{2/3}$ . As we shall argue below, one important implication of this situation is that certain features of the Horava-Witten approximation to the strongly-coupled heterotic dynamics, such as the global supersymmetry-breaking mechanism of Horava [16], may be artefacts of the approximation, that are not present in the four-dimensional effective supergravity. It is nevertheless instructive to compactify explicitly the fifth dimension and see the structure of the four-dimensional supergravity in the consistent lowest-order approximation in  $\kappa^{2/3}$ .

One issue to be discussed within this framework is the transmission of supersymmetry breaking by the five-dimensional bulk supergravity [10, 11, 8]<sup>1</sup>. The five-dimensional supergravity formulation is particularly suitable for studying the dynamical supersymmetry breaking of the residual four supercharges and its transmission between spatially-separated gauge sectors, for the following reasons. Since we know completely the structure of the simplest five-dimensional supergravity, we can study the breaking of supercharges within a fully-consistent supersymmetric model. Secondly, we can identify already in five dimensions all the massless degrees of freedom of the effective four-dimensional supergravity model. The bulk fields have well-defined properties under the  $Z_2$  orbifold parity, and therefore one can write down  $Z_2$ -invariant couplings with the walls. As we see in more detail later, the supersymmetry breaking on the visible wall occurs locally, without any need to average over the fifth dimension, once some fields or their derivatives acquire vev's that are non-vanishing locally on the wall, as a result of equations of motion in the fifth dimension.

In the present letter, we complete the previous work first by adding explicitly to the bulk Lagrangian studied in [10] the most relevant Lagrangian terms on the four-dimensional walls, as obtained by dimensional reduction of the original Horava-Witten wall Lagrangian from

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<sup>1</sup>Discussions of alternative approaches to supersymmetry breaking, e.g., the Scherk-Schwarz mechanism, averaging over the fifth dimension, anomaly mediation and invoking a tower of Kaluza-Klein states can be found in [17, 19, 20, 21]. These have features that differ from the mechanism discussed here.

$d = 10 \rightarrow 4$  on a Calabi-Yau manifold. Next, we reduce the fifth dimension by solving the equations of motion and integrating out the components of the five-dimensional fields which depend on the fifth coordinate. The equations of motion are determined by the structure of the five-dimensional theory, and it turns out that we can solve them explicitly only in the linearized approximation. In this way, we identify the five-dimensional origins of various parts of the four-dimensional effective action, in particular those that play a role in gaugino condensation. As already mentioned, this explicit construction is complete only to lowest order in  $\kappa^{2/3}$  and, therefore, the four-dimensional supergravity obtained in this approximation is different from that considered in Section 2, which includes fully the threshold corrections to the gauge kinetic functions that arise in the strongly-coupled heterotic  $E_8 \times E_8$  string. Finally, we address the subtle question how the nonlinearities appearing in the solutions of the five-dimensional equations of motion for the bulk moduli fields fit into the canonical structure of the four-dimensional supergravity.

## Supersymmetry Breaking in Four Dimensions

We begin by following the first route described in the Introduction. We recall that one obtains from the Horava-Witten model [2] the Kähler functions and the gauge kinetic functions for the standard and non-standard embeddings, consistently to order  $\kappa^{2/3}$ , that is with the threshold corrections to the gauge kinetic functions included. This is sufficient to reconstruct directly the parts of the scalar potential that are relevant for seeing the supersymmetry-breaking structure in the effective four-dimensional supergravity theory arising from the strongly-coupled heterotic string below the scale  $m_5 = \frac{1}{R_5}$ . In this case, it is fully adequate to work entirely within the four-dimensional supergravity framework, as we assume in this Section.

We first recall the way the vev of gaugino bilinears  $\langle \lambda^a \lambda^b \rangle$  enters the effective scalar potential of the four-dimensional supergravity [22]<sup>2</sup>. Using the canonical normalization in four dimensions for the gravitational, gauge and gaugino kinetic terms, the relevant part of the Lagrangian is

$$V = e^K g^{i\bar{j}} (D_i W + \frac{1}{4} e^{-K/2} \partial_i f_{ab} \langle \lambda^a \lambda^b \rangle) (D_{\bar{j}} \bar{W} + \frac{1}{4} e^{-K/2} \partial_{\bar{j}} \bar{f}_{ab} \langle \bar{\lambda}^a \bar{\lambda}^b \rangle) + \dots, \quad (1)$$

where  $g^{i\bar{j}}$  is the inverse Kähler metric and rest of the notation is standard [22].

Comparing (1) with the well-known general expression  $V = g_{i\bar{j}} F^i F^{\bar{j}} - 3e^G$  for the four-dimensional potential in terms of the  $F^i$ , the auxiliary fields for the chiral multiplets, we read off the modified expressions for the auxiliary fields in the presence of the condensates

$$F^i = e^{K/2} g^{i\bar{j}} (D_{\bar{j}} \bar{W} + \frac{1}{4} e^{-K/2} \partial_{\bar{j}} \bar{f}_{ab} \langle \bar{\lambda}^a \bar{\lambda}^b \rangle). \quad (2)$$

In the following, we match this expression explicitly to the fermionic bilinears in the effective Lagrangian, since [4] this is a better description when the gauge kinetic function depends on

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<sup>2</sup>We denote by  $\lambda^a$  the gaugino components, where  $a$  is an adjoint group index, and  $i$  labels complex moduli fields.

more than one modulus, and the gaugino bilinears are among the terms obtainable directly from the Calabi-Yau reduction.

We recall briefly the situation in the weakly-coupled heterotic string. There, at tree level, the gauge kinetic function is universal:  $f = S$ , and the Kähler function for the illustrative case of a single universal modulus  $T$  is  $K = -\log(S + \bar{S}) - 3\log(T + \bar{T})$ . In this case, the full potential

$$V = g_{S\bar{S}}|F^S|^2 + g_{T\bar{T}}|F^T|^2 - 3|W|^2 e^K \quad (3)$$

reduces to

$$V = g_{S\bar{S}}|F^S|^2, \quad (4)$$

since  $g_{T\bar{T}}|F^T|^2 = 3|W|^2 e^K$ . This relation is equivalent to the vanishing of the perfect square containing the gaugino condensates in the ten-dimensional effective action of the weakly-coupled heterotic superstring [23]. In this way, we find

$$F^S = 0, \quad F^T \neq 0 \quad (5)$$

At the level of quantum corrections, there are additional contributions to the gauge kinetic functions which depend on the modulus  $T$ , and additional terms required by the space-time modular invariance appear in terms containing gaugino bilinears, and hence in the effective superpotential describing condensation. However, in all the models studied so far, the vacuum relation  $F^S \ll F^T$  persists, and supersymmetry breaking occurs along the  $T$  direction in the moduli space. The technical reason is that the dependence on  $S$  factorizes out in simple modular-invariant models of condensation, and the equations of motion, i.e., dynamics, tell us that  $F^S$  is small. The vev of the modulus  $T$  is rather small in these models, close to unity in supergravity units.

The structure of the supersymmetry-breaking sector is significantly modified in the strongly-coupled regime. The Kähler function for the universal moduli  $S, T$  is the same, but the classical, or tree level, gauge kinetic functions are changed, and are different for different walls:  $f_{1,2} = S \pm \xi_0 T$  where

$$\xi_0 = -\frac{\pi\rho_0}{2(4\pi)^{4/3}\kappa^{2/3}} \frac{1}{8\pi^2} \int_X \omega_K \wedge (tr F^{(1)} \wedge F^{(1)} - \frac{1}{2} tr R \wedge R). \quad (6)$$

Here  $\omega_K$  is the Kähler (1,1)-form, and the topological integral over Calabi-Yau space can be parametrized in terms of gauge and gravitational instanton numbers characterizing the embedding [24]:

$$\xi_0 = \frac{n_{F1} - \frac{1}{2}n_R}{32\pi^3}, \quad (7)$$

The interesting region of moduli space is where  $S = \mathcal{O}(2)$  and  $T = \mathcal{O}(80)$  [24, 25]. Hence, we are not interested in mechanisms which generate minima of the potential at  $T \approx 1$ , but need some new mechanism which generates a minimum in the region of current interest. We do not discuss any specific mechanism here, but just state the possibilities opened up by the current form of the kinetic functions.

First, we note that  $S$  and  $T$  enter the kinetic functions, and hence any nonperturbative potential, in quite a symmetric way. The relative coefficient  $\xi_0$  (7) which weights the contribution of  $T$  changes from model to model. In the elliptic-fibration models of [25], this number is

smaller than 0.025. It could in principle be either much larger or much smaller in more general constructions. Because of the greater symmetry between  $S$  and  $T$ , there is no obvious reason why  $F^S$  should be much smaller than  $F^T$  in the generic case. In the strongly-coupled case, we obtain

$$V = e^K (S + \bar{S})^2 \left| -\frac{W}{S+\bar{S}} + \frac{1}{4} e^{-K/2} (\Lambda_1^3 + \Lambda_2^3) \right|^2 + e^K \frac{(T+\bar{T})^2}{3} \left| -\frac{3W}{T+\bar{T}} + \frac{1}{4} \xi_0 e^{-K/2} (\Lambda_1^3 - \Lambda_2^3) \right|^2 - 3e^K |W|^2. \quad (8)$$

and the result for the  $F$  terms is <sup>3</sup>:

$$F^S = \frac{1}{4} (S + \bar{S})^2 (\Lambda_1^3 + \Lambda_2^3) \quad (9)$$

$$F^T = \frac{1}{12} (T + \bar{T})^2 \xi_0 (\Lambda_1^3 - \Lambda_2^3) \quad (10)$$

It is clear that supersymmetry is unbroken:  $F^S = F^T = 0$  if and only if both condensates vanish. Even if there is only one condensate, both  $F^S$  and  $F^T$  are non-zero. Moreover, if condensates on both walls are switched on simultaneously, no matter in what proportion, supersymmetry is always broken in four dimensions. In particular, even when the two condensates are switched on with the same magnitude, and opposite signs, supersymmetry is formally broken <sup>4</sup>, contrary to [16]. A further consequence of  $F^S \neq 0$  is a non-zero scalar mass, which arises from (1) upon substituting the correction  $-\delta K_S = \pm \xi_0 |A^q|^2 / (S + \bar{S})^2$  <sup>5</sup>, which gives soft scalar masses proportional to  $\pm \xi_0 F^S$ .

Finally, we examine the ratio of the two  $F$  terms

$$\frac{F^S}{F^T} = \frac{3}{\xi_0} \frac{\Lambda_1^3 + \Lambda_2^3}{\Lambda_1^3 - \Lambda_2^3} \left( \frac{S}{T} \right)^2. \quad (11)$$

In the present region of moduli space, the ratio of  $S/T$  is of the order 1/40 or so, so it would not require very much fine-tuning to arrange the magnitudes of the condensates in such a way that the ratio  $F^S/F^T$  is of the order of unity or larger. To make the possibility of the mixed  $S, T$ -moduli-driven scenario more plausible, we look at the ratio  $F^S/F^T$  more carefully. As pointed out in [24], one can easily express the expectation value of  $T$  through the observable quantities  $T = \left( \frac{M_P}{M_{GUT}} \right)^2 \frac{\alpha_{GUT}^{1/3}}{2^{17/3} \pi^3}$ . Then we can express  $S$  as  $S = \frac{1}{4\pi\alpha_{GUT}} - \xi_0 T$ . As a result, we obtain the ratio of the  $F$  terms as a function of  $\xi_0$ :

$$\frac{F^S}{F^T} = \frac{3}{\xi_0} \left( \frac{2^{11/3} \pi^2 M_{GUT}^2}{\alpha_{GUT}^{4/3} M_P^2} - \xi_0 \right)^2 \frac{\Lambda_1^3 + \Lambda_2^3}{\Lambda_1^3 - \Lambda_2^3}. \quad (12)$$

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<sup>3</sup>Here we suppress the possibility of a constant superpotential contribution, which could arise as a vev of the gauge and/or gravitational Chern-Simons forms on either wall, whose inclusion does not change the general picture.

<sup>4</sup>The magnitude of the breaking is to be determined from the vacuum solution of the effective potential, but we would expect  $m_{3/2} = \mathcal{O}(\Lambda^3/M_P^2)$ .

<sup>5</sup>This correction arises from the correction to the metric of the Calabi-Yau space, i.e., to the factor of  $\sqrt{g_{(6)}}$  that multiplies the kinetic terms of the four-dimensional charged scalars [26].

The prefactor multiplying the condensates can be studied as a function of  $\xi_0$ , when we fix the observables at their MSSM values. One finds that the prefactor vanishes at  $\xi_0 \approx 0.025$ , but grows quickly to values of  $\mathcal{O}(1)$  for larger  $\xi_0$ , and to the values  $\geq 0.07$  at  $\xi_0 \leq 0.01$ . For negative  $\xi_0$ , i.e., in the regime of ‘strong’ unification, the value of the prefactor is always larger than  $1/10$ . Thus, it is possible to obtain quite a large value of  $F^S$ , and even the extreme option of  $S$ -dilaton-driven supersymmetry breaking cannot be excluded in the strongly-coupled heterotic string. This could have interesting consequences, given the promising results of phenomenological investigations of this limit in the weakly-coupled string.

We finish this section with a list of the soft terms we found in the four-dimensional supergravity approach, which is the way the soft terms were obtained in all the papers published so far: [18, 19, 27, 28, 29]. We generalize the earlier results by considering non-standard embeddings in which charged matter is present on both walls, i.e., in both gauge sectors, and we allow for condensates to form on both walls <sup>6</sup>. Restoring powers of the reduced Planck mass  $M$ , the physical gravitino mass in four dimensions is:

$$m_{3/2}^2 = \frac{(S + \bar{S})^{2/3}}{2^{2/3} 3^{1/2} M^4} (\Lambda_1^3 + \Lambda_2^3)^2 + \frac{(T + \bar{T})^2}{432 M^4} \xi_0^2 (\Lambda_1^3 - \Lambda_2^3)^2 \quad (13)$$

and the mixing angle  $\theta$  introduced through the relation  $\frac{F^S}{F^T} = \sqrt{3} \frac{S + \bar{S}}{T + \bar{T}} \tan \theta$  is given by

$$\tan \theta = \sqrt{3} \xi_0^{-1} \frac{S + \bar{S}}{T + \bar{T}} \frac{\Lambda_1^3 + \Lambda_2^3}{\Lambda_1^3 - \Lambda_2^3}. \quad (14)$$

Assuming that the CP-violating phases vanish, we obtain trilinear scalar terms of the form

$$A = \sqrt{3} m_{3/2} \left( \sin \theta \left( -1 \pm \xi_0 \frac{3(T + \bar{T})}{3(S + \bar{S}) \pm \xi_0(T + \bar{T})} \right) + \sqrt{3} \cos \theta \left( -1 + \frac{3(T + \bar{T})}{3(S + \bar{S}) \pm \xi_0(T + \bar{T})} \right) \right) \quad (15)$$

and gaugino masses

$$M_{1/2} = \frac{\sqrt{3} m_{3/2}}{(S + \bar{S}) \pm \xi_0(T + \bar{T})} \left( \sin \theta (S + \bar{S}) \pm \xi_0 \cos \theta \frac{(T + \bar{T})}{\sqrt{3}} \right). \quad (16)$$

Note that there is a difference of sign between the expressions linear in  $\xi_0$  corresponding to different walls. This can have consequences in some of non-standard embedding models, where, e.g., matter with Standard Model hypercharge may exist on both walls. The dilaton-dominated limit corresponds to  $\sin \theta \rightarrow 1$ . We can see from the formulae for the  $A$  terms and gaugino masses that even in this limit there is non-universality between terms containing charged fields from different walls. As for soft scalar masses in this context, one can easily imagine strongly-coupled heterotic string models analogous to weakly-coupled orbifolds with twisted matter, where the scalar masses for many twisted fields are non-zero. Using standard supergravity formulae, one can easily write down the expressions for these masses, in terms of the corresponding Kähler potential  $K$ ,

$$m_{i\bar{j}}^2 = K_{i\bar{j}} \left( m_{3/2}^2 - \frac{3m_{3/2}^2}{3(S + \bar{S}) \pm \xi_0(T + \bar{T})} \left( \pm \xi_0 (T + \bar{T}) \left( 2 - \frac{\pm \xi_0 (T + \bar{T})}{3(S + \bar{S}) \pm \xi_0 (T + \bar{T})} \right) \sin^2 \theta \right. \right. \\ \left. \left. + (S + \bar{S}) \left( 2 - \frac{3(S + \bar{S})}{3(S + \bar{S}) \pm \xi_0 (T + \bar{T})} \right) \cos^2 \theta - \frac{\pm \xi_0 2\sqrt{3}(T + \bar{T})(S + \bar{S})}{3(S + \bar{S}) \pm \xi_0 (T + \bar{T})} \sin \theta \cos \theta \right) \right). \quad (17)$$

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<sup>6</sup>However, we do not consider five-branes in the bulk.

One again notices the characteristic differences of sign between terms from different wall, and the universality of these terms is violated even in the dilaton-driven supersymmetry-breaking limit.

## The Five-Dimensional Connection between Four-Dimensional Worlds

In this Section, we construct explicitly the four-dimensional supergravity Lagrangian that arises from the sequence of compactifications:  $d = 11 \rightarrow 5 \rightarrow 4$ , remembering that the result can be trusted only in the lowest order in  $\kappa^{2/3}$ . In this case, the four-dimensional Lagrangian is born from two spatially-separated gauge sectors. As will be clear from our construction, the result obtained at the lowest order of the expansion in  $\kappa^{2/3}$  should be a four-dimensional supergravity theory in the approximation  $f_{1,2} = S$ .

The general five-dimensional model obtainable through compactification of eleven-dimensional supergravity on the deformed Calabi-Yau space constructed in [1] has already been given in [11]. There were worked out the couplings of some important operators belonging to the Yang-Mills sectors living on the walls, such as gaugino bilinears and scalar trilinear operators derived from the Lorentz Chern-Simons terms, to scalar fields living in the five-dimensional bulk. Here we concentrate on details of these couplings which are relevant for the construction of the effective four-dimensional model, and identifying the operators that violate four-dimensional supersymmetry.

We must first return to the Bianchi identity for  $G_{11ABC}$ , and to the perfect-square structure of Horava [16]. Horava has shown that, for the Lagrangian describing the eleven-dimensional supergravity coupled to two Yang-Mills sectors on the ten-dimensional boundaries to be supersymmetric, one needs among other terms a specific pair of gaugino interaction terms:

$$\begin{aligned} & \sum_{i=1,2} \left( \frac{\sqrt{2}}{96\pi(4\pi\kappa^2)^{2/3}} \int_{M^{(i)}} d^{10}x \sqrt{g} \bar{\chi}^a \Gamma^{ABC} \chi^a G_{ABC11} \right. \\ & \left. - \frac{\delta(x_{(i)}^5)}{96(4\pi)^{10/3}\kappa^{2/3}} \int_{M^{(i)}} d^{10}x \sqrt{g} \bar{\chi}^a \Gamma^{ABC} \chi^a \bar{\chi}^b \Gamma_{ABC} \chi^b \right) \end{aligned} \quad (18)$$

where  $i = 1, 2$  labels the walls and  $x_{(1)}^5 = 0$ ,  $x_{(2)}^5 = \pi\rho_0$ . We write these terms in this form to stress that they should be interpreted as boundary terms. Indeed, the first, non-singular term is simply the well-known coupling of the ten-dimensional gauginos to the antisymmetric tensor field. Since the product  $\delta(x^{11}) \delta(x^{11} - \pi\rho_0)$  vanishes, the combination of terms involving  $G_{11ABC}$  and ten-dimensional gauginos  $\chi$  (18) can be written as a *bulk* perfect-square action:

$$L_{sq} = -\frac{1}{12\kappa^2} \int_{M^{11}} d^{11}x \sqrt{g} \left( G_{11ABC} - \sum_m \frac{\sqrt{2}}{16\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \delta^{(m)}(x^{11}) \text{Tr} \bar{\chi}^{(m)} \Gamma_{ABC} \chi^{(m)} \right)^2, \quad (19)$$

where

$$\begin{aligned} G_{11abc} = & (\partial_{11} C_{abc} \pm 23 \text{ perms}) + \frac{\kappa^2}{\sqrt{2}\lambda^2} \delta(x^{11}) \left( \frac{2}{3} \text{tr} A_a [A_b, A_c] + \text{cycl.} \right) \\ & + \frac{\kappa^2}{\sqrt{2}\lambda^2} \delta(x^{11} - \pi\rho_0) \left( \frac{2}{3} \text{tr} A_a [A_b, A_c] + \text{cycl.} \right). \end{aligned} \quad (20)$$



We have retained in this expression only these parts of the gauge Chern-Simons forms which give rise to couplings between zero modes arising from the compactification of the six-dimensional internal space, and are directly related to the low-energy effective superpotential.

Since, in this Section, we first construct the five-dimensional theory, we perform the Weyl rotation of the metric which gives the canonical Einstein-Hilbert action in five dimensions. The relation between the canonical eleven- and five-dimensional metrics is  $g_{MN}^{(11)} = (e^{-2\sigma} g_{\mu\nu}^{(5)}; e^\sigma g_{ab}^{(0)})$ . In this notation, we take  $e^{3\sigma} = Re(S)$ , where  $S$  is the  $Z_2$ -even scalar from the universal hypermultiplet [10, 11]. This five-dimensional moduli  $S, T, U$  should not be confused with the four-dimensional chiral superfields encountered in the previous Section, which we denote by  $\tilde{S}, \tilde{T}, \tilde{U}$  when necessary in this Section. In what follows, we shall use the same symbols both for other five-dimensional moduli, e.g.,  $\sigma, \gamma$ , and for the corresponding four-dimensional quantum fields, whenever it is obvious from the context which we actually have in mind. We need the decompositions of eleven-dimensional spinors  $\chi_L(x, y) = \chi_L(x) \otimes \eta_+(y)$ , where  $\eta_+$  is the covariantly-constant spinor of positive chirality <sup>7</sup>. The corresponding decomposition of  $\Gamma$  matrices is  $\Gamma_{(11)}^M = \{\gamma_{(11)}^m \otimes \mathbf{1}; i\bar{\gamma} \otimes \gamma^k\}$  in tangent indices, where  $\bar{\gamma} \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$ ,  $\Omega \equiv \gamma^7 = i\prod_k \gamma^k$ ,  $\bar{\gamma}\Omega = \Gamma^{11}$ . The relation between  $\gamma$  matrices with world indices in eleven- and five-dimensional normalizations is  $\Gamma_{(11)}^\mu = e^\sigma \gamma_{(5)}^\mu$ ,  $\Gamma_{(11)}^M = e^{-\sigma/2} \Gamma_{(0)}^M$ , where the subscript (0) denotes the fiducial metric on the zeroth-order (undeformed) Calabi-Yau manifold.

If we restrict ourselves to a Calabi-Yau space with  $h_{(2,1)} = 0$ , then the decomposition of gauge fields with compact indices which defines matter fields in **27**s of  $E_6$  is  $A_i = A^{kp}(x)\omega_i^{kj}(y)T_{jp}$ , where the  $\omega_i^{kj}(y)$  are harmonic (1,1)-forms, and the  $T_{jp}$  are generators of  $E_6$ . We note the following properties of the generators:  $\text{Tr}T_{ip}T_{jq}T_{kr} = \epsilon_{ijk}d_{pqr}$ ,  $\text{Tr}T_{jp}T_{j'p'} = \delta_{jj'}\delta_{pp'}$ , and the appropriate expansions for the field strength:  $F_{\mu a} \rightarrow \partial_\mu C^{Kp}\omega_a^{Kj}T_{jp}$ ,  $F_{\nu b} \rightarrow \partial_\nu C^{Lp'}\omega_b^{Lj'}T_{j'p'}$ . Finally, we shall use the following decomposition of the regular part of  $G_{abc11}$  in terms of massless Calabi-Yau modes [11]

$$(G_{11})_{abc} = 2\partial_{11}C_0\bar{\Omega}_{abc} + h.c. \quad (21)$$

With these conventions and definitions, the result of the reduction of the perfect square down to five dimensions is

$$\mathcal{L}_{sq} = -\frac{V_0}{12\kappa^2} \int d^5x \sqrt{g_{(5)}} e^{-3\sigma} g^{55} \left( 2\partial_5 C_0 + 4\frac{\kappa^2}{\sqrt{2}\lambda^2} \delta^{(m)} P^{(m)} - \frac{1}{32\pi} \left(\frac{\kappa}{4\pi}\right)^{2/3} e^{\frac{9}{2}\sigma} (g_{55})^{3/4} \delta^{(m)} \bar{\epsilon}_m^{(4)} \epsilon_m^{(4)} \right)^2 \quad (22)$$

where  $P^{(m)}(A) = \lambda_{KLM}d_{pqr}A^{Kp}A^{Lq}A^{Mr}$ ,  $m = 1, 2$  labels the walls, and we have used the canonical normalization for gauginos in four dimensions. Here  $e^{3\sigma(x, x^5)}$  is the five-dimensional variable measuring the volume of the Calabi-Yau space along the orbifold interval, in units of the fiducial volume  $V_0$ . This is the real part of the  $Z_2$ -even scalar from the universal hypermultiplet:  $Re(S) = e^{3\sigma(x^5)}$ . The relation between  $S$  and the four-dimensional fields  $\tilde{S}, \tilde{T}$  is

$$e^{3\sigma(x^5)} = e^{3\sigma} + \xi_0 e^\gamma \left(1 - \frac{2x^5}{\pi\rho_0}\right). \quad (23)$$

In the above expression,  $e^{3\sigma} = Re(\tilde{S})$ , and  $e^\gamma = Re(\tilde{T}) = \sqrt{g_{55}(x^\mu)}$  are functions of the four noncompact coordinates.

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<sup>7</sup>We choose the basis in which negative-chirality spinors on the walls are projected out by  $Z_2$  parity.

To construct the effective theory in four dimensions, one has to integrate out the components of the five-dimensional fields which do not correspond to massless degrees of freedom in four dimensions, and the natural way to do this is through the solution of the equations of motion along the dimension which one wishes to compactify. To the lowest order in  $\kappa^{2/3}$ , i.e., to the zeroth order in  $\xi_0$ , the equation is

$$\partial_5^2 C_0 = \sum_m \left( -\frac{\sqrt{2}\kappa^2}{\lambda^2} P^{(m)} + \frac{1}{64\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} e^{\frac{9}{2}\sigma} (g_{55})^{3/4} \bar{\epsilon}_m^{(4)} \epsilon_m^{(4)} \right) \partial_5 \delta^{(m)} \quad (24)$$

The solution to this equation which obeys the periodicity condition on the full circle and is antisymmetric across the fixed points of the  $S^1/Z_2$  has a finite discontinuity at each of these points. Its derivative develops  $\delta$ -function singularities at  $x^5 = 0, \pi\rho_0$ , which cancel other  $\delta$ -function terms coming from the expansion of the formal ‘square’. The regular part of the derivative, which is continuous everywhere, is

$$\partial_5 C_0 = \frac{1}{2\pi\rho_0} \sum_{m=1,2} \left( -\frac{\sqrt{2}\kappa^2}{\lambda^2} P^{(m)} + \frac{1}{64\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} e^{\frac{9}{2}\sigma_m} (g_{55m})^{3/4} \bar{\epsilon}_m^{(4)} \epsilon_m^{(4)} \right). \quad (25)$$

where the subscript  $m = 1, 2$  denotes the restriction of the given function to the  $m$ 'th wall. We note that the coefficients of the gaugino bilinears above would differ in higher order in  $\xi_0$  (see (23)). The effective four-dimensional Lagrangian is obtained by substituting (25) into Lagrangian (22) and then integrating over the coordinate  $x^5$ .

We need to comment on the next-order corrections to the above solution. As we stressed earlier, higher-order corrections to (25) cannot be reliably calculated from the Lagrangian (22). It turns out that, in order to find the corrections reliably and to reconstruct the complete four-dimensional Lagrangian at higher order in  $\xi$ , it would be necessary to go beyond the linear order in the  $x^5$  dependence of the field  $C_0$ , hence beyond the order to which the effective Lagrangian can be trusted. Secondly, the compactification  $11 \rightarrow 5$  leads in general to a nonlinear  $\sigma$ -model structure which goes beyond the simple expression (22): as pointed out in [11], in five dimensions the perfect square (22) is a part of the larger nonlinear  $\sigma$ -model structure

$$L_{coupling} = -\frac{1}{2} g_{xy} g^{55} (\partial_5 \sigma^x - \mathcal{L} \delta(x^5 - \pi\rho) \delta^{xx_0}) (\partial_5 \sigma^y - \mathcal{L} \delta(x^5 - \pi\rho) \delta^{yy_0}), \quad (26)$$

where we assume that the four-dimensional gaugino condensate is proportional to the Calabi-Yau  $(3, 0)$ -form  $\Omega_{ijk}$ . The operators  $\mathcal{L}$  contain the terms trilinear in matter fields and bilinear in gauginos. The scalars  $\sigma^x$  are components of the five-dimensional hypermultiplets, including the universal one:  $(S, C_0); (Z, C_1); \dots$ . The approximate expression for the metric is

$$g = \begin{bmatrix} \frac{1}{(S+S)^2} & \frac{-2(C_0+\bar{C}_0)}{(S+S)^2} & 0 & \frac{-2(C_1+\bar{C}_1)}{(S+S)^2} \\ \frac{-2(C_0+\bar{C}_0)}{(S+S)^2} & \frac{2}{S+S} & \frac{2(C_1+\bar{C}_1)}{S+S} & \frac{2(Z+\bar{Z})}{S+S} \\ 0 & \frac{2(C_1+\bar{C}_1)}{S+S} & 1 & \frac{2(C_0+\bar{C}_0)}{S+S} \\ \frac{-2(C_1+\bar{C}_1)}{(S+S)^2} & \frac{2(Z+\bar{Z})}{S+S} & \frac{2(C_0+\bar{C}_0)}{S+S} & \frac{2}{S+S} \end{bmatrix}, \quad (27)$$

when we expand the quaternionic metric up to linear order in all the fields except  $S$ . There exist additional sources for the field  $S$  on the walls, coming from the additional ten-dimensional terms

which can be read from the supersymmetrization of the ten-dimensional Green-Schwarz terms:  $F_{(i)}^2 \rightarrow F_{(i)}^2 - \frac{1}{2}R_{mn}R^{mn}$  [30]. These terms are the dominant source of the vacuum domain-wall configuration, but they do not contain observable fields. Hence, as far as the matter fields and gaugino couplings are concerned, the relevant couplings to moduli are in (26). We reiterate that nonlinearities are to be expected in the solution of the full theory in five dimensions, since it contains nonlinear terms associated with gauging, as well as with the nonlinear  $\sigma$  model structure.

It is useful to summarize certain properties of the zeroth-order result (25). First, we recall [10, 11] that the order parameter for supersymmetry breaking in the microscopic five-dimensional vacuum is the non-vanishing vev of  $\partial_5 C_0$  if we compactify on a Calabi-Yau space with  $h_{(2,1)} = 0$ : see (29) below for the general case with  $h_{(2,1)} > 0$ . Secondly, (25) expresses this quantity in terms of fields charged under the gauge group, which are legitimate zero modes in the effective four-dimensional theory. We see that the terms corresponding to superpotentials generated on separate walls enter this expression additively, which is simply the minimal structure for building the effective four-dimensional supergravity. This is not surprising, knowing the way the parts of the superpotential add up in the effective Lagrangian derived from the weakly-coupled string, but the low-energy superpotential could in principle be a non-trivial chiral function of  $W^1$  and  $W^2$ . Thirdly, the terms corresponding to gaugino bilinears enter the expression (25) with coefficients that are equal to the order to which we have solved the corresponding equation of motion. We shall now discuss some of these points in more detail.

In the five-dimensional approach which we assumed in this Section, following [10, 11], it is straightforward to identify the terms that violate the four low-energy supersymmetries, namely, the terms containing couplings of the charged fields to the derivative  $\partial_5 C_0$ , assuming this derivative acquires a vev. The full set of supersymmetry-breaking terms, which are all soft, can be summarized as

$$\mathcal{L}_{soft} = -\frac{2V_0}{3\kappa^2} \sum_{m=1,2} \int_{M_4^{(m)}} d^4x \sqrt{g_m^{(4)}} e^{-3\sigma_m} (g^{55m})^3 \langle \partial_5 C_0 |_m \rangle \times \left[ \frac{\sqrt{2}\kappa^2}{\lambda^2} P^{(m)} - \frac{1}{64\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} e^{\frac{9}{2}\sigma_m} (g_{55m})^{3/4} \bar{\epsilon}_m^{(4)} \epsilon_m^{(4)} \right] \quad (28)$$

where the subscripts  $m$  denote the values of the corresponding five-dimensional fields on the  $m$ 'th wall. Modulo certain details to be discussed below, we see in (28) soft terms that are proportional to  $P^{(m)}$  and trilinear in matter scalars, which were unnoticed in early papers, and soft gaugino masses, in both the hidden and the visible sectors. Terms which are absent above include nonchiral soft scalar masses. This absence of soft scalar masses is directly related to the form of the bulk-wall coupling, i.e., to the form of the perfect-square coupling, which was a consequence of eleven-dimensional supersymmetry, in the order to which the Horava-Witten Lagrangian was constructed. The deeper reason for the absence of explicit scalar masses is that we are only compactifying the eleven-dimensional field theory and not the full strongly-coupled string theory, i.e., we do not perform the equivalent of the orbifold compactification in the weakly-coupled string, since the full theory is currently inaccessible only in the strongly-coupled limit.

In order to see how the soft terms found above would look in the presence of the non-universal hypermultiplets in the bulk, i.e., for Calabi-Yau spaces with Hodge number  $h_{(2,1)} > 0$ ,

we recall [11] that one has to make the replacement

$$2\partial_5 C_0 \rightarrow \frac{2}{1 - |Z^a|^2} (\partial_5 C_0 + \partial_5 C_a Z^a) \quad (29)$$

in the expression (28), where  $a = 1, \dots, h_{(2,1)}$  labels  $Z_2$ -odd ( $C_a$ ) and  $Z_2$ -even ( $Z^a$ ) scalars from the non-universal hypermultiplets. The presence of the derivatives  $\partial_5 C_a$  signals that, in addition to the  $F$  terms of the universal moduli  $\tilde{S}, \tilde{T}$  there will be  $F$  terms of the non-universal  $(2, 1)$   $\tilde{U}$  moduli participating in the breaking of supersymmetry at low energy.

We now discuss the interpretation of the solution (25) from the point of view of the low-energy supergravity Lagrangian. First of all, we note once more that, if we switch off the gaugino bilinears, then what is left in  $\partial_5 C_0$  is the scalar component of the superpotential, which happens to be *the sum* of the superpotentials from individual walls. This is exactly what we expect from the point of view of the four-dimensional supergravity, but it could not be taken for granted from the beginning. Our procedure is the only way to recover all the supergravity terms involving the scalar component of the superpotential, which we take as a clear sign that integrating out the fifth dimension, as done here, is the canonical way of recovering the correct four-dimensional effective theory. We now consider the effect of switching on the gaugino condensates. It is clear that they can be regarded as effectively non-dynamical parts of the superpotential in the infrared limit. The consistent way to summarize these observations is to write

$$\partial_5 C_0 = -\frac{1}{256\pi^2\rho_0} \left(\frac{\kappa}{4\pi}\right)^{2/3} \sum_m W^{(m)} \quad (30)$$

where (using  $g_{55} = e^{2\gamma}$ ):

$$W^{(m)} = 64\sqrt{2}P^{(m)} - 2e^{\frac{3}{2}(3\sigma_m + \gamma_m)} \langle \bar{\epsilon}_m^{(4)} \epsilon_m^{(4)} \rangle \quad (31)$$

is the effective low-energy superpotential generated on the  $m$ 'th wall. This identification clearly can be done at the level of the lowest-order solution for  $\partial_5 C_0$ . There, the condensates enter with exactly the same coefficients, and one can conclude that the four-dimensional scalar potential obtained from the solution (25) is that the one of the four-dimensional supergravity with  $f_{1,2} = \tilde{S}$ : this leads to

$$F^{\tilde{S}} \propto W^1 + W^2 - \frac{1}{4}(\tilde{S} + \bar{\tilde{S}})^{3/2}(\tilde{T} + \bar{\tilde{T}})^{3/2}(\Lambda_1^3 + \Lambda_2^3), \quad (32)$$

which agrees with (25) and (31) at the lowest order, i.e., with  $\sigma_m \rightarrow \tilde{\sigma}$ ,  $\gamma_m \rightarrow \tilde{\gamma}$ . In this case, we immediately recover the original conclusion drawn in eleven dimensions [16], namely that two condensates on opposite walls with the same magnitudes and opposite phases result in unbroken supersymmetry. This is in contrast to the result obtained previously in four-dimensional supergravity with gauge kinetic functions linear in  $\xi_0$ . However, the rederivation of the result of [16] in five dimensions on the basis of (25) in the latter case, with higher order corrections switched on, is misleading, since it is the dynamics which must choose the physical configuration of fields from the family of all those possible, which we have not yet fully taken into account. We come back to an explanation of this point at the end of this Section.

Actually, the above is not the only way in which supersymmetry breaking due to condensates can be modified. One should note that the reduction of the ten-dimensional Chern-Simons

forms (including the gravitational ones) can result not only in trilinear terms  $P^{(m)}$ , but also in constant terms, corresponding to non-zero fluxes of the gauge and tangent vacuum bundles. These terms, which are constant from the four-dimensional point of view, might also cancel condensates, causing the vev of  $\partial_5 C_0$  to vanish. To decide what is the actual vev controlling supersymmetry breaking is a dynamical problem, which can be addressed in the context of the results of the present paper. Unfortunately, as predicted already in [11], the effective potential which one obtains has a runaway direction.

Lacking clearer options, we assume whenever necessary that, if  $\partial_5 C_0$  is non-zero, then it is of the order of the effective condensation scale  $\Lambda^3/2\pi\rho_0$ . In this connection, we observe on the basis of the exact expression (28) that soft terms born on a given wall are proportional to  $\partial_5 C_0|_m$ . i.e., the derivative of  $C_0$  computed locally on that wall. There is no five-dimensional averaging involved: soft terms are generated by *local* interactions on the walls.

This observation is important, as it leads to the next consistency check of the implicit assumption of the existence of the effective four-dimensional supergravity. The obvious place to read off the effective superpotential in a given four-dimensional supergravity model is the gravitino mass term, which is  $e^{K/2}W$  in the canonical normalization for the gravitino in four dimensions. When one reduces the only eleven-dimensional term which can give rise to the gravitino mass term <sup>8</sup> to lower dimensions, one finds

$$-\frac{\sqrt{2}}{48\kappa^2} \int d^{11}x \sqrt{g} \bar{\psi}_\mu \Gamma^{\mu\nu ABC11} \psi_\nu G_{11ABC} \quad (33)$$

and so obtains as the mass-like coefficient in four dimensions the integral

$$\langle e^{-\frac{3}{2}(\gamma+\sigma)} \partial_5 C_0 \rangle = \frac{1}{\pi\rho_0} \int_0^{\pi\rho} dx^5 e^{-\frac{3}{2}(\gamma+\sigma)} \partial_5 C_0. \quad (34)$$

We recall first that  $\frac{1}{4}e^{-\frac{3}{2}(\gamma+\sigma)}$  is indeed  $e^{K/2}$ . In order to be consistent with the soft terms found previously, which are proportional to the local values of the derivative  $\partial_5 C_0$  on the respective walls, one must demand that

$$\langle \partial_5 C_0 \rangle = \partial_5 C_0|_{(m=1)} = \partial_5 C_0|_{(m=2)} \quad (35)$$

One natural and acceptable, but not of course unique, solution to this condition is  $\partial_5 C_0$  constant, which is exactly the case for the solution of lowest order in  $\kappa^{2/3}$  which we consider here. The obvious question is whether there is any need to consider situations when the vacuum solution for  $C_0$  is not exactly linear in  $x^5$ . The answer is that there exists a natural source of such nonlinearities - the nonlinear couplings from the non-trivial Kähler metric for the bulk moduli, i.e., the non-trivial  $\sigma$  model discussed in [10, 11]. The incorporation of such nonlinear vacuum solutions into the standard four-dimensional supergravity form lies beyond the scope of this paper: one may speculate that it could involve field-theoretical loop corrections.

Finally, we would like to discuss in some detail the relation between the five- and four-dimensional pictures of supersymmetry breaking. To this end, we first illustrate the way the

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<sup>8</sup>We focus on the highest-helicity components, as the lower-helicity massive components must come from the super-Higgs effect.

corrections of higher order in  $\kappa^{2/3}$  differentiate between the terms coming from different walls. We study the simplest higher-order couplings related to Witten's deformation, in the operators bilinear in gaugino fields. The supergravity Lagrangian with  $f_{1,2} = \tilde{S} \pm \xi_0 \tilde{T}$  gives

$$\frac{e^{K/2}}{4} W g^{i\bar{j}} K_i \partial_{\bar{j}} \bar{f}_{ab} \langle \bar{\lambda}^a \bar{\lambda}^b \rangle + h.c. = -\frac{e^{K/2}}{2} W (\Lambda_1^3 (e^{3\sigma} + \xi_0 e^\gamma) + \Lambda_2^3 (e^{3\sigma} - \xi_0 e^\gamma)) + h.c. \quad (36)$$

where  $\Lambda_i^3$  is the four-dimensional condensate on the  $i$ 'th wall. The first term on the right-hand side of (36) looks like the contribution from the first wall, and the second like the one from the other wall. Indeed, these terms are very similar to the two expressions for the gaugino bilinears coming from different walls which we obtained in (28), once we take into account the linear correction to the volume:

$$\sqrt{g_6(x^5)} \rightarrow \sqrt{g_6^{(0)}(x^5)} \left(1 + \xi_0 e^\gamma \left(1 - \frac{2x^5}{\pi \rho_0}\right)\right), \quad (37)$$

which expresses the difference between the volumes of the Calabi-Yau spaces on the two walls. Moreover, we note that  $\partial_5 C_0 \propto W^{(1+2)}$  at the zeroth order in  $\kappa^{2/3}$ .

Pursuing explicitly this quest for higher-order operators cannot give reliable results because of the problems discussed above the equation (26). However, we stress that the procedure of reducing from five to four dimensions which we have applied here already has many general features of the exact reduction, and illustrates the way the two spatially-separated gauge sectors are glued together. This is the case not only in the Horava-Witten models with spatially separated 9-branes, but also, we believe, in more general situations like those in Type-I/Type-IIB orientifold models.

As the next step towards a better understanding of the relation between the five- and four-dimensional pictures, we return to the four-dimensional expression for the auxiliary fields controlling supersymmetry breaking. We recall that the condition of unbroken supersymmetry  $F^{\tilde{S}} = F^{\tilde{T}} = 0$  is fulfilled if and only if both condensates vanish. On the other hand, in general<sup>9</sup> in the five-dimensional picture:  $\partial_5 C_0 = \alpha_1 \Lambda_1^3 + \alpha_2 \Lambda_2^3$ , with  $\alpha_1 = \alpha_2$  in the lowest-order solution. Hence, in five dimensions, unbroken supersymmetry does not imply  $\Lambda_1^3 = \Lambda_2^3 = 0$  as in four dimensions. The resolution of the puzzle lies in the dynamics. One has to remember that, among the possible values of the parameter that controls supersymmetry breaking, the allowed ones are those which extremize the energy functional. In the present case, the relevant contributions to the energy are the gradient terms corresponding to the derivatives with respect to  $x^5$ :

$$KE_5 = \int_{M^5} \frac{2}{S + \bar{S}} (2\partial_5 C_0 + \dots)^2 + \int_{M^5} \frac{1}{(S + \bar{S})^2} (\partial_5 S)^2 + \dots \quad (38)$$

After integrating  $C_0(x^5)$  and  $S(x^5)$  out using their equations of motion, these terms become potential-energy terms from the point of view of four dimensions. Then, setting  $\partial_5 C_0 = 0$  does not extremize the full energy functional. Since we use dynamical equations of motion to integrate out non-zero mode parts of  $C_0$  and  $S$ , the information that  $\partial_5 C_0 = 0$  is not their solution when one includes all the corrections, becomes encoded in the structure of the effective

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<sup>9</sup>Namely, including higher-order corrections in  $\kappa^{2/3}$  (or  $\xi_0$ ).

four-dimensional Lagrangian. In fact, the property that not only the perfect square but also other derivatives with respect to  $x^{11}$  are important can be seen already in eleven dimensions, when one makes the Ansatz for the metric reflecting the canonical choice of the four-dimensional moduli  $\sigma$  and  $\gamma^{10}$ . When one expands the eleven-dimensional Einstein-Hilbert action in terms of these variables, one obtains

$$-\frac{1}{2} \int_{M^{11}} d^{11}x \sqrt{g} R^{(11)} = -\frac{1}{2} \int_{M^{11}} d^{11}x \sqrt{g_{(4)}} e^{-\gamma-2\sigma} \left( -\frac{1}{2} e^{\gamma+2\sigma} (3\gamma^\mu \gamma_\mu + 9\sigma^\mu \sigma_\mu + 2\Box\gamma + 4\Box\sigma) \right. \\ \left. + e^{-\sigma} (6\partial_{11}\gamma\partial_{11}\gamma + 16\partial_{11}\gamma\partial_{11}\sigma + 30\partial_{11}\sigma\partial_{11}\sigma - 4\partial_{11}^2\gamma - 10\partial_{11}^2\sigma) \right) \quad (39)$$

which contains not only four-dimensional kinetic terms but also terms quadratic in derivatives with respect to  $x^{11}$ .

It is the total sum of both types of both these and the terms coming from the walls should be extremized. The fact that gradient energy is, in general, non-zero in the five-dimensional bulk is reflected in the non-vanishing values of the four dimensional  $|F^S|^2$  and  $|F^T|^2$  contributions to the energy density.

## Conclusions

We have obtained in this paper soft supersymmetry-breaking operators in perturbative non-standard embeddings in the Horava-Witten model of the effective low-energy field-theory limit of the strongly-coupled heterotic string. The important feature that we have studied in detail is that, in these models, charged matter lives in two sectors with different gauge kinetic functions on two ten-dimensional walls which are spatially separated. We have also considered the possibility that gauginos may condense on both walls. We have also studied how such two sectors combine with each other to form the effective supergravity in four dimensions. We have shown how the soft supersymmetry-breaking terms are born in such a five-dimensional picture, and shown that this picture is consistent, at the level of the lowest-order solution to the equation of motion along the fifth dimension, with the effective four-dimensional results. We have also shown that integration over the fifth dimension gives the structure of a four-dimensional supergravity with the gauge kinetic functions  $f_{1,2} = S$  at the lowest order in  $\kappa^{2/3}$ . We have found non-universality of the soft terms on different walls, which is due to sign differences in the gauge kinetic functions and in the corrections to the kinetic functions of charged scalars.

We have argued that mixed  $F^S/F^T$ , and perhaps even  $F^S$ -dominated, supersymmetry breaking may occur in the class of models discussed here, and, we believe, also in their counterparts in more general constructions with different gauge sectors separated in higher dimensions. We also believe that other considerations in this paper should be helpful in more general situations, like type I/type IIB orientifold models with gauge sectors located on different branes.

During the completion of this project, we learnt of the forthcoming work of J.P. Derendinger and R. Sauser [14], which addresses the problem of constructing the effective low-energy effective supergravity in also in the presence of five-branes in the eleven-dimensional bulk. We thank

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<sup>10</sup>In the real basis  $x^i$ , we now take for the metric on the internal six-dimensional space the metric  $g_{ij} = \frac{1}{2} e^\sigma \delta_{ij}$ .

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